

# Artificial Intelligence

## Lec 9: Searching with Non-Deterministic Actions, Environments with Multiple Agents

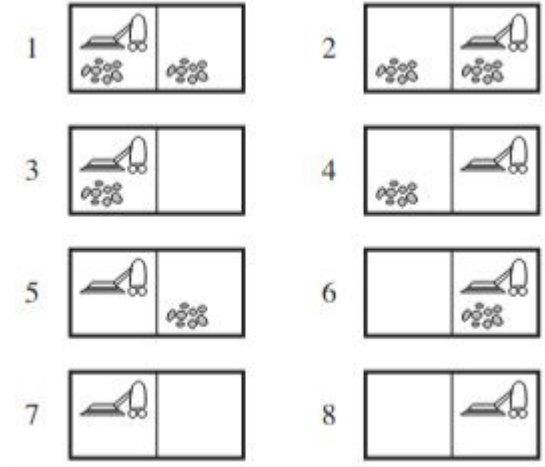
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# Searching with Non-Deterministic Actions

- Till now, we have assumed that the **environment** is **fully observable and deterministic**.
- And the agent knows what the **exact effects of each action** are
  - can “**exactly determine**” which **state** results **from** any sequence of **actions** and always knows which state it is in.
- Therefore, percepts or observations from the environment do not provide any additional information after each action.
- In a **non-deterministic environment**, there can be **many** possible **outcomes** or **effects** of an action.
- When the environment is non-deterministic, percepts become useful.
  - percepts tell the agent which of the possible outcomes of its actions has actually occurred.
  - Therefore, the agent's future actions will depend on the percepts
- **Solution** to a problem is **not a sequence** but a **contingency plan** (also known as a strategy) that specifies what to do depending on what percepts are received

# Environment with Non-Deterministic Actions: Example

- The **erratic vacuum world** has two squares with dirt in them, and the vacuum cleaner is either in the left or right square.
- There are three actions—Left, Right, and Suck
- The state space has eight states and the goal is to clean up all the dirt (states 7 and 8).
- **Trivially solvable** with the **search algorithms** we know till now, **if** the **environment** is **observable, deterministic, and completely known**
  - The **solution** is an **action sequence**
- Now suppose that we introduce **nondeterminism** in the form of a powerful but erratic vacuum cleaner. E.g.
  - Suck action applied to a dirty square, cleans the square and sometimes cleans up dirt in an adjacent square, too.
  - Suck action applied to a clean square, the action sometimes deposits dirt on the carpet.

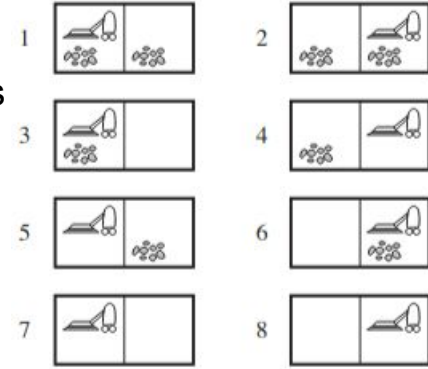


Can you predict  
the exact  
sequence of  
actions now?

No

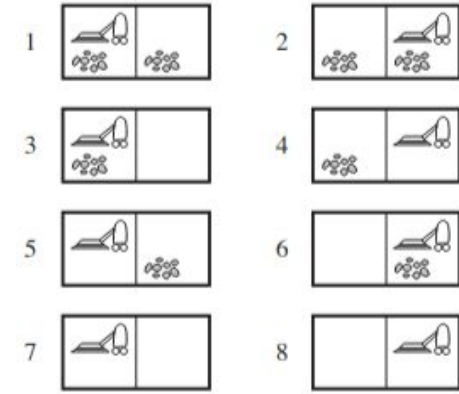
# Environment with Non-Deterministic Actions: Example

- Instead of **defining the transition model** by a RESULT function that returns a single state, we use a RESULTS function **that returns a set of possible outcome states**.
  - e.g., in the erratic vacuum world, the Suck action in state 1 leads to a state in the set {5, 7}, i.e., the dirt in the right-hand square may or may not be vacuumed up.
- We also need to **generalize** the notion of a **solution** to the problem.
  - e.g., if we start in state 1, there is **no single sequence of actions** that solves the problem.
  - Instead, we need a **contingency plan** such as the following:  
[Suck, if State = 5 then [Right, Suck] else [ ]]
- **Solutions** for nondeterministic problems can contain **nested if-then-else statements**.
  - this means that they are **trees rather than sequences**.
  - this allows the **selection of actions based on contingencies arising during execution**.
- Many problems in the real, physical world are contingency problems because exact prediction is impossible.



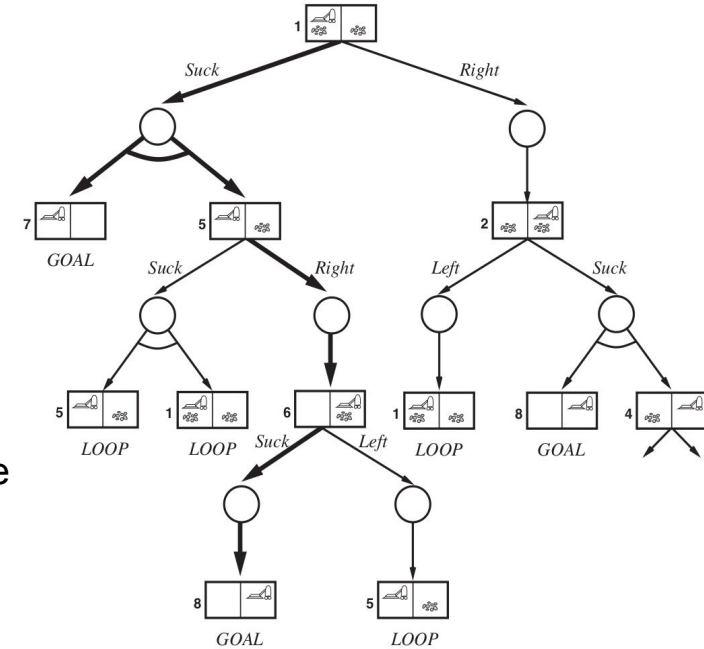
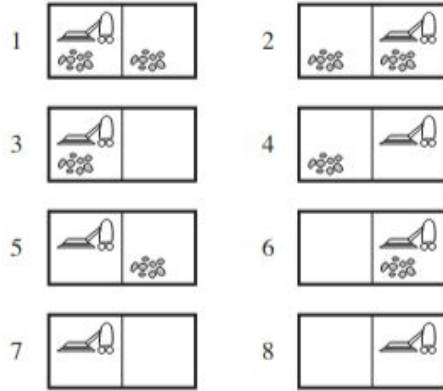
# AND – OR Search Trees

- We need a search tree first to find solutions in this setting: AND-OR Search Trees
  - Contains OR nodes and AND nodes
- In a deterministic environment, the only branching is introduced by the agent's own choices in each state
- Nodes where the agent chooses a deterministic action to move to another state are called OR-Nodes
  - e.g. at an OR NODE node the agent chooses Left or Right or Suck
- In a nondeterministic environment, branching is also introduced by the **environment's choice of outcome for each action**. These are the AND nodes.
  - e.g. the Suck action in state 1 leads to a state in the set {5, 7}, so the agent would need to find a plan for state 5 and for state 7



# AND – OR Search Trees

- A solution for an AND – OR search problem is a subtree that
  - has a goal node at every leaf,
  - specifies one action at each of its OR nodes,
  - includes every outcome branch at each of its AND nodes.
  - sometimes has a trimmed subtree in case of revisiting a node



# Searching for solution

Recursive, depth-first algorithm for AND – OR graph search

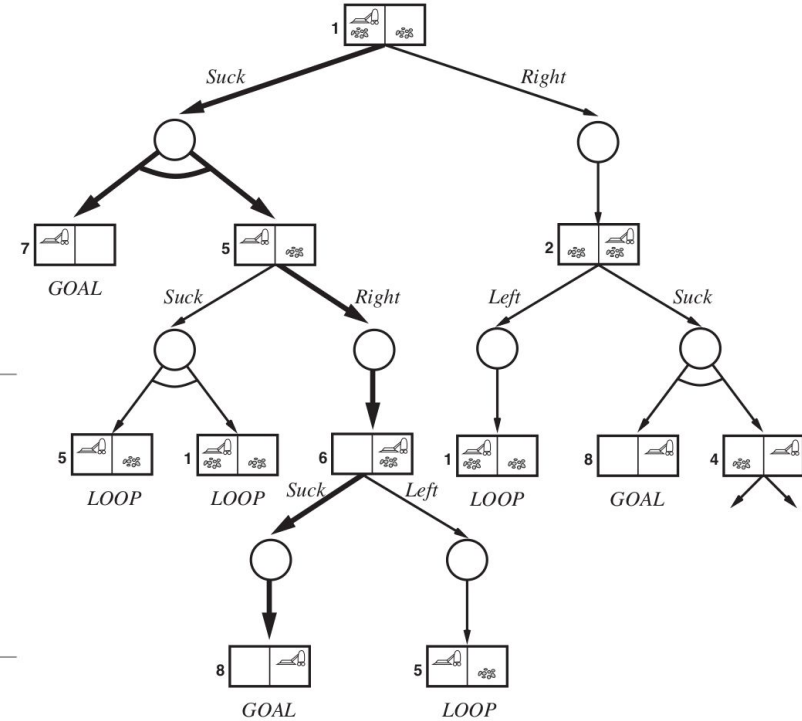
**function** AND-OR-GRAPH-SEARCH(*problem*) **returns** a conditional plan, or failure  
 OR-SEARCH(*problem*.INITIAL-STATE, *problem*, [])

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**function** OR-SEARCH(*state*, *problem*, *path*) **returns** a conditional plan, or failure  
**if** *problem*.GOAL-TEST(*state*) **then return** the empty plan  
**if** *state* is on *path* **then return failure**  
**for each** *action* **in** *problem*.ACTIONS(*state*) **do**  
     *plan*  $\leftarrow$  AND-SEARCH(RESULTS(*state*, *action*), *problem*, [*state* | *path*])  
     **if** *plan*  $\neq$  failure **then return** [*action* | *plan*]  
**return failure**

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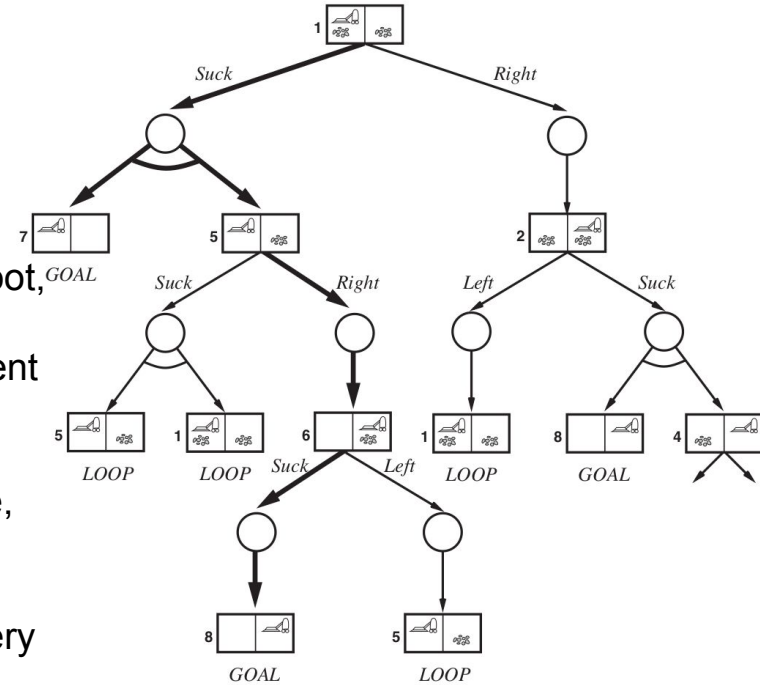
**function** AND-SEARCH(*states*, *problem*, *path*) **returns** a conditional plan, or failure  
**for each** *s<sub>i</sub>* **in** *states* **do**  
     *plan<sub>i</sub>*  $\leftarrow$  OR-SEARCH(*s<sub>i</sub>*, *problem*, *path*)  
     **if** *plan<sub>i</sub>* = failure **then return failure**  
**return** [*if s<sub>1</sub> then plan<sub>1</sub> else if s<sub>2</sub> then plan<sub>2</sub> else ... if s<sub>n-1</sub> then plan<sub>n-1</sub> else plan<sub>n</sub>*]



## Searching for solution

## Recursive, depth-first algorithm for AND – OR graph search

- If the current state is identical to a state on the path from the root, then it returns with failure.
  - This doesn't mean that there is no solution from the current state.
  - it simply means that if there is a solution, it must be reachable from the earlier incarnation of the current state, so the new incarnation can be discarded.
- With this check, we ensure that the algorithm terminates in every finite state space, because every path must reach a goal, a dead end, or a repeated state

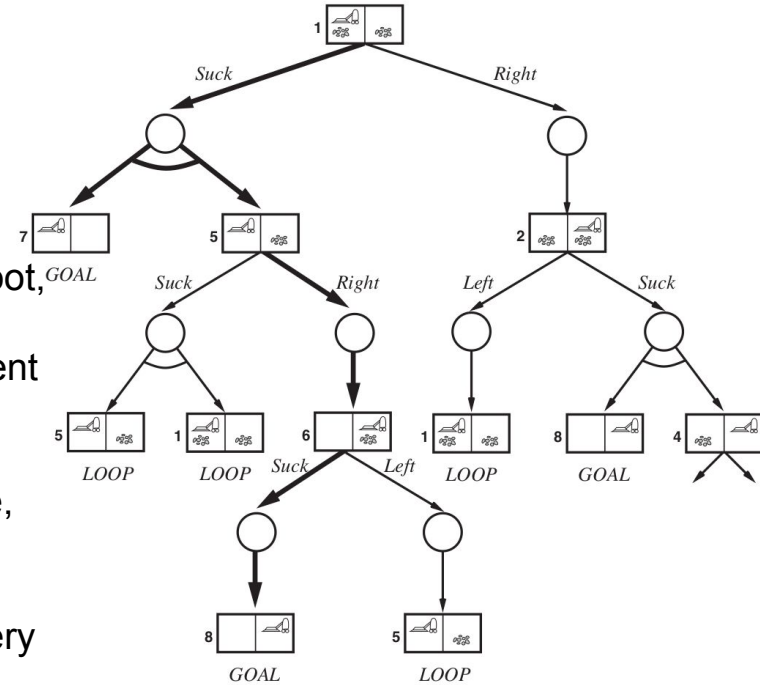




# Searching for solution

Recursive, depth-first algorithm for AND – OR graph search

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Solution is not just a sequence of action. More like a tree.

# Environments with Multiple Agents

- Until now, we have been looking into environments having a single agent interacting with the environment.
- Multiagent environments are environments in which **multiple** agents **interact with each other** to achieve their **individual goals**.
  - Compete or Cooperate or Both
- In chess, the opponent entity B is trying to maximize its performance measure, which, by the rules of chess, minimizes agent A's performance measure.
  - Thus, it is a **competitive** multiagent environment.
- In the taxi-driving environment, avoiding collisions maximizes the performance measure of all agents
  - So it is a partially **cooperative** multiagent environment.
  - Also partially competitive because, for example, only one car can occupy a parking space.

# Environments with Multiple Agents

- Multiple agents lead to more complex environments/ecosystems
  - Inspired by evolution
  - games, robotics, generative adversarial networks (GANs)
- We'll focus on games, but multi-agent ideas come up in many areas of AI
- Why games?
  - They are usually good reasoning problems, formal (with fixed rules) and non-trivial
  - Direct comparison with humans and other computer systems easy, e.g. AlphaGo

# Types of Games

- Many different kinds of games!
  - Deterministic or stochastic?
    - Is there any randomness, like rolling a die?
    - Chess, Backgammon, Ludo
  - Zero sum?
    - Purely competitive or just a general multiplayer environment
  - Perfect information (can you see the state)?
    - Poker – Cards of opponent not visible
    - Chess – All pieces visible
- Want algorithms for calculating a strategy (policy) which recommends a move from each state

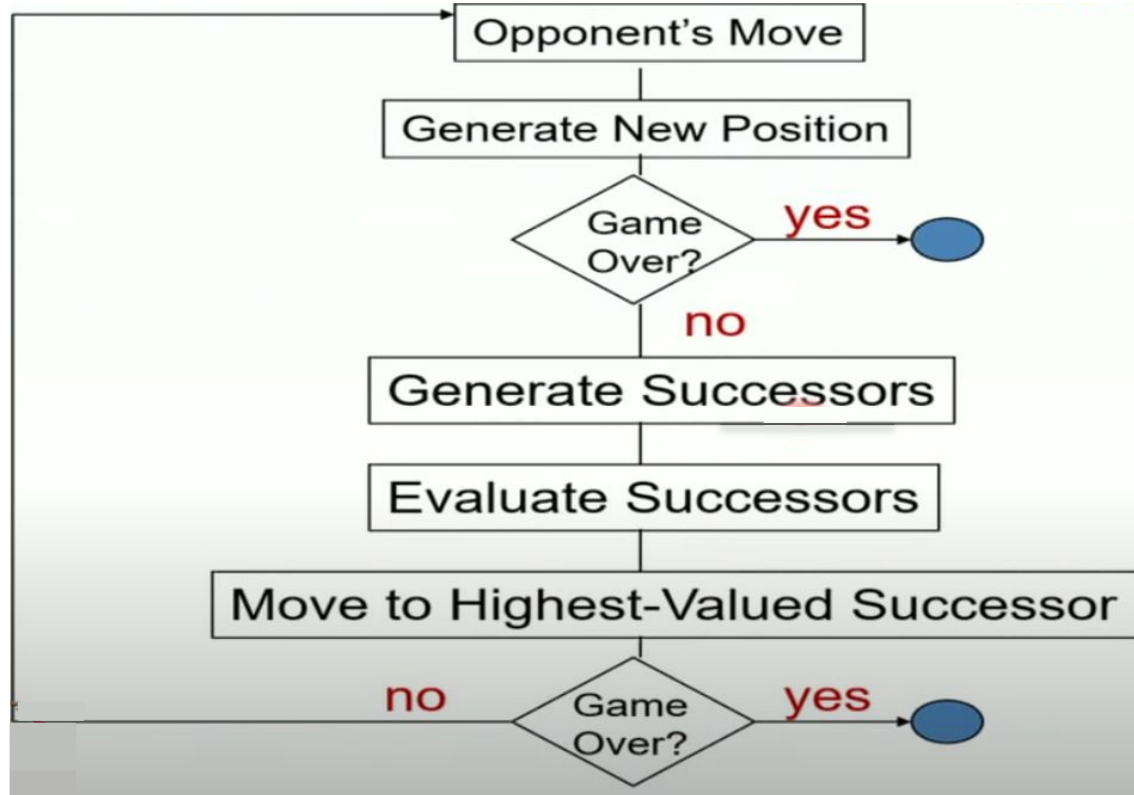


# Games vs General Search Problems

Player 1 cannot plan a simple sequence of actions to reach the goal, considering only his/her possible moves.

- After player 1 makes a move, now player 2 **takes control** and makes the next move  
[Unpredictable opponent]

# Two Player Game



# Deterministic Games

- Many possible formalizations, one is:
  - States:  $S$  (start at  $s_0$ )
  - Players:  $P=\{1\dots N\}$  (usually take turns)
  - Actions:  $A$  (may depend on player / state)
  - Transition Function:  $S \times A \rightarrow S$
  - Terminal Test:  $S \rightarrow \{\text{true}, \text{false}\}$
  - Terminal Utilities:  $S \times P \rightarrow R$ 
    - e.g. Win/Draw/Lose/Maximize Money
- Solution for a player is a policy:  $S \rightarrow A$



# Zero-Sum Games

- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes).
  - Lets us think of a single value that one maximizes and the other minimizes.
  - Adversarial, pure competition



- General Games
  - Agents have independent utilities (values on outcomes).
  - Cooperation, indifference, competition, and more are all possible.



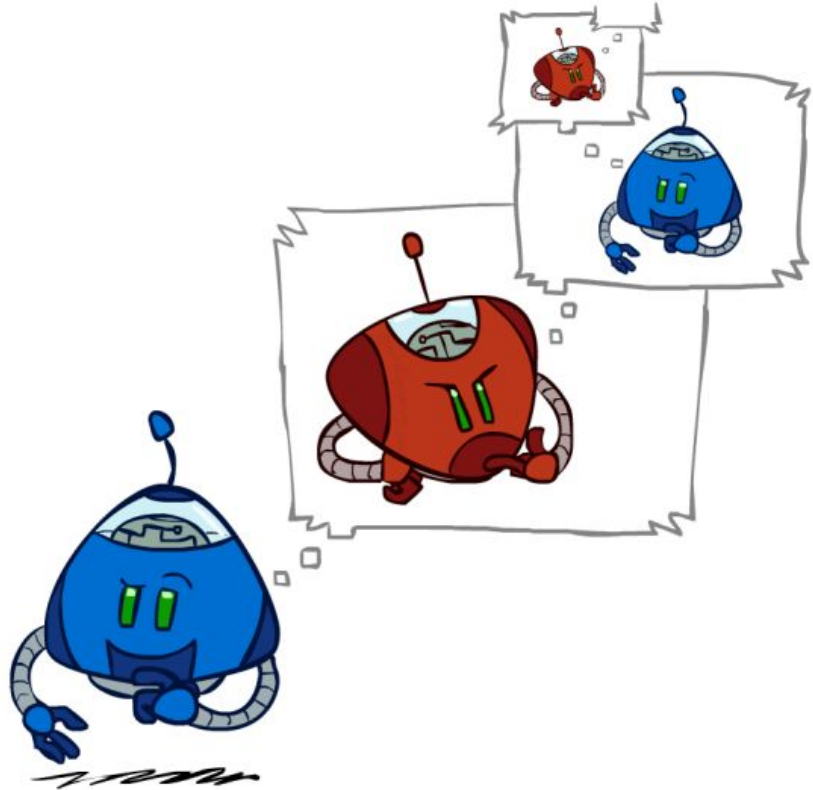


# Solving Zero-Sum Games

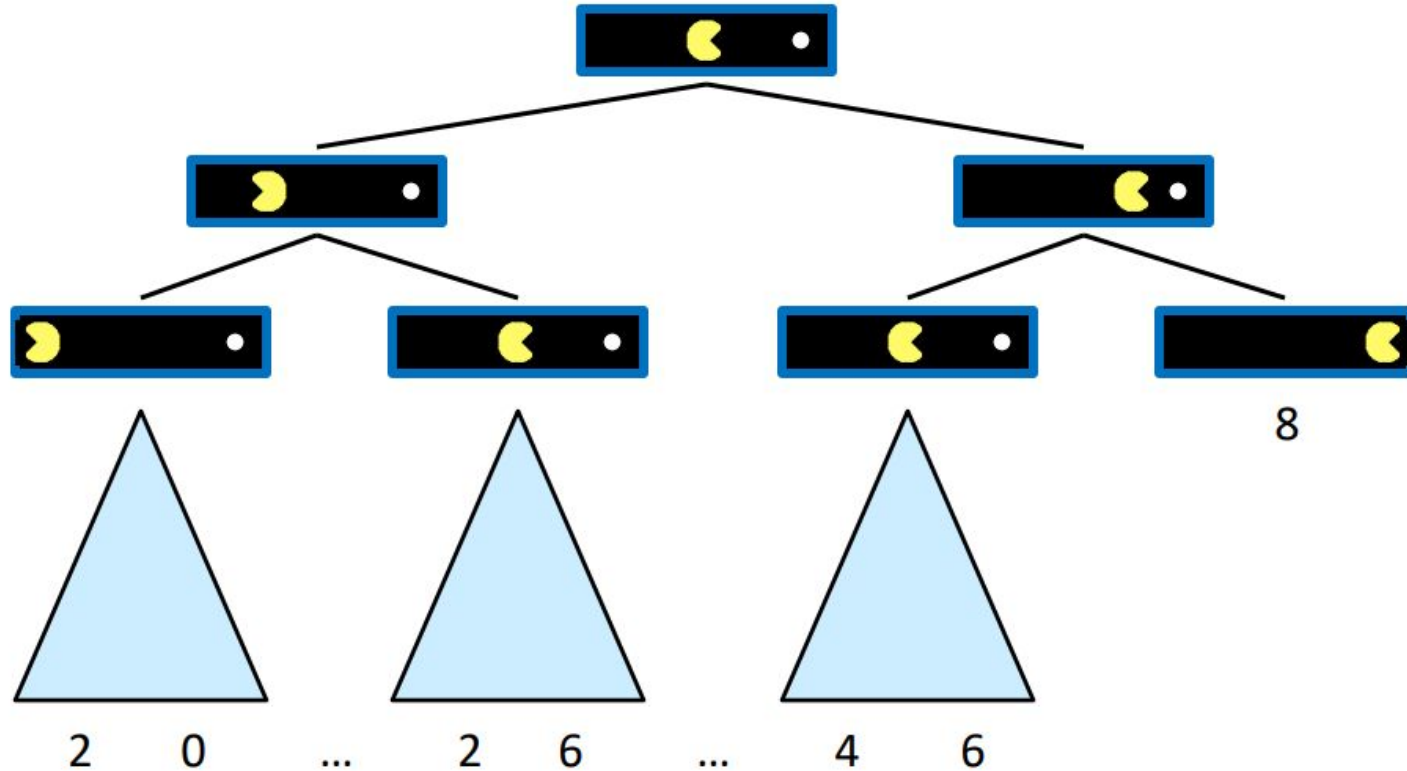
Similar to search

- but the player is not just looking at the future of its actions.
- but also thinking about what actions the other player would take as well.
- but if the other player was thinking about what actions it would take and so on

A recursive chain of “what if”

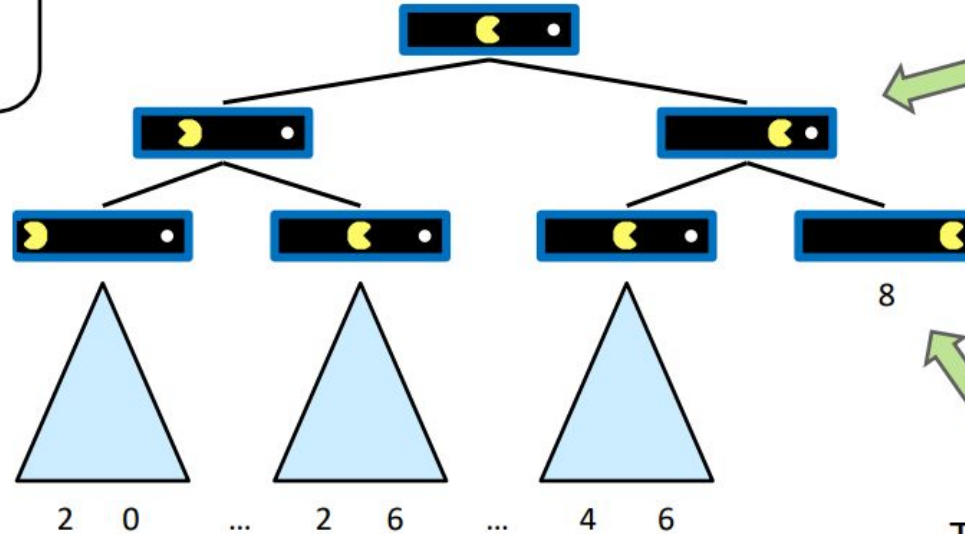


# Single-Agent Search Trees



# Value of a State

Value of a state:  
The best achievable  
outcome (utility)  
from that state



Non-Terminal States:

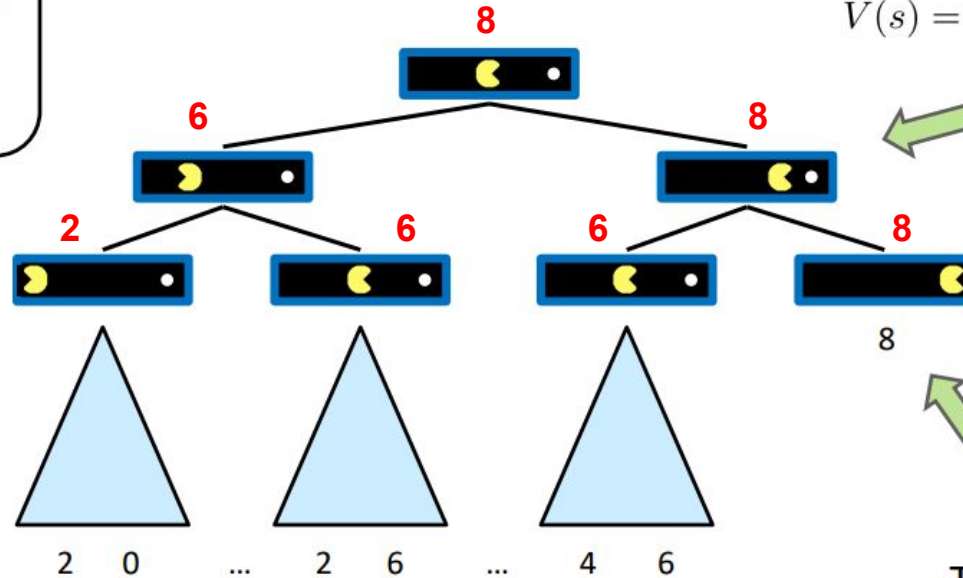
$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

Terminal States:

$$V(s) = \text{known}$$

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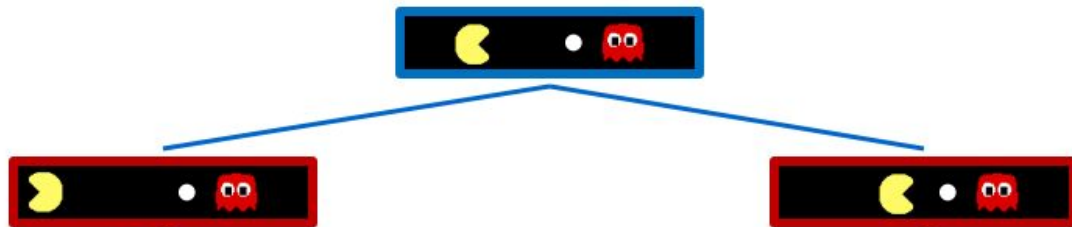
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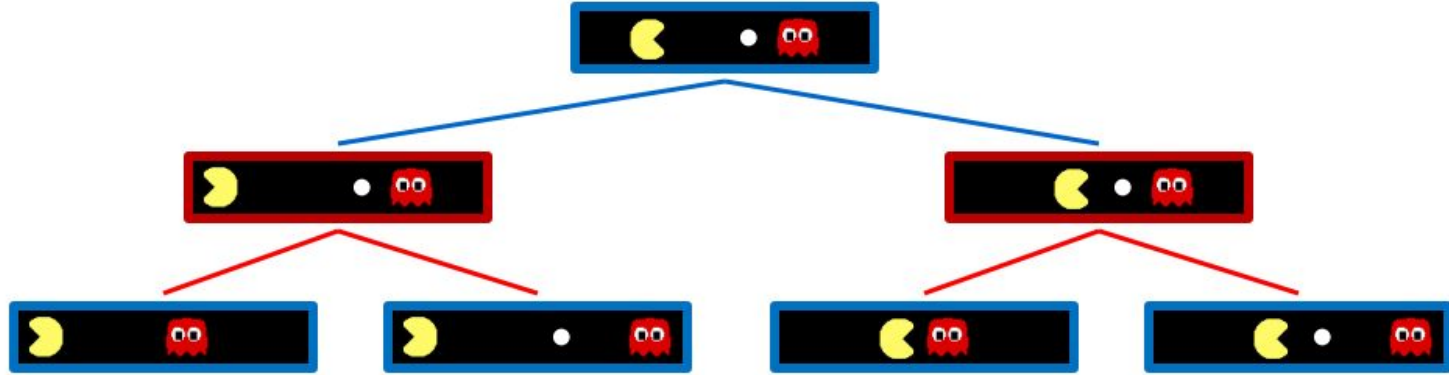
# Adversarial Game Trees



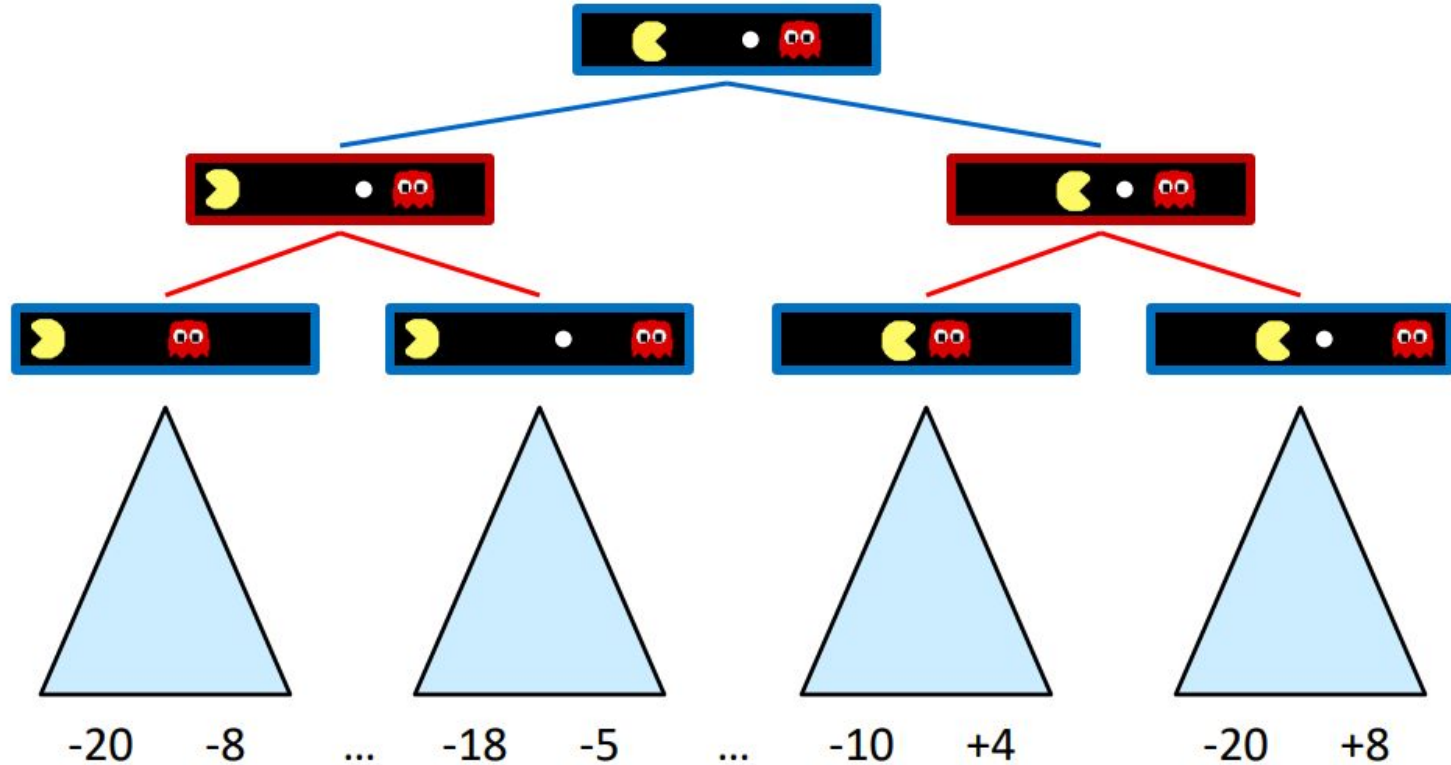
# Adversarial Game Trees



# Adversarial Game Trees



# Adversarial Game Trees





# Optimal Decision in Games

- In a **normal search** problem, the **optimal solution** would be a **sequence of actions** leading to a **goal state**.
  - a terminal state that is a win.
- In **adversarial search** with players MIN and MAX, MIN has something to say about the optimal solution for MAX, and vice versa.
- MAX therefore must find a contingent **strategy**, which specifies MAX's move in the initial state.
  - then MAX's moves in the states resulting from every possible response by MIN,
  - then MAX's moves in the states resulting from every possible response by MIN to those moves,
  - and so on.
- This is somewhat similar to the AND – OR search algorithm.
- Roughly speaking, an **optimal strategy** leads to **outcomes** that are at least **as good as** any other **strategy** when one is playing an **infallible opponent**.

# Optimal Decision in Games

- Given a **game tree**, the **optimal** strategy can be determined from the **minimax value of each node**.
- The **minimax** value of a node is the **utility** (for player MAX) of being in the **corresponding state**, **assuming** that both **players play optimally** from there to the end of the game.
- The minimax value of a terminal state is just its utility (known).
- Given a choice,
  - **player MAX** prefers to **move** to a state of **maximum** value,
  - whereas player **MIN** prefers a state of **minimum** value

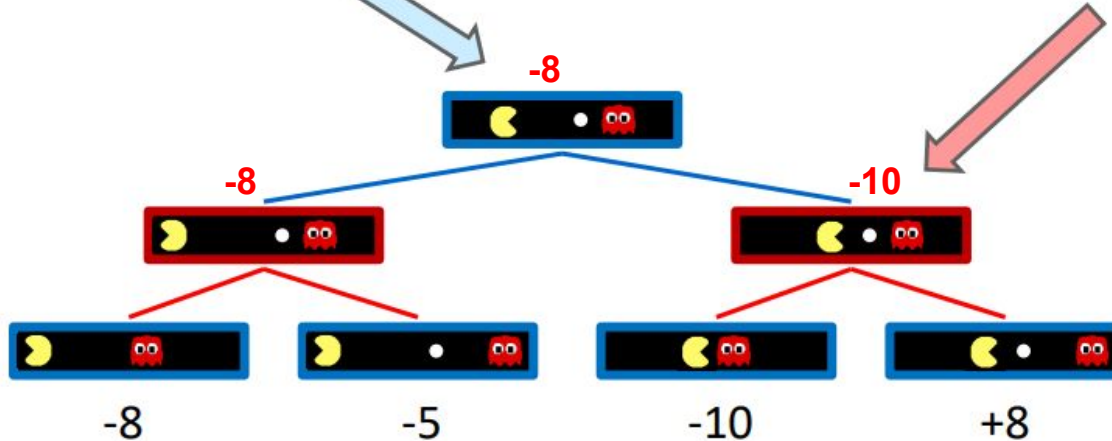
# Optimal Decision in Games: Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

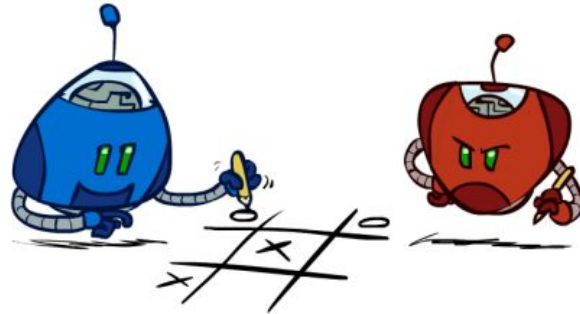
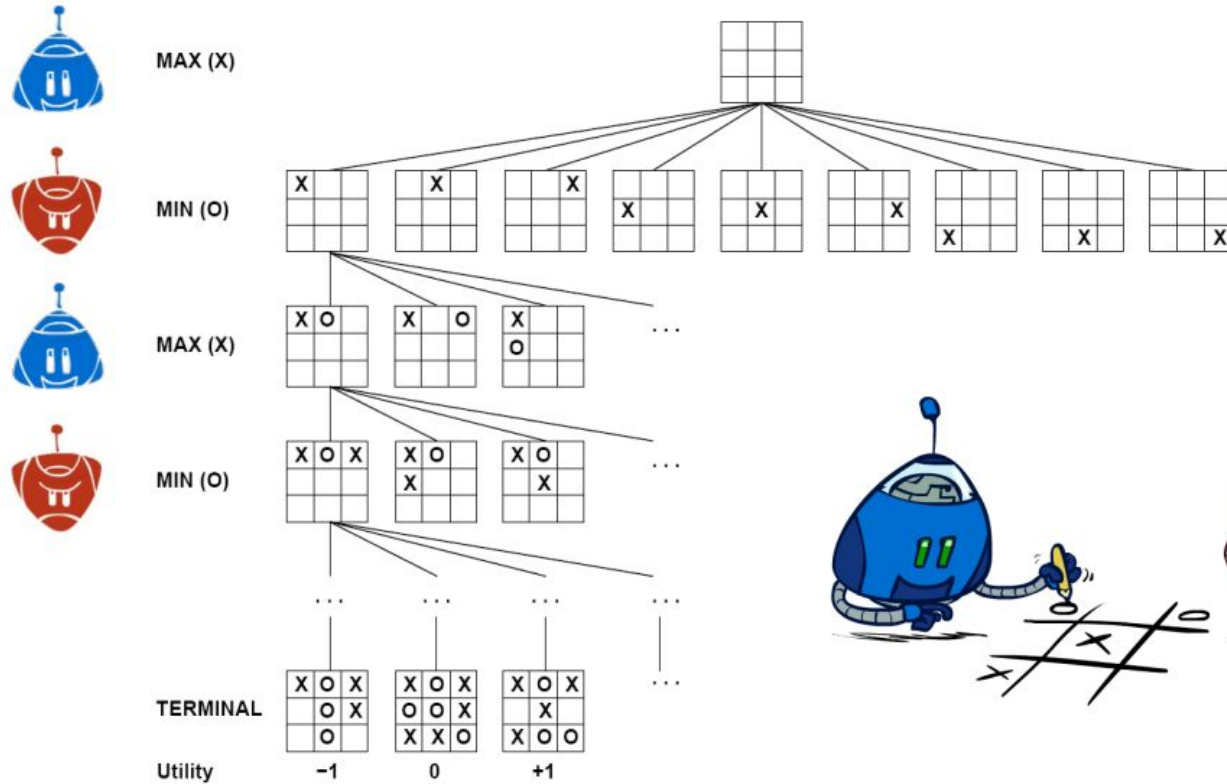
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

$$V(s) = \text{known}$$

# Tic-Tac-Toe Game Tree



# Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value:
    - the best achievable utility **against a rational (optimal) adversary**

