Examining Hausdorff dimension and Scaling behaviour with Worm algorithm

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Fractals

Scaling Mass

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A Measure of Roughness

hi

Box Counting Method

hi

Hausdorff Dimension

hi

Algorithms Used For Generating Graph Patterns

Worm Algorithm

Idea is to sample non-zero contributions of the partition function at $T = T_c$. Express them in a way as to form 'loops'.

Hoshen Kopelman Labeling and Graph Dividing

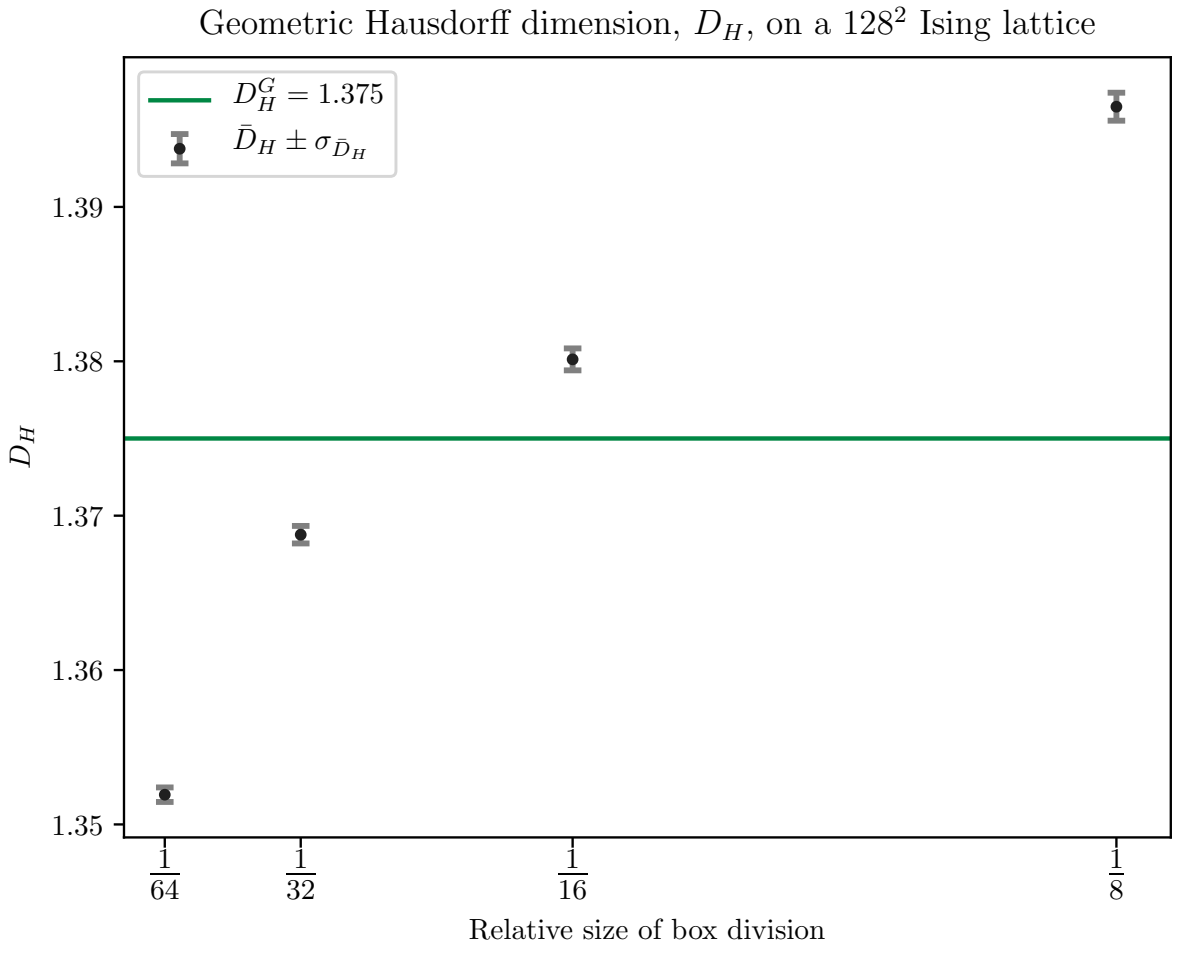
Idea is to sample non-zero contributions of the partition function at $T = T_c$. Express them in a way as to form 'loops'.

Ising Model

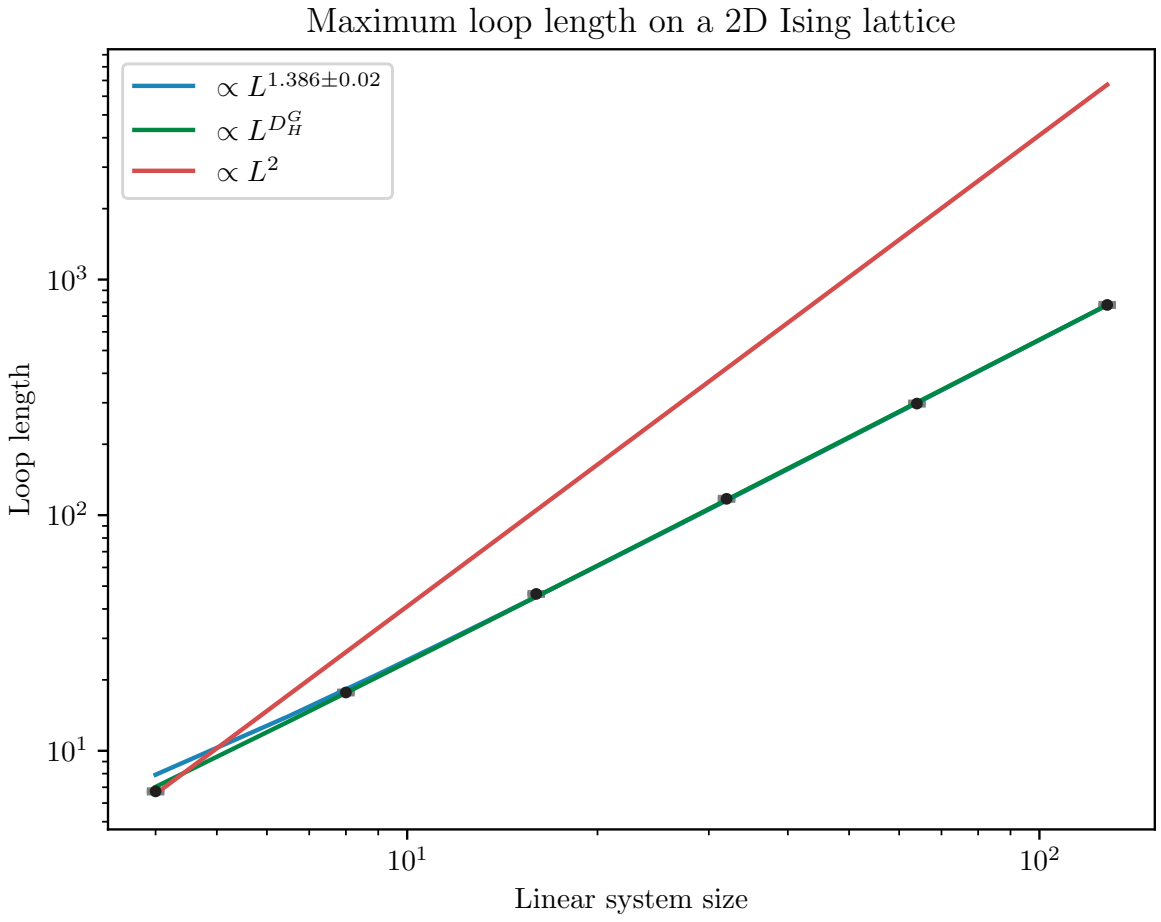
Ising Loop Expansion

hi

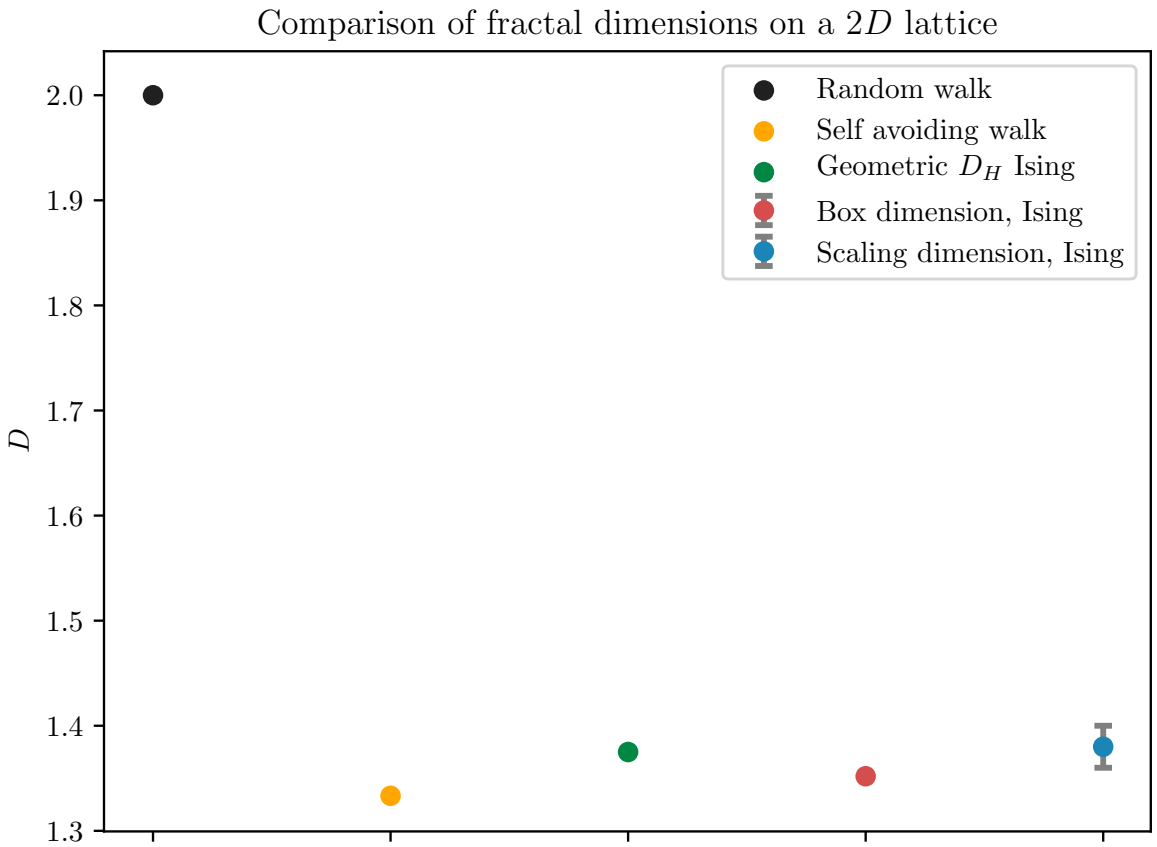
Box Dimension



Scaling Dimension

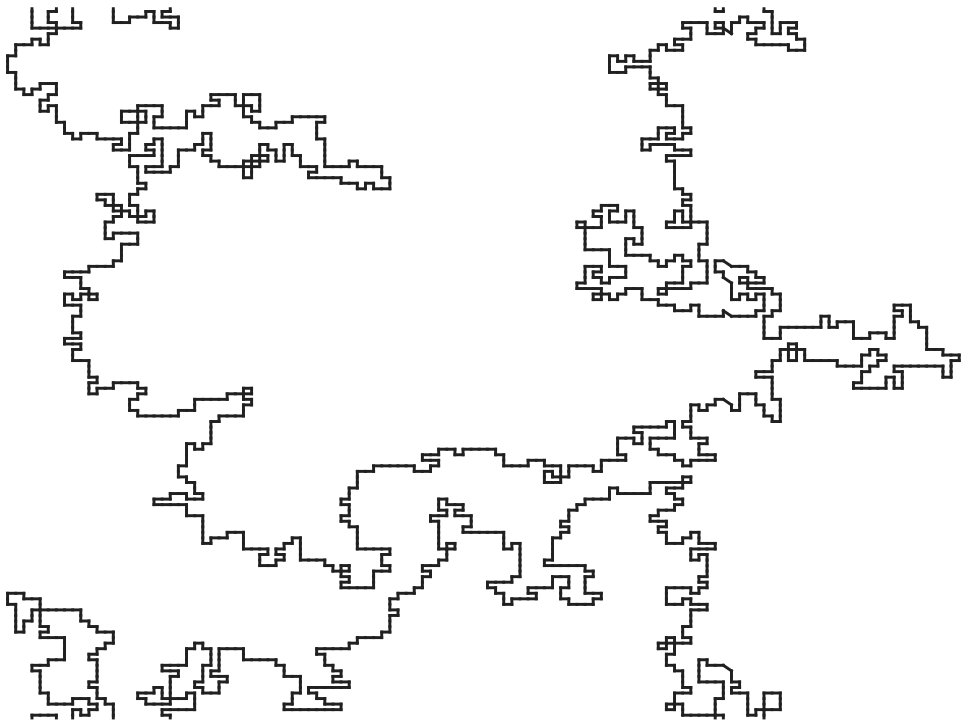


Comparison of Dimensions $2D$ Ising



$2D$ Ising Animation

Largest Ising Loop on a 128^2 Lattice



XY Model

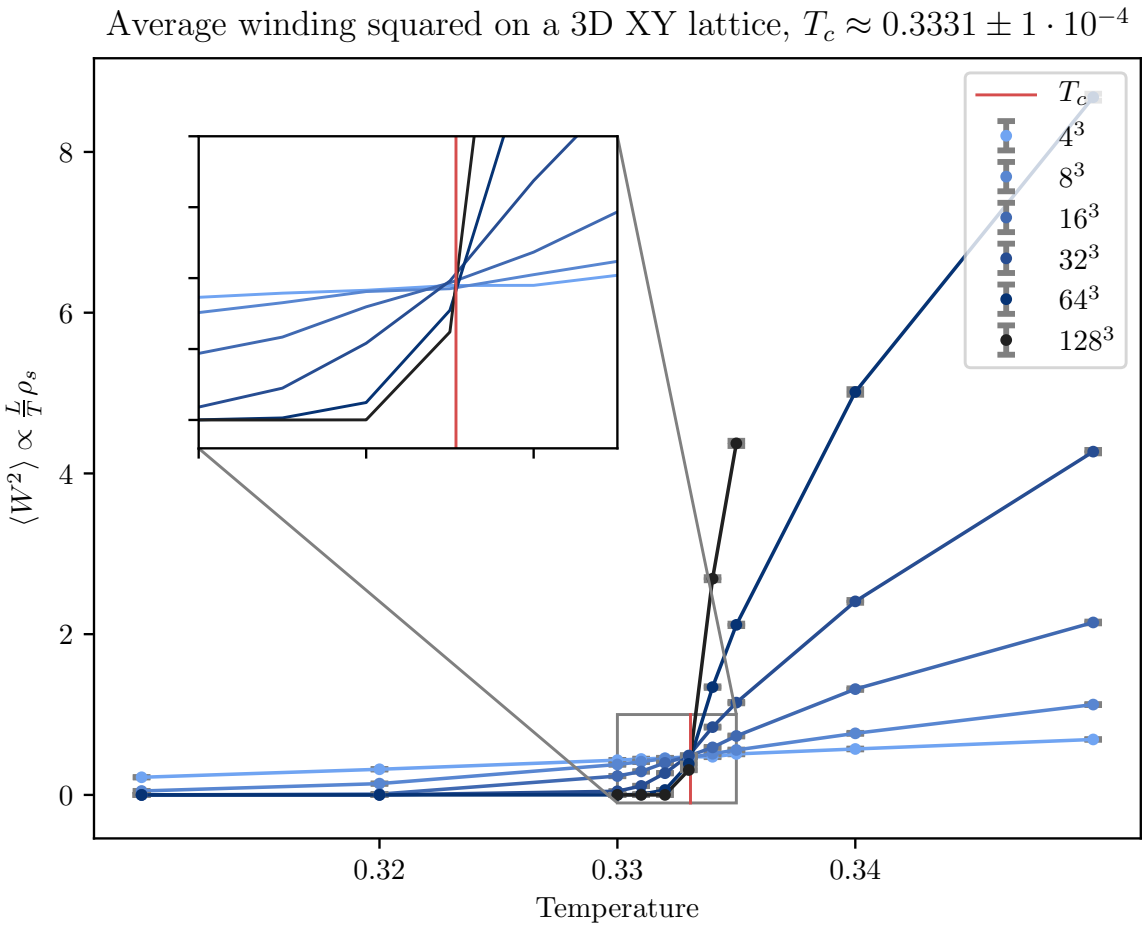
XY Model

Rotors... Loop Expansion. Adds complexity as a direction and weight. Use Villain approximation $\Rightarrow$ Displaces T_c $\Rightarrow$ Use winding number to 'find' T_c.

Winding Number

Explain how winding numbers can show T_c. Maybe illustrations.

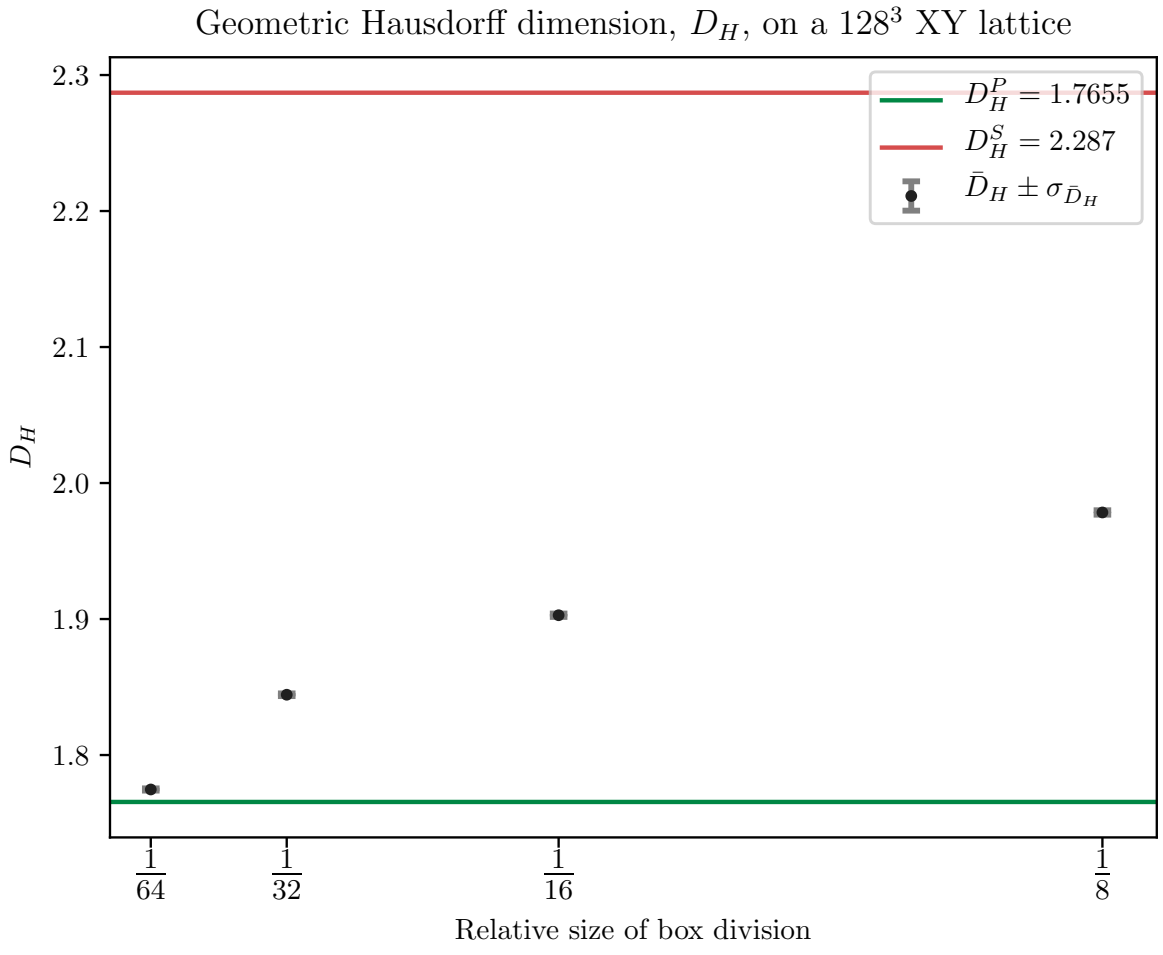
Winding Number



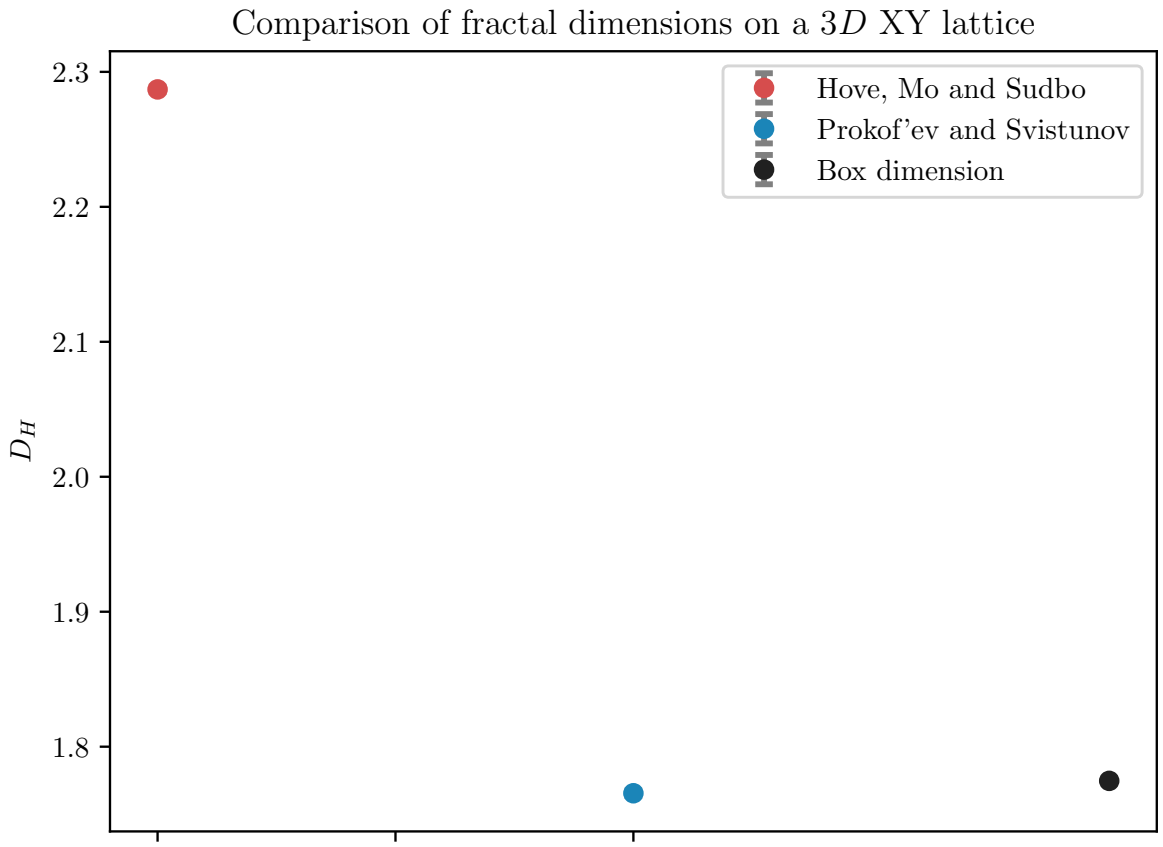
$3D$ **XY Model Hausdorff Dimension**

- Hove, Mo and Sudbo: $D_H = 2.287 \pm 2 \cdot 10^{-3}$
- Prokof'ev and Svistunov Comment: $D_H = 1.7655 \pm 2 \cdot 10^{-3}$

Box Counting Method $3D$ XY



Comparison of Dimensions $3D$ XY



3*D* **XY Animation**

Summary

	D_H
Box	1.35193(5)
Scaling	1.38(2)
D_H^G	1.375
SAW	1.33
Random Walk	2

Table 1: $2D$ Ising

	D_H
Box	1.77468(4)
Prokof'ev	1.765(2)
Sudbo	2.287(2)

Table 2: $3D$ XY