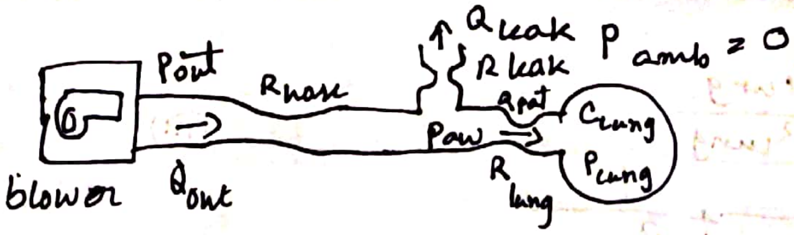
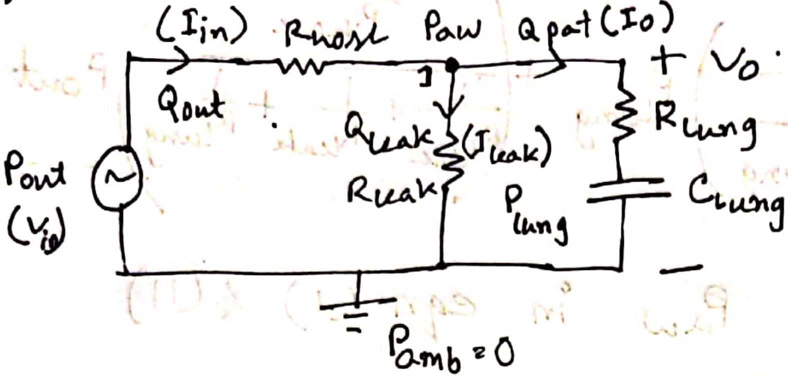
Date :



Equivalent electrical circuit -



$P \Rightarrow$ Voltage, $Q \Rightarrow$ Current, $C \Rightarrow$ Capacitance, $R \Rightarrow$ Resistance

$$Q_{out} = \frac{P_{out} - P_{aw}}{R_{hose}} \; ; \; Q_{leak} = \frac{P_{aw}}{R_{leak}}$$

$$Q_{pat} = Q_{out} - Q_{leak} = \frac{P_{aw} - P_{lung}}{R_{lung}} \quad \text{--- (I)}$$

$$\frac{dP_{lung}}{dt} = \dot{P}_{lung} = \frac{Q_{pat}}{C_{lung}} = \frac{P_{aw} - P_{lung}}{C_{lung} R_{lung}} \quad \text{--- (II)}$$

Applying KCL at node 1.

$$\frac{P_{out} - P_{aw}}{R_{hose}} = \frac{P_{aw}}{R_{leak}} + \frac{P_{aw} - P_{lung}}{R_{lung}}$$

$$\Rightarrow \frac{P_{out}}{R_{hose}} - \frac{P_{aw}}{R_{hose}} = \frac{P_{aw}}{R_{leak}} + \frac{P_{aw}}{R_{lung}} - \frac{P_{lung}}{R_{lung}}$$

$$\Rightarrow \frac{P_{aw}}{R_{hose}} + \frac{P_{aw}}{R_{leak}} + \frac{P_{aw}}{R_{lung}} = \frac{P_{out}}{R_{hose}} + \frac{P_{lung}}{R_{lung}}$$

$$\Rightarrow P_{aw} = \frac{\frac{P_{out}}{R_{hose}} + \frac{P_{lung}}{R_{lung}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}} \quad \text{--- (III)}$$

$$\Rightarrow P_{aw} = \left(\frac{\frac{1}{R_{lung}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}}\right) P_{lung} + \left(\frac{\frac{1}{R_{hose}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}}\right) P_{out} \quad \text{--- (IV)}$$

Substituting the value of P_{aw} in eqn (I) & (II)

$$Q_{pat} = \frac{\frac{1}{R_{lung}} P_{lung} + \frac{1}{R_{hose}} P_{out}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} - \frac{1}{R_{lung}} P_{lung}$$

$$= \frac{\frac{1}{R_{lung}} - \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} P_{lung} + \frac{\frac{1}{R_{hose}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} P_{out}$$

$$Q_{pat} = \left(-\frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}\right) P_{lung} + \left(\frac{\frac{1}{R_{hose}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}\right) P_{out} \quad \text{--- (V)}$$

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$$\dot{P}_{Lung} = \frac{\frac{1}{R_{lung}} P_{Lung} + \frac{1}{R_{hose}} P_{out}}{C_{lung} R_{Lung} \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}} \right)} - \frac{P_{Lung}}{C_{lung} R_{Lung}}$$

$$= \frac{\frac{1}{R_{lung}} - \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}} \right)}{C_{Lung} R_{lung} \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}} \right)} P_{Lung} + \frac{\frac{1}{R_{hose}}}{C_{lung} R_{lung} \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}} \right)} P_{out}$$

$$\therefore \dot{P}_{lung} = \left(- \frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{R_{lung} C_{lung} \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}} \right)} \right) P_{Lung} + \left(\frac{\frac{1}{R_{hose}}}{R_{Lung} C_{lung} \left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}} \right)} \right) P_{out} \quad \text{——— (VI)}$$

Here, input $\Rightarrow P_{out}$, output $\Rightarrow \begin{bmatrix} P_{aw} \\ Q_{pat} \end{bmatrix}$, state $\Rightarrow P_{lung}$

From eqn (IV) to (VI) we find:

$$\dot{P}_{Lung} = A_h P_{Lung} + B_h P_{out} \quad \text{——— (7)}$$

$$\begin{bmatrix} P_{aw} \\ Q_{pat} \end{bmatrix} = C_h P_{lung} + D_h P_{out} \quad \text{——— (8)}$$

where,

$$A_h = -\frac{\frac{1}{R_{nose}} + \frac{1}{R_{leak}}}{R_{lung}C_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{nose}} + \frac{1}{R_{leak}}\right)}$$

$$B_h = \frac{\frac{1}{R_{nose}}}{R_{lung}C_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{nose}} + \frac{1}{R_{leak}}\right)}$$

$$C_h = \left[\frac{\frac{1}{R_{lung}}}{\frac{1}{R_{lung}} + \frac{1}{R_{leak}} + \frac{1}{R_{nose}}} - \frac{\frac{1}{R_{nose}} + \frac{1}{R_{leak}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{nose}} + \frac{1}{R_{leak}}\right)}\right]$$

$$D_h = \left[\frac{\frac{1}{R_{nose}}}{\frac{1}{R_{nose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}} \quad \frac{\frac{1}{R_{nose}}}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{nose}} + \frac{1}{R_{leak}}\right)}\right]$$

We know that,

$$Y(s) = \left[C_n (sI - A_n)^{-1} B_n + D_n \right] u(s)$$

where $Y(s)$ is the output matrix and $u(s)$ is the input matrix.

so, $$\frac{Y(s)}{u(s)} = H(s) = C_h (sI - A_h)^{-1} B_h + D_h$$

first, calculating the values of A_h, B_h, C_h, D_h using the Parameters given in Table 1.

$$A_h = -5.443 \quad , \quad B_h = 5.063$$

$$C_h = \begin{bmatrix} 0.4557 \\ -108.86 \end{bmatrix} \qquad D_h = \begin{bmatrix} 0.5063 \\ 101.266 \end{bmatrix}$$

so, $$(sI - A_h)^{-1} = \left[s + 5.443 \right]^{-1} = \left[\frac{1}{s + 5.443} \right]$$

And, $$C_h (sI - A_h)^{-1} B_h = \begin{bmatrix} 0.4557 \\ -108.86 \end{bmatrix} \left[\frac{1}{s+5.443} \right] \left[5.063 \right]$$

$$= \begin{bmatrix} \dfrac{2.3072}{s+5.443} \\ -\dfrac{551.1582}{s+5.443} \end{bmatrix}$$

$$\text{So, } H(s) = C_n(sI - A_n)^{-1}B_n + D_n$$

$$= \begin{bmatrix} \dfrac{2.3072}{s+5.443} \\ -\dfrac{551.1582}{s+5.443} \end{bmatrix} + \begin{bmatrix} 0.5063 \\ 101.266 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} \dfrac{0.5063(s+10)}{s+5.443} \\ \dfrac{101.266(s+0.322\times10^{-3})}{s+5.443} \end{bmatrix}$$