Exercise 1

IH: (induction hypothesis)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

IB: $n := 1$ (induction base)

$$\sum_{i=1}^{n} i = \sum_{i=1}^{1} i = 1 = \frac{1 \cdot (1+1)}{2} = \frac{n \cdot (n+1)}{2}$$

IS: $n := n+1$ (induction step)

$$\begin{aligned}
\sum_{i=1}^{n+1} i &= (n+1) + \sum_{i=1}^{n} i \\
&\overset{IH}{=} (n+1) + \frac{n \cdot (n+1)}{2} \\
&= \frac{2 \cdot (n+1)}{2} + \frac{n \cdot (n+1)}{2} \\
&= \frac{(n+2) \cdot (n+1)}{2} \\
&= \frac{((n+1)+1) \cdot (n+1)}{2}
\end{aligned}$$

Exercise 2

IH: (induction hypothesis)

$$\sum_{i=1}^{d} \left(2^{d-i} \cdot i\right) \leq 2^{d+1} - d - 2$$

IB: $d := 1$ (induction base)

$$\sum_{i=1}^{d} \left(2^{d-i} \cdot i\right) = 2^0 \cdot 1 = 1 \leq 1 = 2^2 - 1 - 2 = 2^{d+1} - d - 2$$

IS: $d := d + 1$ (induction step)

$$\begin{aligned}
\sum_{i=1}^{d+1} \left(2^{(d+1)-i} \cdot i\right) &= 2 \cdot \sum_{i=1}^{d+1} \left(2^{d-i} \cdot i\right) \\
&= 2 \cdot \sum_{i=1}^{d} \left(2^{d-i} \cdot i\right) + 2 \cdot \left(2^{d-(d+1)} \cdot (d+1)\right) \\
&= 2 \cdot \sum_{i=1}^{d} \left(2^{d-i} \cdot i\right) + 2 \cdot \left(2^{-1} \cdot (d+1)\right) \\
&= 2 \cdot \sum_{i=1}^{d} \left(2^{d-i} \cdot i\right) + (d+1) \\
&\overset{IH}{\leq} 2 \cdot \left(2^{d+1} - d - 2\right) + (d+1) \\
&= 2^{(d+1)+1} - 2d - 4 + d + 1 \\
&= 2^{(d+1)+1} - d - 3 \\
&= 2^{(d+1)+1} - (d+1) - 2
\end{aligned}$$