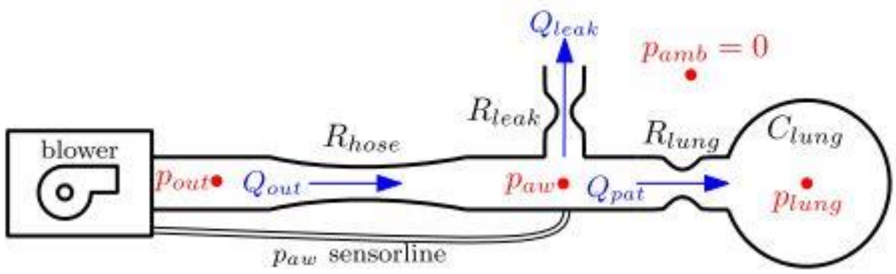
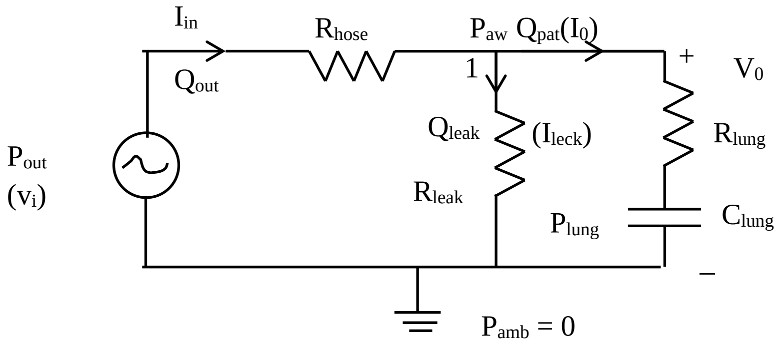
Task 1:

Determine the transfer function of the respiratory system shown in Fig. 3



Equivalent Electrical Circuit -



P $\Rightarrow$ Voltage

Q $\Rightarrow$ Current

C$\Rightarrow$ Capacitance

R $\Rightarrow$ resistance

$$Q_{out} = \frac{P_{out} - P_{aw}}{R_{hose}}$$

$$Q_{leak} = \frac{P_{aw}}{R_{leak}}$$

$$Q_{pat} = Q_{out} - Q_{leak} = \frac{P_{aw} - P_{lung}}{R_{lung}} \text{ ....(i)}$$

$$\frac{dP_{lung}}{dt} = \dot{P}_{lung} = \frac{Q_{pat}}{C_{lung}} = \frac{P_{aw} - P_{lung}}{C_{lung}R_{lung}} \text{ ....(ii)}$$

Applying KCL at node 1

$$\frac{P_{out} - P_{aw}}{R_{hose}} = \frac{P_{aw}}{R_{leak}} + \frac{P_{aw} - P_{lung}}{R_{lung}}$$

$$\Rightarrow \frac{P_{out}}{R_{hose}} - \frac{P_{aw}}{R_{hose}} = \frac{P_{aw}}{R_{leak}} + \frac{P_{aw}}{R_{lung}} - \frac{P_{lung}}{R_{lung}}$$

$$\Rightarrow \frac{P_{aw}}{R_{hose}} + \frac{P_{aw}}{R_{leak}} + \frac{P_{aw}}{R_{lung}} = \frac{P_{out}}{R_{hose}} + \frac{P_{lung}}{R_{lung}}$$

$$\Rightarrow P_{aw} = \frac{\frac{P_{out}}{R_{hose}} + \frac{P_{lung}}{R_{lung}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}} \text{ ....(iii)}$$

$$\Rightarrow P_{aw} = \left(\frac{\frac{1}{R_{lung}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}}\right) P_{lung} + \left(\frac{\frac{1}{R_{hose}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}}\right) P_{out} \text{ ....(iv)}$$

Substituting the value of P_{aw} in eqn (i) & (ii)

$$Q_{pat} = \frac{\frac{1}{R_{lung}}P_{lung} + \frac{1}{R_{hose}}P_{out}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} - \frac{1}{R_{lung}} P_{lung}$$

$$= \frac{\frac{1}{R_{lung}} - \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} P_{lung} + \frac{\frac{1}{R_{hose}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} P_{aw}$$

$$Q_{pat} = \left(-\frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}\right) P_{lung} + \left(\frac{\frac{1}{R_{hose}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}\right) P_{out} \text{ ....(v)}$$

$$P_{lung} = \frac{\frac{1}{R_{lung}}P_{lung} + \frac{1}{R_{hose}}P_{out}}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} - \frac{P_{lung}}{C_{lung}R_{lung}}$$

$$= \frac{\frac{1}{R_{lung}} - \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} P_{lung} + \frac{\frac{1}{R_{hose}}}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} P_{out}$$

$$P_{lung} = \left(-\frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}\right) P_{lung} + \left(\frac{\frac{1}{R_{hose}}}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}\right) P_{out} \text{ ....(vi)}$$

Here, input $\Rightarrow P_{out}$, output $\Rightarrow \begin{bmatrix} P_{aw} \\ Q_{pat} \end{bmatrix}$, state $\Rightarrow P_{lung}$

From eqn (iv) to (vi) we find:

$P_{lung} = A_h P_{lung} + B_h P_{out}$ ....(vii)

$\begin{bmatrix} P_{aw} \\ Q_{pat} \end{bmatrix} = C_h P_{lung} + D_h P_{out}$ ....(viii)

Where,

$$A_h = -\frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}$$

$$B_h = \frac{\frac{1}{R_{hose}}}{C_{lung}R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}$$

$$C_h = \begin{bmatrix} \frac{\frac{1}{R_{leak}}}{\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}} \\ -\frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} \end{bmatrix}$$

$$D_h = \begin{bmatrix} \frac{\frac{1}{R_{hose}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}} \\ -\frac{\frac{1}{R_{hose}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} \end{bmatrix}$$

We know that,

$Y(s) = \left[C_n \,(sI - A_h)^{-1}_{B_h} + D_h\right]u(s)$

where Y(s) is the output matrix and u(s) is the input matrix

so, $\frac{Y(s)}{u(s)} = H(s) = C_h(SI - A_h)^{-1}B_h + D_h$

Using the values of parameters given in table 1 of the reference paper, H(s) was calculated using the code given below.

Rlung	5/1000
Clung	20
Rleak	60/1000
Rhose	4.5/1000
ω_n	$2\pi30$

<u>Code:</u>

```
clc;
clearvars;
Rlung = 5/1000;
Clung = 20;
Rleak = 60/1000;
Rhose = 4.5/1000;

Ah = -((1/Rhose)+(1/Rleak))/(Rlung*Clung*((1/Rlung)+(1/Rhose)+(1/Rleak)))
Bh = (1/Rhose)/(Rlung*Clung*((1/Rlung)+(1/Rhose)+(1/Rleak)))
Ch = [(1/Rlung)/((1/Rlung)+(1/Rhose)+(1/Rleak));...
    -((1/Rhose)+(1/Rleak))/(Rlung*((1/Rlung)+(1/Rhose)+(1/Rleak)))]
Dh = [(1/Rhose)/((1/Rlung)+(1/Rhose)+(1/Rleak));...
    (1/Rhose)/(Rlung*((1/Rlung)+(1/Rhose)+(1/Rleak)))]
s = tf('s');
Hs = Ch*inv((s*eye(1) - Ah))*Bh + Dh
```

So, the calculated H(s) is:

$$H(s) = \begin{bmatrix} \frac{0.5063s + 5.063}{s + 5.443} \\ \frac{101.3s + 1.137e-13}{s + 5.443} \end{bmatrix}$$

Task 2:

Determine the overall transfer function of the closed loop control system shown in Fig. 4.

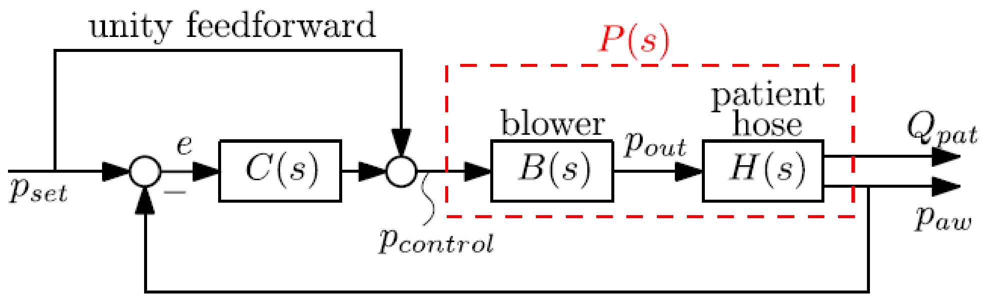
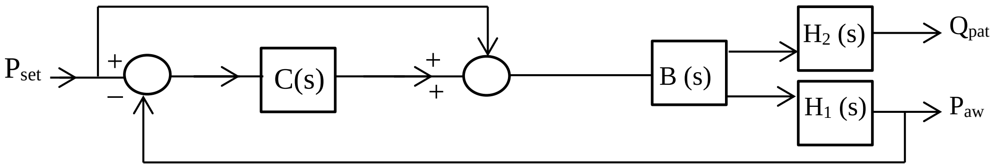


Fig. 4. Closed-loop control scheme with a linear controller $C(s)$.

Changing the diagram to:

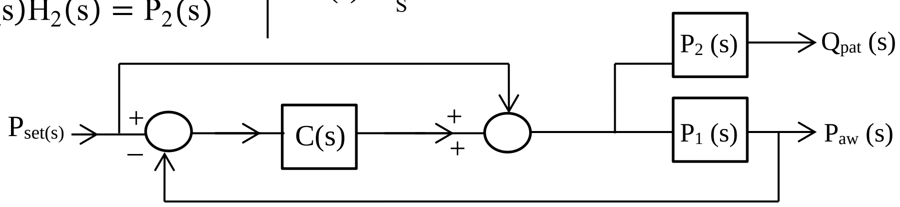


Let

$$B(s)H_1(s) = P_1(s)$$
$$B(s)H_2(s) = P_2(s)$$

$$C(s) = \frac{Ki}{s}$$

So,



Now,

$$P_{aw}(s) = P_1(s)[P_{set}(s) + C(s)(P_{set}(s) - P_{aw}(s))]$$
$$= P_1(s)P_{set}(s) + P_1(s)C(s)P_{set}(s) - P_1(s)C(s)P_{aw}(s)$$
$$\Rightarrow P_{aw}(s) + P_1(s)C(s)P_{aw}(s) = P_{set}(s)(P_1(s) + P_1(s)C(s)\ \ )$$
$$\Rightarrow \frac{P_{aw}(s)}{P_{set}(s)} = \frac{P_1(s)+C(s)P_1(s)}{1+C(s)P_1(s)} = T_1(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again,

$$Q_{pat}(s) = P_2(s)[P_{set}(s) + C(s)(P_{set}(s) - P_{aw}(s)]$$
$$= P_2(s)P_{set}(s) + P_2(s)C(s)P_{set}(s) - P_2(s)C(s)P_{aw}(s)$$
$$= P_2(s)P_{set}(s) + P_2(s)C(s)P_{set}(s) - P_2(s)C(s)P_{set}(s)T_1(s)$$
$$= P_{set}(s)[P_2(s) + P_2(s)C(s) - P_2(s)C(s)T_1(s)]$$
$$\Rightarrow \frac{Q_{pat}(s)}{P_{set}(s)} = P_2(s)[1 + C(s)(1 - T_1(s))]$$
$$= P_2(s)\left[1 + C(s) \times \frac{1+C(s)P_1(s)-P_1(s)-C(s)P_1(s)}{1+C(s)P_1(s)}\right]$$
$$= P_2(s)\left[1 + \frac{C(s)-C(s)P_1(s)}{1+C(s)P_1(s)}\right]$$
$$= P_2(s) \times \frac{1+C(s)P_1(s)+C(s)-C(s)P_1(s)}{1+C(s)P_1(s)}$$
$$\frac{Q_{pat}(s)}{P_{set}(s)} = \frac{P_2(s)+C(s)P_2(s)}{1+C(s)P_1(s)} = T_2(s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

By running the following code:

```
clc;
clearvars;

ki = 1;
wn = 2*3.1416*30;
zeta = 1;
s = tf('s');
Hs = [(0.5063*s + 5.063)/(s + 5.443);...
    (101.3*s + 1.137e-13)/(s + 5.443)];
Bs = (wn^2)/(s^2+(2*zeta*wn*s)+wn^2);
Ps = Hs.*Bs;
Cs = ki/s;
Closed_Paw = (Ps(1)+(Cs*Ps(1)))/(1+(Cs*Ps(1)));
CLosed_Qpat = (Ps(2)+(Cs*Ps(2)))/(1+(Cs*Ps(1)));
```

We found $T_1(s)$ to be:

```
Closed_Paw =


  1.799e04 s^9 + 1.396e07 s^8 + 4.135e09 s^7 + 5.68e11 s^6 + 3.387e13 s^5 + 5.755e14 s^4

                                              + 3.83e15 s^3 + 1.002e16 s^2 + 6.728e15 s

  -------------------------------------------------------------------------------------

  s^11 + 1147 s^10 + 5.515e05 s^9 + 1.428e08 s^8 + 2.119e10 s^7 + 1.753e12 s^6

                + 7.044e13 s^5 + 8.957e14 s^4 + 4.759e15 s^3 + 1.052e16 s^2 + 6.728e15 s
```

And $T_2(s)$ to be:

```
Closed_Qpat =


  3.599e06 s^9 + 2.757e09 s^8 + 7.997e11 s^7 + 1.057e14 s^6 + 5.721e15 s^5 + 5.794e16 s^4

                                              + 1.869e17 s^3 + 1.346e17 s^2 + 151.1 s

  -------------------------------------------------------------------------------------

  s^11 + 1147 s^10 + 5.515e05 s^9 + 1.428e08 s^8 + 2.119e10 s^7 + 1.753e12 s^6

                + 7.044e13 s^5 + 8.957e14 s^4 + 4.759e15 s^3 + 1.052e16 s^2 + 6.728e15 s
```

Task 3:

Sketch the root locus of the control system shown in Fig. 4 for 0<ki< of the integral controller C(s).

Comparing equations 1 and 2 with,

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)} \equiv \frac{P_1(s) + C(s)P_1(s)}{1 + \frac{Ki}{s}P_1(s)} = T_1(s)$$

And,

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)} \equiv \frac{P_2(s) + C(s)P_2(s)}{1 + \frac{Ki}{s}P_1(s)} = T_2(s)$$

We Find that, for both closed loop transfer functions,

$$KG(s)H(s) \equiv Ki\frac{P_1(s)}{s}$$

So, the root locus will be drawn for $\frac{P(s)}{s}$ for $k_i > 0$

Code for plotting root locus:

```
clc;
clearvars;

s = tf('s');
Ps = [(1.799e04*s+1.799e05)/(s^3+382.4*s^2+3.758e04*s+1.934e05);...
    (3.599e06*s+4.04e-09)/(s^3+382.4*s^2+3.758e04*s+1.934e05)];

figure(1)
rlocus(Ps(1)/s);
axis([-200 5 -205 205]);
```

Root Locus: