

Penguins Go Parallel: a grammar of graphics framework for generalized parallel coordinate plots

AUTHOR 1^{1*} AUTHOR 2² AUTHOR 3³

¹ University 1; ² University 2; ³ University 3

October 7, 2022

Abstract

Parallel coordinate plots (PCP) are a **valuable** tool in exploratory data analysis of high-dimensional numerical data. The use of PCPs is limited when working with categorical variables or a mix of categorical and continuous variables. In this paper, we propose generalized parallel coordinate plots (GPCP) to extend the ability of PCPs from just numeric variables to dealing seamlessly with a mix of categorical and numeric variables in a single plot. In this process we find that existing solutions for categorical values only, such as hammock plots or parsets become edge cases in the new framework. By focusing on individual observations rather than a marginal frequency we gain additional flexibility. The resulting approach is implemented in the R package ggpcp.

XXX polyline or poly-line?

1 Introduction

Few approaches in data visualization exist that are truly high-dimensional. Most visualizations are projections of data into two or three dimensions enhanced by facetting or additional mappings to plot aesthetics, such as point size and color. Parallel coordinate plots are one of the exceptions: in parallel coordinate plots we can actually visualize an arbitrary number of variables to get a visual summary of a high-dimensional data set. In a parallel coordinate plot, each variable takes the role of a vertical (or parallel) axis; giving the visualization its name. Multivariate observations are then plotted by connecting their respective values on each axis across all axes using polylines (cf. Figure 1). For just two variables this switch from orthogonal axes to parallel axes is equivalent to a switch from the familiar Euclidean geometry to the projective space. In the projective space, points take the role of lines, while lines are replaced by points, i.e. points falling on a line in the Euclidean space correspond to lines crossing in a single point in the projective space. This duality provides a good basis for interpreting geometric features observed in a parallel coordinate plot [Inselberg, 1985].

The origins of parallel coordinate plots date back to the 19th century and are, depending on the source, either attributed to d’Ocagne [1885] or Gannett [1880]. Modern era parallel coordinate plots go back to Inselberg [1985] and Wegman [1990]. Parallel coordinate plots are used in an exploratory setting as a way of getting a high-level overview of the marginal distributions involved, identifying outliers in the data, and finding potential clusters of points. In the absence of those, Parallel Coordinate Plots are often criticized for the amount of clutter they produce, resembling a game of mikado (also known as pickup-sticks – if you are not familiar with the game, imagine spilling a box of spaghetti) rather than organized data. This clutter is sometimes mitigated by the use of α -blending [Miller and Wegman, 1991], density estimation [Heinrich and Weiskopf, 2009], or edge-bundling parallel coordinate plots [McDonnell and Mueller, 2008]. For a detailed overview of these and other techniques see Heinrich and Weiskopf [2013].

*Corresponding author: xxx

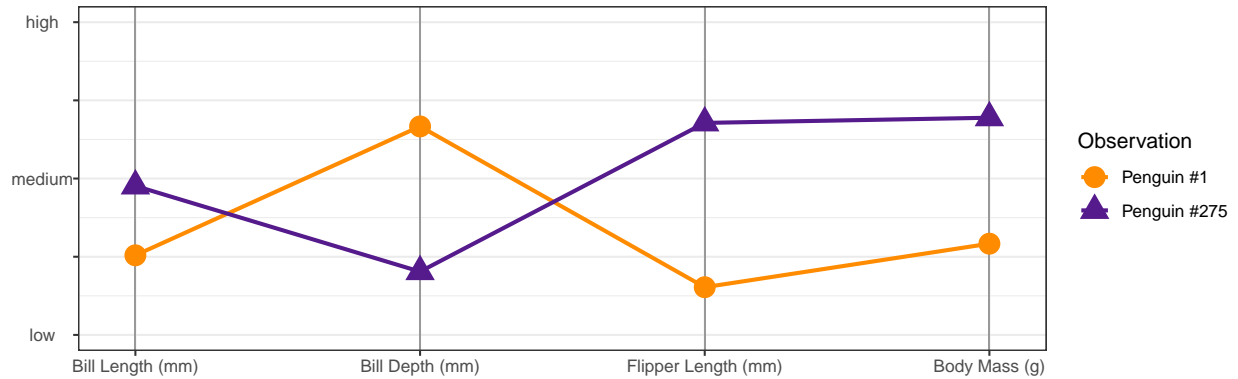


Figure 1: Sketch of a parallel coordinate plot of two observations in four dimensions. Each dimension is shown as a vertical axis, observations are connected by polylines from one axis to the next. Two penguins from the Palmer Penguin data set (see section 5) were sampled for this example.

While parallel coordinate plots are a powerful tool using categorical variables alongside quantitative variables is a great challenge. In current solutions, levels of categorical variables are transformed to numbers and variables are then used as if they were numeric. This introduces ties into the data, and the resulting parallel coordinate plot becomes uninformative, as it only shows a mesh of lines from each level of one variable to each level of the next variable. Some versions of parallel coordinate plots have been specifically developed to deal with categorical data: parallel set plots [Kosara et al., 2006], Hammock plots [Schonlau, 2003], and common angle plots [Hofmann and Vendettuoli, 2013], but these solutions do not accommodate quantitative variables. Instead, they are intended for use with tabular data and show bands of observations from one categorical variable to the next. Hammock plots and common angle plots mitigate effects of the sine-illusion [Day and Stecher, 1991, VanderPlas and Hofmann, 2015] on parallel sets plots. An attempt to combine categorical and numeric variables in a parallel coordinate plot is introduced in the categorical parallel coordinate plots of Pillhöfer and Unwin [2013] by treating factor variables as numeric. Similar to parallel sets, this approach is also based on marginal frequencies for the categorical variables. Categorical parallel coordinate plots are the closest of these variations to our solution, but the extracat package has not been updated recently and is no longer on CRAN.

The remainder of the paper is organized as follows: section 2 introduces the ggpcp syntax and explains the improvements in ggpcp over other parallel coordinate plot software packages. section 3 describes the data processing for parallel coordinate plots and how this wrangling is separated from the plot rendering in ggpcp. section 4 discusses the rendering of parallel coordinate plots and factors such as plotting order and tie-breaking which are important for the design of PCPs. section 5 provides three examples which highlight the use of generalized PCPs in exploratory settings.

2 Motivation and Package Usage

An important motivation for the ‘ggpcp’ package is that other implementations of parallel coordinate plots for categorical variables make it difficult to follow a single observation across the chart. ‘ggpcp’ alleviates this difficulty with two new developments: careful treatment of categorical variables to prevent line intersections at vertical axes, which maintains the visual ability to follow individual cases across the chart, and methods for ordering observations within categorical variables to reduce the amount of visual clutter. Together, these features allow for easier perception of lines in generalized parallel coordinate plots: by reducing the number of intersecting lines at pivot points along the vertical axes, we allow our brains to leverage the gestalt principle of good continuation to follow one line across the plot. Reducing the number of line crossings at non-axis points simplifies the plot, reducing the overall cognitive load

required to "untangle" (literally and metaphorically) the individual observations.

Listing 1: A demonstration of *ggpcp*'s data wrangling and plotting API.

```
1 pcp <- penguins %>% # data management:
2   pcp_select(sex, 3:6, species) %>% # variable selection, see section 3.1
3   pcp_scale(method="uniminmax") %>% # setting scaling method, see section 3.2
4   pcp_arrange() %>% # arranging categorical data
5   ggplot(aes_pcp()) + # plotting the chart:
6     geom_pcp_axes() + # vertical lines for axes
7     geom_pcp(aes(colour = species)) # parallel coordinate line segments
```

XXX the results from the code should be shown somewhere, so the code is not so unmotivated.

In addition, *ggpcp* leverages the full *ggplot2* philosophy instead of using highly specific wrapper functions, allowing users to focus on the data, rather than the names of various parameters used for customization. *ggpcp* adopts tidy conventions for data wrangling, separating the necessary data manipulation to generate a parallel coordinate plot from the visual rendering. Scaling, ordering of cases, and the arrangement of the parallel axes are completed using *pcp_select*, *pcp_arrange*, and *pcp_scale*, respectively; the resulting data frame is then passed directly into the familiar *ggplot()* call. During the plotting state, the only modification from default *ggplot2* syntax is the use of *aes_pcp()* in place of *aes()*; this is necessary to handle the multiple axes in a parallel coordinate plot while maintaining the ability to map all other variables of the original data frame to aesthetics such as linetype and color. The user has complete control over layers such as PCP lines (*geom_pcp*), labels (*geom_pcp_labels*), and boxes around categorical variables (*geom_pcp_boxes*), but there are additional advantages to the use of *ggplot2*. Users can also augment their parallel coordinate plots with additional information, such as boxplots or violin plots, with standard *ggplot2* syntax.

One of the strengths of *ggplot2* is its handling of small multiple plots with *facet_grid* and *facet_wrap*; these functions are fully supported in *ggpcp*.

In addition, basing *ggpcp* on *ggplot2* expands the functionality available to users without much additional code, thanks to other packages such as *plotly* [Sievert, 2020] which leverage *ggplot2* to create interactive graphics for the web and *gridSVG* [Murrell and Potter, 2020] which exposes vectorized graphics to allow programmable changes, such as elementary interactions with graphical elements.

3 Data management

One of the ideas behind this re-implementation of parallel coordinate plots is to expose parallel coordinate plots at a functional level. Rather than using a single function with parameters controlling every aspect, we separate the data management from the visual rendering. In particular, we separate out the data management into three parts:

1. Variable selection and reshaping data,
2. Scaling of axes, both at the individual level and in the relationship of the axes to each other, and
3. Treatment of ties in categorical axes.

XXX Cite subfigure above with code XXX

The modularization of the data wrangling process has the additional advantage of laying out the necessary elements in successive steps. Some of these steps are optional - e.g. scaling variables might not be necessary, if all variables are already on the same scale (i.e. method 'raw' in *GGally*); similarly, using *pcp_arrange* to break ties is only necessary if there are any factor variables, and if we actively want to spread these observations out. In addition, by exposing these elements of the *pcp* data wrangling process, we allow users to create additional functions for handling these tasks.

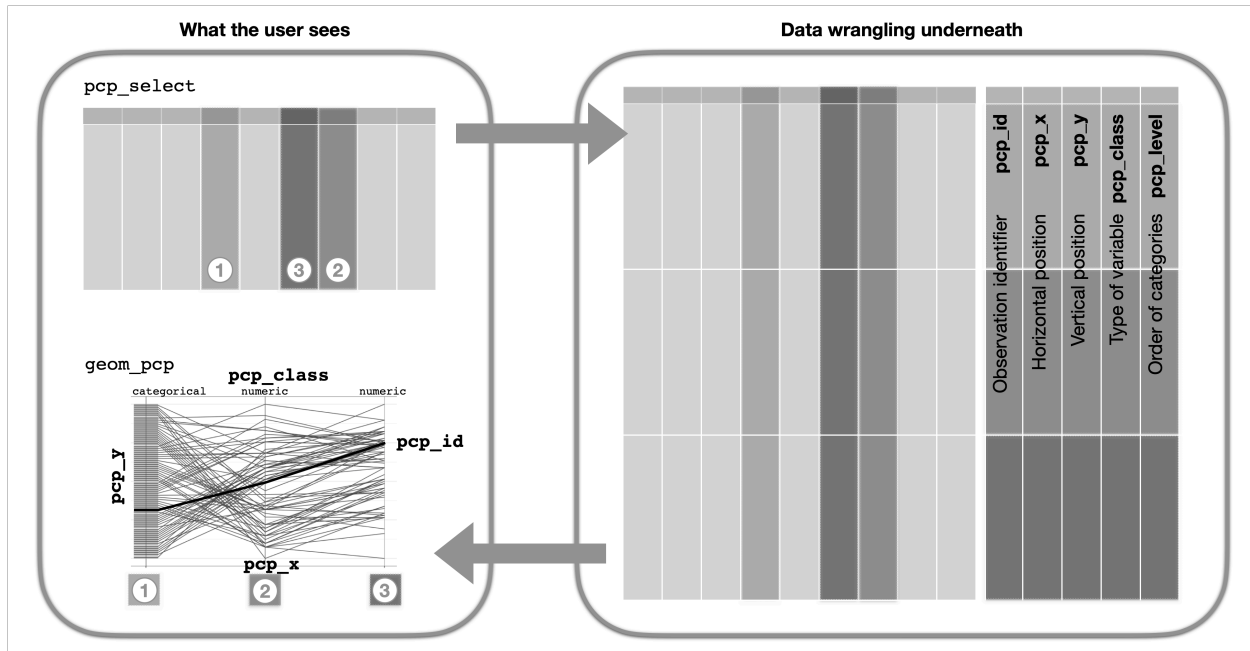


Figure 2: The user selects a set of three variables (top left). On the right, an overview of the data wrangling step before a parallel coordinate plot can be drawn (bottom left). Note that the order in which variables are selected is reflected in the order in which variables are included in the parallel coordinate plot.

The treatment of ties is an aspect not generally addressed in the original parallel coordinate plots of Inselberg [1985] and Wegman [1990]. We have found a need to deal with ties, because ties are visually the main obstacle of allowing the viewer to follow an observation from axis to axis through the high-dimensional space. If we can track a single observation through the high-dimensional space, we have the ability to look beyond the two-variable associations of adjacent axes. This allows users to more easily summarize main trends and identify observations which do not follow those trends. When ties cannot be separated and users cannot follow individual observations, higher-dimensional insights are next to impossible.

3.1 Variable Selection and Order of the Variables

One of the biggest strengths of the Grammar of Graphics is its mapping between data variables and visual aesthetics. In standard plots any mapping is a function between one data variable and one aesthetic. In a parallel coordinate plot, this one-to-one mapping between data and plot aesthetics is seemingly turned into a one-to-many mapping between arbitrarily many data variables to the x axis. By transforming the wide form of the data set into a long form [Wickham, 2014, 2021], we get to a version of the dataset in which we achieve a one-to-one mapping to a now discrete x axis consisting of the (names of the) original data variables.

From the user's perspective this data reshaping is purely a form of a data selection, while the data wrangling is going on behind the scenes in `pcp_select`:

`pcp_select(data, ...)` selects the variables to be included in the parallel coordinate plot. Variables can be specified by any combination of the following methods:

- position, e.g. 1:4, 7, 5, 4,
- name, e.g. class, age, sex, aede1:aede3 or
- using pattern selectors, e.g. `starts_with("aede")`, see `?tidyselect::select_helpers`

Variables can be selected multiple times and will then be included in the data and the resulting plot

multiple times. Note that the order in which variables are selected determines the order in which the corresponding axis is drawn in the parallel coordinate plots. `pcp_select` transforms the selected variables to long form and embellishes the data set with a number of additional variables. All of the newly created and added variables start with the prefix `pcp_`:

- `pcp_id`: integer variable identifying each observation in the original dataset. This variable is used as the grouping variable to identify which values should be connected by a line segment in the parallel coordinate plot.
- `pcp_x`: discrete variable consisting of the names of the selected variables in the order that they were selected - this is the order in which the variables will be included in the plot.
- `pcp_y`: numeric variable containing the values of all of the selected variables. In case a selected variable is not numeric, it is converted to a factor variable and the (numeric) factor levels are saved in `pcp_y`.
- `pcp_class`: character variable containing the class information of a selected variable.
- `pcp_level`: character variable containing the factor levels of selected data variables. In case of numeric variables, the data values are stored (in textual form). [The ordering of factor variables will be discussed below but is implemented using this added variable.](#)

As a consequence of these design decisions, users have several ways of performing different tasks within the flow of generating data for a parallel coordinate plot. For instance, users can reorder variables using `pcp_select` or after variable selection using the `pcp_x` variable.

[Users can transform individual variables, reordering factors or reversing an axis, using a mutate statement before variable selection, as shown in Listing 2, which generates Figure 3.](#) Univariate transformations like these may be useful to reduce the overall visual complexity of a parallel coordinate plot by reducing the number of anticorrelated axes in sequence.

Listing 2: Mutations before the `pcp_select` stage can be used to reorder factors or transform numerical variables, as shown in Figure 3.

```

1 penguins %>%
2   mutate(
3     species_inv = factor(species, levels = rev(levels(.$species))),
4     neg_bill_depth_mm = -bill_depth_mm) %>%
5   pcp_select(species, neg_bill_depth_mm, 3, 6, 5, species_inv) %>%
6   pcp_scale(method="uniminmax") %>%
7   pcp_arrange() %>%
8   ggplot(aes_pcp()) +
9     geom_pcp_axes() +
10    geom_pcp(aes(colour = species), alpha = 0.6) +
11    geom_pcp_labels()
```

Ordering of levels in factor variables (XXX not sure where to put this, yet): What we are doing with the ordering of levels in factor variables, is to stick with the basic interpretation of factor variables as a type of variable that has both labels and an ordering of those labels. Whenever we assign a numeric value to the ordering, we refer to the associated score, which is an integer value from one to the number of categories, if not specified explicitly otherwise. Ordered factors are plotted from the lowest level upwards. If a factor legend is included, it will need to be reversed to match this order. Use `guides(color = guide_legend(reverse=TRUE))` as shown in the example in subsection 5.2.

3.2 Scaling

`pcp_scale(data, method)` scales the values on each axis and determines the relative relationship of the axes to each other.

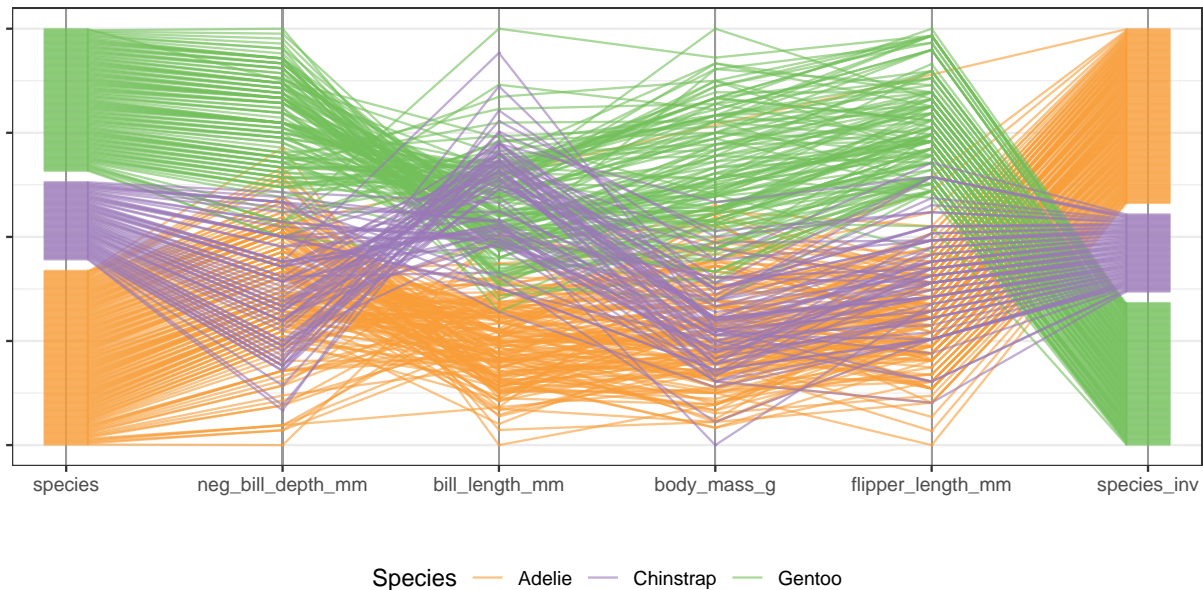


Figure 3: The left-most axis shows species, where the right-most axis shows the same variable with categories reordered; similarly, *neg_bill_depth_mm* shows the bill depth, multiplied by -1 to invert the order of data points.

`method` is a character string specifying the method to be used when transforming the values of each variable onto a common y axis. By default, the method `uniminmax` is chosen, which univariately scales each variable onto a range of [0,1] with the minimum at 0 and the maximum at 1. `globalminmax` maps the values across all axes onto a an interval of [0,1]. This method should only be used if the values across all variables are comparable. The method `robust` normalizes values univariately by mapping the median value to 0.5 and a robust 95% confidence interval (based on the median absolute deviation) to an interval of 0 to 1.

Figure 4 shows two of the scaling methods applied to the olive oil data [Forina et al., 1983, Wickham et al., 2011]: Measurements of fatty acids in 572 olive oils from three different regions in Italy are visualized as parallel coordinate plots. Similar to the findings in Cook and Swayne [2007], we see that eicosenoic acid is only found in increased quantities in olive oils from Southern Italy. Quantities of oleic and linoleic acids allow a separation between olive oils from Sardinia and Northern Italy. Both scaling methods enable us to find these conclusions. While `uniminmax` scaling uses the space allotted to the chart most efficiently, the robust normalization method emphasizes the heavy tails and skewness of some of the measurements, such as the percentages of stearic and arachidic fatty acids.

4 Visual Rendering

4.1 Breaking ties on categorical axes

XXX How to deal with ties has a huge visual impact. In order to get to that we need to do some more data wrangling. This subsection is somewhere between data management and visualization.

XXX it might be good to separate the section into 'how' (data) and 'why' (visual aspects).

Assigning all cases of the same level to the same point on the axis of a categorical variable makes it impossible to distinguish them. The approach of 'ggpcp' is to draw boxes proportional to the numbers of cases in each level and to order the cases within their boxes so that visual clutter is minimised and individual cases can be followed. This idea is implemented in the 'ggpcp' function `pcp_arrange(data, method, space)`. The two implemented methods are "from-left" and "from-right", meaning that ties are broken



Figure 4: Two scaling methods showing fatty acid compositions of olive oils from different regions in Italy, areas within each region are colored using similar hues within region (green for Northern Italy, purple for Sardinia, and tans for Southern Italy).robust scale: median to 0 check units XXX The two scaling methods roughly allow the same conclusions.

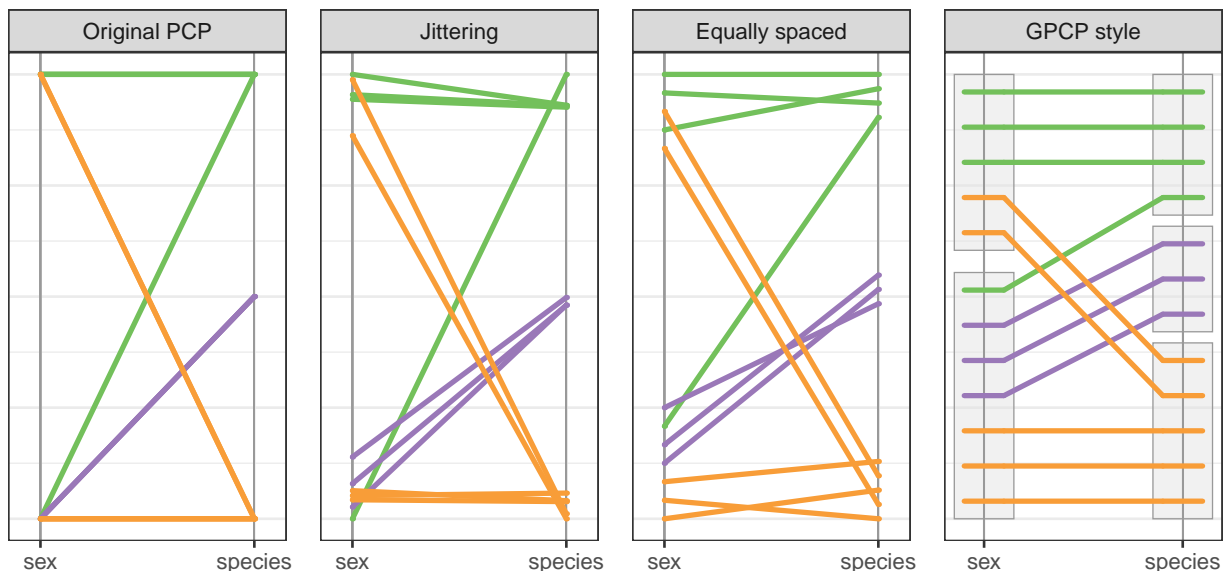


Figure 5: Using 12 randomly sampled penguins from the Palmer penguin data, we show four different approaches of dealing with categorical variables: the panel on the left shows the typical net of lines resulting from categorical variables in regular parallel coordinate plots. In the other three panels, ties in categorical levels are broken using different approaches (from left to right): jittering, equi-spaced line segments and ordered equi-spaced line segments are shown.

using a hierarchical ordering determined by variables values from the left or the right, respectively. The parameter space specifies the amount of the y axis to use for space between levels of categorical variables. By default, 5% of the axis is used for spacing.

XXX hierarchical ordering seems to minimize line crossings - but I don't have a proof. If we have a reference on ordering metrics this might go well here

Figure 5 shows several approaches of dealing with categorical variables in parallel coordinate plots. The left-most panel shows two categorical variables and the typical net of lines that forms between them in an original parallel coordinate plot. The other three panels show three different approaches of breaking the ties resulting from the categorical variables, with our favored solution shown on the right: all observations are spaced out evenly. This results in a natural visualization of the marginal frequencies along each axis (additionally enhanced by the lightly greyed boxes grouping observations in the same category). The ordering of the observations within the level is such that a minimal number of line crossings occurs between the axes. This method of dealing with categorical variables is the one we propose in the generalized parallel coordinate plot. While it is aesthetically pleasing, it also allows us, in the spirit of the original parallel coordinate plots, to follow an individual observation from left to right through the plot even for categorical variables. The other two solutions in the middle panels of Figure 5 show two intermediary solutions of breaking ties in categorical variables: jittering and equi-spaced (unordered) values.

4.2 Plotting Order of line segments

XXX We need to include some code somewhere to also show the 'how'.

XXX Order of levels and order of variables are important. Drawing pcps depends on what your goal might be.

One of the primary advantages of the generalized approach to dealing with categorical variables is the ability to follow a single observation throughout the plot. As the number of observations increases, this becomes less feasible because of overplotting of line segments, particularly for larger data sets. As more observations and line segments are drawn, more lines cross each other, increasing the effort required to

follow a poly-line from one side of the plot to the other. Coloring by groups improves the readability of plots. However, the order of drawing the cases may affect what can be seen due to overplotting. As a countermeasure, the order in which line segments are plotted should be carefully chosen. The parameter `overplot` offers the option "small-on-top" where groups are plotted in order of size from largest to smallest so that the smallest is on top. The default setting "none" is very flexible, but requires the user to specify the order they want before plotting—or just accept the current order of the dataset.

XXX additional counter-measures: use α -blending

5 Examples

5.1 Palmers Penguins

Several aspects in Parallel Coordinate Plots depend on orderings: the order of variables along the x axis, the order of levels in a categorical variable, the orderings of cases within categorical variable levels, and the order in which lines are drawn. Orderings should therefore (a) have good defaults, and (b) be easily changeable.

The top of Figure 6 shows a generalized parallel coordinate plot of the Palmer penguins data [Horst et al., 2020]. The numeric data consists of body measurements of three species of penguins: bill length, bill depth, flipper length, body weight. What can be seen is that Adelie penguins generally have smaller bill lengths than the other two species, while Gentoo penguins can be distinguished by their relatively large flipper lengths. The bottom of Figure 6 shows the effect of re-ordering the levels of both the 'species' and the 'island' variables in the generalized parallel coordinate plots. This re-ordering of factor levels has the effect of emphasizing that Gentoo penguins and Chinstrap penguins are each found on only one island, while Adelie penguins are found on all three islands.

Only after levels of 'island' and 'species' are re-ordered, we can see that for each species the numbers of penguins in the three years of the study (the study ID variable) were roughly the same.

Distinguishing species

As already explained, `ggpcp` is part of the `ggplot2` family, so faceting is easy. This has been done in Figure 6 to show that the patterns are similar for both males and females.

figure 6 and 7 look quite similar, but are quite different - visual emphasis? Maybe use `colors[1]` and `colors[3]`?

In addition to the ordering of factor levels, it is also important to carefully order the variables on the x-axis, as shown in Figure 7, where the variables have been re-ordered to allow the viewer identify which body measurements distinguish the species. Besides the re-ordering, the axis for bill depth has been reversed. Both changes help to separate the species. Gentoo penguins have the lowest bill depth, while generally having the longest flippers and being heaviest. Reversing the axis for bill depths, aligns the smallest bill depths with the longest flippers, thereby moving Gentoo penguins closer together as a group.

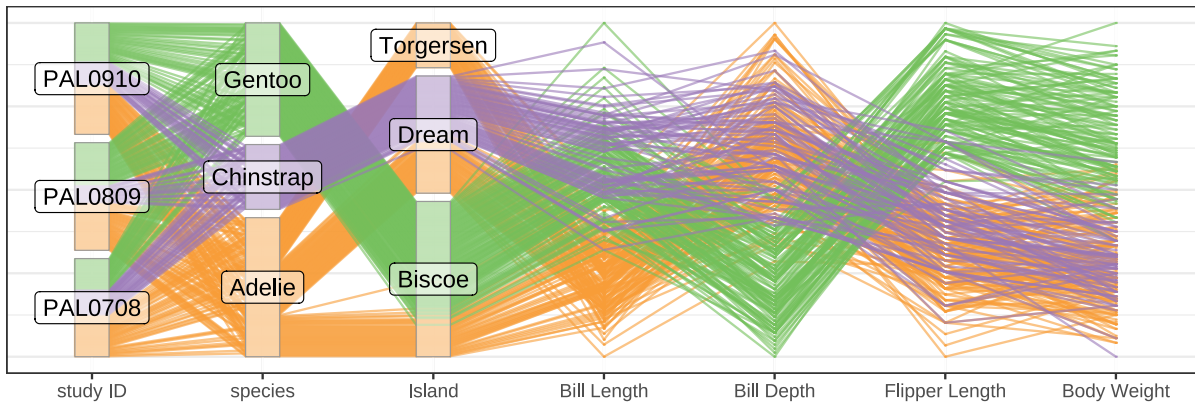
The plot shows that the Gentoo penguins are bigger, that Gentoo and Chinstrap are both only found on single islands, and, finally, that Adelie and Chinstrap are distinguished by the lengths of their bills.

Note that we are using facetting by penguins' sex in Figure 7. While all of the results are the same for the two sexes, any variability of body measures due to sex is removed from the plot by facetting. This makes the results stand out more.

Interestingly, some potential outliers that were not visible previously, now become visible. Note for example the two Gentoo males with particularly short flippers.

Determining sex

Original order of levels and variables



Levels reordered to emphasize relationship between islands and species

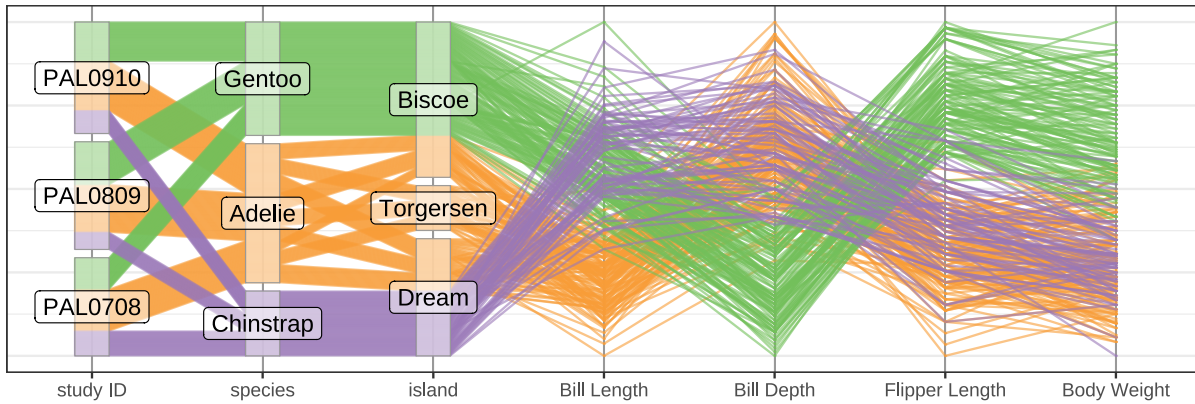


Figure 6: Both of the levels of the island and the species variable re-ordered to reflect that two of the species are each only found on one island.

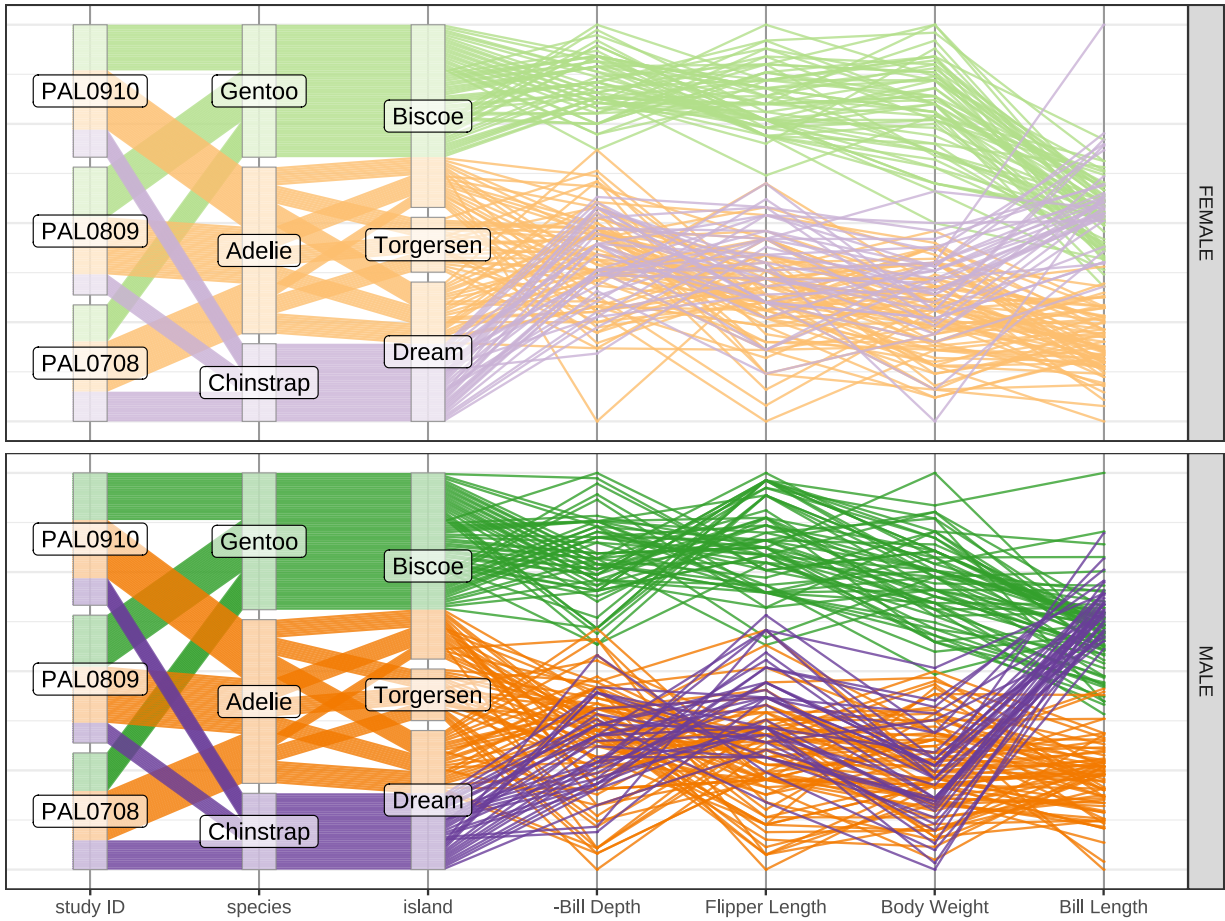


Figure 7: Changing the order of the variables along the x-axis emphasizes the differences in body measurements between the species.

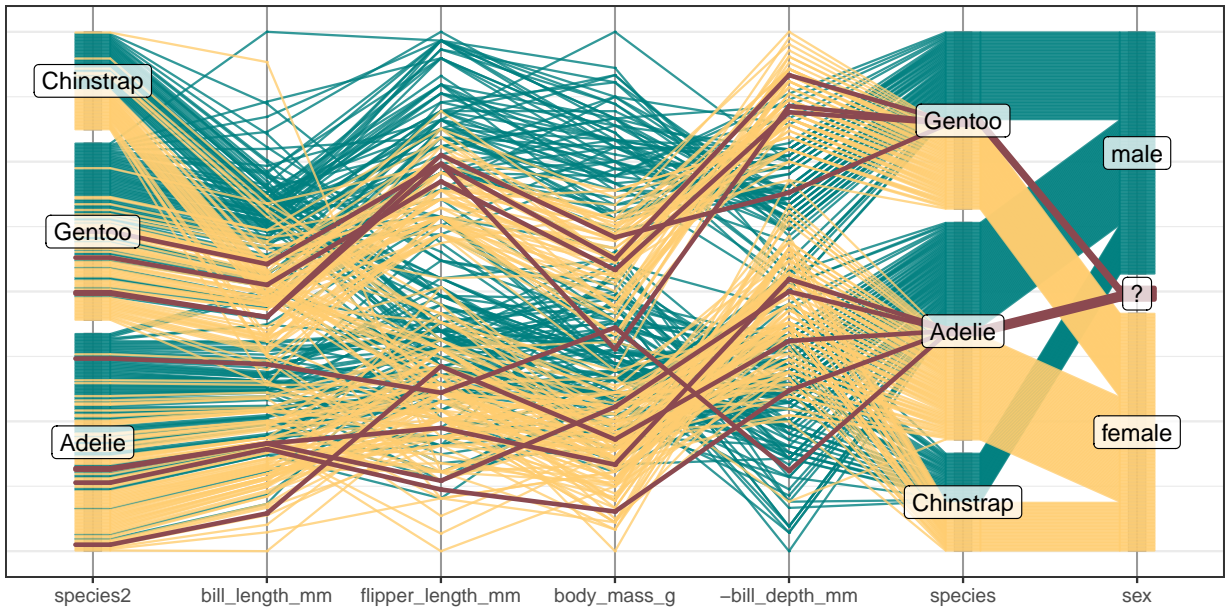


Figure 8: Generalized Parallel Coordinate Plot of the Palmer penguins data with sex of penguin mapped to color. Dark lines represent penguins for which sex could not be determined. We see that researchers were able to sex all of the Chinstrap penguins. Note that species is included twice (with different order of the levels). XXX reason for that here or in the text?

Instead of coloring by species, we can color by sex and compare the distributions of the measurements for males and females, as is done in Figure 8. This also helps to investigate which sex might be the few penguins whose sex is not given.

Figure 8 shows that within each species, the males tend to be larger in size and heavier than the females. For several of the penguins, sex could not be determined because either the sexing primer did not amplify or no blood sample was obtained [Gorman et al., 2014]. These penguins are represented by dark lines. Comparing these penguins' body measurements to those of the other penguins, we can make suggestions regarding their sex. In Figure 9 we explore this idea a bit further. This figure is based on the same data as Figure 8, however, we exclude Chinstrap penguins as researchers were able to sex all of those penguins. The body measurements of all sexed penguins are summarised by two ribbons for each sex and species. The inner ribbons are bounded by the 25% and the 75% percentile values on each axis. The lighter ribbon covers 95% of observations on each variable. We use these ribbons to reduce the noise introduced by individual lines. Body measurements of the unsexed animals are represented as line segments on top of the ribbons. This helps us to evaluate and assess the lines drawn for individual, unsexed penguins within the context of the marginal distributions (in this case their putative sex and species).

While we facet both by species and sex, note that the axes are re-scaled within each species to make use of the full range in y . However, we use the same scale between the two sexes of each species. This different treatment of faceting variables is achieved by the use of the grouping variable. The listing below shows the code for prepping the data shown in Figure 9. By grouping on species but not on sex (see line 9), data is being rescaled within species but the same scaling is used across males and females. Measurements for unsexed animals are shown as line segments on top of inter-quantile ribbons of both sexes. Viewers are encouraged to draw a conclusion about an animal's sex based on their values within the (2d density) context of their species and putative sex. Statistically, this comparison relates to a likelihood ratio test: the viewer is asked to make an assessment of the likelihood to observe the measurements of an animal under each of the two competing hypotheses of sex.

Listing 3: Code to prepare data for Figure 8 by relabelling penguins with NA sex as '?' and ordering sex so that penguins of unknown sex are between the male and female labels.

```

1 penguins_pcp <- penguins %>%
2   filter(species != "Chinstrap") %>%           # no unsexed animals in Chinstrap
3   mutate(
4     sex = ifelse(is.na(sex), "?", as.character(sex)), # make assignment more readable
5     sex = factor(sex, levels = c("female", "?", "male"))
6   ) %>%
7   filter(!is.na(body_mass_g)) %>%
8   pcp_select(6:5, 3:4) %>%
9   group_by(species) %>%                       # re-scale by species
10  pcp_scale() %>%
11  pcp_arrange()

```

Chinstrap penguins are excluded (line 2) because all of their individuals in the data have a sex assigned. The general pattern of measurements of the Gentoo penguins suggests that three of the four individuals with missing sex information are female (the three with the lowest bill depth). The fourth animal has an exceptionally deep bill, however, all other measurements suggest that this animal, too, is female. For further evidence, we find from the original data that their nest partners are all sexed as male; this additional information is shown in Figure 9. While assuming that nest partners are male and female is not a perfect method, in particular, for penguins, which have been shown to live in same-sex partnerships, in all three of the studies considered for this data only nests with breeding successes have been considered. More details can be found in Gorman et al. [2014]. For Adelie penguins determining sex is not quite as clear-cut, but based on body mass and bill length measurements the three lightest penguins might be female, while the heaviest one could be male. The fifth penguin exhibits measurements that are neither typically male nor typically female. Trying to confirm these putative assignments is a bit more tricky, because four of the five unsexed penguins are nest partners. The lightest unsexed penguin is the partner of a sexed male penguin. The putative assignments do not contradict the hypothesis that each nest is host to a male and a female penguin. Using this assumption, the last unsexed penguin would be resolved as the male of the nest.

5.2 Getting a second, third, ... and seventh opinion

Figure 10 shows data from Agresti [2002] published as part of the poLCA package [Linzer and Lewis, 2011]. Seven pathologists were asked to assess the same 118 slides for the presence or absence of carcinoma in the uterine cervix. Binary responses for each slide were recorded (yes/no). Pathologists all agreed on about 25% of slides, which they considered to be carcinoma free, and a further 12.5% of slides, which were considered to show carcinoma by all pathologists. *For the remaining 62.5% of slides there was some disagreement and it is clear that this disagreement is not random. The pathologists have been ordered from left to right from the fewest number of overall carcinoma diagnoses made to the highest number. This shows a strong level of agreement between adjacent axes.*

Note, in this example we do not need to scale the variables. Aside from the actual scale the values are ordered in the same way.

Landis and Koch [1977] (Table 1) allow us a closer look at this data. *The pathologists evaluated the slides using five levels from 1 to 5, given as: (1) Negative, (2) Atypical Squamous Hyperplasia, (3) Carcinoma in Situ, (4) Squamous Carcinoma with Early Stromal Invasion, and (5) Invasive Carcinoma. Agresti [2002] classified levels 1 and 2 as "no" and levels 3 to 5 as "yes".* Figure 11 gives an overview of this more detailed data. The different pathologists are drawn in the same order as in Figure 10. The results for each scan are colored by the overall average score (rounded to the closest integer). Compared to the previous figure, Figure 11 shows more variability between pathologists' evaluations, but only few scans have vastly different scores assigned to them. Pathologist C, in particular, rates two scans as negative, that

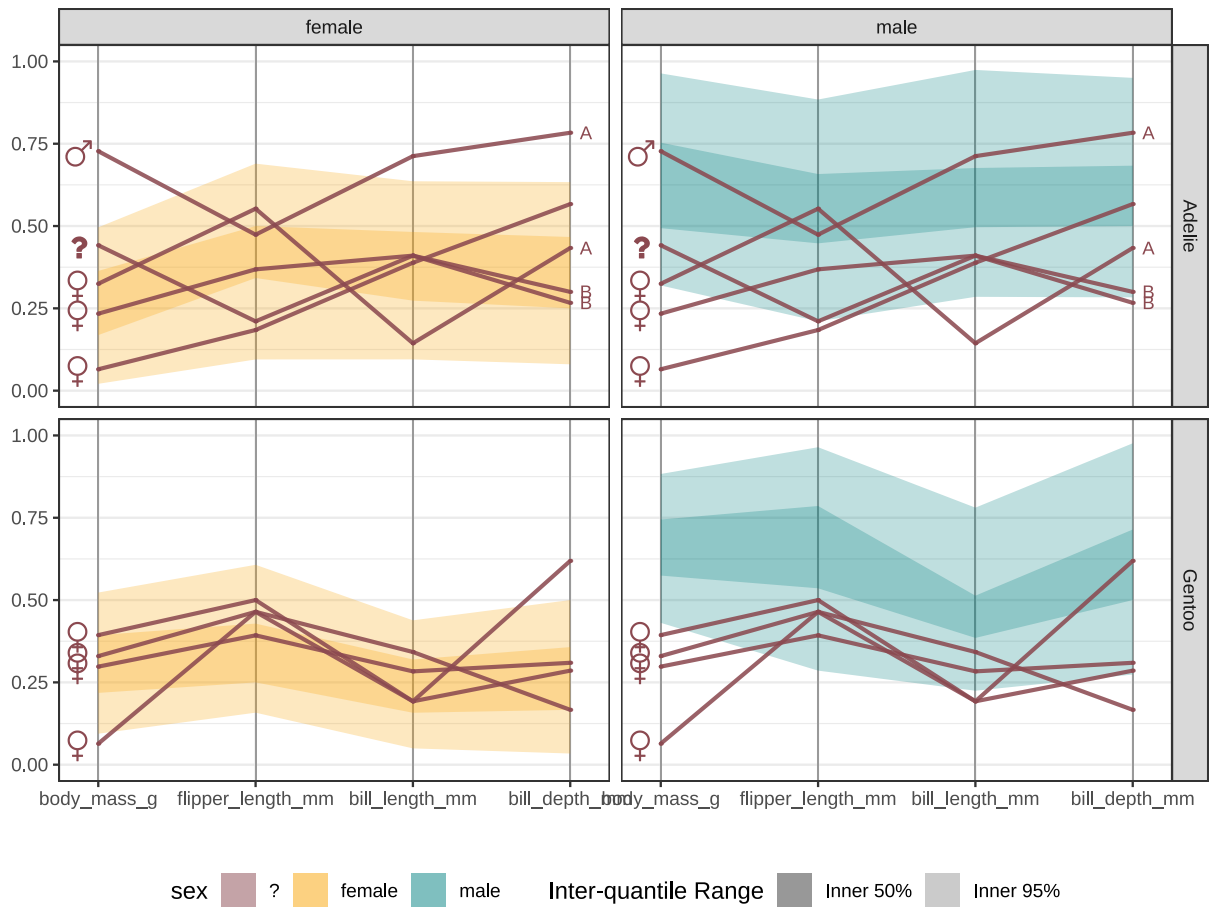


Figure 9: Closer investigation of non-sexed Adelle and Gentoo penguins. The `group_by` call before `pcp_scale` is responsible for scaling by species while the same scale is kept across sex within species. Penguins without assigned sex (based on blood markers) are drawn on top of both sexes. The labels to the left of the ribbons are our best guess at a penguin's sex based on body measurements of other penguins of the same species. The letters on the right indicate nests – two penguins with the same letter share the same nest.

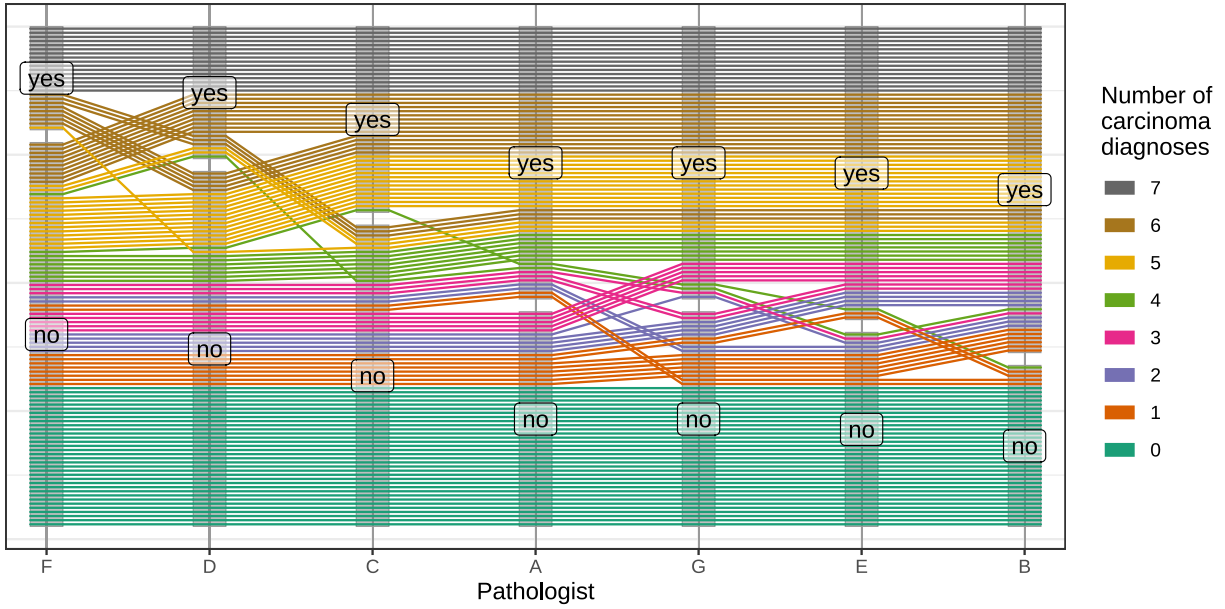


Figure 10: Pathologists' diagnoses of absence (no) or presence (yes) of carcinoma in the uterine cervix based on 118 slides. Each slide is shown by a poly-line.

all other pathologists rate as quite advanced cancer. Mostly, the variability between pathologists scores stems from a difference in applying the scores rather than from an actual difference of opinions. The similarity in evaluations is particularly striking between pathologists A, G, E, and B.

XXX very unpolished idea, but I think worthwhile exploring XXX The generalized parallel coordinate plot gives us a visual tool for assessing the similarity between evaluations by different pathologists that moves beyond a mere correspondence of scores to an analysis that is based on ranks. Tie-breaks in the ranks are featured as the y coordinate. When the poly-lines additionally are relatively flat, this means that pathologists can either agree on the score of these scans or agree at least on the relative severity of carcinomas in the scans. Obviously, we can assess 'flat-ness' of the poly-lines as the variance of the calculated variable 'pcp_y'.

5.3 Clustering with PCPs

Let's revisit the penguins data for an example of working with clusters in the framework of generalized parallel coordinate plots. We use k -means clustering on all numeric body measurements and investigate which observations are generally captured in each of the clusters.

k -means clustering assigns cluster labels arbitrarily based on random cluster centers. In order to maintain a persistent ordering over different values of k we reorder the clusters by the value of `body_mass_g`. This helps us to compare between k and $k + 1$ clusters. Figure 13 shows the numeric measurements along with the assigned clusters, with categorical variables species and sex on the right. Each line is colored by the assigned cluster, allowing us to determine how the categorical variables relate to the quantitative variables and the resulting clusters. When $k = 2$, Figure 13a shows that the largest difference in the observed data is between Gentoo penguins and the other two species. When $k = 3$, in Figure 13b, the additional cluster separates the Adelie and Chinstrap penguins into two groups with a few misclassifications; this additional cluster is based on the length of the bill (which we can follow due to the clear connection between data values in the generalized PCP). Adding a fourth cluster, as in Figure 13c splits Adelie penguins into males and females, though again there are some penguins that are misclassified. The addition of a fifth cluster in Figure 13d splits Chinstrap penguins into male and female. Once we add a sixth cluster in Figure 13e, we finally split the Gentoo penguins by sex as well, though again this clustering is not perfect.

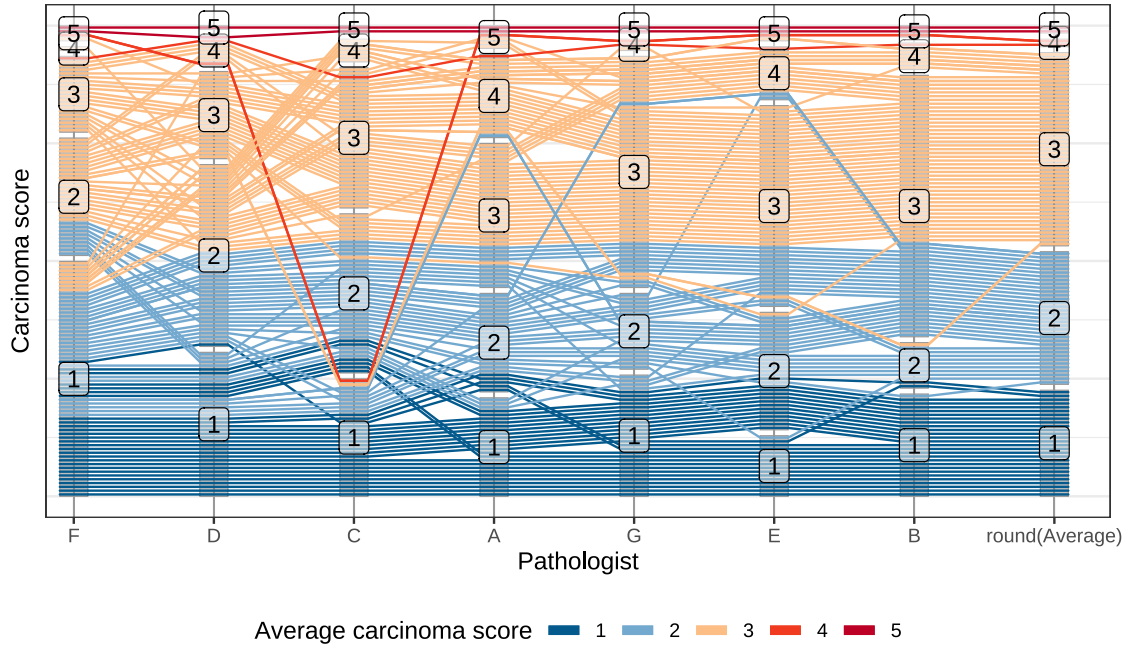


Figure 11: Closer look at pathologists' evaluations on a more detailed scale from 1 (Negative) to 5 (Invasive Carcinoma). Rounded average scores are mapped to color to help distinguish severity of scan evaluations.

What is clear from this exercise is that Adelie and Chinstrap penguins are much more similar to each other than they are to Gentoo penguins, but that there is still noticeable sexual dimorphism within each species.

We also see from the figure that some of the separation into sexes is lost from one clustering to the next. This is typical for non-hierarchical clustering algorithms. Rather than refining a previous cluster, a switch from k clusters to $k + 1$ clusters starts the clustering process anew. If the signal in the data to separate into k clusters is not strong or is ambiguous, we will see this reflected in the results; observations might be quite arbitrarily put together into groups, or a group of observations might be split into multiple clusters. In the Hartigan-Wong [Hartigan and Wong, 1979] algorithm used here for the clustering, points are assigned to random clusters in the initialization. In order to assess the effect of this non-deterministic start on the results, it is good practice to investigate the cluster stability by repeating the clustering multiple times for the same number of classes k (if $k > 1$). Figure 14 shows a comparison of the results from multiple runs of the k -means algorithm for $k = 6$. The lines in this figure are colored by species and sex. We see that the splits by species are relatively stable – there are only a few cases across all results in which individuals end up in clusters with individuals from another species, and if they do, it is the same individuals across different results. Splits by sex show more variability: Chinstrap penguins rarely split into male/female clusters, while Gentoo penguins shows a relatively stable separation into males and females. The Adelie population has subsets of individuals that are separated into males only, females only, and a third, more variable subset of a combination of the two.

For the conclusion? XXX Using generalized parallel coordinate plots and item sorting allows us to assess the stability of clustering results.

6 Discussion

generalized parallel coordinate plots offer a constructive solution to incorporating categorical variables in parallel coordinate plots. In particular, they preserve the key idea of being able to follow individual

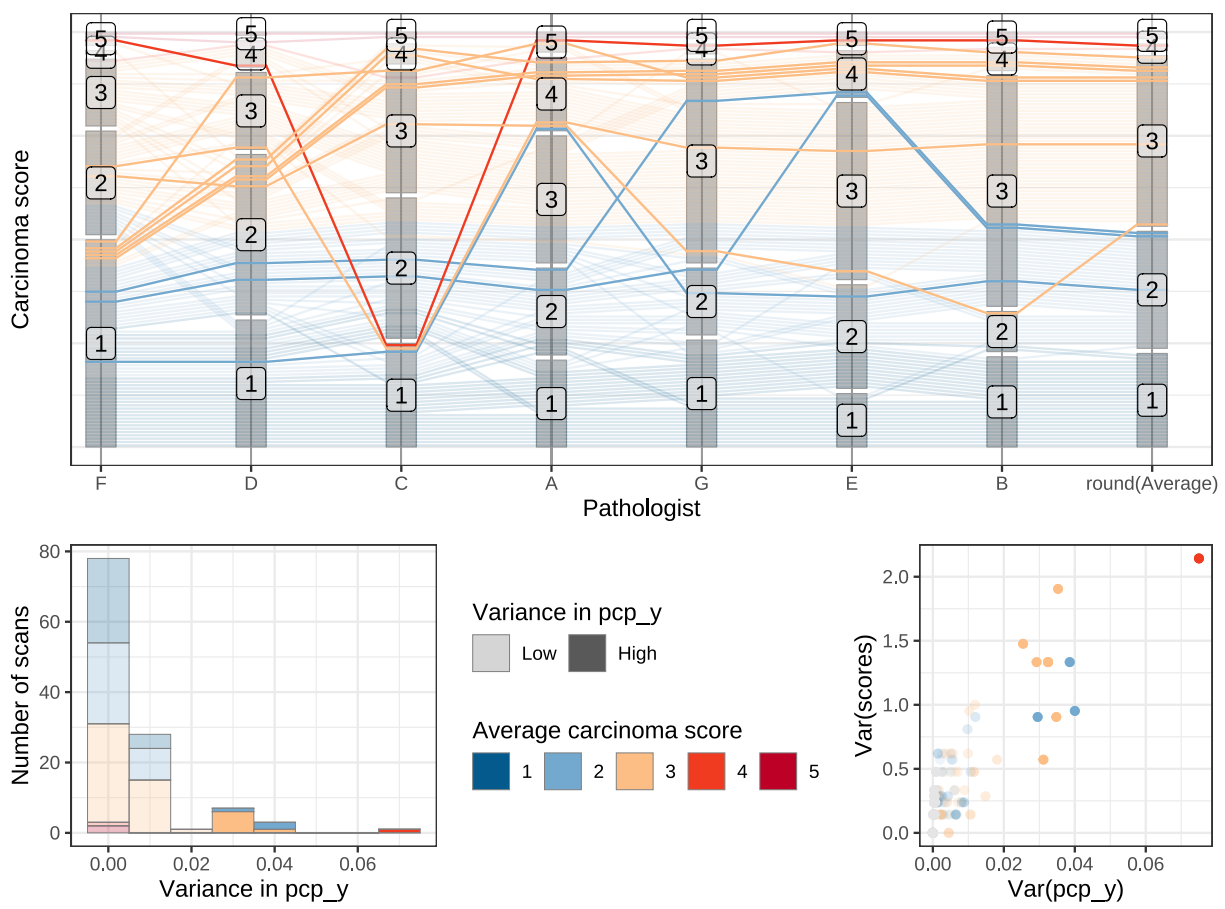


Figure 12: Highlighting lines with high variability

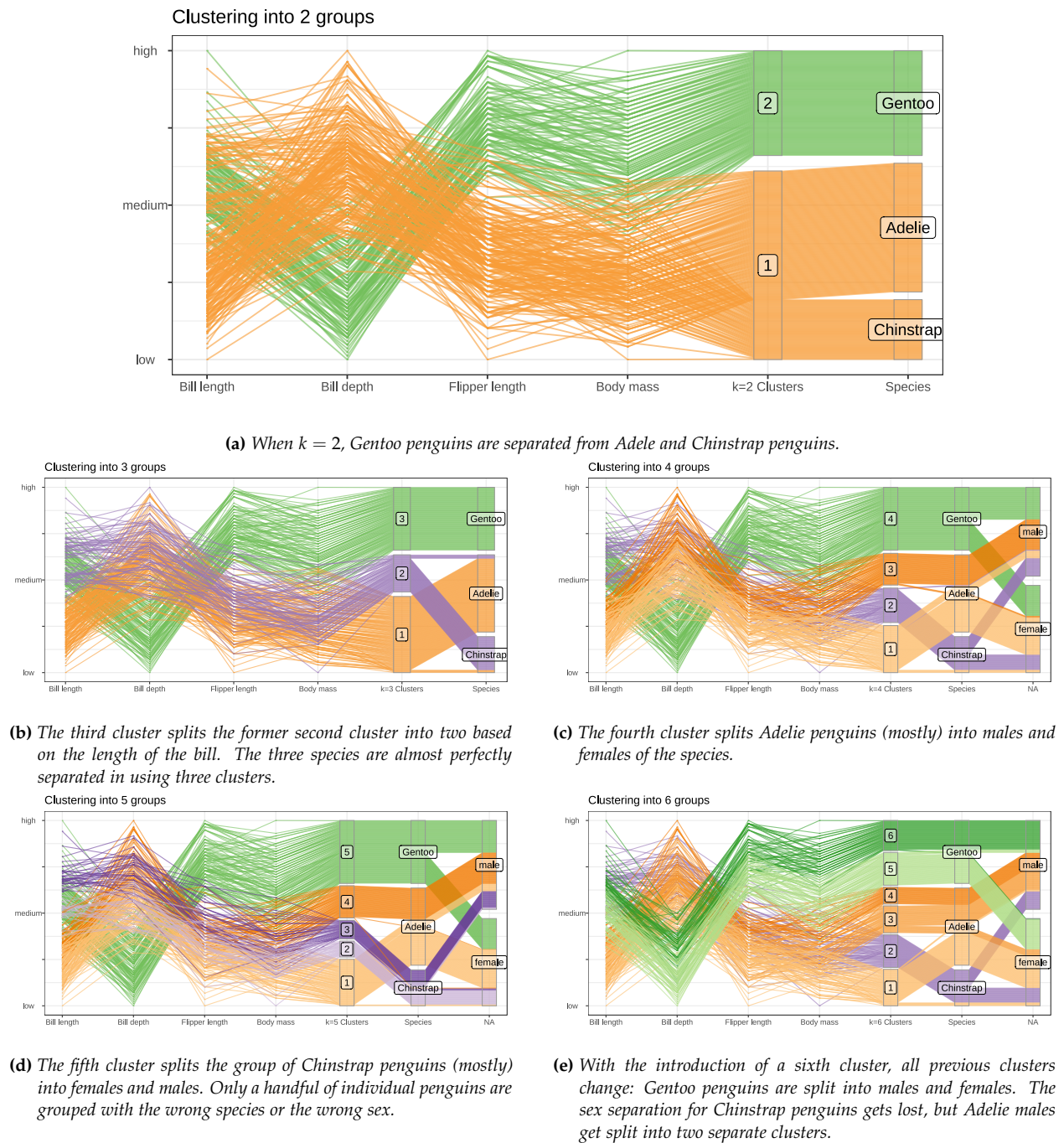


Figure 13: An overview of the use of parallel coordinate plots to examine which variables contribute to clustering and to identify individuals who are misclassified.

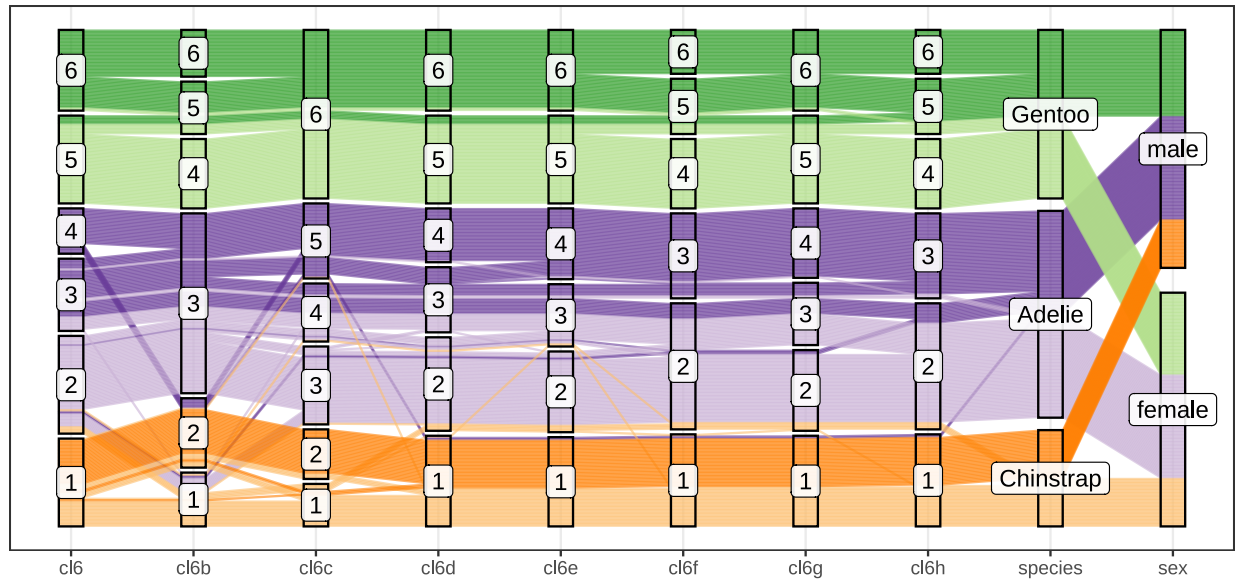


Figure 14: Comparison of eight k -means runs for $k = 6$. Color of lines is given by species and sex. The differences between the clusters are introduced by the different random seeds in the initial cluster centers.

cases across the plot. As always with parallel coordinate plots the use of color to distinguish groups, the ordering of the axes, and the ordering of drawing the lines play key roles. With generalized parallel coordinate plots two additional kinds of ordering have a part to play, the ordering of levels of categorical variables and the ordering of cases within those levels. The ideas described here have been implemented in the R package `ggpcp`. There are two central features. Data management has been separated from visual rendering, and the package follows the full `ggplot2` philosophy. Both these features contribute to the power and flexibility of the software.

References

- A. Agresti. *Categorical Data Analysis*. John Wiley & Sons, Hoboken, 2 edition, 2002.
- D. Cook and D. F. Swayne. *Interactive and Dynamic Graphics for Data Analysis With R and GGobi*. Springer Publishing Company, Incorporated, 1st edition, 2007. ISBN 0387717617.
- R. H. Day and E. J. Stecher. Sine of an illusion. *Perception*, 20:49–55, 1991.
- M. d’Ocagne. Coordonnées parallèles et axiales : Méthode de transformation géométrique et procédé nouveau de calcul graphique déduits de la considération des coordonnées parallèles. *Gauthier-Villars*, page 112, 1885. URL <https://archive.org/details/coordonnesparal00ocaggoog/page/n10>.
- M. Forina, C. Armanino, and S. Lanteri. Classification of olive oils from their fatty acid composition. *Food Research and Data Analysis*, pages 189–214, 01 1983.
- H. Gannett. General summary showing the rank of states by ratios 1880, plate 71. In *Scribner’s statistical atlas of the United States, showing by graphic methods their present condition and their political, social and industrial development*. Charles Scribner’s Sons, New York, 1880.
- K. B. Gorman, T. D. Williams, and W. R. Fraser. Ecological sexual dimorphism and environmental variability within a community of antarctic penguins (genus *pygoscelis*). *PLOS ONE*, 9(3):1–14, 03 2014. doi: 10.1371/journal.pone.0090081. URL <https://doi.org/10.1371/journal.pone.0090081>.

- J. A. Hartigan and M. A. Wong. Algorithm as 136: A k-means clustering algorithm. *Applied Statistics*, 28(1):100, 1979. doi: 10.2307/2346830. URL <http://dx.doi.org/10.2307/2346830>.
- J. Heinrich and D. Weiskopf. Continuous Parallel Coordinates. *IEEE Transactions on Visualization and Computer Graphics*, 15(6):1531–1538, 2009. doi: 10.1109/TVCG.2009.131. URL <http://ieeexplore.ieee.org/document/5290770/>.
- J. Heinrich and D. Weiskopf. State of the Art of Parallel Coordinates. In M. Sbert and L. Szirmay-Kalos, editors, *Eurographics 2013 - State of the Art Reports*. The Eurographics Association, 2013. doi: 10.2312/conf/EG2013/stars/095-116.
- H. Hofmann and M. Vendettuoli. Common Angle Plots as Perception-True Visualizations of Categorical Associations. *IEEE Transactions on Visualization and Computer Graphics*, 19(12):2297–2305, Dec. 2013. doi: 10.1109/TVCG.2013.140. URL <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6634157>.
- A. M. Horst, A. P. Hill, and K. B. Gorman. *palmerpenguins: Palmer Archipelago (Antarctica) penguin data*, 2020. URL <https://allisonhorst.github.io/palmerpenguins/>. R package version 0.1.0.
- A. Inselberg. The plane with parallel coordinates. *The Visual Computer*, 1(2):69–91, Aug. 1985. doi: 10.1007/BF01898350. URL <http://link.springer.com/10.1007/BF01898350>.
- R. Kosara, F. Bendix, and H. Hauser. Parallel Sets: interactive exploration and visual analysis of categorical data. *IEEE Transactions on Visualization and Computer Graphics*, 12(4):558–568, 2006. doi: 10.1109/TVCG.2006.76. URL <http://ieeexplore.ieee.org/document/1634321/>.
- J. R. Landis and G. G. Koch. An application of hierarchical kappa-type statistics in the assessment of majority agreement among multiple observers. *Biometrics*, 33(2):363–374, Jun 1977. doi: <https://doi.org/10.2307/2529786>.
- D. A. Linzer and J. B. Lewis. polCA: An R package for polytomous variable latent class analysis. *Journal of Statistical Software*, 42(10):1–29, 2011. URL <http://www.jstatsoft.org/v42/i10/>.
- K. T. McDonnell and K. Mueller. Illustrative Parallel Coordinates. *Computer Graphics Forum*, 27(3):1031–1038, May 2008. doi: 10.1111/j.1467-8659.2008.01239.x. URL <http://doi.wiley.com/10.1111/j.1467-8659.2008.01239.x>.
- J. J. Miller and E. J. Wegman. *Computing and Graphics in Statistics*, chapter Construction of Line Densities for Parallel Coordinate Plots, pages 107–123. Springer-Verlag New York, Inc., New York, NY, USA, 1991. ISBN 0-387-97633-7. URL <http://dl.acm.org/citation.cfm?id=140806.140816>.
- P. Murrell and S. Potter. *gridSVG: Export 'grid' Graphics as SVG*, 2020. URL <https://CRAN.R-project.org/package=gridSVG>. R package version 1.7-2.
- A. Pilhöfer and A. Unwin. New Approaches in Visualization of Categorical Data: R Package extracat. *Journal of Statistical Software*, 53(7), 2013. doi: 10.18637/jss.v053.i07. URL <http://www.jstatsoft.org/v53/i07/>.
- M. Schonlau. Visualizing Categorical Data Arising in the Health Sciences Using Hammock Plots. In *Proceedings of the Section on Statistical Graphics, American Statistical Association*, 2003.
- C. Sievert. *Interactive Web-Based Data Visualization with R, plotly, and shiny*. Chapman and Hall/CRC, 2020. ISBN 9781138331457. URL <https://plotly-r.com>.
- S. VanderPlas and H. Hofmann. Signs of the sine illusion—why we need to care. *Journal of Computational and Graphical Statistics*, 24(4):1170–1190, 2015. doi: 10.1080/10618600.2014.951547. URL <https://doi.org/10.1080/10618600.2014.951547>.

E. J. Wegman. Hyperdimensional data analysis using parallel coordinates. *Journal of the American Statistical Association*, 85:664–675, 1990.

H. Wickham. Tidy data. *Journal of Statistical Software, Articles*, 59(10):1–23, 2014. ISSN 1548-7660. doi: 10.18637/jss.v059.i10. URL <https://www.jstatsoft.org/v059/i10>.

H. Wickham. *tidyr: Tidy Messy Data*, 2021. URL <https://CRAN.R-project.org/package=tidyr>. R package version 1.1.3.

H. Wickham, D. Cook, H. Hofmann, and A. Buja. tourr: An R Package for Exploring Multivariate Data with Projections. *Journal of Statistical Software, Articles*, 40(2):1–18, 2011. ISSN 1548-7660. doi: 10.18637/jss.v040.i02. URL <https://www.jstatsoft.org/v040/i02>.

A Example usage

Commented code for Figure 10:

```
1 data(carcinoma, package = "polCA")  
  
2 # Prepping the Dataset  
  
3 carcinoma$total <- rowSums(carcinoma) - 7  
  
4 carcinoma <- carcinoma %>% mutate(  
  
5   across(A:G, .fns = as.factor)  
  
6 )  
  
7 carcinoma %>%  
  
8 # Selecting and scaling variables  
  
9 pcp_select(F, D, C, A, G, E, B, tot) %>%  
  
10 pcp_scale(method="uniminmax") %>%  
  
11 pcp_arrange() %>%  
  
12 # Setting up ggplot for pcp and setting pcp display options  
  
13 ggplot(aes_pcp()) +
```

```

14 geom_pcp_axes() +
15 geom_pcp_boxes(colour="black", alpha=0) +
16 geom_pcp(aes(colour = tot)) +
17 geom_pcp_labels(aes(label = pcp_level), fill="white", alpha = 1) +
18 # Choosing general ggplot display options
19 scale_colour_brewer("Number of\ncarcinoma\ndiagnoses", palette = "Dark2") +
20 theme_bw() +
21 guides(color = guide_legend(reverse=TRUE, override.aes = list(size = 5))) +
22 scale_x_discrete(expand = expansion(add=0.25)) +
23 xlab(NULL) + ylab(NULL) +
24 theme(axis.text.y=element_blank(), axis.ticks.y=element_blank())

```