ICSE 2017 Q8 b

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Question: In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. IF $\angle BAQ = 30^\circ$, prove that:
(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle.
Solution:

$$\angle BAQ = 30^\circ \tag{0.1}$$
$$\Rightarrow \angle BAC = 30^\circ \tag{0.2}$$

Also,

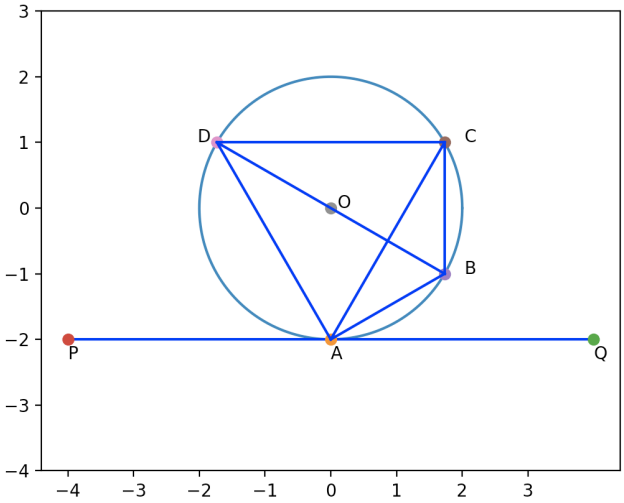
$$\angle CAP = 180^\circ - \angle CAQ \tag{0.3}$$
$$\Rightarrow \angle CAP = 120^\circ \tag{0.4}$$
$$\Rightarrow \angle CAD = \angle PAD = 60^\circ \tag{0.5}$$
$$\Rightarrow \angle BAD = 90^\circ \tag{0.6}$$

So BD is a diameter.

Since angle made by a chord at two different points is equal,

$$\angle ADB = \angle ACB = 30^\circ \tag{0.7}$$

And since $\angle CAB = 30^\circ$, $\triangle ABC$ is an isosceles triangle.



Steps for drawing the diagram:
Finding the coordinates of the points A.

Symbol	Value	Description
θ	30°	Input, $\angle QAB$
r	2	Radius, Input
O	$(0,0)$	Center, Input
P	(-4,-2)	Point on the tangent, Input
Q	$(4,-2)$	Point on the tangent, Input
A	$(0,-2)$	(0, $-r$), calculated
B	$(1,-\sqrt{3})$	$(rsin2\theta, -rcos2\theta)$, calculated
C	$(1,\sqrt{3})$	$(rsin2\theta, rcos2\theta)$, calculated
D	$(-1,\sqrt{3})$	$(-rsin2\theta, rcos2\theta)$, calculated

TABLE 0.1

1) A is on the line segment PQ.
2) The point closest to the circle on the segment lies on a line passing through O and perpendicular to PQ $\Rightarrow$ A $\left(0-2\right)$.

Finding the coordinates of the points B.

1) A $\left(0-2\right)$.
2) $\angle BAQ = 30^\circ$
3) $|AB| = 2$
4) $\Rightarrow$ **B** $\left(1,-\sqrt{3}\right)$

Finding the coordinates of the points C.

1) A $\left(0-2\right)$.
2) $\angle CAQ = 60^\circ$
3) $|AC| = 2\sqrt{3}$
4) $\Rightarrow$ **C** $\left(1,\sqrt{3}\right)$

Finding the coordinates of the points D.

1) A $\left(0-2\right)$.
2) $\angle DAP = 60^\circ$
3) $|AD| = 2\sqrt{3}$
4) $\Rightarrow$ **D** $\left(-1,\sqrt{3}\right)$