Indian Institute of Technology Jodhpur

Statistics for Data Science - MAL 7061

First Semester (2024-25)

Assignment - III

1. The marketing director of a large departmental store wants to estimate the average number of customers who enter the store every 5 minutes. He randomly selects 5-minute intervals and counts the number of arrivals. The figures are 4, 5, 3, 4, 7, 5, 6, 7, 6, 3, 5. Compute a 95% confidence interval for mean.

2. A market researcher wants to perform a test with the intent of establishing that his company's medium pump bottle of soap has a mean life greater than 40 days. The sample size is 70 and he knows that $\sigma = 5.6$

   (a) If you set the rejection region to be $R : \bar{X} \geq 41.39$, what is the level of significance of your test?

   (b) Find the numerical value of c so that the test $R : \bar{X} \geq c$ has a 5% level of significance.

3. Let $X_1, \cdots, X_n$ represents a random sample from a distribution with pdf

$$f(x) = \frac{x}{\theta} e^{-x^2/(2\theta)}, x > 0$$

   Show that $E(X^2) = 2\theta$. Use this to construct an unbiased estimator of θ based on $\sum X_i^2$. Also, estimate θ from the following $n = 10$ observations on vibratory stress of a turbine blade under specified conditions:

$$16.88 \quad 10.23 \quad 4.59 \quad 6.66 \quad 13.68 \quad 14.23 \quad 19.87 \quad 9.40 \quad 6.51 \quad 10 : 95$$

4. A sample of size 2 is taken from an exponential distribution with mean λ. We propose two estimators

$$(X_1 + X_2)/2 \quad \text{and} \quad \frac{4}{\pi}(X_1 X_2)^{1/2}$$

   In terms of unbiasedness and minimum variance which one is better.

5. Assuming $\sigma = 20$, how large a random sample be taken to assert with probability 0.95 that the sample mean will not differ from the true mean by more than 3 points?

6. Research has shown it is equally dangerous to use hand-held devices while driving. A study of 5000 students revealed that 78 percent of them reported that they talked while driving. Find a 98% confidence interval for the proportion of all students who use cell phone while driving.

7. A study of 8960 births revealed that, among $n = 1103$ births to parents who both smoked, there were 480 males. Construct a real hypotheses with $\alpha = 0.01$ with the intent of establishing that when both parents amoke, the population proportion of male births is less than 0.5.

8. The following measurements of the diameters of Indian mounds at a place were gathered

$$22 \quad 24 \quad 24 \quad 30 \quad 22 \quad 20 \quad 28 \quad 30 \quad 24 \quad 34 \quad 36 \quad 15 \quad 37$$

(a) Do these data substantiate the conjecture that the population mean diameter is larger than 21 feet? Test at $\alpha = 0.01$.

(b) Determine a 90% confidence interval for the population mean diameter of Indian mounds.