Ch. 7: Paired Data

One Quantitative Variable

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7.1: Paired Designs

Introduction: Independent Groups vs. Paired Design

- We compared long-run proportions (probabilities) of two groups in chapter 5
- We compared long-run averages (means) of two groups in chapter 6
- These chapters studied independent groups
    - The observational units in different groups are not related to each other

Paired Design

- Data is observed in pairs; each observational unit is a pair
    - Now, a subject in group 1 is related to its partner in group 2

Two Features in Paired Designs

1. One pair of responses per observational/experimental unit
2. Data is built to compare the two groups for every pair

Here is an example from the book:

27 students will listen to music with lyrics or listen to music without lyrics. While students are listening to the music, they will play a memorization game: They will study a list of 25 common five-letter words for 90 seconds. Then, the students will write down as many of the words as they can remember.

Instead of randomly assigning students to each of the two groups, each student will play the memorization game twice: once listening to the song with lyrics and once without lyrics.

There is a New Response Variable

- This response variable is quantitative
- **Response Variable**: The difference between the two groups
  - Each observational unit pair will have a recorded difference
  - The difference can be positive or negative
- Differences are expected to have **less variablility** than measurements in two independent groups

Types of Paired Designs

- **Repeated Measures**: Measure each subject twice, once under explanatory group 1 and once under explanatory group 2
  - Each horse run a 100 yard dash with the shoes a rancher currently uses vs. a new shoe they may buy
- A person gets a medicine, then a placebo a few days later, or visa-versa

Types of Paired Designs

- **Matching Pairs**: Match individuals based on prior knowledge to create our pairs
  - Match two pigs of the same litter to see which feed will add more fat to the meat
  - Pair people, based on their BMI, to see the difference in the amount of pushups they can do in regular clothes vs. workout clothes.

Paired Designs Benefits

- Paired Designs, just by themselves excludes all confounding variables except two:
  - order of tasks
  - carryover effects
- To eliminate task order confounding variable:
  - Randomize which task is first
  - Randomize group assignment to each subject in a pair
- To eliminate carryover effects, use a rest period (wash-out period) to eliminate a carry-over effect of the first task to the second
  - Wait for a drug to leave the body
  - Rest to recover from fatique

When Should You Use a Paired Design?

- When we expect a strong association between repeated observations within a pair
- Why use Paired Designs?
  - In some scenarios, you will get more informative results than independent groups

Parameter and Statistic

Parameter: μ_D

- The long-run mean difference of __ between group 1 and group 2

Hypotheses:

- $H_0 : \mu_D = 0$
- $H_a : \mu_D >, <, \neq 0$

Statistics:

- Sample mean difference between two groups: $\overline{x_D}$
- Sample standard deviation of the differences: s_D

Quick Note

Observational studies can have paired designs too!

Practice Problems

Student researchers, who were big fans of Jeopardy, wanted to see whether an adult's reaction time is different if they hit a button once they see a flashing light vs. hear a noise. They randomly sampled 55 adults. To start, the adult subjects would have both hands behind their back. Then, the researchers had a computer program display a red circle. When it flashed or when the computer made a loud noise, the subjects were to hit the buzzer as fast as they could. The computer would calculate the reaction time in milliseconds. Researchers recorded the reaction time. Each subject did both tests. The researchers randomly determined which test (flashing button vs. a noise) was completed first.

- Is this an indpendent groups or paired design?
- If it is a paired design, what type is it? **Justify your answer**.

Practice Problems

On March 19th 2019, Lincoln residents and businesses were urged to continue to voluntarily reduce water consumption by 25 percent. UNL's Institute of Agriculture and Natural Resources wanted to see if voluntary water restrictions reduced water consumption.

They delivered fliers to 200 Old Cheney Place apartments. They randomly gave a flier notifying residents of voluntary water restrictions to a treatment group (100 one-bedroom apartments). For the control group, they gave a flier to 100 other apartments and only advertised the CASNR week food drive.

- Is this an indpendent groups or paired design?
- If it is a paired design, what type is it? **Justify your answer**.
- If not, how could you make this a paired design?

Come Up With Your Own Study

7.2: Simulation Based Approach to Analyzing Paired Data

Review

Data is observed in pairs. Each observational unit is a pair

- Now, a subject in group 1 is related to its partner in group 2
- Do a paired design when you expect a strong association between repeated observations within a pair

Response Variable: The difference between the two groups

Parameter: μ_D

- The long-run mean difference of _ __ between group 1 and group 2

Hypotheses:

- $H_0 : \mu_D = 0$
- $H_a : \mu_D >, <, \neq 0$

Statistics:

- Sample mean difference between two groups: $\overline{x_D}$
- Sample standard deviation of the differences: s_D

Review

- Repeated Measures: Each subject is measured twice. Once in each group. Their two measurements equals an observational unit pair
- Matched Pairs: Each observational unit is a pair of subjects that are closely related to each other
  - One subject is in a group. The second subject is in the other

Eliminate Confounding Variable Effects

- Randomize the order of tasks:
  - Repated Measures: randomize which task/group is measured first
  - Matched Pairs: randomize which group goes to one of the subjects in each pair
- Rest period to eliminate the carry-over effects.
  - rest from fatique
  - wait for drugs/food to leave the body

Introduction to Simulation

- Remember, when performing simulations, we are assuming the null hypothesis is true!
  - Therefore, we assume there is no association between the explanatory and response variables
  - In other words, we assume $\mu_D = 0$
- Therefore, the two measurements in a pair are different by chance.
  - We can swap the two!

Simulation Process

Simulation Process:

1. For each pair…
2. Flip a coin
   a. If tails, keep the order of the measurements the same as observed data
   b. If heads, swap the measurements
3. Calculate the new difference for each pair
4. Average the differences = $\overline{x_D}$
5. Repeat steps 1 through 4 many times

Let's Do An Example

Can a person's dominant hand click at a different speed than their other hand? Last semester, I asked each of my STAT 218 students to see how many clicks they can do in 5 seconds. I randomly assigned which hand each person used first. I didn't do a wash-out period for the students.

Pair	DominantHand	WeakHand	Difference
1	32	27	5
2	45	32	13
3	33	34	-1
4	34	26	8
5	32	24	8

Average difference of the 5 people = 6.6

Do two simulation reps and calculate the new sample average differences

Use the Matched Pairs Applet

Simulation in Matched Pairs Applet:

1. Download Excel file
2. Copy and Paste Excel File data into the left-hand box.
3. "Use Data" button
4. Click "Randomize" checkbox
5. Input number of simulations (like 1000)
6. Randomize Button
7. P-value using "Count Samples: (Beyond, greater than, or less than) Observed Statistic"
8. P-value is the proportion in the red text

Work with the Same Data

- Download ch7SpeedTest.csv from Canvas Chapter 7
- There are 26 observational units
- $\overline{x_D}$ should equal 2.808
- s_D should equal 4.561
- Find the p-value knowing these two hypotheses
  - $H_0 : \mu_D = 0$
  - $H_a : \mu_D > 0$

We Used the Same Simulation Process

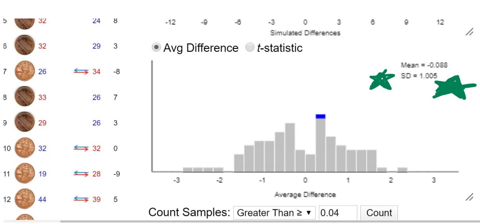
1.Statistic: Compute the statistic in the sample: $\overline{x_D}$

2.Simulate: Identify a chance model that reflects the null hypothesis. We tossed a coin for each pair. If heads, swap the measurements. Repeat many times.

3.Strength of evidence: How many statistics in our 1,000 repetitions were as extreme or more extreme than our observed $\overline{x_D}$

You Can Still...

- You can still find the following with your simulations:
  - Standardized Statistic
  - Confidence Intervals



Use SD of $\bar{x}_D$ from your simulations

7.3: Thoery-Based Approach to Analyzing Paired Data

Validity Conditions

- When the sample size is large enough, the shape of the null distribution is approximately normal
  - The null distribution is centered at the hypothesized value of μ_D
  - The variability of the null distribution depends on the sample size (larger samples will have less variability) and on the variability in the sample differences
- **Validity Conditions**
  - Need at least 20 pairs $(n_D \geq 20)$
  - Distribution of the sample's differences isn't strongly skewed

Standardized Statistic

$t = \frac{\overline{x_D} - \mu_{D0}}{\frac{s_D}{\sqrt{n_D}}}$

$\overline{x_D}$ = observed mean difference of the pairs

s_D = the observed standard deviation of the differences

n_D = number of observed pairs

μ_{D0} = 0 because of the null hypothesis

Standardized Statistic

Same strength of evidence guidelines as before

t-value range	Interpretation
$\lvert t\rvert < 1.5$	**little or no** evidence against H_0
$1.5 \leq \lvert t\rvert < 2$	**moderate** evidence against H_0
$2 \leq \lvert t\rvert < 3$	**strong** evidence against H_0
$3 \leq \lvert t\rvert$	**very strong** evidence against H_0

Theory-Based Confidence Interval

- Confidence Interval: $\overline{x_D} \pm multiplier * \frac{s_D}{\sqrt{n_D}}$
- We are 95% confident the true mean difference of {response variable} between {group 1} and {group 2} is within the interval **[] and []**.
- Is 0 in the interval?
  - No? Reject H_0
  - Yes? Fail to reject H_0, because it is still a plausible explanation for μ_D

Exploration 7.3

- Do questions 1 through 9 with your group
- We will give periodical guidance for the whole class. Good luck!