Tmux

$\underline{\mathbf{T}}$ erminal $\underline{\mathbf{mu}}$ ltiple $\underline{\mathbf{x}}$ or

Simon Rydell

Systecon

Hello World in tmux



- Random and self avoiding walks
- Spin lattices
- Dual transformations
- Geometric properties
- Ising, XY

Table of contents

- 1. Ways of defining fractal dimension
- 2. Algorithms for Working with Graph Patterns
- 3. Ising Model
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Ways of defining fractal

dimension

Scaling Mass

Here was a figure

Scaling Mass

Here was a figure

$$L \rightarrow \frac{1}{2}L$$

$$L o rac{1}{2}L$$

Scaling Mass

Here was a figure

$$L \to \frac{1}{2}L$$

$$\left(\frac{1}{2}\right)^d M = \frac{1}{4}M$$

$$d = 2$$

$$L \to \frac{1}{2}L$$

$$\left(\frac{1}{2}\right)^{d} M = \frac{1}{3}M$$

$$d = \log_{2}(3)$$

$$\approx 1.585$$

Box Counting Method

$$N \sim rac{1}{\epsilon^d}$$

Box Counting Method

$$N \sim rac{1}{\epsilon^d}$$

$$d = \lim_{\epsilon \to 0^+} rac{\ln N(\epsilon)}{\ln 1/\epsilon}$$

Hausdorff Dimension

$$\mathcal{H}_d(A) = \lim_{\delta \to 0} \inf \left\{ \sum_{B \in \mathcal{B}} (\operatorname{diam}(B))^d \right\}$$
$$\operatorname{dim}_H(A) = \inf \{ d > 0 : \mathcal{H}_d(A) = 0 \}$$

Algorithms for Working with

Graph Patterns

Worm Algorithm

• Graph configurations

Here was a figure

• Metropolis steps

Labeling and Dividing the Graph

 Hoshen-Kopelman Algorithm

Here was a figure

• Graph Dividing

Ising Model

Ising Loop Expansion

$$Z \propto \sum_{\{S\}} \left(1 + \mathsf{tanh}(\mathcal{K}) \sum_{l=1} S_i S_j + \mathsf{tanh}^2(\mathcal{K}) \sum_{l=2} (S_i S_j) (S_{i'} S_{j'}) + \ldots \right)$$

Ising Loop Expansion

Scaling Dimension Ising Cluster

Here was a figure [1] B. Duplantier, Conformally Invariant Fractals and Potential Theory, Physical Review Letters 84, 1363 (2000).

Box Dimension Ising Cluster

Here was a figure [1] B. Duplantier, Conformally Invariant Fractals and Potential Theory, Physical Review Letters 84, 1363 (2000).

Comparison of Dimensions 2*D* **Ising**

Largest Ising Loop on a 128² Lattice

2D Ising Animation

XY Model

XY Loop Expansion - Hamiltonian & Partition Function

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$
 $Z = \prod_i \int \frac{\mathrm{d}\theta_i}{2\pi} \prod_{\langle ij \rangle} e^{K \cos(\theta_i - \theta_j)}$

XY Loop Expansion

$$Z \sim \int rac{\mathrm{d} heta_i}{2\pi} \mathrm{e}^{i\sum_{\langle ij
angle} j_{\langle ij
angle}(heta_i- heta_j)} \ \sim \delta_{0,\sum_{\langle ij
angle} j_{\langle ij
angle}}$$

3D XY Animation - 4³ system

Villain Model

$$E = \frac{1}{2} \sum_i j_i^2$$

$$\lim_{\beta \to \infty} \beta_d = 0 \qquad \lim_{\beta \to 0} \beta_d = \infty$$

Winding Number

$$\rho_s = L^{2-d} T \langle W_\mu^2 \rangle$$

Winding Number

3D XY Model Hausdorff Dimension

- Hove, Mo and Sudbo (2000): $D_H = 2.287 \pm 4 \cdot 10^{-3} [1]$
- Prokof'ev and Svistunov Comment (2006): $D_H = 1.765 \pm 2 \cdot 10^{-3}$ [2]

- [1] J. Hove, S. Mo and A. Sudb, Hausdorff Dimension of Critical Fluctuations in Abelian Gauge Theories, Physical Review Letters 85, 2368 (2000).
- [2] N. Prokofev and B. Svistunov, Comment on Hausdorff Dimension of Critical Fluctuations in Abelian Gauge Theories, Physical Review Letters 96 (2006).

bit.ly/2qGMGnR

Box Counting Method 3*D* **XY**

Comparison of Dimensions 3*D* **XY**

3D XY Animation - Largest cluster

Summary

	D_H
Box	1.35193(5)
Scaling	1.386(2)
D_H^G	1.375 [1]
SAW	1.33

Table	1:	2D	Ising
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	D_H
Box	1.77468(4)
Prokof'ev	1.765(2) [2]
Sudbo	2.287(2) [3]

Table 2: 3D XY

- [1] B. Duplantier, Conformally Invariant Fractals and Potential Theory, Physical Review Letters 84, 1363 (2000).
- [2] N. Prokofev and B. Svistunov, Comment on Hausdorff Dimension of Critical Fluctuations in Abelian Gauge Theories, Physical Review Letters 96 (2006).
- [3] J. Hove, S. Mo and A. Sudb, Hausdorff Dimension of Critical Fluctuations in <u>Abelian Gauge Theories, Physical Review</u> Letters 85, 2368 (2000). bit.ly/2qGMGnR

Extra slides: Box Dimension 64² Ising

Extra slides: Susceptibility 2D Ising

Extra slides: Largest Ising cluster

Extra slides: Self avoiding walk

Extra slides: Graph Dividing Algorithm

Extra slides: Graph Dividing Algorithm