

ICSE 2017 Q8 b

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0.1. Solution:

$$\angle BAQ = 30^\circ$$

$$\Rightarrow \angle BAC = 30^\circ$$

$$\text{also } \angle CAP = 180^\circ - \angle CAQ \Rightarrow \angle CAP = 120^\circ$$

$$\Rightarrow \angle CAD = \angle PAD = 60^\circ$$

$$\Rightarrow \angle BAD = 90^\circ$$

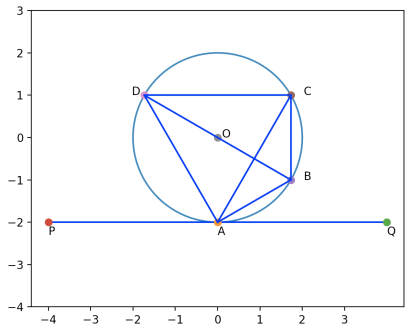
$\Rightarrow BD$ is a diameter

$$\angle ADB = \angle ACB = 30^\circ$$

[Angle made a chord at two different points]

$$\text{Also } \angle CAB = 30^\circ$$

$\Rightarrow \triangle ABC$ is an isosceles triangle



Steps for drawing the diagram:

Symbol	Value	Description
r	2	Radius
O	(0, 0)	Center
P	(-4, -2)	Point on the tangent
Q	(4, -2)	Point on the tangent
A	(0, -2)	Point of contact of the tangent
B	$(1, -\sqrt{3})$	Given in Question
C	$(1, \sqrt{3})$	Given in Question
D	$(-1, \sqrt{3})$	Given in Question

TABLE 0.1.1

Finding the coordinates of the points A.

- A is on the line segment PQ.
- The point closest to the circle on the segment lies on a line passing through O and

perpendicular to PQ $\Rightarrow A(0, -2)$.

Finding the coordinates of the points B.

- $A(0, -2)$.
- $\angle BAQ = 30^\circ$
- $|AB| = 2$
- $\Rightarrow B(-1, \sqrt{3})$

Finding the coordinates of the points C.

- $A(0, -2)$.
- $\angle CAQ = 60^\circ$
- $|AC| = 2\sqrt{3}$
- $\Rightarrow C(1, \sqrt{3})$

Finding the coordinates of the points D.

- $A(0, -2)$.
- $\angle DAP = 60^\circ$
- $|AD| = 2\sqrt{3}$
- $\Rightarrow D(1, -\sqrt{3})$