Stat 982: Module 1, Homework 2

Due on September 27, 2022

Problem 1

(Truncated exponential families) Suppose that distributions P_η have R-N derivatives with respect to a σ-finite measure μ of the form $f_\eta(x) = K(\eta)\exp\left(\sum_{i=1}^k \eta_i T_i(x)\right) h(x)$. For a measurable set A with $P_\eta(A) > 0$, consider the family of distributions Q_η^A on A with R-N derivatives with respect to μ

$$g_\eta(x) \propto f_\eta(x) I[x \in A]$$

Argue that this is an exponential family and say what you can about the natural parameter space for this family in comparison to that of the P_η family.

Problem 2

Suppose that $(X_1, Y_1), (X_2, Y_2), ..., (X_n, Y_n)$ are iid random vectors, and that X_i and Y_i are independently distributed as $N(\mu, \sigma_1^2)$ and $N(\mu, \sigma_2^2)$, respectively, with $\theta = (\mu, \sigma_1^2, \sigma_2^2) \in \mathbb{R}^1 \times (0, \infty) \times (0, \infty)$. Let $\overline{X}$ and S_X^2 be the sample mean and variance for the X_i's and $\overline{Y}$ and S_Y^2 be the sample mean and sample variance for the Y_i's. Show that $(\overline{X}, \overline{Y}, S_X^2, S_Y^2)$ is minimal sufficient but not boundedly complete. What is a first-order ancillary statistic here?

Problem 3

Suppose that $X_1, X_2, ..., X_n$ are iid P_θ for $\theta \in \mathbb{R}^1$, where if $\theta \neq 0$ the distribution P_θ is $N(\theta, 1)$, while P_0 is $N(\theta, 2)$. Show that $\overline{X} = \frac{1}{n}\sum_{i=1}^n$ is complete but not sufficient for θ.

Problem 4

Go through each line of the proof of the Factorization theorem and it's correlary and explain each step in a way that makes it clear that you understand it.