Artificial Intelligence

Lec 3 - Informed Search

Pratik Mazumder

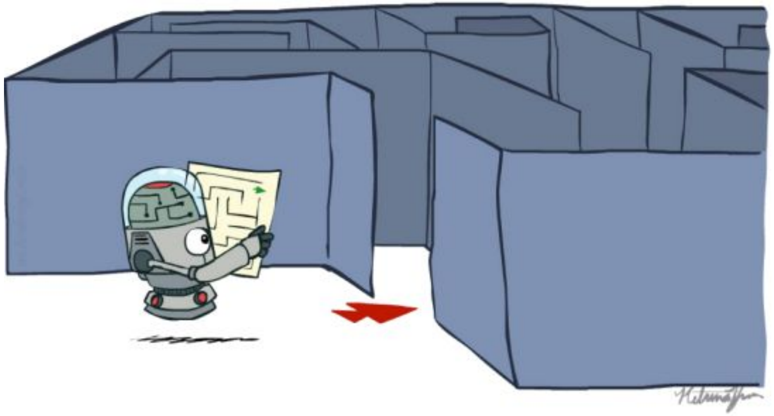
Content taken from various sources including
CS188 Intro to AI at UC Berkeley

Recap: Search

- **Search problem:**
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test

- **Search tree:**
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)

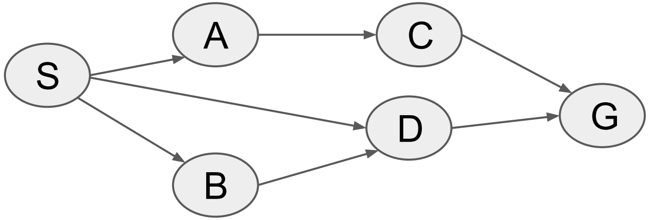
- **Search algorithm:**
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans



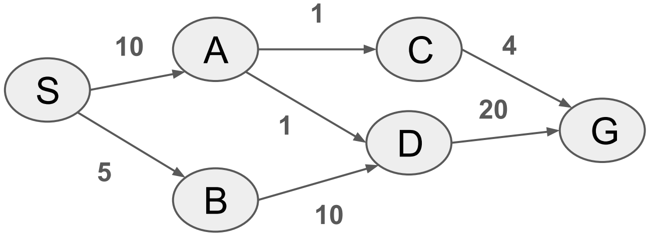
Practice

Which chooses the shorter plan? BFS, DFS, UCS

Ques1



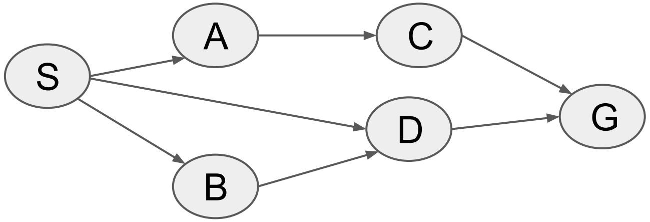
Ques2

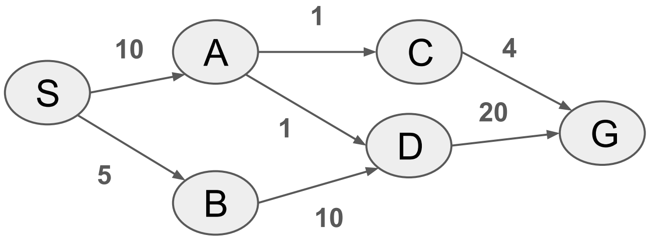


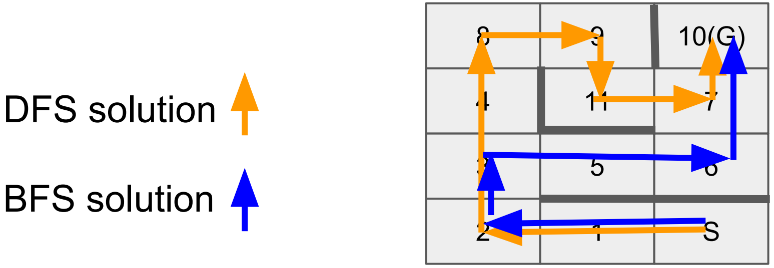
Ques3

Practice

Which chooses the shorter plan? BFS, DFS, UCS

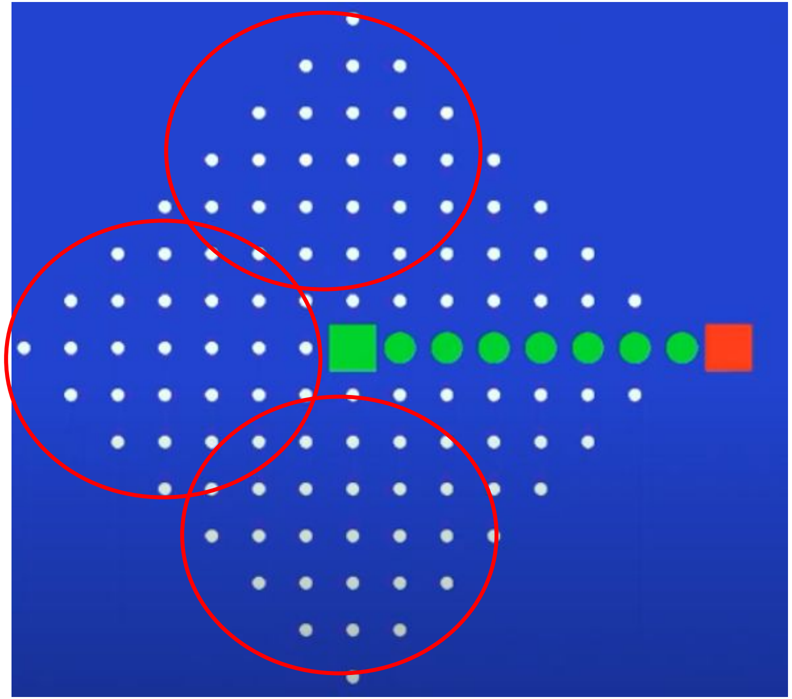






Uninformed Search

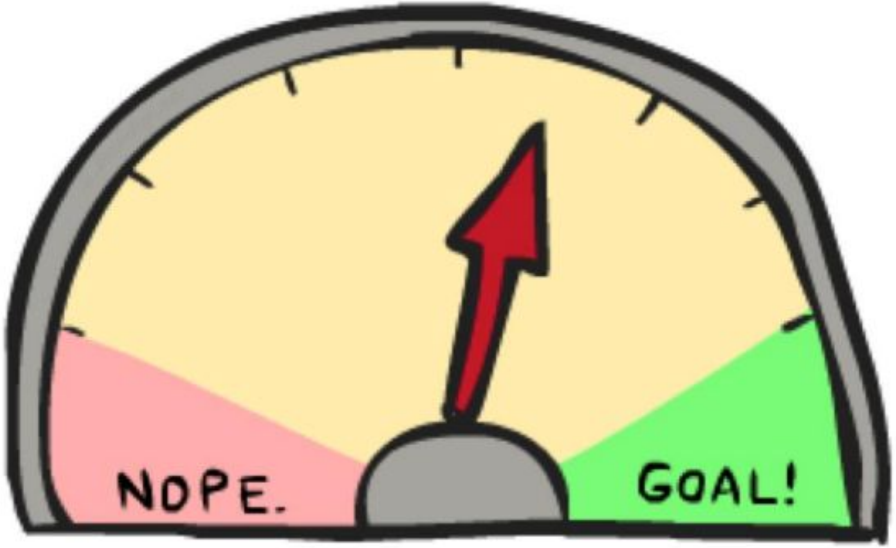
- BFS and UCS give complete plans.
- BFS gives optimal plans in terms of no. of actions.
- UCS gives optimal plans in terms of the cost involved.



The bad aspect:

- Explores options in every “direction”
- No information about goal location

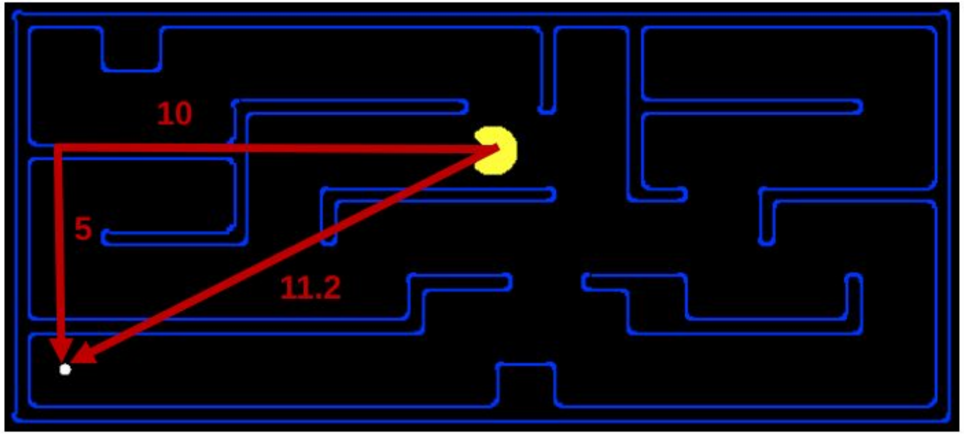
Informed Search

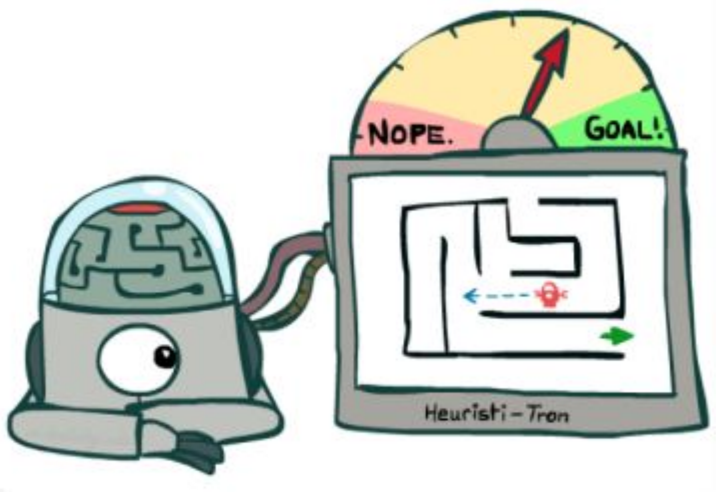


Search Heuristics

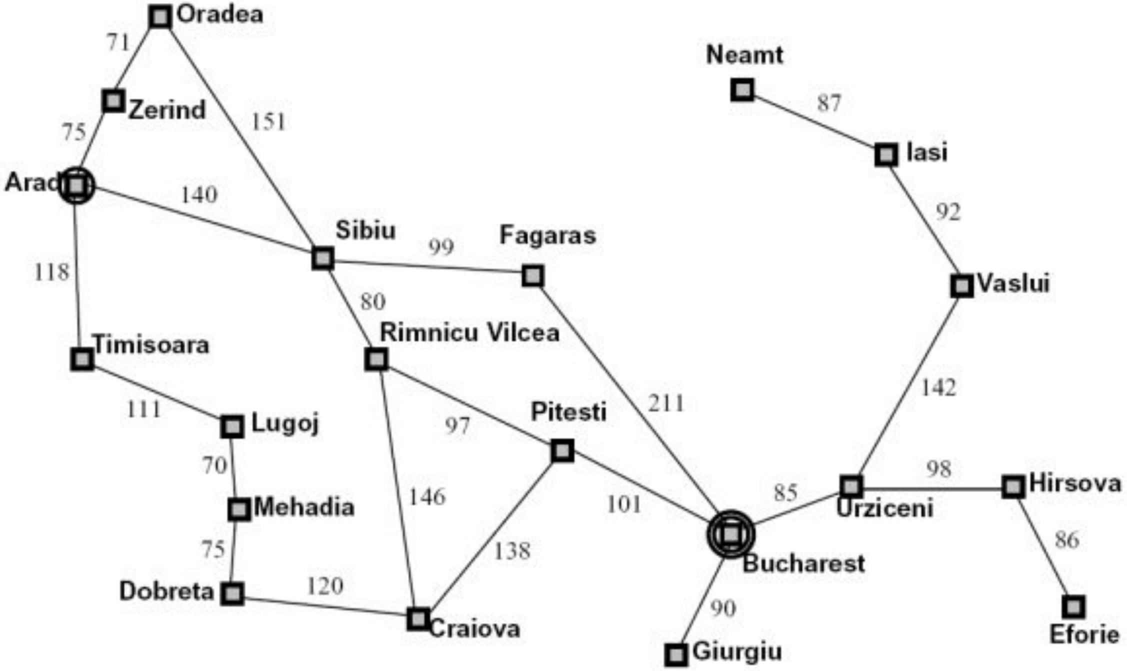
A heuristic is:

- A function that **estimates** how **close** a state is **to a goal**
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance for pathing
- Guesses - may not be the actual distance





Example: Heuristic Function



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
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Oradea	380
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Rimnicu Vilcea	193
Sibiu	253
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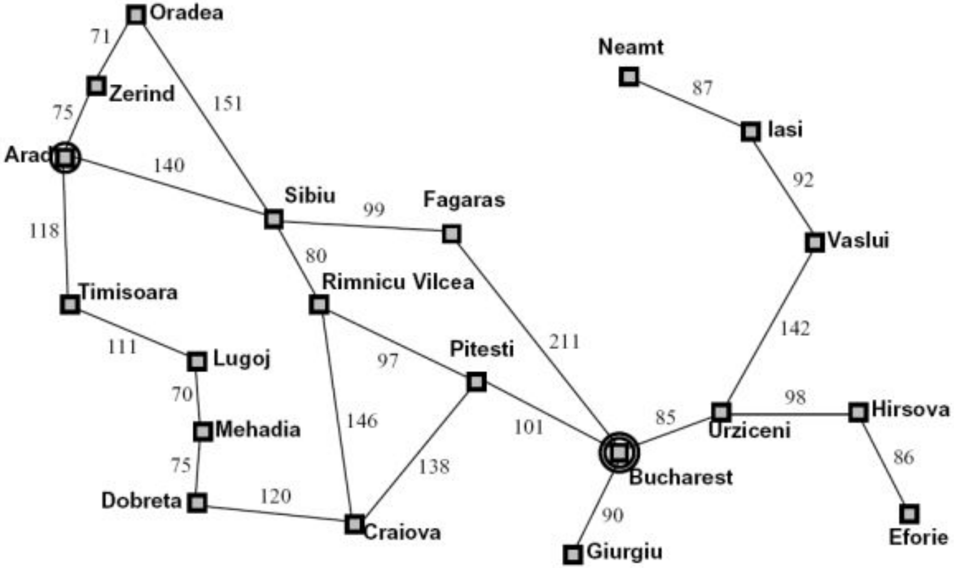
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Greedy Search

Greedy Search

Strategy: Expand the **node that seems closest to a goal state**



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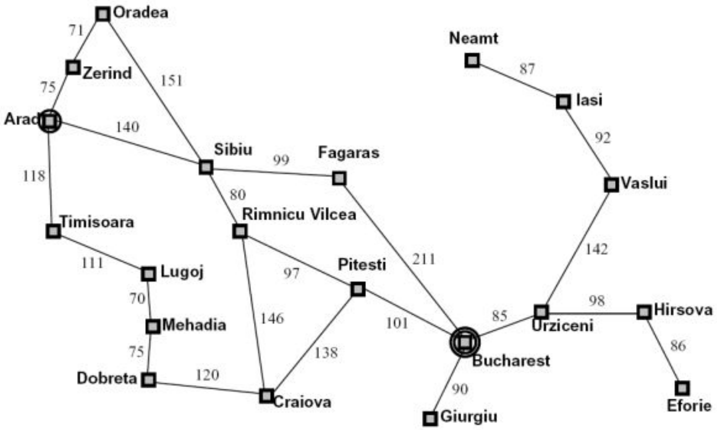
h(x)

Greedy Search



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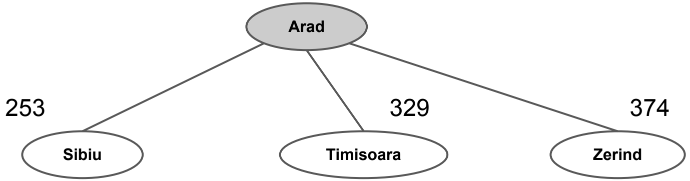
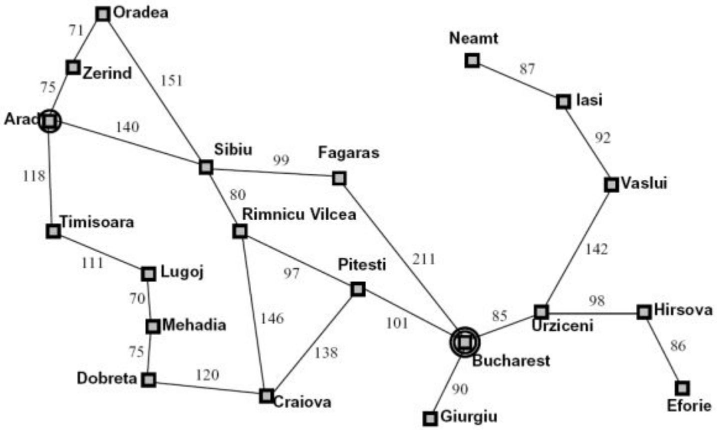
h(x)

<u>Fringe</u>

Arad, 366

Greedy Search





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h(x)

Fringe

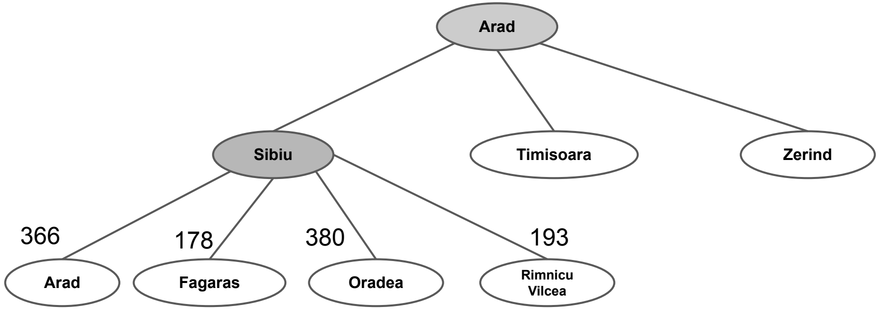
~~Arad, 366~~

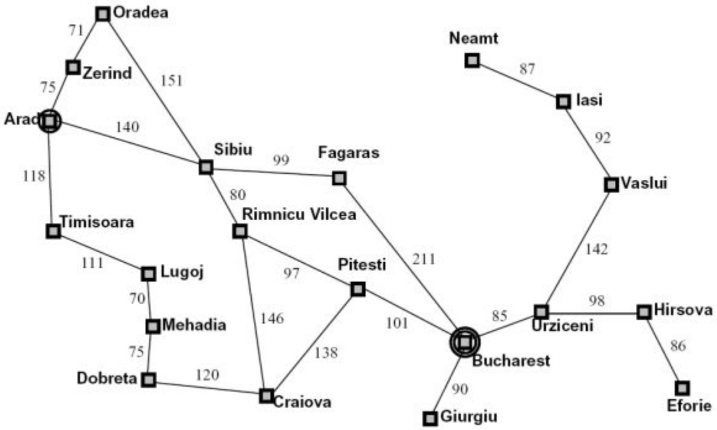
Arad→Sibiu,253

Arad→Timisoara,329

Arad→Zerind,374

Greedy Search





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h(x)

Fringe

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Arad→Sibiu→Arad,366

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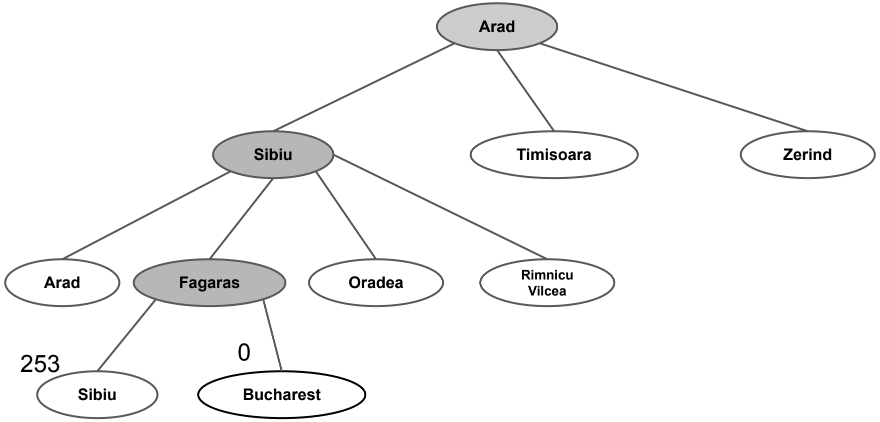
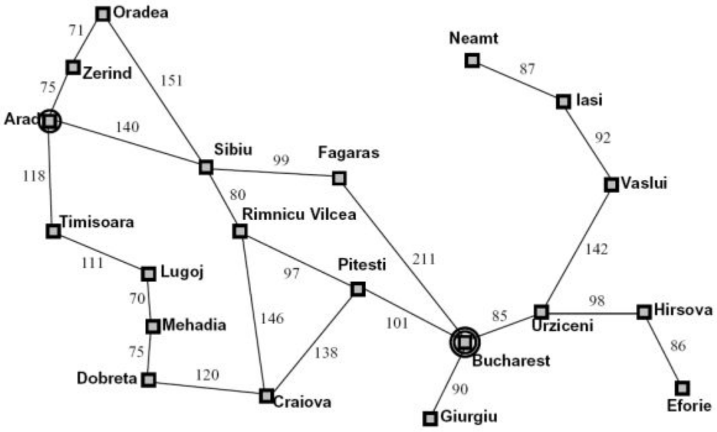
Arad→Sibiu→Rimnicu Vilcea,193

Greedy Search





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h(x)

Fringe

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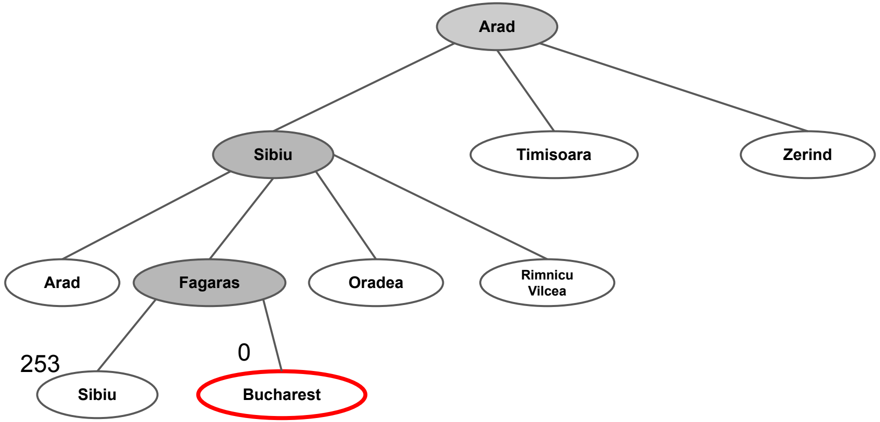
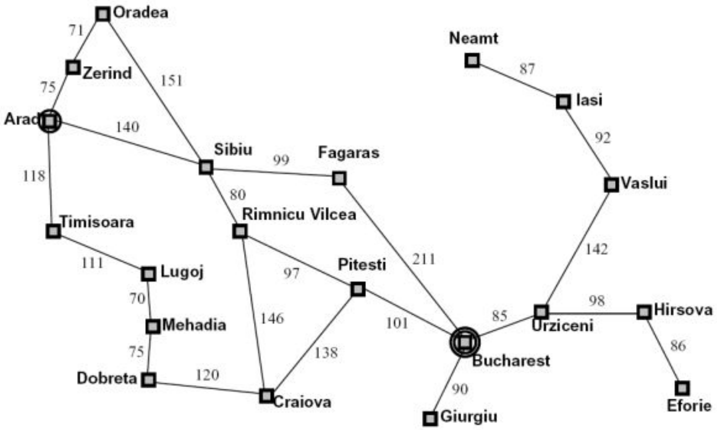
Arad→Sibiu→Fagaras→Sibiu,253
Arad→Sibiu→Fagaras→Bucharest,0

Greedy Search





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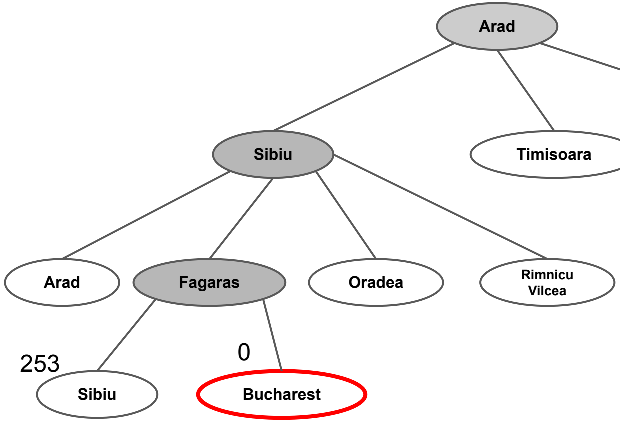
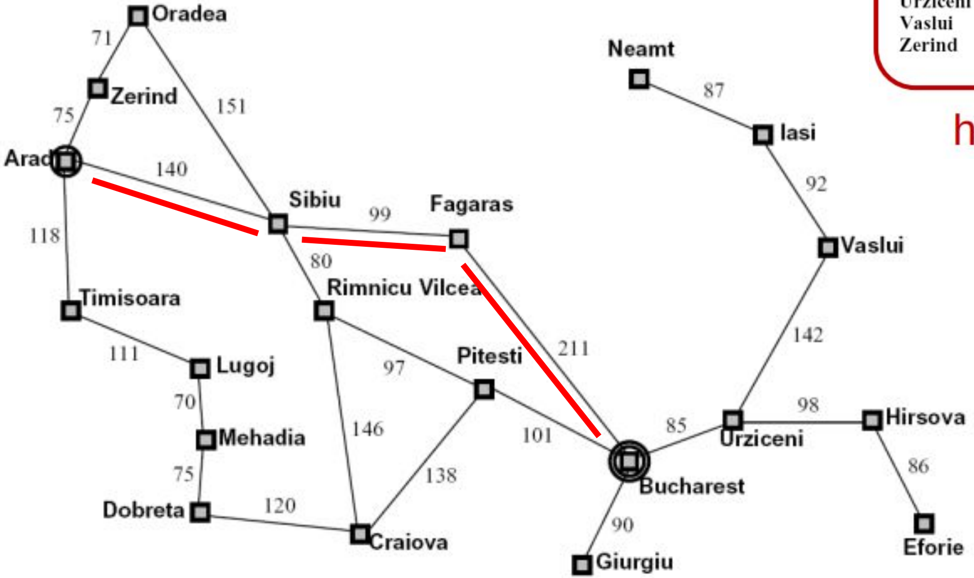
solution Arad→Sibiu→Fagaras→Bucharest,0

Greedy Search





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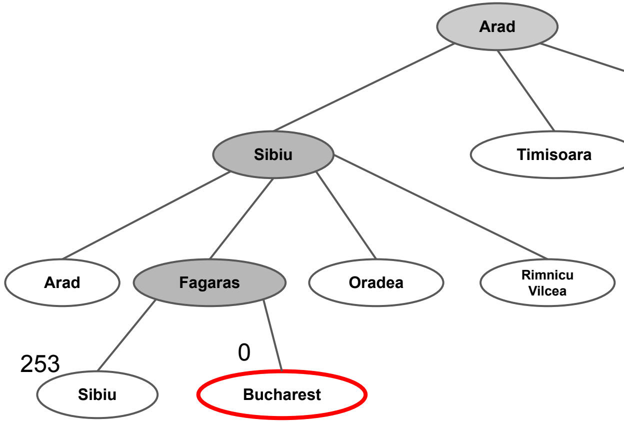

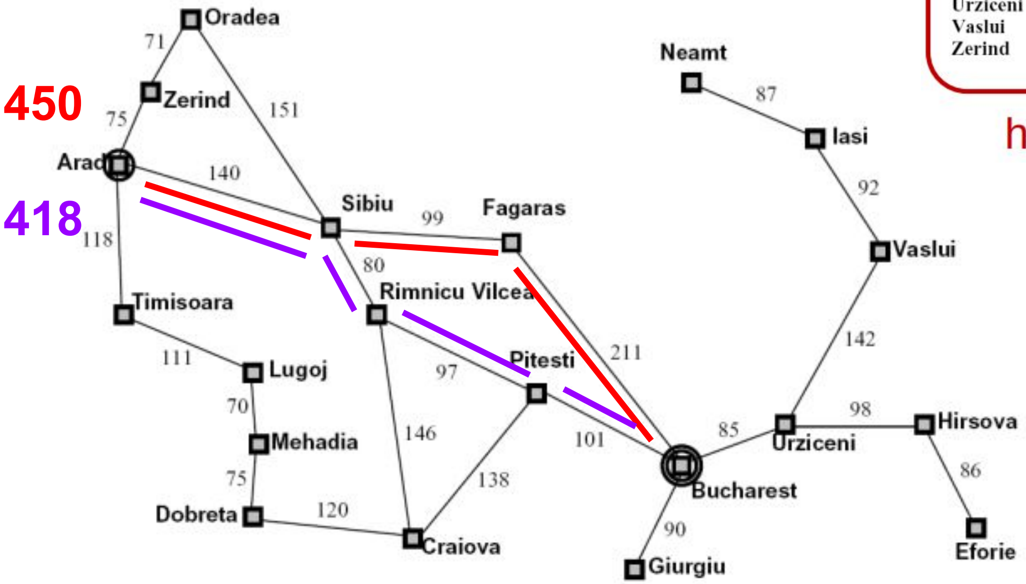

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h(x)

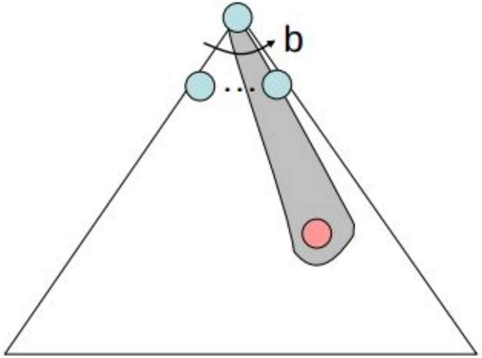
Greedy Search

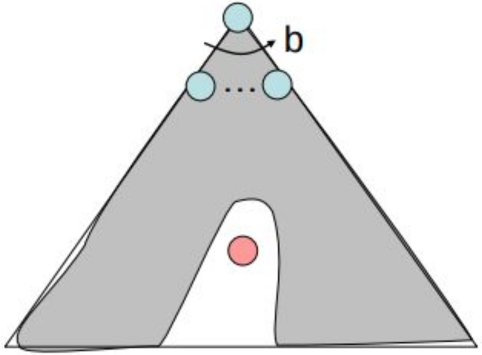
Strategy: expand a node that **you think** is closest to a goal state

Heuristic: **estimate** of **distance** to **nearest goal** for each state

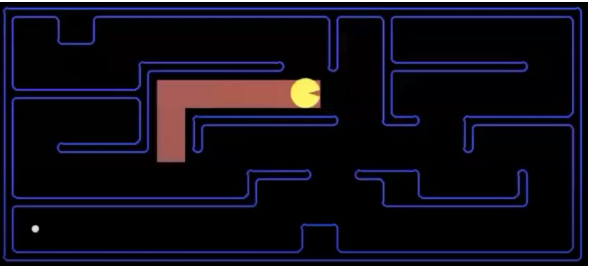
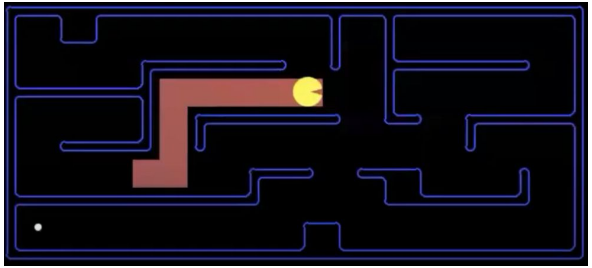
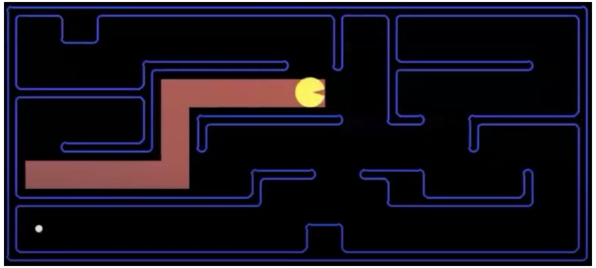
A common case: Best-first takes you straight to the (wrong) goal - wrong in terms of non optimal

Worst-case: like a badly-guided DFS

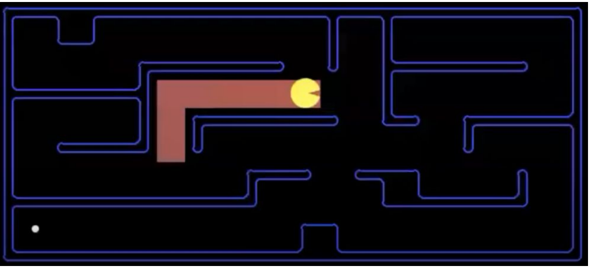


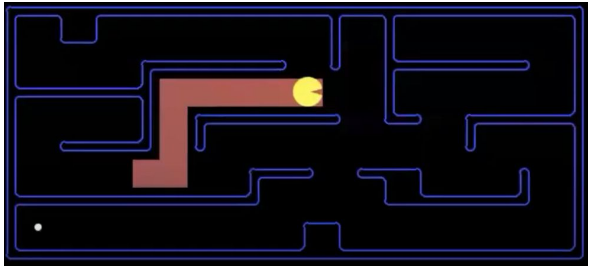


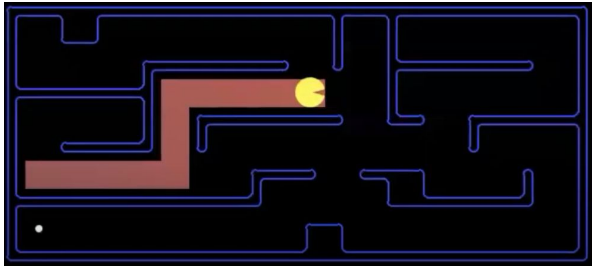
Greedy Search

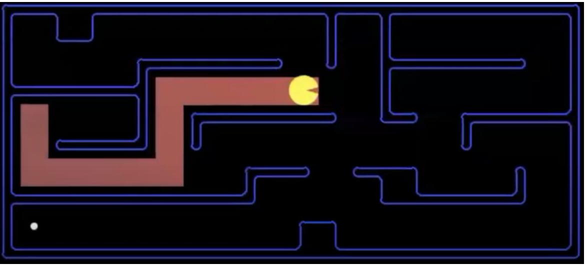
  

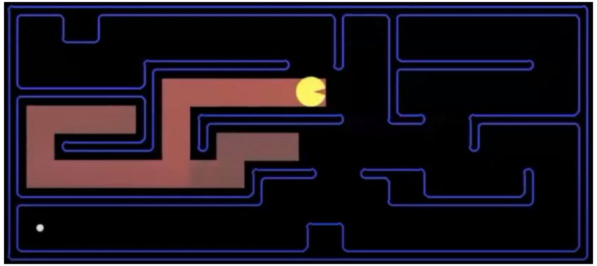
Greedy Search

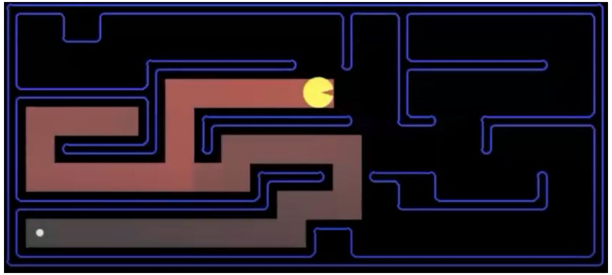












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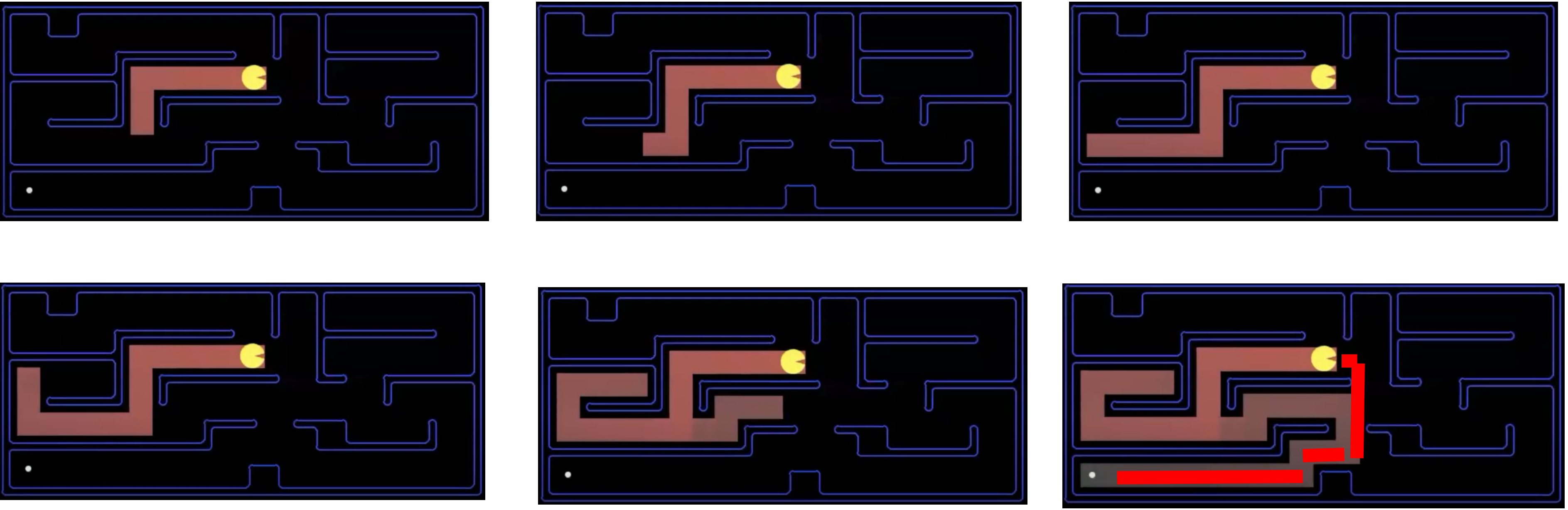


A* Search

A* Search

UCS

Greedy

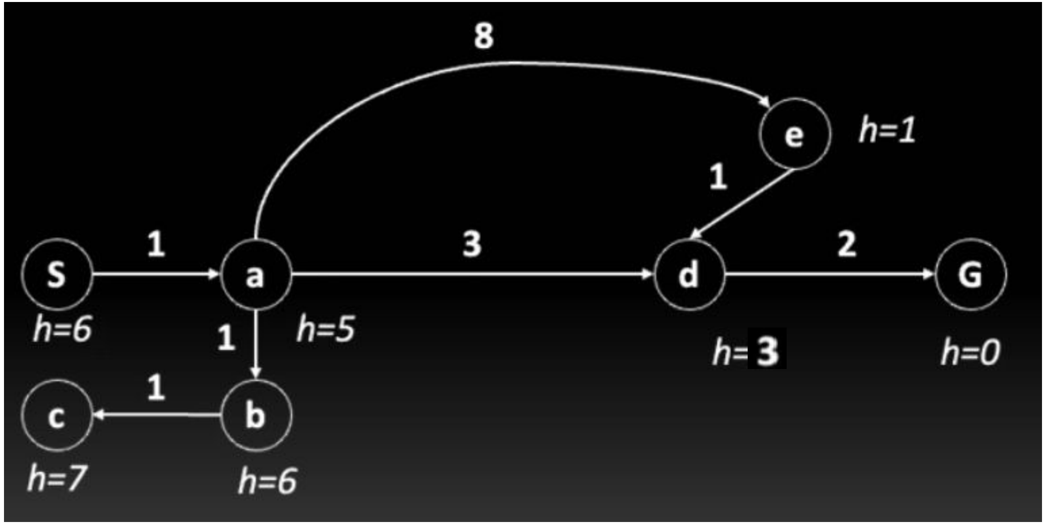
A* Search

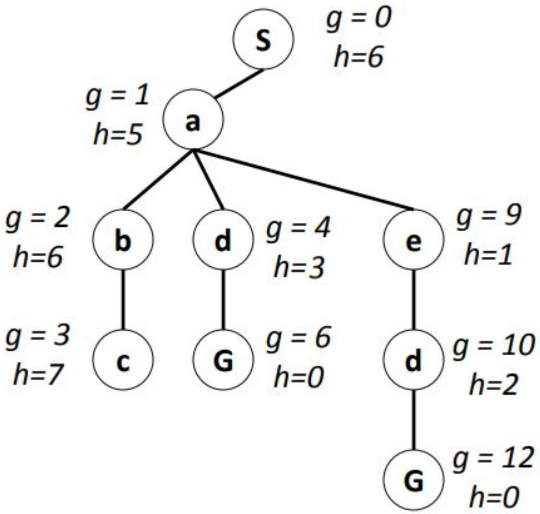
UCS

Greedy

A*

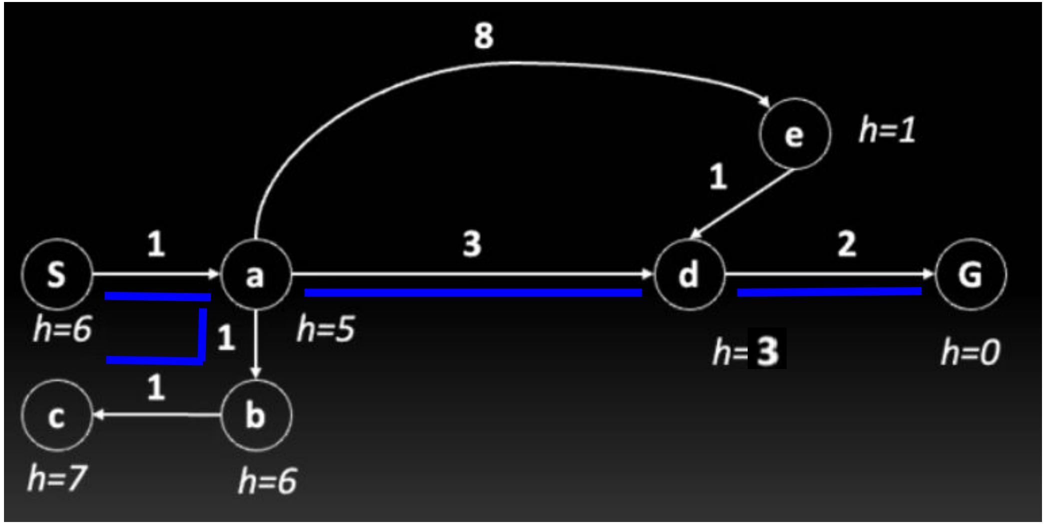
Combining UCS and Greedy

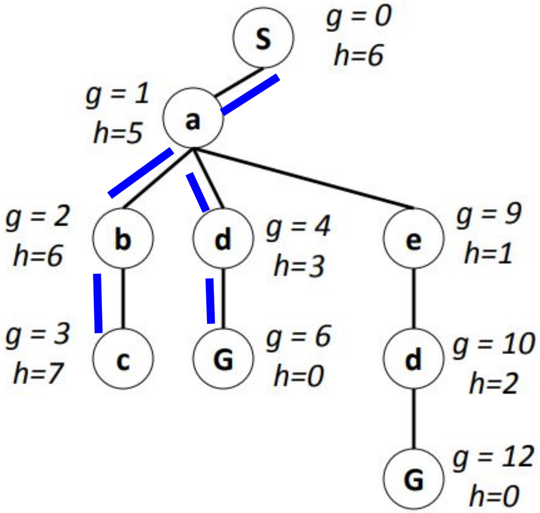




Combining UCS and Greedy

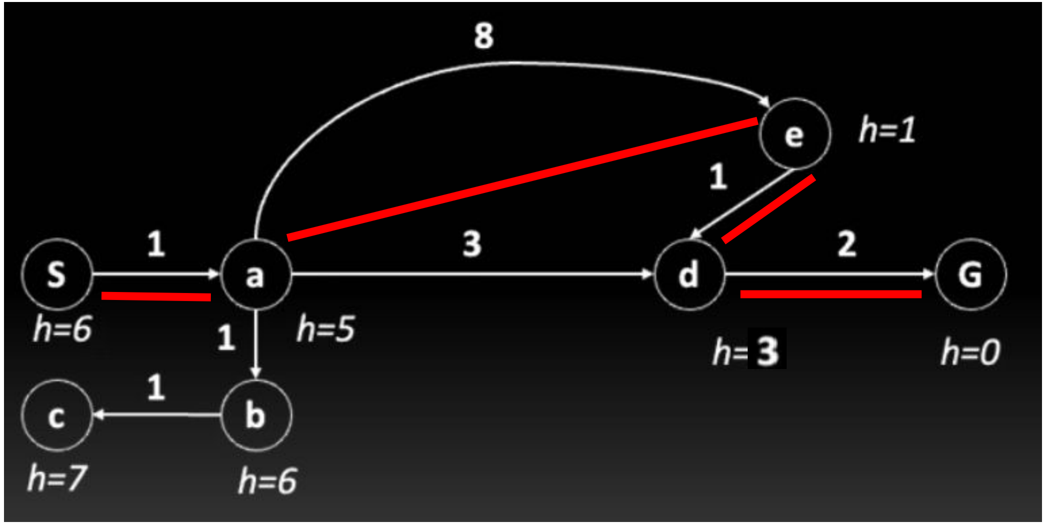
- Uniform-cost orders by path cost, or backward cost g(n)

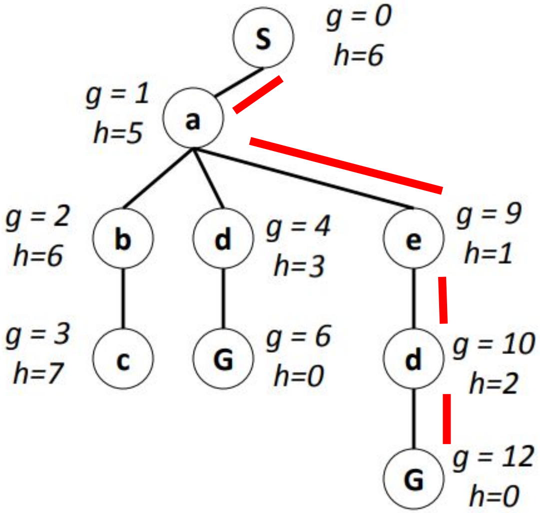




Combining UCS and Greedy

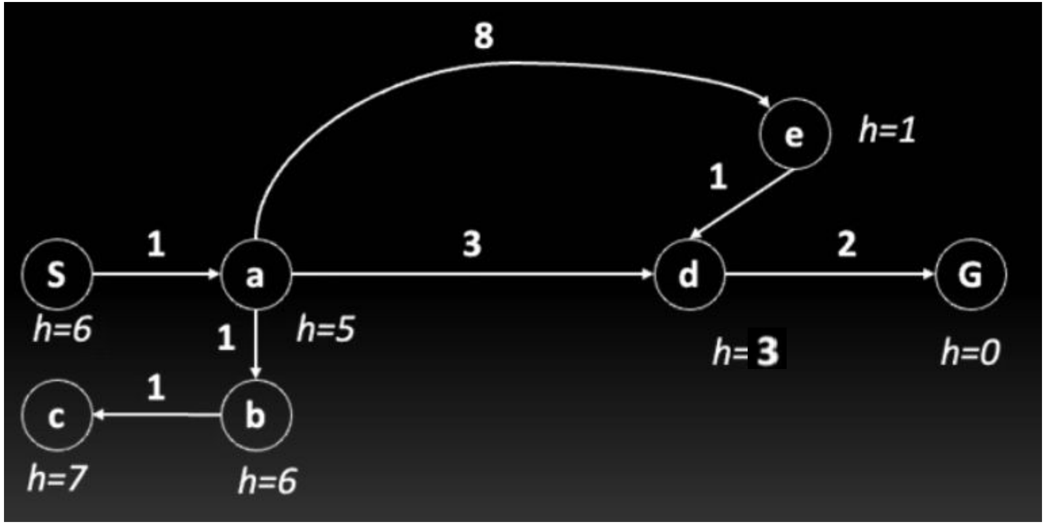
- Uniform-cost orders by path cost, or backward cost g(n)
- Greedy orders by goal proximity, or forward cost h(n)

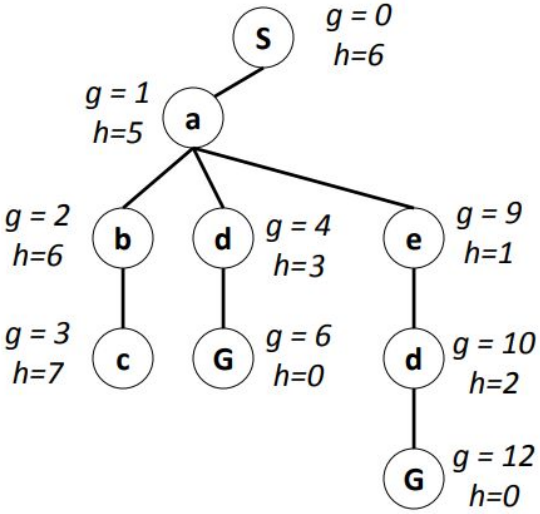




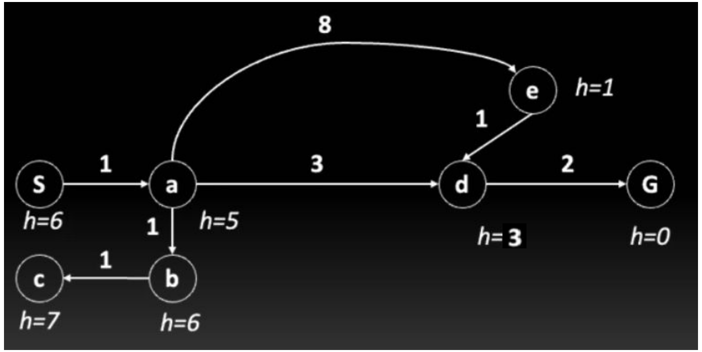
Combining UCS and Greedy

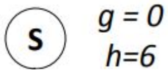
- Uniform-cost orders by path cost, or backward cost g(n)
- Greedy orders by goal proximity, or forward cost h(n)
- A* Search orders by the sum: f(n) = g(n) + h(n)





A* : Combining UCS and Greedy

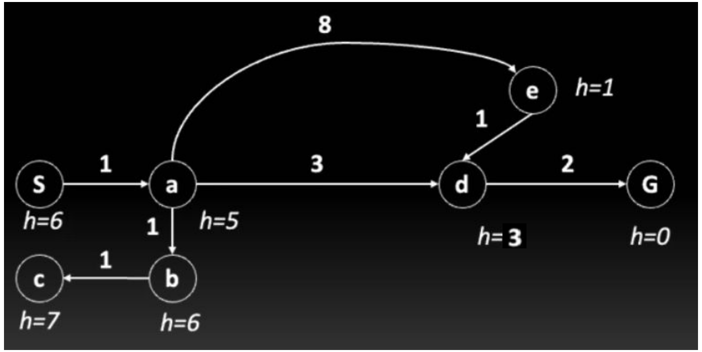


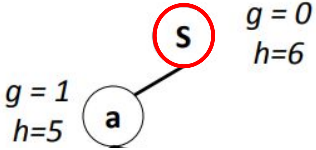


Fringe

S,f=6

A* : Combining UCS and Greedy



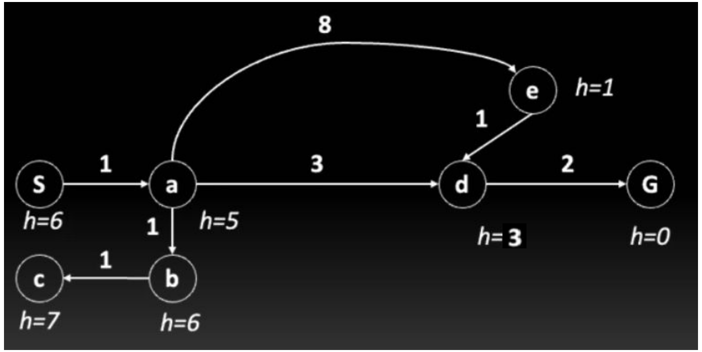


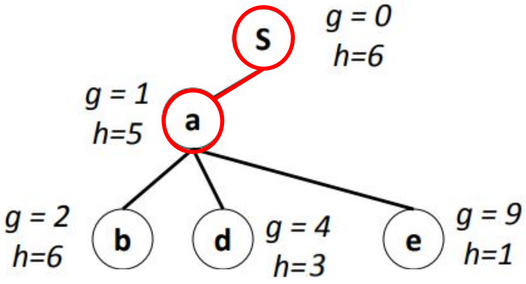
Fringe

~~S,f=6~~

S→a,f=6

A* : Combining UCS and Greedy





<u>Fringe</u>

~~S,f=6~~

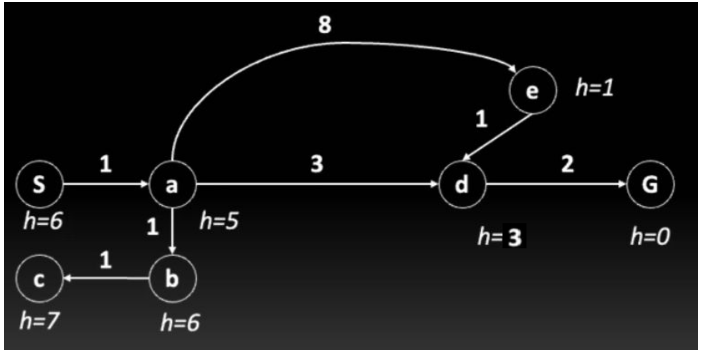
~~S→a,f=6~~

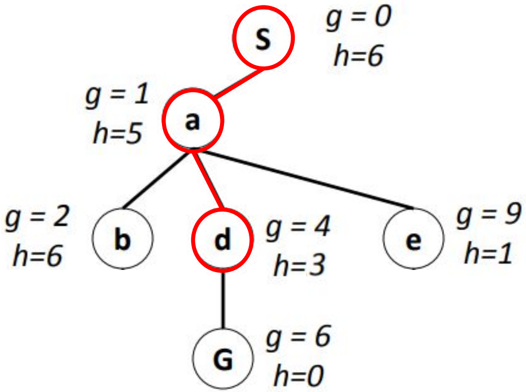
S→a→b,f=8

S→a→d,f=7

S→a→e,f=10

A* : Combining UCS and Greedy





Fringe

~~S,f=6~~

~~S→a,f=6~~

S→a→b,f=8

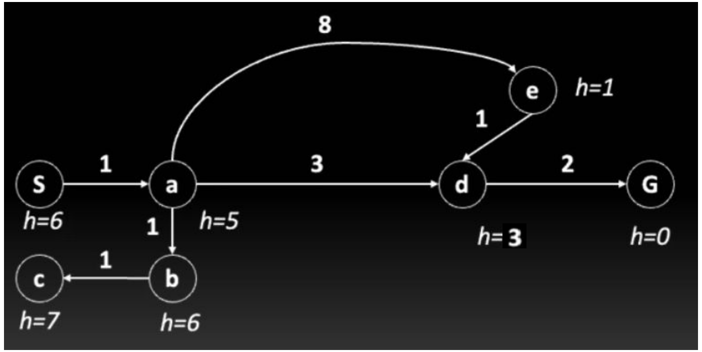
~~S→a→d,f=7~~

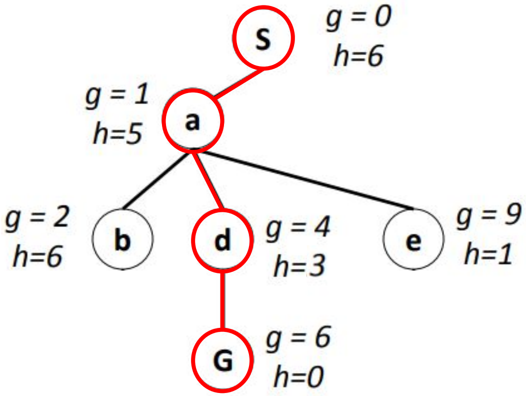
S→a→e,f=10

S→a→b→c,f=10

S→a→d→G,f=6

A* : Combining UCS and Greedy





<u>Fringe</u>

~~S,f=6~~

~~S→a,f=6~~

S→a→b,f=8

~~S→a→d,f=7~~

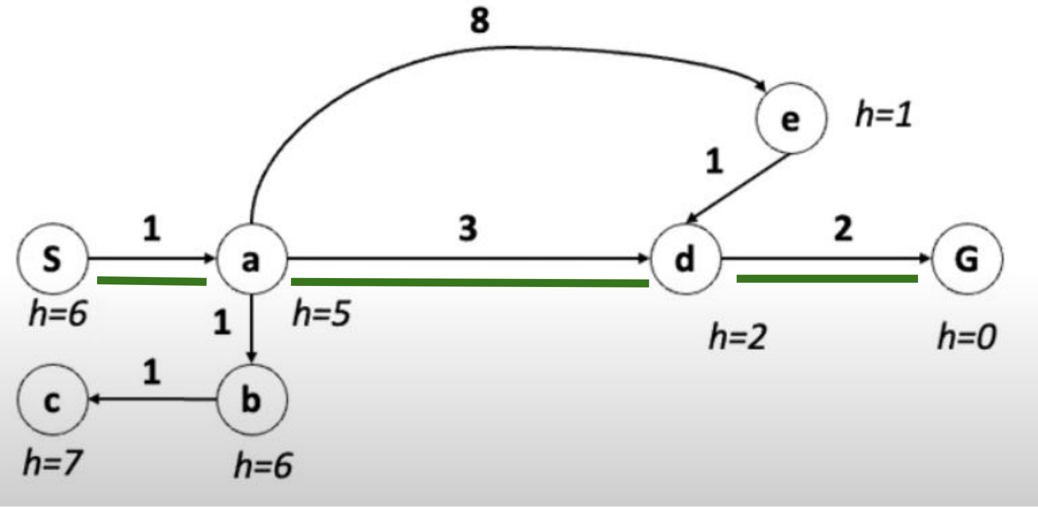
S→a→e,f=10

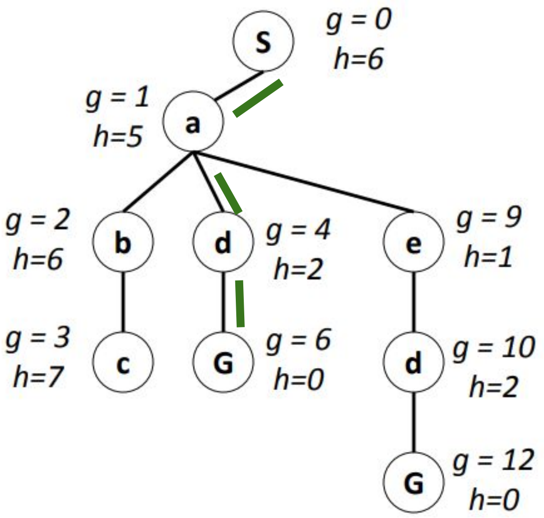
S→a→b→c,f=10

solution S→a→d→G,f=6

Combining UCS and Greedy

- Uniform-cost orders by path cost, or backward cost g(n)
- Greedy orders by goal proximity, or forward cost h(n)
- A* Search orders by the sum: f(n) = g(n) + h(n)



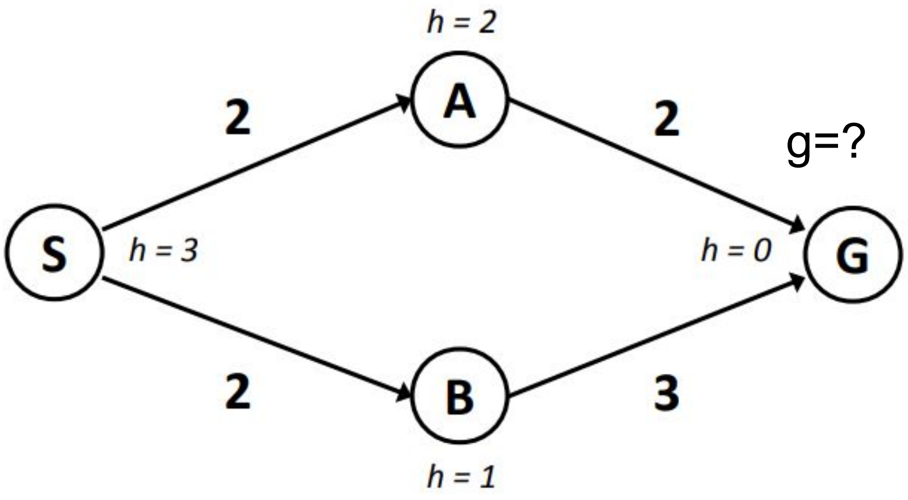


When should A* terminate?

Should we stop when we enqueue/insert a goal state in the fringe?

No, only stop when we dequeue/remove a goal from the fringe

- Insert S [f=3]
- Expand S
  - Insert S->B [f=3]
  - Insert S->A [f=4]
  - Fringe contains S->B[f=3],S->A[f=4]
- Remove S
- Expand S->B
  - Insert S->B->G [f=5]
  - Fringe contains S->A[f=4], S->B->G[f=5]
- Remove S->B
- Expand S->A
  - Insert S->A->G[f=4]
  - Fringe contains S->B->G[f=5], S->A->G[f=4]



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- Insert S [f=3]
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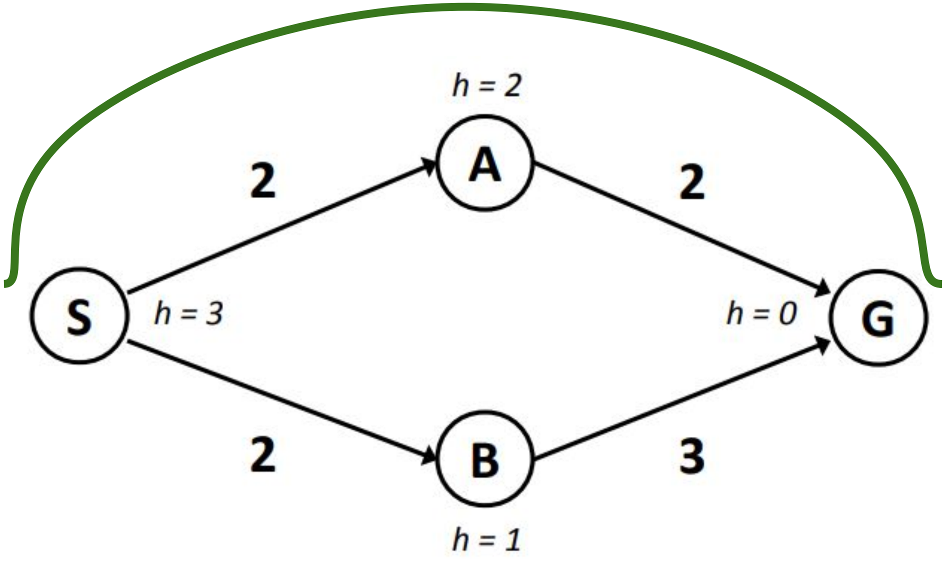
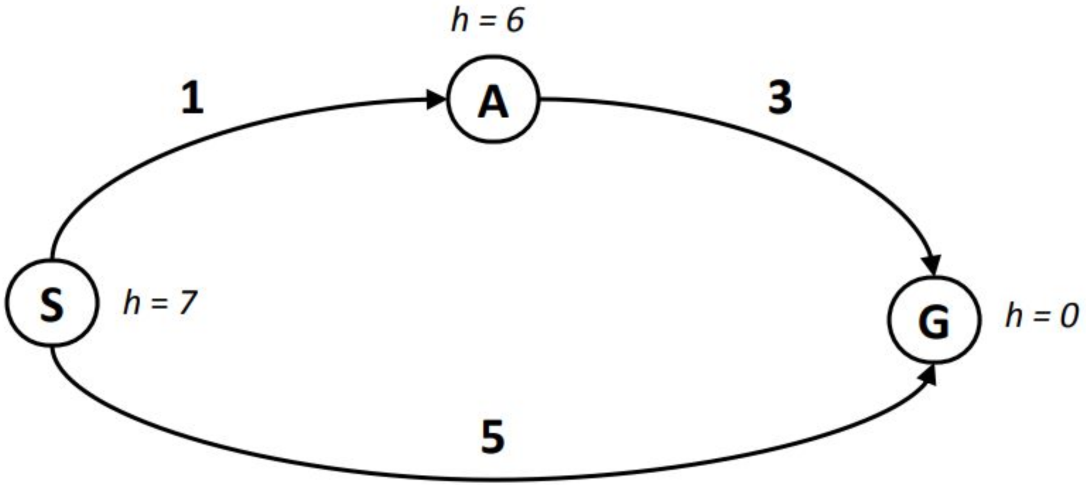
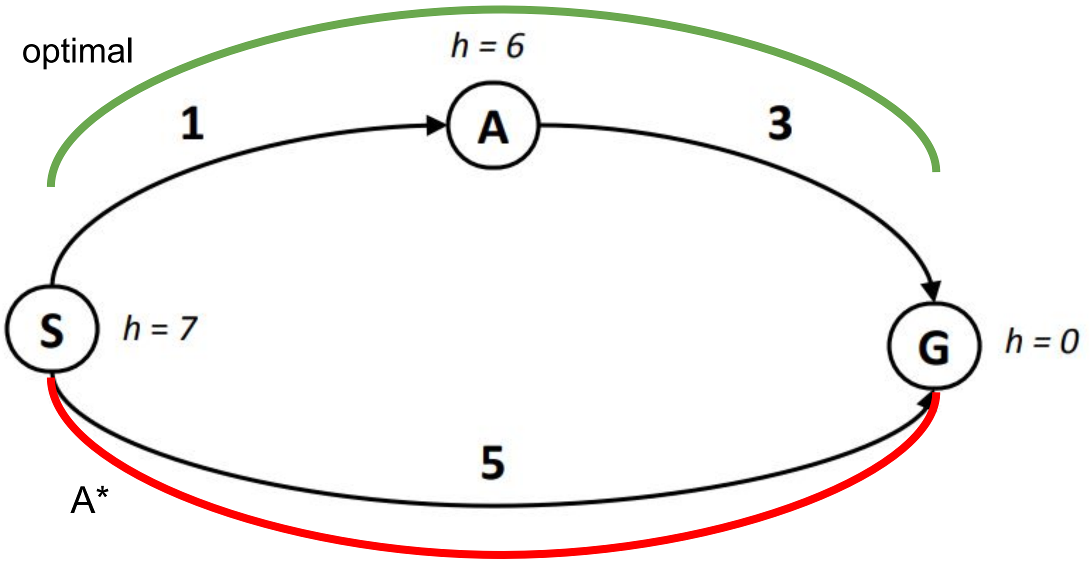
    - Fringe contains S->B[f=3],S->A[f=4]
- Remove S
- Expand S->B
    - Insert S->B->G [f=5]
    - Fringe contains S->A[f=4], S->B->G[f=5]
- Remove S->B
- Expand S->A
    - Insert S->A->G[f=4]
    - Fringe contains S->B->G[f=5], S->A->G[f=4]



Is A* Optimal?



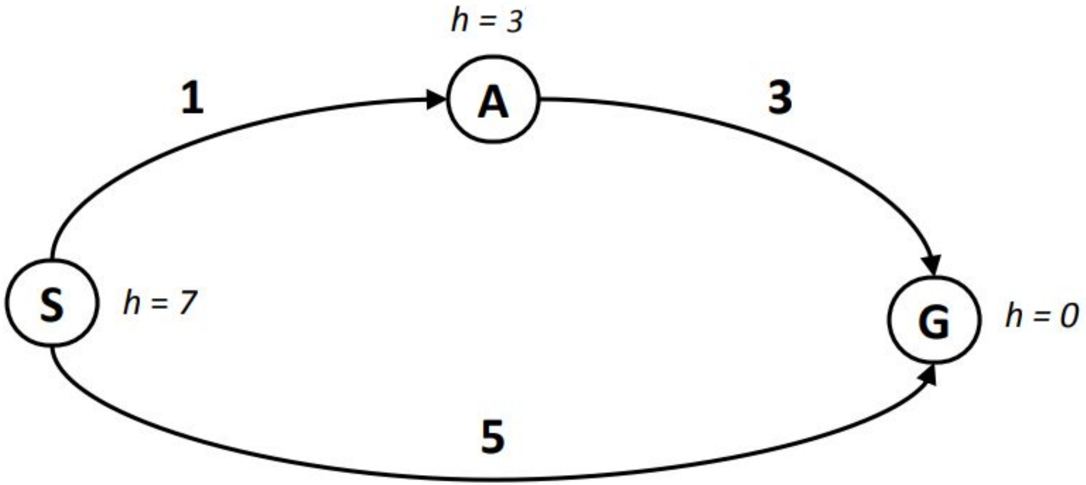
Is A* Optimal?



Actual bad goal cost < estimated good goal cost

We need estimates to be less than actual costs!

Is A* Optimal?



Actual bad goal cost < estimated good goal cost

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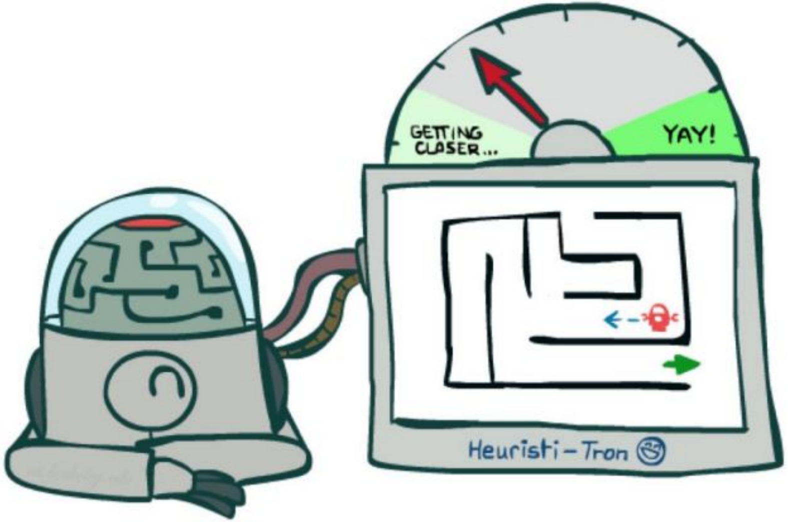
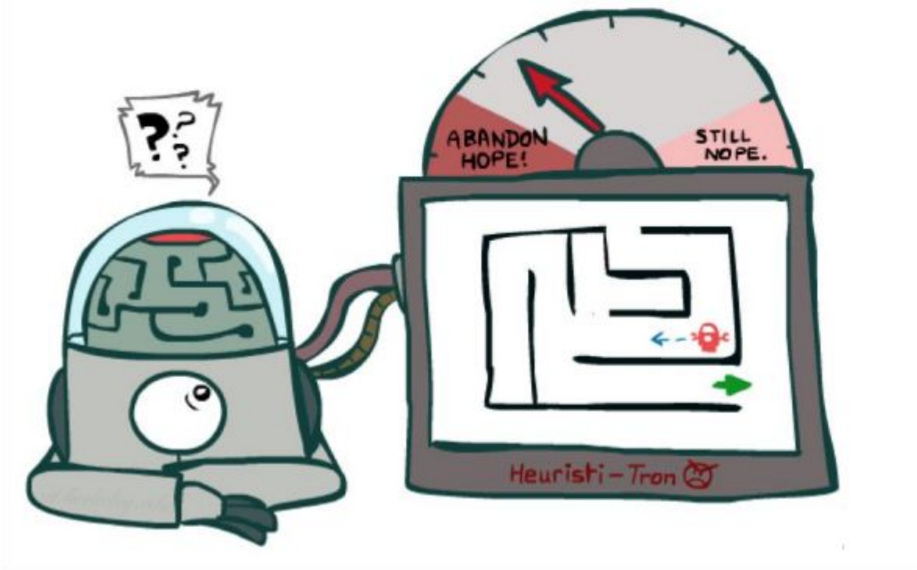
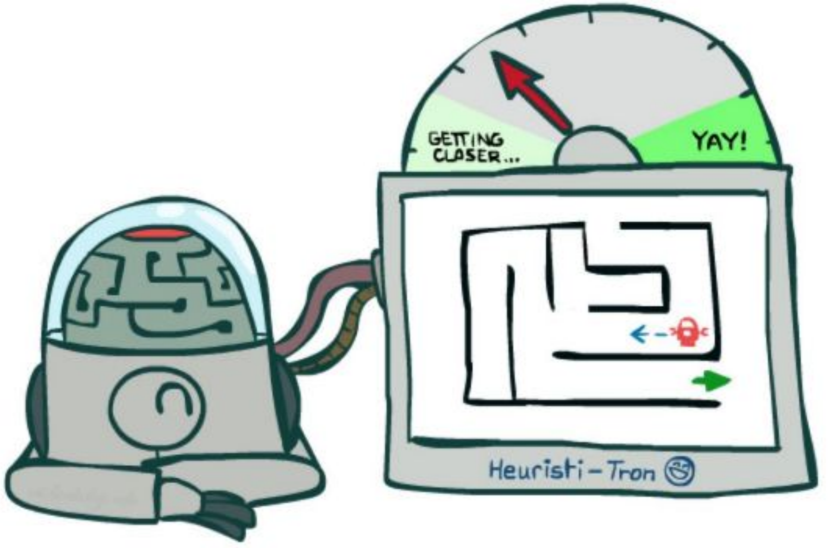
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Admissible Heuristics

- A heuristic *h* is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

  Where $h^*(n)$ is the true cost to a nearest goal

- Coming up with admissible heuristics is most of what's involved in using A* in practice.

- Needs to decided per problem

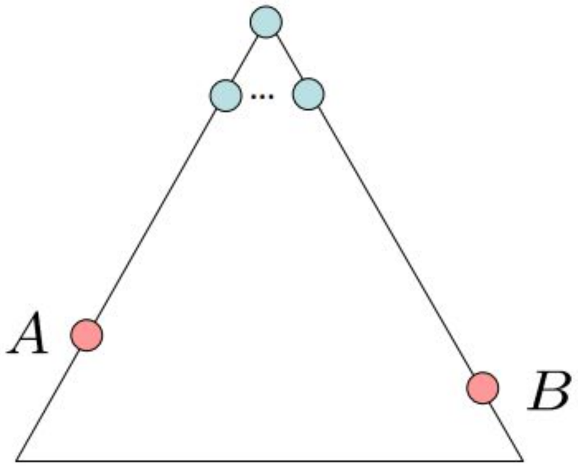
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

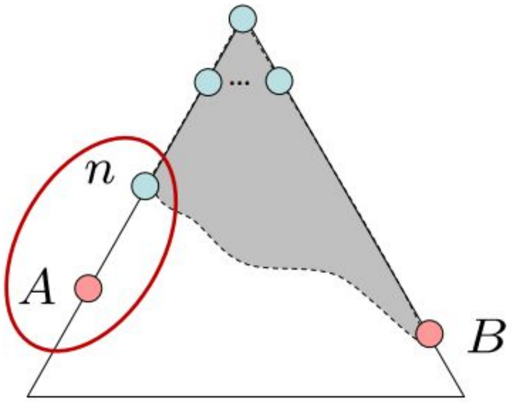
- A will exit the fringe before B



Optimality of A* Tree Search

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)



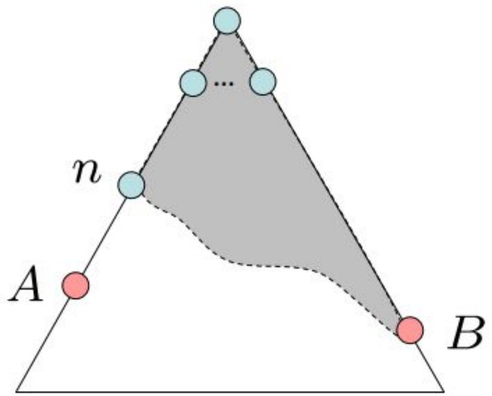
$$f(n) = g(n) + h(n)$$ Definition of f-cost

$$f(n) \leq g(A)$$ Admissibility of h

$$g(A) = f(A)$$ h = 0 at a goal

Optimality of A* Tree Search

1. f(n) is less than or equal to f(A)
   - Definition of f-cost says:
     f(n) = g(n) + h(n) = (path cost to n) + (est. cost of n to A)
     f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)



Optimality of A* Tree Search

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   - Definition of f-cost says:
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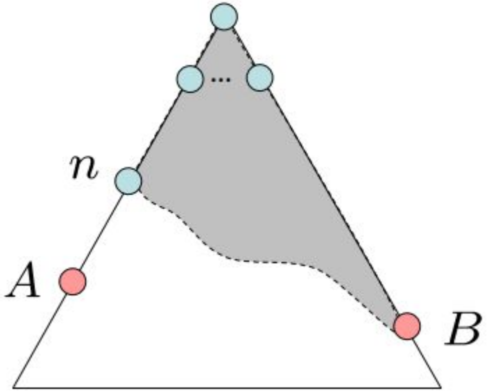
     f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)
   - The admissible heuristic must underestimate the true cost
     h(A) = (est. cost of A to A) = 0



Optimality of A* Tree Search

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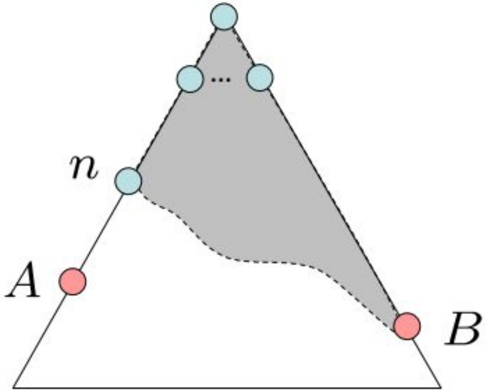
     f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)
   - The admissible heuristic must underestimate the true cost
     h(A) = (est. cost of A to A) = 0
   - So now, we have to compare:
     f(n) = g(n) + h(n) = (path cost to n) + (est. cost of n to A)
     f(A) = g(A) = (path cost to A)



Optimality of A* Tree Search

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   - The admissible heuristic must underestimate the true cost
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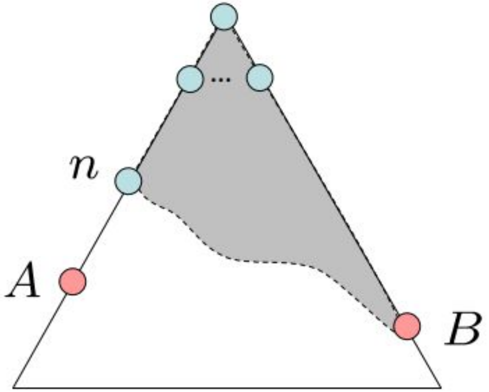
   - So now, we have to compare:
     f(n) = g(n) + h(n) = (path cost to n) + (est. cost of n to A)
     f(A) = g(A) = (path cost to A)
   - h(n) must be an underestimate of the true cost from n to A
     (path cost to n) + (est. cost of n to A) ≤ (path cost to A)



Since, path cost to A = path cost to n + path cost from n to A

Optimality of A* Tree Search

1. f(n) is less than or equal to f(A)
   - Definition of f-cost says:
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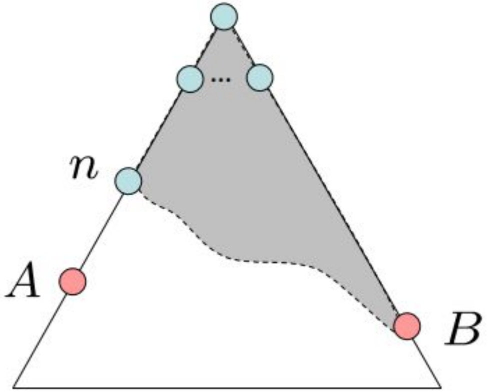
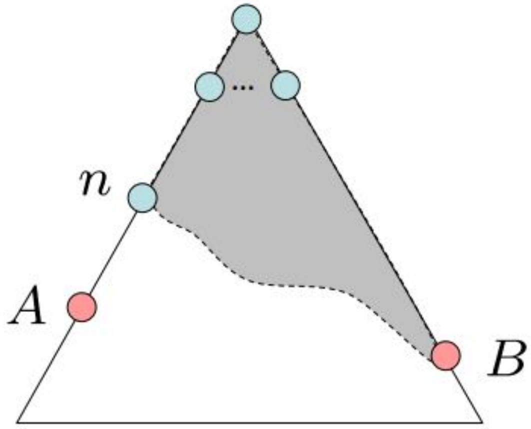
   - h(n) must be an underestimate of the true cost from n to A
     (path cost to n) + (est. cost of n to A) ≤ (path cost to A)
     g(n) + h(n) ≤ g(A)
     f(n) ≤ f(A)



Optimality of A* Tree Search

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



$g(A) < g(B)$	B is suboptimal
$f(A) < f(B)$	h = 0 at a goal

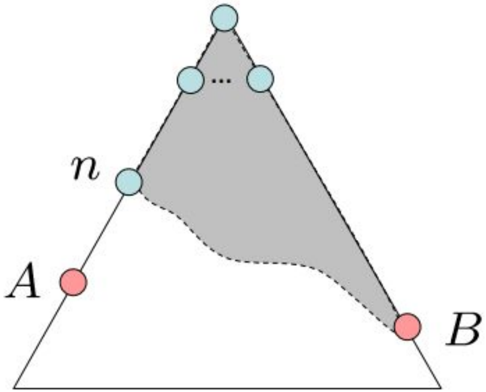
Optimality of A* Tree Search

2. f(A) is less than f(B)

   - We know that:

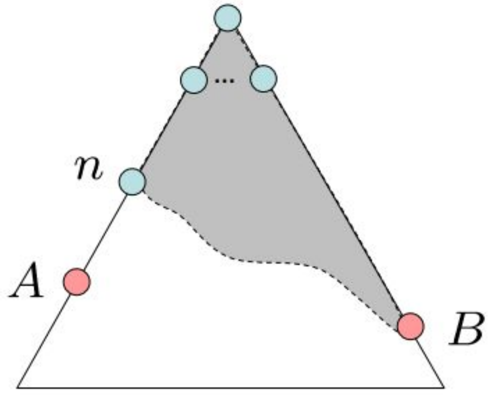
     f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)

     f(B) = g(B) + h(B) = (path cost to B) + (est. cost of B to B)



Optimality of A* Tree Search

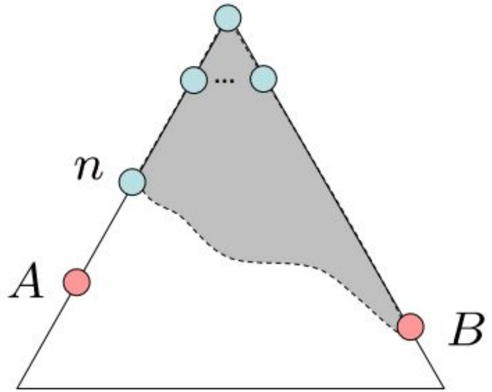
2. f(A) is less than f(B)
   - We know that:
     f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)
     f(B) = g(B) + h(B) = (path cost to B) + (est. cost of B to B)
   - The heuristic must underestimate the true cost:
     h(A) = h(B) = 0
   - So now, we have to compare:
     f(A) = g(A) = (path cost to A)
     f(B) = g(B) = (path cost to B)



Optimality of A* Tree Search

2. f(A) is less than f(B)

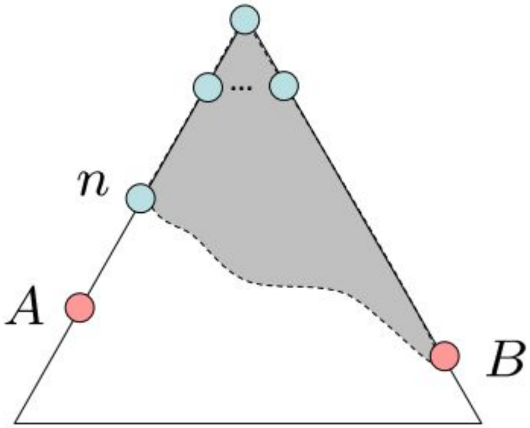
- We know that:
  f(A) = g(A) + h(A) = (path cost to A) + (est. cost of A to A)
  f(B) = g(B) + h(B) = (path cost to B) + (est. cost of B to B)
- The heuristic must underestimate the true cost:
  h(A) = h(B) = 0
- So now, we have to compare:
  f(A) = g(A) = (path cost to A)
  f(B) = g(B) = (path cost to B)
- We assumed that B is suboptimal! So
  (path cost to A) < (path cost to B)
  g(A) < g(B)
  f(A) < f(B)



Optimality of A* Tree Search

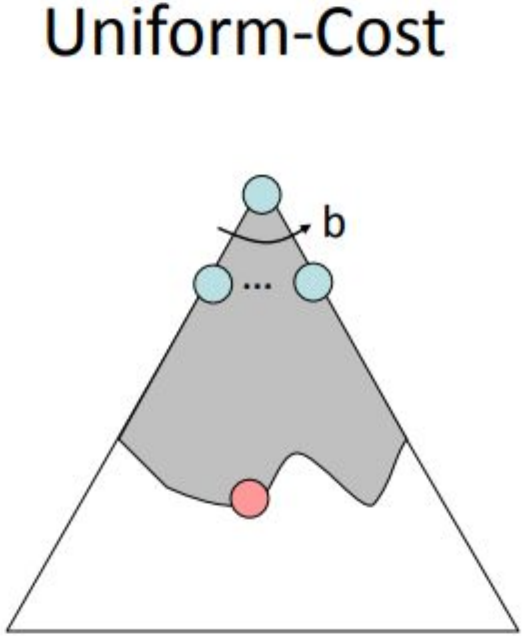
Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

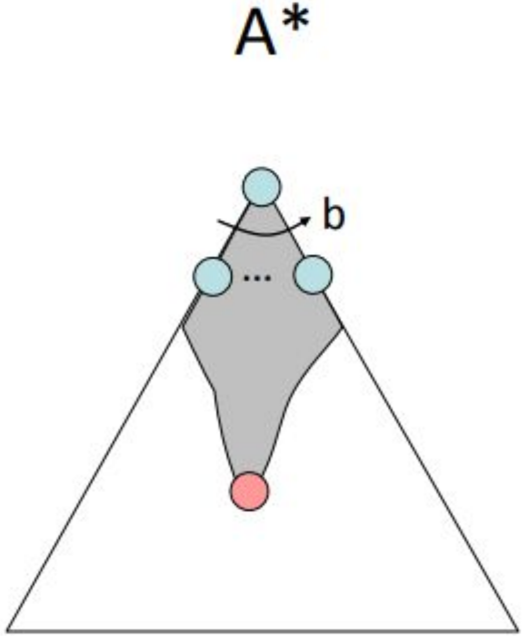


$$f(n) \leq f(A) < f(B)$$

Properties of A*



Contour-wise



Still Contour-wise but contour defined by f

Practice

Is the heuristics function admissible?
Find the solution using A*