

ICSE 2017 Q8 b

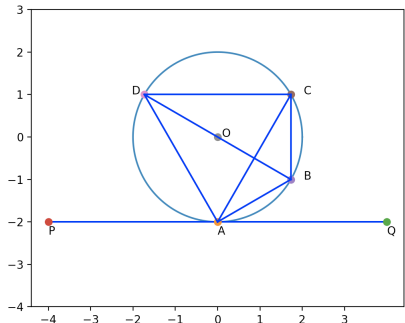
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0.1. **Question:** In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. IF $\angle BAQ = 30^\circ$, prove that:

- (i) BD is a diameter of the circle.
- (ii) ABC is an isosceles triangle.

0.2. **Solution:**

$\angle BAQ = 30^\circ$
 $\Rightarrow \angle BAC = 30^\circ$
 also $\angle CAP = 180^\circ - \angle CAQ \Rightarrow \angle CAP = 120^\circ$
 $\Rightarrow \angle CAD = \angle PAD = 60^\circ$
 $\Rightarrow \angle BAD = 90^\circ$
 $\Rightarrow BD$ is a diameter
 $\angle ADB = \angle ACB = 30^\circ$
 [Angle made a chord at two different points]
 Also $\angle CAB = 30^\circ$
 $\Rightarrow \triangle ABC$ is an isosceles triangle



drawing the diagram:

Finding the coordinates of the points A.

- a) A is on the line segment PQ.
- b) The point closest to the circle on the segment lies on a line passing through O and perpendicular to PQ $\Rightarrow A(0, -2)$.

Finding the coordinates of the points B.

Symbol	Value	Description
θ	30°	Input, $\angle QAB$
r	2	Radius, Input
O	(0,0)	Center, Input
P	(-4,-2)	Point on the tangent, Input
Q	(4, -2)	Point on the tangent, Input
A	(0, -2)	(0, -r), calculated
B	$(1, -\sqrt{3})$	$(r\sin 2\theta, -r\cos 2\theta)$, calculated
C	$(1, \sqrt{3})$	$(r\sin 2\theta, r\cos 2\theta)$, calculated
D	$(-1, \sqrt{3})$	$(-r\sin 2\theta, r\cos 2\theta)$, calculated

TABLE 0.2.1

- a) A(0. -2).
- b) $\angle BAQ = 30^\circ$
- c) $|AB| = 2$
- d) $\Rightarrow B(1, -\sqrt{3})$

Finding the coordinates of the points C.

- a) A(0. -2).
- b) $\angle CAQ = 60^\circ$
- c) $|AC| = 2\sqrt{3}$
- d) $\Rightarrow C(1, \sqrt{3})$

Finding the coordinates of the points D.

- a) A(0. -2).
- b) $\angle DAP = 60^\circ$
- c) $|AD| = 2\sqrt{3}$
- d) $\Rightarrow D(-1, \sqrt{3})$

Steps for