Homework 6

October 17, 2018

1. Write a function `reduce: ('a * 'a -> 'a) -> a' list -> 'a list` that behaves like `foldl` except that it takes the first element of the list as the initial value.

   For example, `reduce (op -) [1,2,3]` evaluates to `3 - (2 - 1) = 2`. This method does not apply to empty list.

   For the rest of the questions, use `zip`, `map`, and `reduce` to solve the problems, where `zip` function is from homework 4.

   The following questions are about vectors and matrix. We represent row vectors using lists. For example, `[2,3,5,4]` represents a row vector of four integers. We represent a matrix using a list of lists. For example, the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

   is written as `[ [1,2,3], [4, 5, 6] ]`.

2. Write a function `vectorAdd: int list * int list -> int list` that add two integer vectors of the same size.

   For example, `vectorAdd ([1,2,3], [4,5,6])` should return `[5, 7, 9]`.

3. Write a function `svProduct: int * int list -> int list` that multiple an integer with an integer list.

   For example, `svProduct(2, [1,2,3])` should return `[2,4,6]`.

4. Write a function `vmProduct` that multiple a row vector of size n with a matrix with n rows and m columns to produce a vector of size m. For example, `vmProduct([1,2,3], [[1,1], [2,1], [3,1]])` should return `[14, 6]`. Or,

$$\begin{aligned} [1\ \ 2\ \ 3] \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} &= 1 \times [1\ \ 1] + 2 \times [2\ \ 1] + 3 \times [3\ \ 1] \\ &= [1\ \ 1] + [4\ \ 2] + [9\ \ 3] \\ &= [14\ \ 6] \end{aligned}$$

   This function uses the functions `svProduct` and `vectorAdd` defined earlier.

5. Write a function `matrixProduct` that multiple a $m \times n$ matrix with a $n \times k$ matrix to obtain a $m \times k$ matrix. For example

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix}$$

where

$$v_1 = [1 \;\; 2 \;\; 3] \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 6 \end{bmatrix}$$

and

$$v_2 = [1 \;\; 1 \;\; 1] \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 3 \end{bmatrix}$$

That is,

```
matrixProduct([ [1, 2, 3], [1, 1, 1] ], [ [1, 1], [2, 1], [3, 1] ])
  = [ [14, 6], [6, 3] ]
```

This problem will use the function `vmProduct` defined previously.