ICSE 2017 Q8 b

Suryaansh Jain*

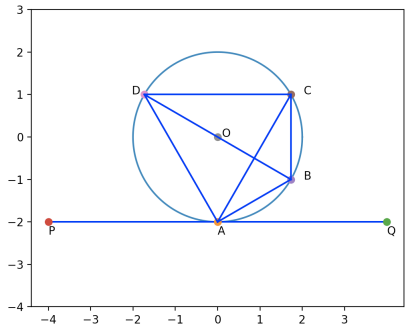
0.1. **Question**: In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. IF $\angle BAQ = 30°$, prove that:
(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle.

0.2. **Solution**:

Symbol	Value	Description
θ	$30°$	Input
r	2	Radius, Input
O	$(0,0)$	Center, Input
P	(-4,-2)	Point on the tangent, Input
Q	$(4,-2)$	Point on the tangent, Input
A	$(0,-2)$	(0, $-r$), calculated
B	$(1,-\sqrt{3})$	$(rsin2\theta, -rcos2\theta)$, calculated
C	$(1,\sqrt{3})$	$(rsin2\theta, rcos2\theta)$, calculated
D	$(-1,\sqrt{3})$	$(-rsin2\theta, rcos2\theta)$, calculated

TABLE 0.2.1

$\angle BAQ = 30°$
$\Rightarrow \angle BAC = 30°$
also $\angle CAP = 180° - \angle CAQ \Rightarrow \angle CAP = 120°$
$\Rightarrow \angle CAD = \angle PAD = 60°$
$\Rightarrow \angle BAD = 90°$
$\Rightarrow BD$ is a diameter
$\angle ADB = \angle ACB = 30°$
[Angle made a chord at two different points]
Also $\angle CAB = 30°$
$\Rightarrow \triangle ABC$ is an isosceles triangle



Steps for drawing the diagram:
Finding the coordinates of the points A.

a) A is on the line segment PQ.
b) The point closest to the circle on the segment lies on a line passing through O and perpendicular to PQ $\Rightarrow A(0,-2)$.

Finding the coordinates of the points B.

a) A(0. -2).
b) $\angle BAQ = 30°$
c) $|AB| = 2$
d) $\Rightarrow B(1,-\sqrt{3})$

Finding the coordinates of the points C.

a) A(0. -2).
b) $\angle CAQ = 60°$
c) $|AC| = 2\sqrt{3}$
d) $\Rightarrow C(1,\sqrt{3})$

Finding the coordinates of the points D.

a) A(0. -2).
b) $\angle DAP = 60°$
c) $|AD| = 2\sqrt{3}$
d) $\Rightarrow D(-1,\sqrt{3})$