#9 Logistic Regression: First-Year GPA

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a)

$$H_0 : \mu_0 = \mu_1$$

$$H_a : \mu_0 \neq \mu_1$$

$$\alpha = 0.05$$

Test: 2-sample difference in means t-test

Random: assume data was collected using simple random sample
Normal: $n_0 = 194 \geq 30$, $n_1 = 25 \ngeq 30$, CLT fails, proceeding with caution
Independent: reasonable to assume that $n \leq 0.1N$

```
t.test(GPA ~ FirstGen, data = gpa)
```

```
##
##  Welch Two Sample t-test
##
## data:  GPA by FirstGen
## t = 2.4474, df = 31.406, p-value = 0.02017
## alternative hypothesis: true difference in means between group 0 and group 1 is not e
## 95 percent confidence interval:
##  0.03820979 0.41912630
## sample estimates:
## mean in group 0 mean in group 1
##        3.122268        2.893600
```

$$t = \frac{(\bar{x}_0 - \bar{x}_1) - (\mu_0 - \mu_1)}{\sqrt{\frac{s_0^2}{n_0} + \frac{s_1^2}{n_1}}}$$

$$t = \frac{(3.122268 - 2.8936) - 0}{\sqrt{\frac{0.4637245^2}{194} + \frac{0.4365001^2}{25}}}$$

$$t = 2.4474$$

$$p = P(t \geq 2.4474) = 0.02017 \text{ on } 31.406 \text{ df}$$

Since our p-value (0.02017) is less than α (0.05), we reject our null hypothesis that the true mean GPA for first-generation college students is equal to that of non-first-generation college students. Therefore, we have enough evidence to conclude that the true mean GPAs for these two groups are not equal.

b)

```r
regres01 <- lm(GPA ~ FirstGen, data = gpa)
summary(regres01)
```

```
##
## Call:
## lm(formula = GPA ~ FirstGen, data = gpa)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.19227 -0.31293  0.04773  0.37773  1.02773
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.12227    0.03308  94.377   <2e-16 ***
## FirstGen1   -0.22867    0.09792  -2.335   0.0204 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4608 on 217 degrees of freedom
## Multiple R-squared:  0.02452,    Adjusted R-squared:  0.02002
## F-statistic: 5.454 on 1 and 217 DF,  p-value: 0.02044
```

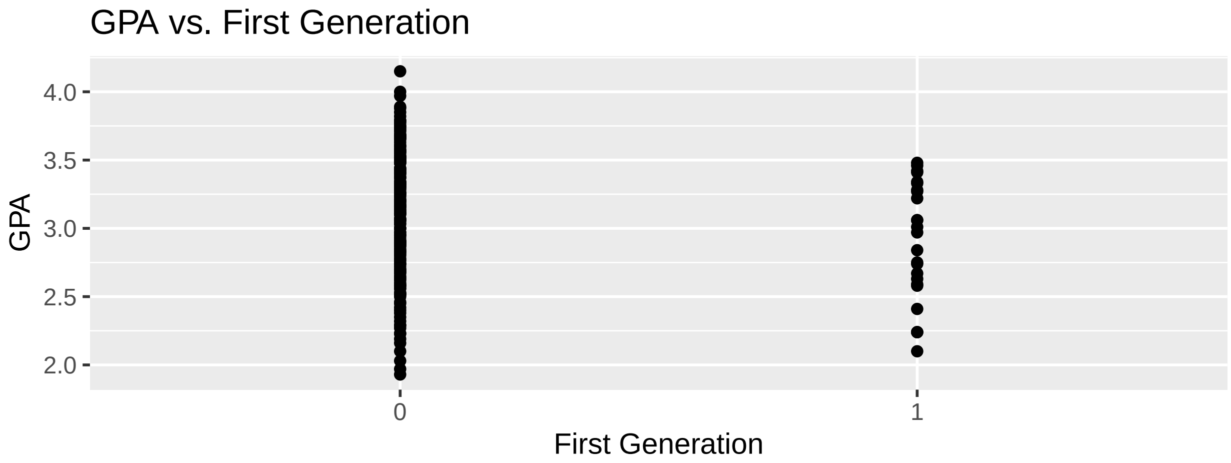
$$H_0 : \beta = 0$$

$$H_a : \beta \neq 0$$

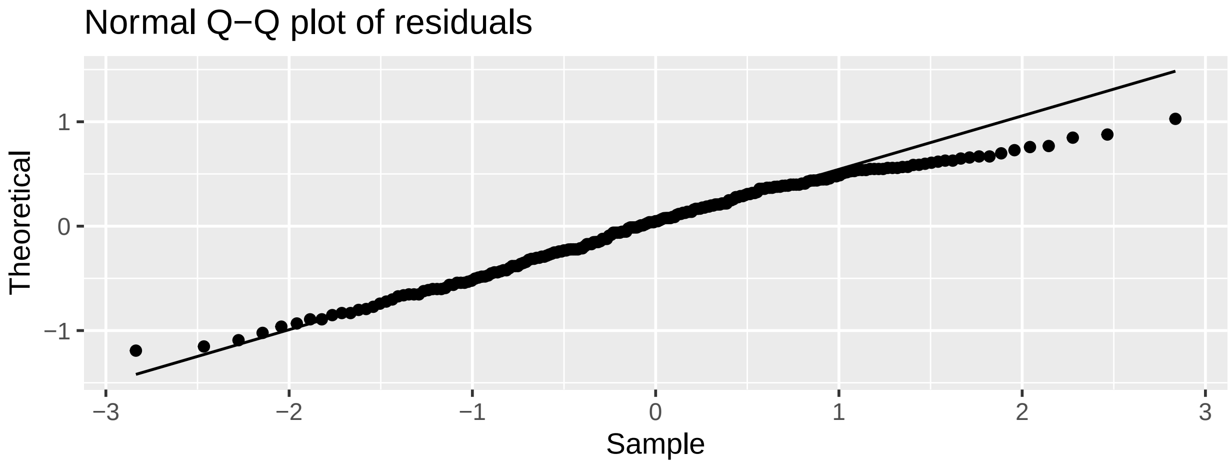
$$\alpha = 0.05$$

Test: Slope model utility t-test

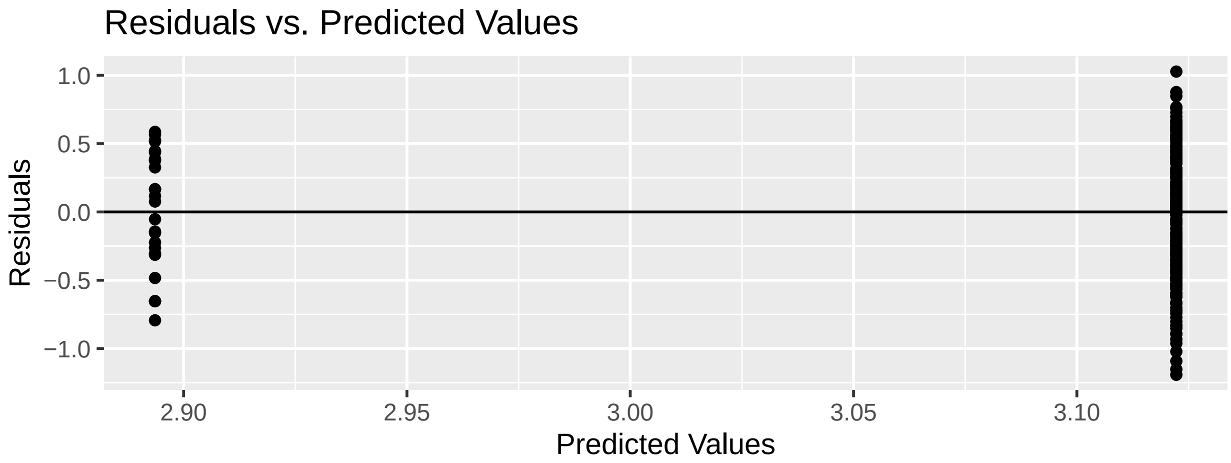
Linear: relationship between GPA and FirstGen does not appear linear, proceeding with caution



Normal: residuals appear normally distributed



Equal Variance: residuals appear randomly scattered about the x-axis



$$t = \frac{b-\beta}{S_b} = \frac{b-\beta}{\frac{\sqrt{\frac{\sum(y-\hat{y})^2}{n-2}}}{\sqrt{\sum(x-\bar{x})^2}}}$$

$$t = \frac{-0.22867 - 0}{0.09792}$$

$$t = -2.335$$

$$p = P(|t| \geq |-2.335|) = 0.02044 \text{ on } 217 \text{ df}$$

Since our p-value (0.02044) is less than α (0.05), we reject our null hypothesis that the true slope equals 0. Therefore, we have enough evidence to support the alternative hypothesis that the true slope does not equal 0. The slope of -0.22867 indicates that the direction of the relationship is negative.

c)

```
regres02 <- glm(FirstGen ~ GPA, data = gpa, family = binomial("logit"))
summary(regres02)
```

```
##
## Call:
## glm(formula = FirstGen ~ GPA, family = binomial("logit"), data = gpa)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.8161  -0.5295  -0.4374  -0.3637   2.2863
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)   1.0751     1.3527   0.795    0.427
## GPA          -1.0381     0.4567  -2.273    0.023 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 155.54  on 218  degrees of freedom
## Residual deviance: 150.28  on 217  degrees of freedom
## AIC: 154.28
##
## Number of Fisher Scoring iterations: 5
```

Let $\alpha = 0.05$

Since our p-value (0.023) is less than α (0.05), we reject our null hypothesis that the true slope of the relationship between the logit transformation of the predicted probability of being a first-generation student and the GPA of the student is 0. Therefore, we have enough evidence to conclude that the true slope is not 0. This conclusion is consistent with those from parts a) and b).

d)

Part 1

$$P(FirstGen|GPA = 4.0) = \frac{e^{1.0751-1.0381(4.0)}}{1 + e^{1.0751-1.0381(4.0)}} = 0.04405638$$

Part 2

$$ln\left(\frac{0.5}{1-0.5}\right) = 1.0751 - 1.0381(GPA)$$
$$1.0381(GPA) = 1.0751 - ln\left(\frac{0.5}{1-0.5}\right)$$
$$GPA = \frac{1.0751 - ln\left(\frac{0.5}{1-0.5}\right)}{1.0381}$$
$$GPA = \frac{1.0751 - 0}{1.0381}$$
$$GPA = 1.035642$$

Part 3

$$ln(2) = 1.0751 - 1.0381(GPA)$$
$$1.0381(GPA) = 1.0751 - ln(2)$$
$$GPA = \frac{1.0751 - ln(2)}{1.0381}$$
$$GPA = 0.3679345$$

e)

Model	**$R^2(adj)$**	**S_e**	**F**	**p**
$\widehat{GPA} = 1.17985 + 0.55501(HSGPA)$	0.196	0.4174	54.15	3.783×10^{-12}
$\widehat{GPA} = 1.160826 + 0.569504(HSGPA) - 0.284592(FirstGen) + 0.003677(HSGPA)(FirstGen)$	0.2235	0.4102	21.91	2.018×10^{-12}
$\widehat{GPA} = 2.0684133 + 0.0016986(SATV)$	0.08842	0.4444	22.15	4.499×10^{-6}
$\widehat{GPA} = 0.2459 + 0.7593(HSGPA) + 1.3010(White) - 0.2923(HSGPA)(White)$	0.2684	0.3981	27.65	2.647×10^{-15}
$\widehat{GPA} = 0.99579 + 0.54004(HSGPA) + 0.29842(White)$	0.2613	0.4001	39.56	2.303×10^{-15}

Note: I did test more models with more predictors but they were too big to put in the table.

The best model I found was:

$$\widehat{GPA} = 0.99579 + 0.54004(HSGPA) + 0.29842(White)$$

I chose this model because it has a good F-statistic and a good adjusted R^2 value.