ICSE 2017 Q8 b

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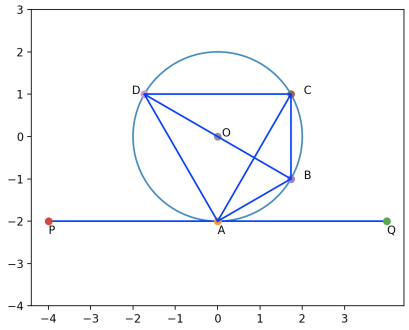
0.1. Solution:

$\angle BAQ = 30°$
$\Rightarrow \angle BAC = 30°$
also $\angle CAP = 180° - \angle CAQ \Rightarrow \angle CAP = 120°$
$\Rightarrow \angle CAD = \angle PAD = 60°$
$\Rightarrow \angle BAD = 90°$
$\Rightarrow BD$ is a diameter
$\angle ADB = \angle ACB = 30°$
[Angle made a chord at two different points]
Also $\angle CAB = 30°$
$\Rightarrow \triangle ABC$ is an isosceles triangle



Steps for drawing the diagram:

Symbol	Value	Description
r	2	Radius, Input
O	$(0, 0)$	Center, Input
P	(-4,-2)	Point on the tangent, Input
Q	$(4, -2)$	Point on the tangent, Input
A	$(0, -2)$	$(0, -r)$, calculated
B	$(1, -\sqrt{3})$	$(rsin2\theta, -rcos2\theta)$, calculated
C	$(1, \sqrt{3})$	$(rsin2\theta, rcos2\theta)$, calculated
D	$(-1, \sqrt{3})$	$(-rsin2\theta, rcos2\theta)$, calculated

TABLE 0.1.1

Finding the coordinates of the points A.

a) A is on the line segment PQ.
b) The point closest to the circle on the segment lies on a line passing through O and perpendicular to PQ $\Rightarrow A(0, -2)$.

Finding the coordinates of the points B.

a) A(0. -2).
b) $\angle BAQ = 30°$
c) $|AB| = 2$
d) $\Rightarrow B(1, -\sqrt{3})$

Finding the coordinates of the points C.

a) A(0. -2).
b) $\angle CAQ = 60°$
c) $|AC| = 2\sqrt{3}$
d) $\Rightarrow C(1, \sqrt{3})$

Finding the coordinates of the points D.

a) A(0. -2).
b) $\angle DAP = 60°$
c) $|AD| = 2\sqrt{3}$
d) $\Rightarrow D(-1, \sqrt{3})$