ICSE 2017 Q8 b

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Question: In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. IF $\angle BAQ = 30°$, prove that:
(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle.
Solution:

$$\angle BAQ = 30° \qquad (0.1)$$
$$\Rightarrow \angle BAC = 30° \qquad (0.2)$$

Also,

$$\angle CAP = 180° - \angle CAQ \qquad (0.3)$$
$$\Rightarrow \angle CAP = 120° \qquad (0.4)$$
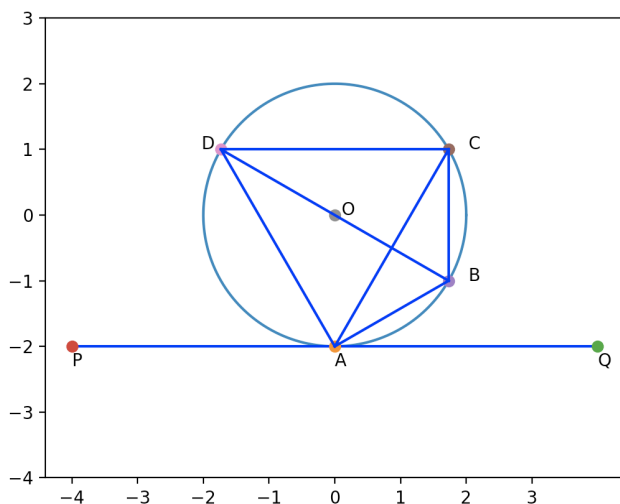$$\Rightarrow \angle CAD = \angle PAD = 60° \qquad (0.5)$$
$$\Rightarrow \angle BAD = 90° \qquad (0.6)$$

So BD is a diameter.
Since angle made by a chord at two different points is equal,

$$\angle ADB = \angle ACB = 30° \qquad (0.7)$$

And since $\angle CAB = 30°$, $\triangle ABC$ is an isosceles triangle.

Symbol	Value	Description
θ	$30°$	Input, $\angle QAB$
r	2	Radius, Input
O	$(0,0)$	Center, Input
P	$(-4,-2)$	Point on the tangent, Input
Q	$(4,-2)$	Point on the tangent, Input
A	$(0,-2)$	$(0, -r)$, calculated
B	$(1,-\sqrt{3})$	$(r\sin 2\theta, -r\cos 2\theta)$, calculated
C	$(1,\sqrt{3})$	$(r\sin 2\theta, r\cos 2\theta)$, calculated
D	$(-1,\sqrt{3})$	$(-r\sin 2\theta, r\cos 2\theta)$, calculated

TABLE 0.1

Finding the coordinates of the points A.
1) A is on the line segment PQ.
2) The point closest to the circle on the segment lies on a line passing through O and perpendicular to PQ $\Rightarrow$ A $(0 - 2)$.

Finding the coordinates of the points B.
1) A $(0 - 2)$.
2) $\angle BAQ = 30°$
3) $|AB| = 2$
4) $\Rightarrow$ **B** $(1, -\sqrt{3})$

Finding the coordinates of the points C.
1) A $(0 - 2)$.
2) $\angle CAQ = 60°$
3) $|AC| = 2\sqrt{3}$
4) $\Rightarrow$ **C** $(1, \sqrt{3})$

Finding the coordinates of the points D.
1) A $(0 - 2)$.
2) $\angle DAP = 60°$
3) $|AD| = 2\sqrt{3}$
4) $\Rightarrow$ **D** $(-1, \sqrt{3})$

Steps for drawing the diagram: