

Hausdorff dimension of clusters at T_c generated by the Worm algorithm

Simon Rydell

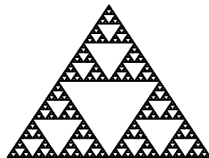
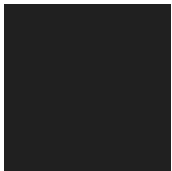
Royal Institute of Technology, Stockholm

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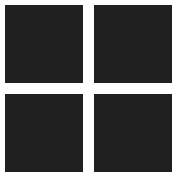
1. Fractals
2. Algorithms for Working with Graph Patterns
3. Ising Model
4. XY Model

Fractals

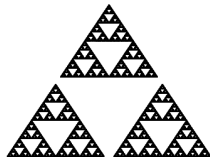
Scaling Mass



Scaling Mass

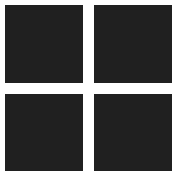


$$L \rightarrow \frac{1}{2}L$$

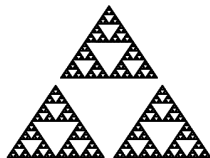


$$L \rightarrow \frac{1}{2}L$$

Scaling Mass

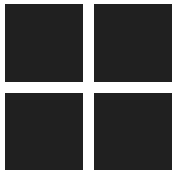


$$L \rightarrow \frac{1}{2}L$$
$$\left(\frac{1}{2}\right)^d M = \frac{1}{4}M$$

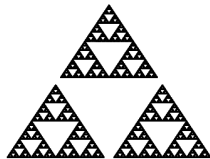


$$L \rightarrow \frac{1}{2}L$$
$$\left(\frac{1}{2}\right)^d M = \frac{1}{3}M$$

Scaling Mass

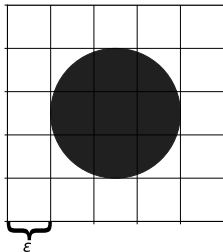


$$L \rightarrow \frac{1}{2}L$$
$$\left(\frac{1}{2}\right)^d M = \frac{1}{4}M$$
$$d = 2$$



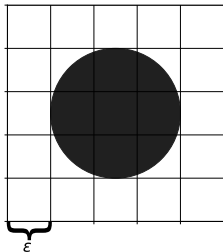
$$L \rightarrow \frac{1}{2}L$$
$$\left(\frac{1}{2}\right)^d M = \frac{1}{3}M$$
$$d = \log_2(3)$$
$$\approx 1.585$$

Box Counting Method



$$N \sim \frac{1}{\epsilon^d}$$

Box Counting Method



$$N \sim \frac{1}{\epsilon^d}$$

$$d = \lim_{\epsilon \rightarrow 0^+} \frac{\ln N(\epsilon)}{\ln 1/\epsilon}$$

$$\mathcal{H}_d(A) = \liminf_{\delta \rightarrow 0} \left\{ \sum_{B \in \mathcal{B}} (\text{diam}(B))^d \right\}$$

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$$\dim_H(A) = \inf \{ d > 0 : \mathcal{H}_d(A) = 0 \}$$

Algorithms for Working with Graph Patterns

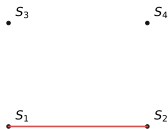
- Graph configurations
- Metropolis Steps

- Hoshen Kopelman Algorithm
- Graph Dividing

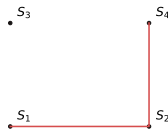
Ising Model

$$Z \propto \sum_{\{S\}} \left(1 + \tanh(K) \sum_{l=1} S_i S_j + \tanh^2(K) \sum_{l=2} (S_i S_j)(S_{i'} S_{j'}) + \dots \right)$$

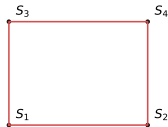
Ising Loop Expansion



a: $(S_1 S_2)$, $L = 1$

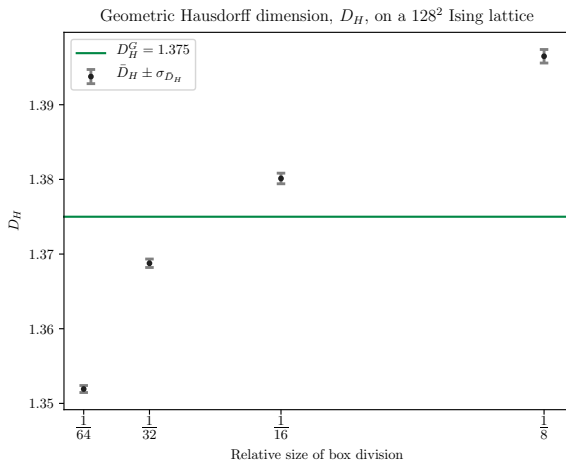


b: $(S_1 S_2)(S_2 S_4)$, $L = 2$

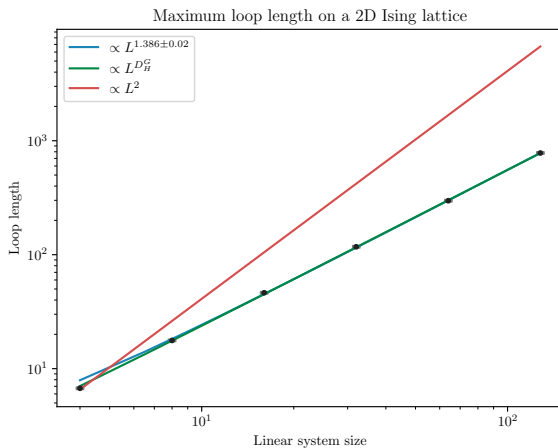


c: $(S_1 S_2)(S_2 S_4)(S_4 S_3)(S_3 S_1)$, $L = 4$

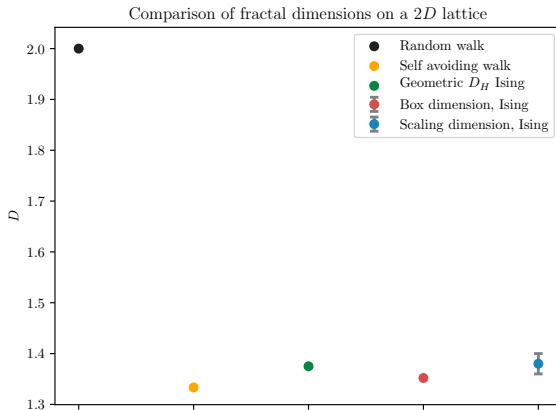
Box Dimension



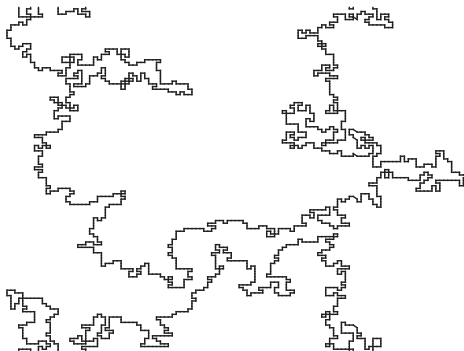
Scaling Dimension



Comparison of Dimensions $2D$ Ising



Largest Ising Loop on a 128^2 Lattice



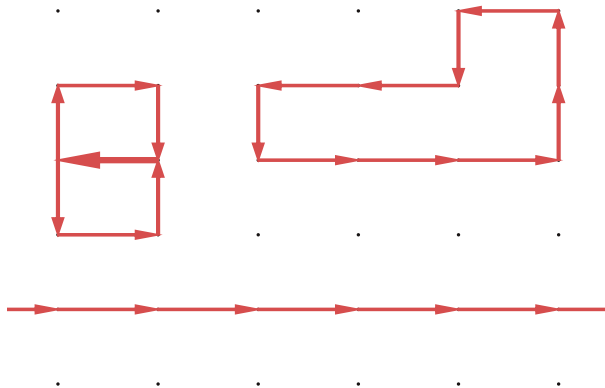
XY Model

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$
$$Z = \prod_i \int \frac{d\theta_i}{2\pi} \prod_{\langle ij \rangle} e^{K \cos(\theta_i - \theta_j)}$$

$$Z \sim \int \frac{d\theta_i}{2\pi} e^{i \sum_{\langle ij \rangle} J_{\langle ij \rangle} (\theta_i - \theta_j)}$$

$$\begin{aligned} Z &\sim \int \frac{d\theta_i}{2\pi} e^{i \sum_{\langle ij \rangle} J_{\langle ij \rangle} (\theta_i - \theta_j)} \\ &\sim \delta_{0, \sum_{\langle ij \rangle} J_{\langle ij \rangle}} \end{aligned}$$

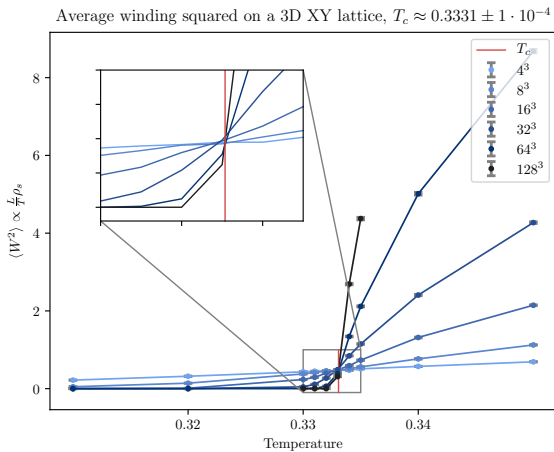
XY Loop expansion



$$E = \frac{1}{2} \sum_i j_i^2$$

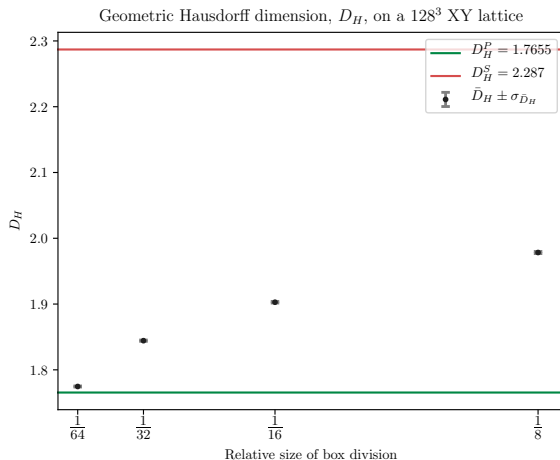
$$\rho_s = L^{2-d} T \langle W_\mu^2 \rangle$$

Winding Number

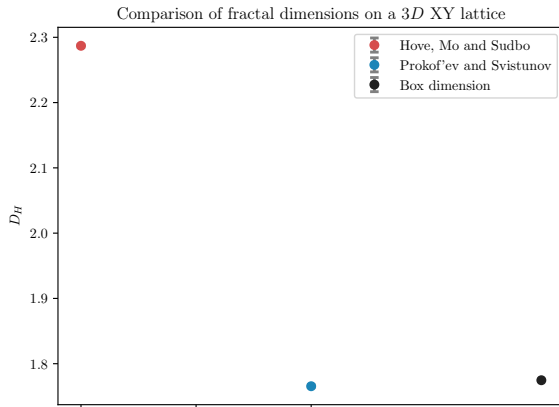


- Hove, Mo and Sudbo (2000): $D_H = 2.287 \pm 4 \cdot 10^{-3}$
- Prokof'ev and Svistunov Comment (2005): $D_H = 1.765 \pm 2 \cdot 10^{-3}$

Box Counting Method 3D XY



Comparison of Dimensions $3D$ XY



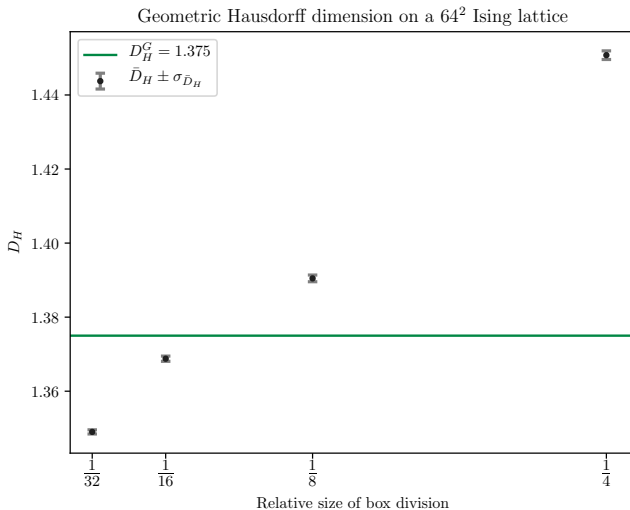
	D_H
Box	1.35193(5)
Scaling	1.38(2)
D_H^G	1.375
SAW	1.33
Random Walk	2

Table 1: 2D Ising

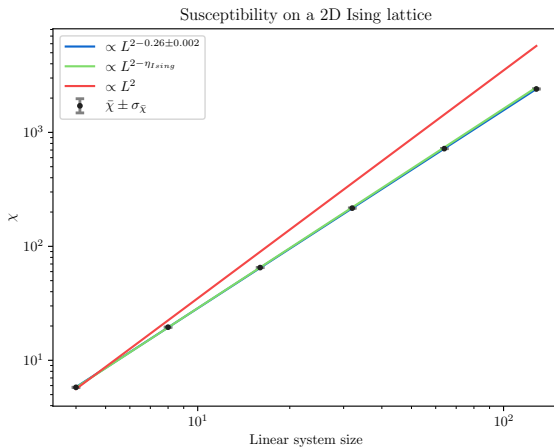
	D_H
Box	1.77468(4)
Prokof'ev	1.765(2)
Sudbo	2.287(2)

Table 2: 3D XY

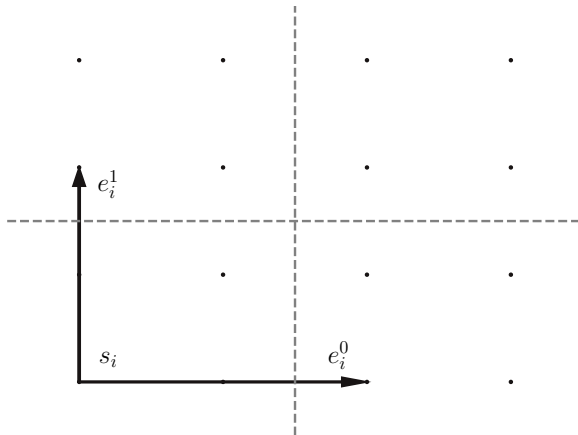
Extra slides: Box Dimension 64^2 Ising



Extra slides: Susceptibility 2D Ising



Extra slides: Graph Dividing Algorithm



Extra slides: Graph Dividing Algorithm

