

# ICSE 2017 Q8 b

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0.1. **Question:** In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of  $\angle CAQ$  and  $\angle PAC$ . IF  $\angle BAQ = 30^\circ$ , prove that:

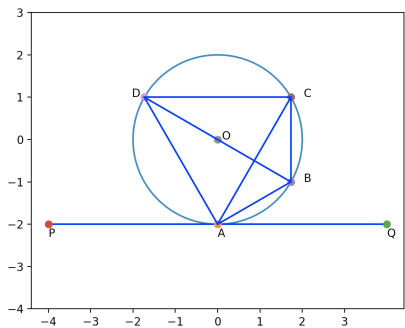
- (i) BD is a diameter of the circle.
- (ii) ABC is an isosceles triangle.

0.2. **Solution:**

Symbol	Value	Description
$r$	2	Radius, Input
$O$	(0, 0)	Center, Input
$P$	(-4, -2)	Point on the tangent, Input
$Q$	(4, -2)	Point on the tangent, Input
$A$	(0, -2)	(0, $-r$ ), calculated
$B$	(1, $-\sqrt{3}$ )	( $r \sin 2\theta$ , $-r \cos 2\theta$ ), calculated
$C$	(1, $\sqrt{3}$ )	( $r \sin 2\theta$ , $r \cos 2\theta$ ), calculated
$D$	(-1, $\sqrt{3}$ )	( $-r \sin 2\theta$ , $r \cos 2\theta$ ), calculated

TABLE 0.2.1

$\angle BAQ = 30^\circ$   
 $\Rightarrow \angle BAC = 30^\circ$   
 also  $\angle CAP = 180^\circ - \angle CAQ \Rightarrow \angle CAP = 120^\circ$   
 $\Rightarrow \angle CAD = \angle PAD = 60^\circ$   
 $\Rightarrow \angle BAD = 90^\circ$   
 $\Rightarrow BD$  is a diameter  
 $\angle ADB = \angle ACB = 30^\circ$   
 [Angle made a chord at two different points ]  
 Also  $\angle CAB = 30^\circ$   
 $\Rightarrow \triangle ABC$  is an isosceles triangle



Steps for drawing the diagram:

Finding the coordinates of the points A.

- a) A is on the line segment PQ.
- b) The point closest to the circle on the segment lies on a line passing through O and perpendicular to PQ  $\Rightarrow A(0, -2)$ .

Finding the coordinates of the points B.

- a) A(0, -2).
- b)  $\angle BAQ = 30^\circ$
- c)  $|AB| = 2$
- d)  $\Rightarrow B(1, -\sqrt{3})$

Finding the coordinates of the points C.

- a) A(0, -2).
- b)  $\angle CAQ = 60^\circ$
- c)  $|AC| = 2\sqrt{3}$
- d)  $\Rightarrow C(1, \sqrt{3})$

Finding the coordinates of the points D.

- a) A(0, -2).
- b)  $\angle DAP = 60^\circ$
- c)  $|AD| = 2\sqrt{3}$
- d)  $\Rightarrow D(-1, \sqrt{3})$