<u>Problem Sheet-2</u>

1. Consider a trial in which a jury must decide between the hypothesis that the defendant is guilty and the hypothesis that he or she is innocent.

   (a) In the framework of hypothesis testing and the India legal system, which of the hypotheses should be the null hypothesis?

   (b) What do you think would be an appropriate significance level in this situation?

2. A sugar refiner packs sugar into bags with an average weight of 1kg. Unfortunately, the filling equipment tends to drift - it is known that the process standard deviation is 0.1. From a sample of 100 bags the sample mean was 1.03. Is the process mean still 1kg? Use a 0.05 level of significance.

3. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of .02. Suppose 10 independent measurements yielded the following pH values:
   $8.18, 8.17, 8.16, 8.15, 8.17, 8.21, 8.22, 8.16, 8.19, 8.18$

   (a) What conclusion can be drawn at the $\alpha = .10$ level of significance?

   (b) What about at the $\alpha = .05$ level of significance?

4. There is some variability in the amount of phenobarbital in each capsule sold by a manufacturer. However, the manufacturer claims that the mean value is 20.0 mg. To test this, a sample of 25 pills yielded a sample mean of 19.7 with a sample standard deviation of 1.3. What inference would you draw from these data? In particular, are the data strong enough evidence to discredit the claim of the manufacturer? Use the 5 percent level of significance.

5. The mean response time of a species of pigs to a stimulus is .8 seconds. Twenty eight pigs were given 2 ounce of alcohol and then tested. If their average response time was 1.0 seconds with a standard deviation of .3 seconds, can we conclude that alcohol affects the mean response time? Use the 5 percent level of significance.

6. An advertisement for a new toothpaste claims that it reduces cavities of children in their cavity-prone years. Cavities per year for this age group are normal with mean 3 and standard deviation 1. A study of $2,500$ children who used this toothpaste found an average of 2.95 cavities per child. Assume that the standard deviation of the number of cavities of a child using this new toothpaste remains equal to 1.

(a) Are these data strong enough, at the 5 percent level of significance, to establish the claim of the toothpaste advertisement?

(b) Do the data convince you to switch to this new toothpaste?

7. The weights of salmon grown at a commercial hatchery are normally distributed with a standard deviation of 1.2 pounds. The hatchery claims that the mean weight of this year's crop is at least 7.6 pounds. Suppose a random sample of 16 fish yielded an average weight of 7.2 pounds. Is this strong enough evidence to reject the hatchery's claims at the

   (a) 5 percent level of significance;

   (b) 1 percent level of significance?

   (c) What is the p-value?

8. Last year, 456 students of IIT Jodhpur had given the CAT examination. These students had an average score of 60 with a standard deviation of 5.6. The national average was 56.5. Test the claim of an administrator that IIT Jodhpur students scored significantly higher than the national average. Let $\alpha = 0.05$.

9. Consider a test of $H_0 : \mu \leq 100$ versus $H_1 : \mu > 100$. Suppose that a sample of size 20 has a sample mean of X = 105. Determine the p-value of this outcome if the population standard deviation is known to equal 5.

10. A medical scientist believes that the average basal temperature of (outwardly) healthy individuals has increased over time and is now greater than 98.6 degrees Fahrenheit (37 degrees Celsius). To prove this, she has randomly selected 100 healthy individuals. If their mean temperature is 98.74 with a sample standard deviation of 1.1 degrees, does this prove her claim at the 5 percent level? What about at the 1 percent level?

11. An oil company claims that the sulfur content of its diesel fuel is at most .15 percent. To check this claim, the sulfur contents of 40 randomly chosen samples were determined; the resulting sample mean and sample standard deviation were .162 and .040. Using the 5 percent level of significance, can we conclude that the company's claims are invalid?

12. A manufacturer of capacitors claims that the breakdown voltage of these capacitors has a mean value of at least 100 V. A test of 12 of these capacitors yielded the following breakdown voltages:
$96, 98, 105, 92, 111, 114, 99, 103, 95, 101, 106, 97$
Do these results prove the manufacturer's claim? Do they disprove them?

13. A car is advertised as having a gas mileage rating of at least 30 miles/gallon in highway driving. If the miles per gallon obtained in 10 independent experiments are

$26, 24, 20, 25, 27, 25, 28, 30, 26, 33$
Should you believe the advertisement? What assumptions are you making?

14. It is claimed that a certain type of bipolar transistor has a mean value of current gain that is at least 210. A sample of these transistors is tested. If the sample mean value of current gain is 200 with a sample standard deviation of 35, would the claim be rejected at the 5 percent level of significance if

    (a) the sample size is 25?

    (b) the sample size is 64?

15. Certain rockets are manufactured with a range of 2,500 meters. It is theorized that the range will be reduced after the rockets are in storage for some time. Six of these rockets are stored for a certain period of time and then tested. The ranges found in the tests are as follows: $2,490, 2,510, 2,360, 2,410, 2,300,$ and $2,440$.
    Does the range appear to be shorter after storage? Test at the 5% significance level. Here, the variation in ranges is also of importance. New rockets have a standard deviation of range measurements equal to 20 kilometers. Does it appear that storage increases the variability of these ranges? Use $\alpha = 0.05$.

16. A machined engine part produced by a certain company is claimed to have a diameter variance no larger than 0.0002 inch. A random sample of ten parts gave a sample variance of 0.0003. Assuming normality of diameter measurements, is there significant evidence to refute the company's claim? Use $\alpha = 0.05$.

17. A gun-like apparatus has recently been designed to replace needles in administering vaccines. The apparatus can be set to inject different amounts of the serum, but because of random fluctuations the actual amount injected is normally distributed with a mean equal to the setting and with an unknown variance σ^2. It has been decided that the apparatus would be too dangerous to use if σ exceeds .10. If a random sample of 50 injections resulted in a sample standard deviation of .08, should use of the new apparatus be discontinued? Suppose the level of significance is $\alpha = .10$. Comment on the appropriate choice of a significance level for this problem, as well as the appropriate choice of the null hypothesis.

18. Aptitude tests should produce scores with a large amount of variation so that an administrator can distinguish between persons with low aptitude and persons with high aptitude. The standard test used by a certain industry has been producing scores with a standard deviation of 5 points. A new test is tried on 20 prospective employees and produces a sample standard deviation of 8 points. Are scores from the new test significantly more variable than scores from the standard? Use $\alpha = 0.05$.

19. A small business has 37 employees. Because of the uncertain demand for its product, the company usually pays overtime per week, whatever is required, and the variance on this figure is about 25. Company officials want to know whether the variance of overtimes hours has changed. Assume hours of overtime are normally distributed, and $\alpha = 0.05$. Given here is a sample of 16 weeks of overtime data (in hours per week).

$$57, 56, 52, 44, 46, 53, 44, 44, 48, 51, 55, 48, 63, 53, 51, 50.$$

20. A manufacturer claims that the lifetime of a certain brand of batteries produced by his factory has a variance of 5000 (hours)^2. A sample of size 26 has a variance of 7200 (hours)^2. Assuming that it is reasonable to treat these data as a random sample from a normal population. Let us test the manufacturer's claim at the 0.02 level.

21. A pharmaceutical house produces a certain drug item whose weight follows normal distribution with mean 5.7 and standard deviation 0.5 milligrams. The company's research team has proposed a new method of producing the drug. However, this entails some costs and will be adopted only if there is strong evidence that the standard deviation of the weight of the items will drop to below 0.5 milligrams. If a sample of 10 items is produced and has the following weights, should the new method be adopted? Let $\alpha = 0.05$.

$$5.728, 5.731, 5.722, 5.719, 5.727, 5.724, 5.726, 5.718, 5.723, 5.722.$$

22. The U.S. Farmers' Production company builds large harvesters. For a harvester to be properly balanced when operating, a 25 pound plate is installed on its side. The machine that produces these plates is set to yield plates that average 25 pounds weight. The distribution of plates produced from the machine is normal. However, the shop supervisor is worried that the machine is out of adjustment and is producing plates that do not average 25 pounds. To test his concern, he randomly selects 20 of the plates produced the day before and weights them. Below shows the weights obtained. $22.6, 22.2, 23.2, 27.4, 24.5, 27, 26.6, 28.1, 26.9, 24.9, 26.2, 25.3, 23.1, 24.2, 26.1, 25.8, 30.4, 28.6, 23.5, 23.6$. Do the test at 5% level of significance.

23. It is known that if a signal value μ is sent from location A, then the value received at location B is normally distributed with mean μ and standard deviation 2. That is, the random noise added to the signal is an $N(0, 4)$ random variable. There is reason for the people at location B to suspect that the signal value $\mu = 8$ will be sent today. Test this hypothesis at 10% level of significance, if the same signal value is independently sent five times and the average value received at location B is 9.5.

24. The mean breaking strength of a certain type of fibre is required to be at least 200 psi. Past experience indicates that the standard deviation of breaking strength is 5 psi. If a sample of 8

pieces of fibre yielded breakage at the following pressure: $210, 195, 197, 199, 198, 202, 196, 195$. Would you conclude, at the five percent level of significance that the fibre is unacceptable? What about at the 10% level of significance?

25. Figures released by the U.S. Department of Agriculture show that the average size of farms has increased since 1940. In 1940, the mean size of a farm was 174 acres; by 1997, the average size was 471 acres. Between those years, the number of farms decreased but the amount of tillable land remained relatively constant, so now farms are bigger. Suppose a agribusiness researcher believes the average size of farms has now increased from the 1997 mean figure of 471 acres. To test this notion, she randomly sampled 23 farms across the U.S. and ascertained the size of the each farm from country records. The data she gathered is as follows: $445, 489, 474, 505, 553, 477, 454, 463, 466, 557, 502, 449, 438, 500, 466, 477, 557, 433, 545, 511, 590, 561, 560$. Use a test at 5% level of significance, and judge her claim. Assume that the number of acres per farm is normally distributed in the population.