

Homework 10: Curried Sorting Algorithms and Vector Operations in Scala

Stephen Wagstaff
CS 431
November 18, 2018

Sorting

1. Write a function `merge_sort(f: (Int, Int) => Boolean)` (`lst: List[Int]`): `List[Int]` that takes a list, and return a sorted list.

```
def merge_sort(f: (Int, Int) => Boolean) (initialList: List[Int]): List[Int] = {
  def split: (List[Int], List[Int]) = {
    val cut = initialList.length/2
    ( initialList take cut , initialList drop cut )
  }
  def merge(leftList: List[Int], rightList: List[Int]): List[Int] = {
    (leftList, rightList) match {
      case (Nil, _) => rightList
      case (_, Nil) => leftList
      case (leftElement::leftRemaining, rightElement::rightRemaining) =>
        if (f(leftElement, rightElement)) leftElement::merge(leftRemaining, rightList)
        else rightElement::merge(leftList, rightRemaining)
    }
  }
  initialList match{
    case Nil => Nil
    case x::Nil => List(x)
    case _ =>
      val (left, right) = split
      merge (merge_sort (f) (left), merge_sort (f) (right))
  }
}
```

2. Write a function `selection_sort(f: (Int, Int) => Boolean)` (`lst: List[Int]`): `List[Int]` that takes a list and return a sorted list.

```
def selection_sort(f: (Int, Int) => Boolean) (initialList: List[Int]): List[Int] = {
  def select(selectList: List[Int]): List[Int] = {
    selectList match {
      case Nil => Nil
      case lastElement::Nil => List(lastElement)
      case firstElement::list =>
        val testElement::remaining = select(list)
        if(f(testElement, firstElement)) select(testElement::firstElement::remaining)
        else firstElement::testElement::remaining
    }
  }
  initialList match {
    case Nil => Nil
    case _ => select(initialList)
  }
}
```

3. Write a function `insertion_sort(f: (Int, Int) => Boolean)` (`lst: List[Int]`): `List[Int]` that takes a list and return a sorted list.

```
def insertion_sort(f: (Int, Int) => Boolean) (initialList: List[Int]): List[Int] = {
  def insert(element: Int, list: List[Int]): List[Int] ={
    (element, list) match {
```

```

    case (_, Nil) => List(element)
    case (_, head::rest) =>
      if(f(element, head)) element::head::rest
      else head::insert(element, rest)
  }
}
def sort(sofar: List[Int], list: List[Int]): List[Int] = {
  list match {
    case Nil =>sofar
    case element::rest => insert(element, sort(sofar,rest))
  }
}
initialList match {
  case Nil => Nil
  case _ => sort(Nil, initialList)
}
}

```

Vector and Matrix Operations

1. Write a function `vectorAdd: (List[Int], List[Int]) => List[Int]` that add two integer vectors of the same size.
 - For example, `vectorAdd (List(1,2,3), List(4,5,6))` should return `List(5, 7, 9)`.

```

def vectorAdd(vector1: List[Int], vector2: List[Int]): List[Int] = {
  (vector1, vector2) match {
    case (Nil, Nil) => Nil
    case (Nil, _) => vector2
    case (_, Nil) => vector1
    case (_, _) => vector1.zip(vector2).map({case(int1, int2)=> int1+int2})
  }
}

```

2. Write a function `svProduct: (Int, List[Int]) => List[Int]` that multiple an integer with an integer list.
 - For example, `svProduct(2, List(1,2,3))` should return `List(2,4,6)`.

```

def svProduct(multInt: Int, intList:List[Int]): List[Int]= {
  (multInt, intList) match{
    case(_, Nil) => Nil
    case(_, _) => intList.map((y: Int) => multInt*y)
  }
}

```

3. Write a function `vmProduct: (List[Int], List[List[Int]]) => List[Int]` that multiple a row vector of size n with a matrix with n rows and m columns to produce a vector of size m.

- For example, `vmProduct(List(1,2,3), List(List(1,1), List(2,1), List(3,1)))` should return `List(14, 6)`. Or,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = 1 \times \begin{bmatrix} 1 & 1 \end{bmatrix} + 2 \times \begin{bmatrix} 2 & 1 \end{bmatrix} + 3 \times \begin{bmatrix} 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 \end{bmatrix} + \begin{bmatrix} 9 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 6 \end{bmatrix}$$

This function uses the functions `svProduct` and `vectorAdd` defined earlier.

```

def vmProduct(vector: List[Int], matrix: List[List[Int]]): List[Int] = {
  vector.zip(matrix)
    .map({case(int:Int, list:List[Int])=>svProduct(int,list)})
    .reduce(vectorAdd)
}

```

4. Write a function `matrixProduct: (List[List[Int]], List[List[Int]]) => List[List[Int]]` that multiple a $m \times n$ matrix with a $n \times k$ matrix to obtain a $m \times k$ matrix.

- For example

$$\begin{array}{ccccccccc} 1 & & 2 & & 3 & & 1 & & 1 \\ 1 & & 1 & & 1 & \times & 2 & & 1 = v1 = 14 \ 6 \\ & & & & 3 & & 1 & & v2 = 6 \ 3 \end{array}$$

where

$$v1 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 6 \end{bmatrix}$$

and

$$v2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 3 \end{bmatrix}$$

That is,

```
matrixProduct(List(List(1,2,3), List(1,1,1)),
List(List(1,1), List(2,1), List(3,1)))
= List( List(14, 6), List(6, 3) )
```

This problem will use the function `vmProduct` defined previously.

```
def matrixProduct(matrix1: List[List[Int]], matrix2: List[List[Int]]): List[List[Int]] = {
  matrix1.map({row: List[Int] => vmProduct(row, matrix2)})
}
```