MAL7051: Matrix Theory
quiz 1

Timing: 50 minutes **Date: August 10 , 2023** **Maximum Marks: 5**

Answer all questions. Give complete details in your answers.

Question 1: Show that $(AB)^T = B^T A^T$. 2 marks

Question 2: Show that the matrices

$$\begin{pmatrix} 1 & 7 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 4 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 & 0 \\ -2 & 4 & 0 \\ -5 & -4 & -1 \end{pmatrix}$$

are similar. 1 mark

Question 3: Diagonalize the matrix

$$\begin{pmatrix} 4 & -3 & 0 \\ 2 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}.$$

In other words, show that the matrix A is diagonalizable and find the matrices P(invertible) and D (diagonal) such that $A = PDP^{-1}$. 2 marks

MAL7051: Matrix Theory
quiz 2

Timing: 50 minutes **Date: September 4 , 2023** **Maximum Marks: 5**

Answer all questions. Give complete details in your answers.

Question 1: Suppose S spans $\mathbb{R}^n$. Show that any maximum number of linearly independent vectors in S form a basis of $\mathbb{R}^n$ $1\frac{1}{2}$ marks

Question 2: Suppose $S = \{w_1, w_2, \cdots, w_r\}$ is an orthogonal basis for a subspace W of $\mathbb{R}^n$. Show that one may extend S to an orthogonal basis for $\mathbb{R}^n$. $1\frac{1}{2}$ mark

Question 3: Suppose $E = \{v_1, v_2, \cdots, v_n\}$ is an orthonormal basis of $\mathbb{R}^n$. Prove that for any $v \in \mathbb{R}^n$, we have

$$v = \langle v, v_1\rangle v_1 + \langle v, v_2\rangle v_2 + \cdots + \langle v, v_n\rangle v_n.$$

1 mark

Question 4: Show that every square real matrix has a polar decomposition. 1 mark

MAL7051: Matrix Theory
Minor 1

Timing: 60 minutes **Date: September 6 , 2023** **Maximum Marks: 30**

Answer all questions. Give complete details in your answers.

Question 1: Let A be a real matrix. Show that the following are equivalent.

(i) A is orthogonal.

(ii) The rows of A form an orthonormal subset of $\mathbb{R}^n$.

5 marks

Question 2. The solution set W of a homogeneous system $AX = 0$ in n unknowns is a subspace of $\mathbb{R}^n$. 2 marks

Question 3.

(i) Determine whether or not each of the following form a basis of $\mathbb{R}^3$.

(a) $(1,2,3),(1,3,5),(1,0,1),(2,3,0)$ 1 mark

(b) $(1,1,2),(1,2,5),(5,3,4)$ 2 marks

(ii) Suppose $v_1, v_2, \cdots, v_m$ are linearly independent vectors in $\mathbb{R}^n$. Prove that the set

$$S = \{a_1v_1, a_2v_2, \cdots, a_mv_m\}, a_i(\neq 0) \in \mathbb{R}$$

is linearly independent. 3 marks

Question 4.

(i) Let $\{v_1, v_2, \cdots, v_n\}$ be a basis of $\mathbb{R}^n$ and let $u_1, u_2, \cdots, u_n$ be any vectors in $\mathbb{R}^m$. Show that there exists a unique linear mapping $F : \mathbb{R}^n \to \mathbb{R}^m$ such that $F(v_i) = u_i$ for $1 \leq i \leq n$. 3 marks

(ii) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map for which $F(1,2) = (2,3)$ and $F(0,1) = (1,4)$. Find a formula for F; that is, find $F(a,b)$. 3 marks

Question 5. Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. Is A diagonalizable? If yes, find P such that $D = P^{-1}AP$ is diagonal.

5 marks

Question 6. Find the singular value decomposition for

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}.$$

6 marks

(2 1 1) (3, 1)

Indian Institute of Technology Jodhpur

Subject: Statistical Inference

Quiz-1

Total Marks: 20 Time: 50 minutes

1. Let $\{1, 2, 4, 8, 4, 6, 1.2, 6, 2, 1, 2.1, 2.2, 3.5\}$ be a random sample drawn from a population having probability distribution given by the density $f(x;\theta) = 2x/\theta^2, \quad 0 < x \leq \theta$. Find the MLE of θ. [5]

2. Independent random samples are taken from the output of two machines on a production line. The weight of each item is of interest. From the first machine, a sample of size 16 is taken, with sample mean weight of 120 grams and a sample variance of 4. From the second machine, a sample of size 13 is taken, with a sample mean weight of 130 grams and a sample standard deviation of 1.5. It is assumed that the weights of items from the first machine are normally distributed with mean μ_1 and variance σ_1^2 and that the weights of items from the second machine are normally distributed with mean μ_2 and variance σ_2^2. [6 + 5]

   (a) Find a 95 percent confidence interval for $\mu_1 - \mu_2$, the difference in population means;

   (b) Find a 99 percent confidence interval for $\mu_1 - \mu_2$, when it is known in advance that the population variances are equal.

3. The daily dissolved oxygen concentration for a water stream has been recorded over 28 days. If the sample average of the 28 values is 3.5 mg/liter and the sample variance is 4.1 mg/liter, determine a value which, with 95 percent confidence, exceeds the mean daily concentration. [4]

End

$\sigma^2 \to$ Test chi-Test $\alpha/2$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

Table ST2. Tail Probability Under Standard Normal Distribution[a]

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2231	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1984	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0017	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

Source: Adapted with permission from P. G. Hoel, *Introduction to Mathematical Statistics*, 4th ed., Wiley, New York, 1971, p. 391.

[a] This table gives the probability that the standard normal variable Z will exceed a given positive value z, that is, $P\{Z > z_\alpha\} = \alpha$. The probabilities for negative values of z are obtained by symmetry.

Indian Institute of Technology Jodhpur

Subject: Statistical Inference
Course Code: MAL7021
Quiz-II

Total Marks: 10 | Time: 45 minutes

Answer all the questions.

1. The mean life of a sample of 9 light bulbs was observed to be 1309 hours with a standard deviation of 420 hours. A second sample of 16 bulbs chosen from a different batch showed a mean life of 1205 hours with a standard deviation of 390 hours. Test at 5% level of significance to see whether there is a significant difference between the means of the two batches. Assume that the population variances are the same. [$t_{23,0.025} = 2.069$, $t_{24,0.025} = 2.064$, $t_{25,0.25} = 2.064$] [4]

2. All cigarettes presently on the market have an average nicotine content of at least 1.6 mg per cigarette. A firm that produces cigarettes claims that it has discovered a new way to cure tobacco leaves that will result in the average nicotine content of a cigarettes being less than 1.6 mg. To test this claim, a sample of 20 of this firm's cigarettes were analyzed. It is known that the standard deviation of a cigarette's nicotine content is 0.8 mg. If the average nicotine content of the 20 cigarettes is 1.54, then answer the following questions. [$3 + 1 + 2$]

   (*a*) What is the p-value?

   (*b*) What conclusion can be drawn at the 3.8% level of significance?

   (*c*) If a test with significance level α is used, then: (i) Give the range of α for which it rejects the null hypothesis, (ii) Give the range of α for which it accepts the null hypothesis.

   [$z_{0.025} = 1.96$, $z_{0.05} = 1.65$, $z_{0.368} = 0.336$, $z_{0.336} = 0.346$]

Indian Institute of Technology Jodhpur

Subject: Statistical Inference

End Sem

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Total Marks: 20 Time: 60 minutes

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1. A method for measuring the pH level of a solution yields a measurement value that is normally distributed with a mean equal to the actual pH of the solution and with a standard deviation equal to .05. An environmental pollution scientist claims that two different solutions come from the same source. If this were so, then the pH level of the solutions would be equal. To test the plausibility of this claim, 10 independent measurements were made of the pH level for both solutions, with the following data resulting. [5]
Measurements of solution A: $6.25, 6.11, 6.2, 6.30, 6.25, 6.26, 6.24, 6.39, 6.22, 6.08$
Measurements of solution B: $6.37, 6.25, 6.33, 6.27, 6.54, 6.31, 6.28, 6.49, 6.34, 6.27$

   (a) Do the data disprove the scientist's claim? Use the 1% level of significance.

   (b) Does the decision change if a test with 10% level of significance is used.

2. A manufacturer of capacitors claims that the breakdown voltage of these capacitors has a mean value of at least 100 V. A test of 12 of these capacitors yielded the following breakdown voltages: $96, 98, 105, 92, 100, 104, 99, 103, 95, 100, 102, 97$. Do these results prove the manufacturer's claim? Use a test with 5% level of significance. [5]

3. Let $0.7, 0.4, 0.3, 0.5, 0.1, 0.8$ be a random sample drawn from a population having probability density function given by [5]

$$f(x|\mu) = \mu(1-x)^{\mu-1}, \quad 0 < x < 1, \quad \mu > 1.$$

   Find the MLE for μ.

4. A random sample of 20 HDFC Bank VISA cardholders accounts indicated a sample mean debt of $\$1,200$ with a sample standard deviation of $\$800$. [5]

   (a) Construct a 98 percent confidence interval estimate of the average debt of all cardholders;

   (b) Find the smallest value v that, with 95 percent confidence, exceeds the average debt per cardholder.

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Table ST2. Tail Probability Under Standard Normal Distribution[a]

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
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0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
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0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
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0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1984	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0017	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

Source: Adapted with permission from P. G. Hoel, *Introduction to Mathematical Statistics*, 4th ed., Wiley, New York, 1971, p. 391.

[a] This table gives the probability that the standard normal variable Z will exceed a given positive value z, that is, $P\{Z > z_\alpha\} = \alpha$. The probabilities for negative values of z are obtained by symmetry.

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TABLE A2 *Values of* $\chi^2_{\alpha,n}$

n	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	11.070	12.832	13.086	16.750
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.484	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.772	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Other Chi-Square Probabilities:
$\chi^2_{.9,9} = 4.2$ $P(\chi^2_{16} < 14.3) = .425$ $P(\chi^2_{11} < 17.1875) = .8976$.

TABLE A3 *Values of $t_{\alpha,n}$*

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576

Other t Probabilities:
$P\{T_8 < 2.541\} = .9825$ $P\{T_8 < 2.7\} = .9864$ $P\{T_{11} < .7635\} = .77$ $P\{T_{11} < .934\} = .81$ $P\{T_{11} < 1.66\} = .94$ $P\{T_{12} < 2.8\} = .984$.

TABLE A3 *Values of* $t_{\alpha,n}$

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576

Other t Probabilities:
$P\{T_8 < 2.541\} = .9825$ $P\{T_8 < 2.7\} = .9864$ $P\{T_{11} < .7635\} = .77$ $P\{T_{11} < .934\} = .81$ $P\{T_{11} < 1.66\} = .94$ $P\{T_{12} < 2.8\} = .984.$

TABLE A2 *Values of* $x^2_{\alpha,n}$

n	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	11.070	12.832	13.086	16.750
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.484	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.772	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Other Chi-Square Probabilities:
$x^2_{.9,9} = 4.2$ $P\{x^2_{16} < 14.3\} = .425$ $P\{x^2_{11} < 17.1875\} = .8976.$

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Indian Institute of Technology Jodhpur

Advanced Data Structures and Algorithms (CSL 7560)

Data Structures and Algorithmic Techniques (CSL 7561) Instructor: Pallavi Jain

Friday 8^{th} September, 2023

Time: 1 hour | *Minor 1* | Maximum Marks: 15

Instructions: In True/False, wrong answer is worth -0.5.

1. A red-black tree must have at least one red node. **(True or False)** [1]

2. Suppose we want to maintain a sequence S of n numbers to support, besides the usual dictionary operations insert, search, and delete, $find(k, S)$, which finds the k-th smallest element in the sequence.
   (a) How would you augment a balanced binary search tree to support this operation in $O(\log n)$ time? [2]
   (b) Explain, how insert and delete operations can still be maintained in $O(\log n)$ time? [2]

3. How do you determine whether a graph is connected? What is the complexity of your algorithm? [2]

4. In a Red-Black tree, can a red node have exactly one black child? Justify your answer. [2]

5. Let $A[1, \ldots, n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called a *bad pair* of A.
   (a) List the five bad pairs of the array $\langle 2, 3, 8, 6, 1 \rangle$. [1]
   (b) What array with elements from the set $\{1, 2, \ldots, n\}$ has the most bad pairs? How many does it have? [2]
   (c) Design an algorithm to count the number of bad pairs in an array of size n in time $O(n \log n)$? [Hint: you can try to use order-statistic tree.] [3]

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Indian Institute of Technology Jodhpur

Advanced Data Structures and Algorithms (CSL 7560)

Data Structures and Algorithmic Techniques (CSL 7561) Instructor: Pallavi Jain

Tuesday 17th October, 2023

Time: 1 hour | *Minor 2* | Maximum Marks: 15

1. For the following problems, mention whether they are solvable in polynomial time or NP-complete. No proofs are necessary. For problems where there is an additional parameter k, it is an integer, and it is part of the input, and could be as large as n. **The wrong answer is worth -0.5.**
   (a) Given an undirected graph on n vertices and two designated vertices s and t, is there a path of length *at most* k between s and t? [1]
   (b) Given an undirected graph on n vertices and two designated vertices s and t, is there a path of length *at least* k between s and t? [1]
   (c) Given a graph G, a set $S \subseteq V(G)$ is called a *vertex cover* of G if for every edge $uv \in E(G)$, either u or v is in S. Given an undirected graph on n vertices, does it have a vertex cover on 15 vertices? [1]
   (d) Given a graph G, a set $S \subseteq V(G)$ is called a *clique* if G has an edge between every pair of vertices in S. Given a bipartite graph, does it have a clique on k vertices? [1]
   (e) Given a graph G, partition the vertex set into sets X and Y such that X and Y are independent sets in G. [1]
   (f) Find the smallest independent set in a graph. [1]

2. Show ONE of the following problems is NP-complete.
   (a) Given an undirected graph and an integer k, does it have at most k vertices that cover all cycles in the graph? I.e. the removal of the k vertices makes the graph acyclic. [4]
   (b) Given a set S of positive integers, is there a way to partition S into two subsets S_1 and S_2 that have the same sum? [4]

   You can assume that the following two problems are NP-Complete.

   - VERTEX COVER: Given a graph G and an integer k, find a vertex cover of G of size at most k.
   - SUBSET SUM: Given a set S of positive integers and a target integer T, is there a subset of S whose sum is T?

3. Consider the flow network D given in Figure 1.
   (a) Draw the residual network R of D [1]
   (b) Show an augmenting path in the residual network R. [1]
   (c) Show an augmented flow for D. [1]

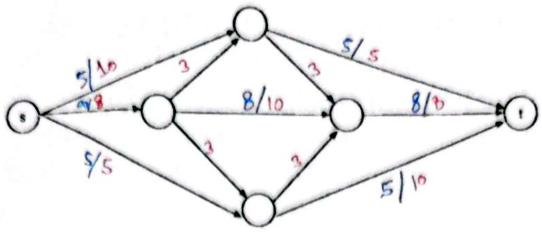


Figure 1: The red colored number shows capacity and blue colored numbers show flow sent on each edge.

OR

Let G be a graph of maximum degree two that has weights on the vertices. Design a polynomial time algorithm to find a maximum weight independent set of G. [3]

4. Consider the following modification to the generic Ford-Fulkerson augmenting path algorithm. Instead of maintaining a residual graph, just reduce the capacity of edges along the augmenting path. In particular, whenever we saturate an edge, just remove it from the graph. Does this algorithm compute a maximum flow? Justify your answer. The pseudocode is in Algorithm 1. [2]

```
for every edge e ∈ E(G), f(e) = 0
while there is a path from s to t do
    let P be an arbitrary path from s to t ;
    let F be minimum capacity of any edge in P ;
    for every edge e in P do
        f(e) = f(e) + F ;
        if c(e) = F then
            remover e from G
        else
            c(e) = c(e) − F
        end if
    end for
end while
```

Algorithm 1: Algorithm for Max Flow

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
Indian Institute of Technology Jodhpur

Advanced Data Structures and Algorithms (CSL 7560)

Instructor: Pallavi Jain

Tuesday 5th December, 2023

Time: 2 hours | *End Semester Examination* | Maximum Marks: 35

1. For the following problems, mark **True** or **False**. No proofs are necessary. For problems where there is an additional parameter k, it is an integer, and it is part of the input, and could be as large as n. **The wrong answer is worth -0.5.**
   (a) In a bipartite graph $G = (V, E)$, a clique on k vertices can be found in $O(1)$ time, if it exists. F [1]
   (b) Given an undirected graph on n vertices and m edges, we can test in $O(n+m)$ time if it is connected? F [1]
   (c) The recurrence $F(n) = n + 2\sqrt{n} \cdot F(\sqrt{n})$ has the solution $\Theta(n \log n)$. F [1]
   (d) A polynomial-time reduction from X to 3-SAT proves that X is NP-hard. T [1]
   (e) k-Center does not admit 1.35 approximation unless P=NP. T [1]
   (f) SAT does not have a polynomial time algorithm unless P=NP. T [1]
   (g) Searching a number in a sorted array of numbers can be done in $O(\log n)$ time. T [1]
   (h) Given an undirected graph on n vertices and two designated vertices s and t, ~~is there~~ there is a simple path of length at most k between s and t? in polynomial time. T [1]
   (i) Given a list of $3n$ numbers, we can find the n-th largest element in $O(n)$ time. F [1]
   (j) We can find a minimum sized cut of a graph G in $O(n^2)$ time deterministically. F [1]

2. Show the red-black trees that result after successively inserting the keys 41, 38, 31, 12, 19, 8 into an initially empty red-black tree. [3]

3. Suppose you are given a directed graph $G = (V, E)$, two vertices s and t, a capacity function $c: E \to \mathbb{N}$, and a second function $f: E \to \mathbb{Z}_{\geq}0$. Describe an algorithm to determine whether f is a maximum (s, t)-flow in G. [3]

4. Design an algorithm to count the number of bad pairs in an array of size n in time $O(n \log n)$. [3]

5. Design an algorithm for 3-SAT that runs in $O(1.6181^n)$ time. [4]

6. This problem asks you to describe polynomial-time reductions between two closely related problems.
   - Subset Sum: Given a set S of positive integers and a target integer T, is there a subset of S whose sum is T?
   - Partition: Given a set S of positive integers, is there a way to partition S into two subsets S_1 and S_2 that have the same sum?

   (a) Describe a polynomial-time reduction from Subset Sum to Partition. [4]
   (b) Describe a polynomial-time reduction from Partition to Subset Sum. [4]

```
M = ∅
while G has at least one edge do
    let uv be any edge in G ;
    add uv to M;
    remove u and v from G;
end while
return M
```

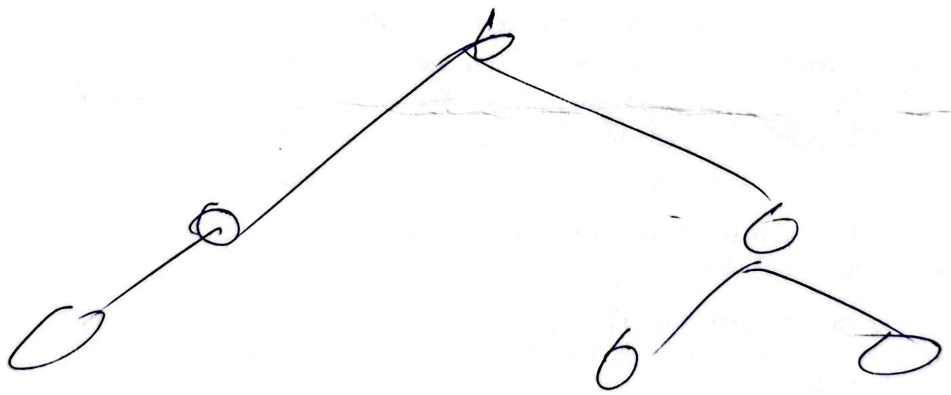
Algorithm 1: Greedy Algorithm

Don't forget to prove that your reductions are correct.

7. Consider Algorithm 1.

   (a) Prove that M is a maximal matching, that is, M is not a subgraph of another matching in G. [1]

   (b) Prove that M contains at most twice as many edges as the smallest maximal matching in G. [3]



Constrained Optimization (MAL7072)

Dr. Md Abu Talhamainuddin Ansary

1. Answer all questions. (5×2.5)

(i) Find the shortest distance point from $(2,3)^T$ to the straight line $3x + 4y = 24$ using Lagrange multiplier method. (No marks for using any other method)

(ii) Is there any $d \in \mathbb{R}^2$ such that $Ad < 0$ where $A = \begin{bmatrix} -1 & -1 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$.

(iii) Consider the problem

$$\begin{aligned} \min\ (x_1+1)^2 + (x_2+1)^2 & \\ s.\,t.\ x_1^2 + x_2^2 &\le 1 \\ -x_1 + x_2 &= 1 \\ x_1 &\le 0 \\ x_2 &\ge 0 \end{aligned}$$

Does $x^* = (-1,0)^T$ satisfy Mangasarian-Fromovitz constraint qualification?

(iv) Construct the dual of the following problem

$$\begin{aligned} \min\ & -2x_1 - 6x_2 \\ s.\,t.\ & 3x_1 + 2x_2 \le 2 \\ & -x_1 + 2x_2 \ge 3 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(v) Consider the problem

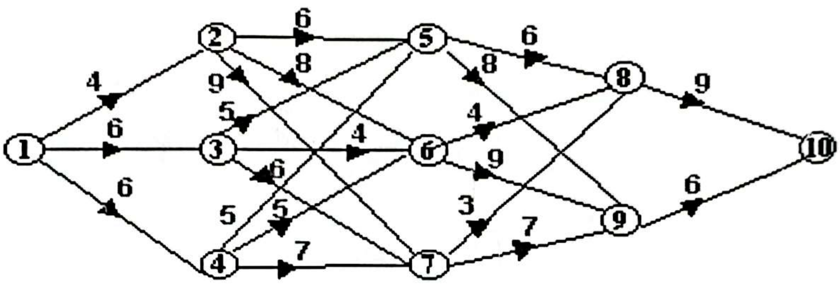
$$\begin{aligned} \min\ & f(x) \\ s.t.\ & g_i(x) \le 0 \quad i = 1,2,\ldots,p \end{aligned}$$

Define barrier function $B(x)$ such that $B(x) > 0$ if $x \in X$ (set of feasible solution) and $B(x) \to \infty$ if $x \to bd(X)$. For $\sigma > \mu > 0$ if $x_\sigma = arg\ \min\ f(x) + \sigma B(x)$ and $x_\mu = arg\ \min\ f(x) + \mu B(x)$ then show that $B(x_\mu) \leq B(x_\sigma)$.

2. Justify whether $x^* = (0,0)^T$ is a KKT point of the following problem or not. (6)

$$\begin{aligned}\min\ & (x_1 + 2.5)^2 + (x_2 + 1.5)^2 \\ s.\,t.\ & (x_1 - 1)^2 + (x_2 - 1)^2 \leq 2 \\ & (x_1 - 1)^2 + (x_2 + 1)^2 \leq 2 \\ & x_1 \geq 0\end{aligned}$$

3. Using dynamic programming technique find the shortest distance path from '1' to '10'. (No marks for using other methods) (6.5)



Quiz 3

Constrained Optimization (MAL7072)

Dr. Md Abu Talhamainuddin Ansary

Answer all questions.

1. Does the system of inequalities $Ad < 0$ has a solution where $A = \begin{bmatrix} 3 & 2 & 1 & 4 \\ 5 & 4 & -1 & 0 \\ -57 & -2 & -1 & -12 \end{bmatrix}$ **(2)**

2. Consider the problem

$$\begin{aligned} \min\ & (x_1-2)^2+(x_2-1)^2 \\ s.t.\ x_1^2+x_2^2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

   Is $x^* = (1,0)$ a Fritz-John point? **(2)**

3. Consider the problem

$$\begin{aligned} \min\ & (x_1-2)^2+(x_2-2)^2 \\ s.t.\ x_1^2+x_2^2 &\leq 25 \\ x_1-3x_2+10 &= 0 \end{aligned}$$

   For $x^* = (1.4, 3.8)^T$, find (λ^*, μ^*) such that $(x^*; \lambda^*, \mu^*)$ be a Lagrangian Saddle point of the problem? **(2)**

4. Consider the problem

$$\begin{aligned} \min \ & 3x_1 + x_2 \\ s.t.\ 2x_1 + 3x_2 & \leq 6 \\ x_1 + x_2 & \geq 1 \\ x_1, x_2 & \geq 0 \end{aligned}$$

Is either $x^* = (1,0)$ or $x^* = (0,1)$ optimal solution or none? **(2)**

5. Suppose x^k be the solution of $\min_x \ \Theta(x, \mu_k) = f(x) + \mu_k P(x)$ and x^{k+1} is the solution of $\min_x \ \Theta(x, \mu_{k+1}) = f(x) + \mu_{k+1} P(x)$ where $\mu_{k+1} > \mu_k > 0$.

Then show that $\Theta(x^k, \mu_k) \leq \Theta(x^{k+1}, \mu_{k+1})$. **(2)**

Indian Institute of Technology Jodhpur
Graph Theory and Applications(CSL7410)
Minor I, Date: 10th Feb 2024

Duration: 1 Hours — Full Marks: 15

Attempt all the questions

1. Let C_n be the cycles on the vertices $\{1, 2, \cdots, n\}$, $n \geq 3$, and let M be its incidence matrix. Then, $det(M)$ equals ...... if n is even and ...... if n is odd. (1)

2. Let G be a connected graph with n vertices and let M be the incidence matrix of G. Then, rank of M is ...... if G is bipartite and ...... otherwise. (1)

3. Let G be a connected graph with vertices $\{1, 2, \cdots, n\}$ and let A be the Adjacency matrix of G. If i, j are vertices of G with $d(i, j) = m$, then, the matrices $I, A, \cdots, A^m$ are linearly dependent or independent and why? (1+1)

4. Let G be a connected graph and d be the diameter of G. Let A, the Adjacency matrix of G has k distinct eigenvalues. Then, what is the relationship between k and d and why? (1+1)

5. An edge is a cut-edge if and only if it doesn't belongs to a .... (1)

6. $K_{t,t}$ has ...... automorphisms. (1)

7. Every graph with n vertices and k edges has at least ...... components. (1)

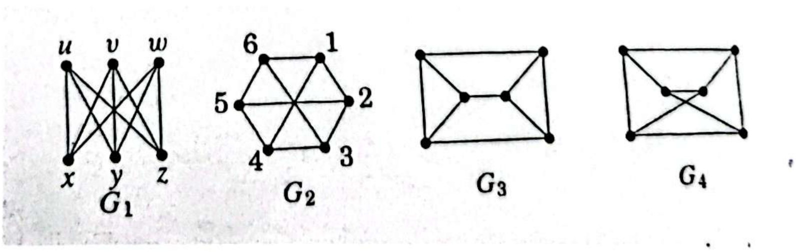
8. If every vertex of G has degree at least ......, then G contains a cycle. (1)

9. Define Petersen Graph and draw the graph. Why there are at least 120 isomorphisms from the Petersen graph to itself? (1+1)

10. Isomorphic or not? and Why?

    $(i)\ \{G_1, G_2, G_3, G_4\}$ $\quad$ $(ii)\ \{\bar{G_1}, \bar{G_2}, \bar{G_3}, \bar{G_4}\}$

    Where



(1.5+1.5)

Best Wishes

Indian Institute of Technology Jodhpur
Graph Theory and Applications(CSL7410)
Minor II, Date: 23rd March 2024

Duration: 1 Hours | Full Marks: 15

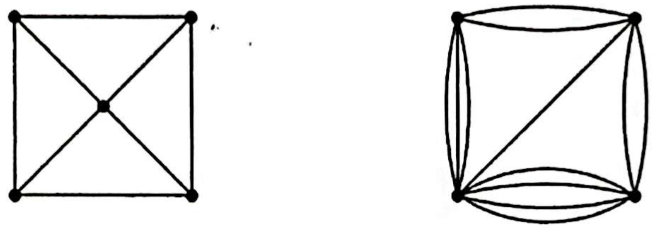
Attempt all the questions

1. Let T be a tree with average degree d. Determine $n(T)$ in terms of d. (2)

2. Every $n-$vertex graph with m edges has at least how many cycles and why? (2)

3. Prove or disprove: if n_i denotes the number of vertices with degree i in a tree, then $\sum in_i$ depends only on the number of vertices. (2)

4. Use contraction recursively to count the spanning trees in the left graph and use the Matrix Tree Theorem to compute the number of spanning trees in the right graph given below.



(2+2)

5. There are five cities in a network. The travel time for traveling directly from i to j is the entry $a_{i,j}$ in the matrix below. The matrix is not symmetric (use directed graphs) and $a_{i,j} = \infty$ indicates that there is no direct route. Determine the least travel time and quickest route from i to j for each pair i, j.

$$\begin{pmatrix} 0 & 10 & 20 & \infty & 17 \\ 7 & 0 & 5 & 22 & 33 \\ 14 & 13 & 0 & 15 & 27 \\ 30 & \infty & 17 & 0 & 10 \\ \infty & 15 & 12 & 8 & 0 \end{pmatrix}$$

(3)

6. Let G be a rooted tree where every vertex has 0 or k children. Given k, for what values of $n(G)$ is this possible? (2)

Best Wishes

Major Exam

Indian Institute of Technology Jodhpur
Graph Theory and Applications(CSL7410)
(May 11, 2024)

Duration: 120 minutes **Maximum Marks: 30**

Instruction: Select all the correct options for each question. Without justification, marks will not be awarded. A negative marking of (−2) will be awarded for wrong answers.

1. Which of the following sequences can be the degree sequence of a graph?

   A) $(3,3,3,3,3)$ B) $(1,2,3,4,5)$ C) $(2,2,3,3,4)$ D) $(0,1,2,3,4)$

   [2]

2. Which of the following properties apply to BFS and DFS?

   A) BFS guarantees to find the shortest path in an unweighted graph.

   B) DFS can be used to find connected components in a graph.

   C) Both BFS and DFS can be used to traverse a tree.

   D) BFS and DFS have the same time complexity for traversal in a connected graph.

   [2]

3. Which of the following statements about graph automorphism are true?

   A) The automorphism group of a cycle graph C_n is isomorphic to the cyclic group C_n.

   B) The automorphism group of a complete graph K_n is the symmetric group S_n.

   C) The automorphism group of a bipartite graph is always trivial.

   D) The automorphism group of a tree is always non-trivial.

   [2]

4. Which of the following statements about the diameter of a graph are true?

   A) The diameter of a directed acyclic graph (DAG) is always finite.

   B) The diameter of a complete binary tree with height h is $2h$.

   C) The diameter of a cycle graph C_n is $n/2$ if n is even and $(n+1)/2$ if n is odd.

   D) The diameter of a connected graph with n vertices is at most $n-1$.

   [2]

5. Which of the following statements about the number of spanning trees in a graph are true?

   A) The number of spanning trees in a cycle graph C_n with n vertices is n.

   B) The number of spanning trees in a complete bipartite graph $K_{m,n}$ with m vertices in one part and n vertices in the other part is $m^{n-1} \times n^{m-1}$.

   C) The number of spanning trees in a graph is always less than or equal to the product of the degrees of all vertices divided by $2n$, where n is the number of vertices.

   D) The number of spanning trees in a connected graph with n vertices is at most n^{n-2}.

   [2]

6. Which of the following statements on the planar graph are true?

A) The maximum number of edges in a simple planar graph with v vertices is $3v - 6$.

B) A planar graph with v vertices can have at most $3v - 5$ edges.

C) The sum of the degrees of all vertices in a planar graph is always less than or equal to $6v - 12$.

D) The maximum number of edges in a planar graph with v vertices and f faces is $3f - 6$.

E) The minimum number of edges in a planar graph with v vertices and f faces is $2v - 4$.

F) In a planar graph with v vertices and e edges, the maximum number of faces is $2e - v + 4$.

[3]

8. Which of the following properties apply to matchings and independent sets in a graph?

A) Every maximum matching in a graph is also a maximum independent set.

B) In a bipartite graph, the size of a maximum matching is equal to the size of a maximum independent set.

C) In a complete graph, the size of a maximum matching is equal to the size of a maximum independent set.

D) In a tree, the size of a maximum matching is less than or equal to the size of a maximum independent set.

E) In a bipartite graph, the size of a maximum matching is equal to the size of a minimum vertex cover.

F) The size of a maximum independent set is always less than or equal to the size of a minimum vertex cover.

G) In a tree, the size of a maximum matching is always equal to the size of a minimum vertex cover.

H) In a complete graph, the size of a maximum independent set is always equal to the size of a minimum vertex cover.

[4]

9. Which of the following properties apply to the chromatic number of a graph?

A) The chromatic number of a graph is always less than or equal to its maximum degree.

B) The chromatic number of a graph is always greater than or equal to its clique number.

C) If a graph is $k-$colorable, then, a vertex removal may reduce its chromatic number by 1.

D) The chromatic number of a cycle graph with n vertices is $\lfloor \frac{n}{2} \rfloor + 1$.

[2]

10. Write the statement of Brooks' theorem and prove it. [1+4]

11. Prove that Petersen graph, K_5 and $K_{3,3}$ are non-planar graphs. [2+2+2]

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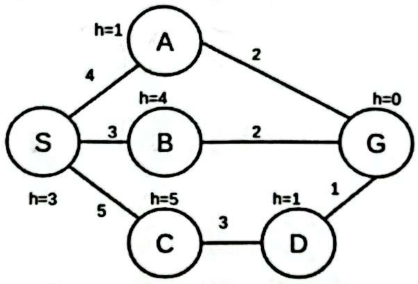
DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
Indian Institute of Technology Jodhpur
Artificial Intelligence (CSL7610/CS323)

September 9, 2023

Time: 1 hour | *Minor 1 Exam* | Maximum Marks: 30

1. Convert the city to city traversal problem given in the figure below to a search problem such that you can apply any uninformed/informed search. Here S is the start city, and G is the target city.

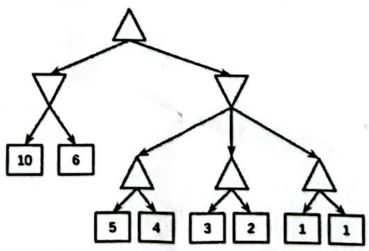


   a) Define all the components of a search problem for this case. [3 Marks]
   b) Apply BFS, DFS and UCS tree search to find the solution. Incase of ties follow alphabetical order. For each approach, show all the intermediate search trees, fringe list and other values as the nodes get expanded and also write the solution sequence of states [6 Marks]
   c) Apply Greedy and A* tree search to find the solution. Incase of ties follow alphabetical order. For each approach, show all the intermediate search trees, fringe list and other values as the nodes get expanded and also write the solution sequence of states. [4 Marks]
   d) Is the solution provided by A* for the above problem optimal? Explain in detail what causes the optimality or non-optimality in this case. [2 Marks]

2. a) Difference between an admissible and a consistent heuristic function? [1 Mark]
   b) Suppose A* uses f(x)=max(g(x),p(x)), where g(x) follows the usual definition. If the path to x from start state s follows s,c,d,x, then p(x)=(h(c)-h(s))+(h(d)-h(c))+(h(x)-h(d)), where h(.) is a consistent and admissible heuristic function. Suppose there is an optimal goal state A and a suboptimal goal state B. Prove that f(A)<f(B). [Donot prove using examples] [2 Marks]
   c) The current algorithm is the same as which other search algorithm and why? [1 Mark]

3. What is the shoulder problem in hill-climbing and how enforced hill-climbing avoids this. [2 Marks]

4. a) Where is alpha-beta pruning used and why? Apply alpha-beta pruning below. Show all the intermediate states and checks involved. Also show its negative effect here [6 Marks]



5. "Expectimax gives the best solution for the MAX player given an optimal MIN player." Explain why or why not? Modify an expectiminimax game tree to get a minimax solution without changing any of the nodes or actions? Show an example game before and after the modification. [3 Marks]

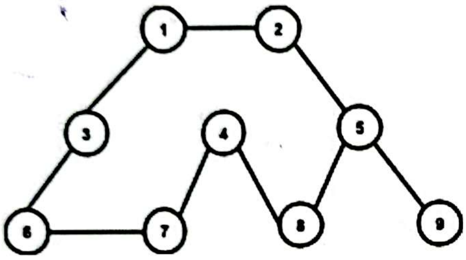
End of exam

CSL7610 AI Minor 2

Total Marks: 30 **Duration:** 60 mins

Use of pencil is NOT allowed. If pencil is used to answer any question, that answer will not be checked.

1. **[8 marks]** Consider the following problem: 8 cricketers arrive at a hotel after their match and have to be allotted rooms for their stay. The cricketers include 3 batsmen: B1, B2, B3, 3 bowlers: W1, W2, W3 and 2 all rounders: A1, A2. There are 9 rooms in the hotel that are arranged as shown in the graph below.



	B1	B2	B3	W1	W2	W3	A1	A2	Order
1	B1	X	X	X	X	X	X	X	1
2	X								
3	X								
4	X								
5	X								
6	X								
7	X								
8	X								
9	X								

Solve the room allotment problem with the following constraints:

C1: The path between the room/nodes of the same types of cricketers should contain more than 1 node, excluding the start and end nodes, e.g., if W2 gets room 2, then no other bowler can be allotted rooms 1, 3, 5, 8, 9

C2: Room number of B2 > Room number of A1

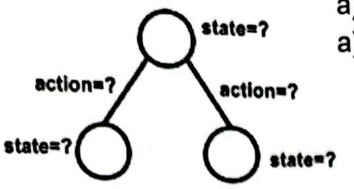
C3: Room number of B3 > Room number of W2

Solve using CSP with backtracking **<u>a) with only forward checking</u>** and **<u>b) with forward checking, arc consistency, and MRV</u>**. In simple forward checking, assign cricketers to rooms in increasing order of the room/node number (1,2,..). In the table, add X to denote that the current room (row) cannot be allotted to this cricketer (column). If multiple cricketers are available to be allotted to a room, then try to allot that room in the order B1>B2>B3>W1>W2>W3>A1>A2. Fill the above table for both of the methods. If more than one room is equally eligible to be allotted next, follow the increasing order of the room number to allot first. **Stop at the first Failure and mention FAILURE**. Failure can be caused by not having any rooms for a specific cricketer or having more than one room that cannot be allotted to anyone. Room 1 is allotted to B1 by default. **A cricketer can be allotted only one room and one room can have only one cricketer. Order column should mention the order in which you tried to allot the room, even if you were not able to assign any cricketer to that room.**

2. **[7 Marks]** Consider a problem where you have to make two dishes D1 and D2. D1 requires the ingredients T1 and T2, whereas D2 requires the ingredient T3. Initially, none of the ingredients and dishes are available. Model this problem as a planning problem using the STRIPS formulation. The actions available are getT1, getT2, getT3, makeD1, makeD2. getT1 can get the ingredient T1 but this cannot be done if T3 or G1 is already present. getT2 can get the ingredient T2 but can only be performed if T1 is already present. getT3 can get the ingredient T3 but this cannot be done if T1 or G2 is already present. makeD1 requires T1 and T2 to be present and it consumes/removes T1 and T2 to produce D1. makeD2 requires T3 to be present, and it consumes/removes T3 to produce D3.

Continued next page...

G → D



a) Represent all the actions and the starting state in STRIPS.

a) Draw the entire state space graph showing all possible states reachable from the starting state and subsequent states due to any valid action. The diagram should clearly show the STRIPS representation for each state. For showing actions, simply the action name can be mentioned.

Note: Use T1,T2,T3,D1,D3 as the corresponding propositions

3. **[5 Marks]** In each case, reduce to a single term using equivalence laws only and no truth table/resolution

a) $A \Rightarrow B \Leftrightarrow B \land A$

b) $((A \Rightarrow B) \land (C \Rightarrow B) \land C) \Rightarrow B$

4. **[5 Marks]** Verify if the query can be inferred from KB showing all the steps and rules (no resolution). Note: Follow the exact mechanism mentioned above each case below.

using FOL Forward Chaining

a) $\forall y \, \forall z \, P(y,z) \land T(y)$

$\forall x \, Q(x)$

$\forall x \, \forall y \, P(x,y) \land Q(y) \Rightarrow R(x) \land S(x,y)$

Query: S(Pam,Tam)

Assuming Pam,Tam are allowed values

using FOL Backward Chaining

b) $\forall x \, P(x) \land R(x) \Rightarrow Q(x)$

$\forall x \, S(x) \Rightarrow P(x)$

$\forall x \, P(x) \Rightarrow R(x)$

S(Pam)

Query: Q(Pam)

5. **[5 Marks]** Verify if the query can be inferred from KB showing all the steps and rules using only PL Natural Deduction (no resolution)

a) $A \land B \Rightarrow ((C \land D) \lor D)$

$A \land D \Rightarrow R$

$A \land B$

Query: R

b) $(\neg A \lor \neg C) \Rightarrow B$

$C \Rightarrow E$

$\neg B$

Query:E

CSL7610/CS323 AI Major Exam

Total Marks: 40 **Duration:** 120 mins

The questions have all the required information available.
Use of pencil is NOT allowed. If pencil is used to answer any question, that answer will not be checked.

1. **[8 marks]** Consider a Bayesian network representing the relationships between weather, traffic, and being late for work. The conditional probability tables are as follows:

W	P(W)
sunny	0.7
rainy	0.3

W	T	P(T|W)
sunny	+t	0.9
sunny	-t	0.1
rainy	+t	0.2
rainy	-t	0.8

W	T	L	P(L|T,W)
sunny	+t	+l	0.9
sunny	+t	-l	0.1
sunny	-t	+l	0.1
sunny	-t	-l	0.9
rainy	+t	+l	0.7
rainy	+t	-l	0.3
rainy	-t	+l	0.3
rainy	-t	-l	0.7

The nodes in the network are Weather (W), Traffic (T), and Late (L). Solve the room allotment problem with the following constraints:

a. Draw the Bayesian Network [2 Marks]
b. What is the probability that it is rainy, there is traffic (+t), and you are late for work (+l) [3 Marks]
c. What is the probability distribution over L with W=sunny, i.e. P(L,W=sunny) [3 Marks]

2. **[5 marks]** Let's consider a Bayesian network (BN) involving five variables: Health (H), Exercise (E), Diet (D), Stress (S), and Heart Disease (A). Suppose, all the conditional probability distributions have been provided to you, namely: P(H), P(E|H), P(D|H), P(S|D,E), P(A|S).
a. Draw the Bayesian Network [2 Marks]
b. What is the name of the algorithm used for checking for conditional independence in a BN [1 Mark]
c. If A=+a is known, check if E & D are conditionally independent using the above method. [2 Marks]

3. **[5 Marks]** Solve the following MDP (described in the table below) using Value Iteration:

Start State	Action	End State	Transition Prob	Reward
A	a	B	0.9	1
A	a	A	0.1	1
A	b	C	0.9	2
A	b	A	0.1	1
B	a	A	0.8	1
B	a	C	0.2	1.5
B	b	B	1	1
C	a	D	0.9	10
C	a	A	0.1	2
C	b	B	1	-1
D	a	D	0	0
D	b	D	1	0

There are 4 states A, B, C, D and 2 actions a and b. The discounting factor is 0.1. Repeat the iterations for either a maximum of 5 iteration steps or as long as the maximum difference between the previous and current expected value of any state is >0.1, whichever comes earlier. **Round off the expected V value obtained after every iteration to 1 decimal position.**

a. Draw the MDP state space with regular and Q nodes [2 Marks]
b. What is the final policy obtained? Show all the steps, and intermediate values. [3 Marks]

4. **[4 Marks]** Solve the following MDP. There are 5 states, A, B, C, D and two actions a and b. The policy is fixed with action a being recommended for all states. The transition probability and reward functions are not available, but the effects of 1 episode of the agent performing actions in the environment have been recorded. In this episode, A goes to B through action a, B goes to C through action a and C goes to D through action a. The rewards obtained are R1, R2, R3, i.e. Episode 1: (A,a,B,R1), (B,a,C,R2), (C,a,D,R3). No transition sample is available from D. Show that finding V(A), V(B), V(C), using Temporal Difference (TD) learning for this single episode is the same as using Direct Evaluation. Use discounting factor=0.1. Also in TD, at each timestep within an episode, the V(A) calculations only looks at the immediate next state after transition which is B in this case. Therefore, repeat the state-wise V calculations in TD till the previous calculation matches the current one. In TD, V(.) is initially 0 for all states. For Direct evaluation note that t starts from 0.

5. **[2 Marks]** Explain under what common condition, likelihood-weighted sampling and rejection sampling are both same as prior sampling.

6. **[3 Marks]** In the following CSP variable value assignment table, **complete** the assignment of values (V1-9) to variables (R1-9) using backtracking, cycle consistency and MRV.

	V1	V2	V3	V4	V5	V6	V7	V8	V9	Order
R1	V1	x	x	x	x	x	x	x	x	1
R2	x	x	x	V4	x	x	x	x	x	2
R3	x	x	x	x	x	x	V7	x	x	3
R4	x	x		x	x		x	x		
R5	x	V2	x	x	x	x	x	x	x	4
R6	x	x	x	x	V5	x	x	x	x	5
R7	x	x	x	x	x	x	x	V8	x	6
R8	x	x		x	x		x	x		
R9	x	x		x	x		x	x		

In case of a tie between values follow the order V1>V2>V3>V4>V5>V6>V7>V8 and in case of tie between variables follow the order R1>R2>R3>R4>R5>R6>R7>R8>R9. **Some variables are already allotted. Don't change any existing entries.** Don't miss out on the Order column. x denotes values that cannot be assigned to current row (variable). The only constraint that you should look out for is that, adjacent variables should not have adjacent values, e.g., if R2 has V2 then R1 and R3 cannot have V1 or V3. Please Note that R1 and R9 are not adjacent variables and V1 and V9 are not adjacent values. One variable can have only one value and vice versa

7. **[4 Marks]** Verify if the query can be inferred from KB showing **all the steps and rules** using Resolution Refutation. (For inference only Resolution inference rule can be used, Don't use truth values or statements like if A is True then B will also be True) **[Solve any one of the two]**

a) A->(B->C)
(A v C) -> D
Query: C

b) A v C
A->B
C->D
¬B
Query: D

8. **[4 Marks]** Verify if the query can be inferred from KB showing **all the steps and rules** using only PL Natural Deduction (no resolution or truth value). (Don't use truth values or statements like if A is True then B will also be True) **[Solve any one of the two]**

a) P v (Q ∧ R)
(P v R) -> S
Query: S

b) A v (A ∧ B)
¬C->¬A
C->D
Query: D

9. **[5 Marks]** Suppose you have an array of size N=5 and you have to search for a number D. The starting state is the middle position (3). You can start checking from the starting state using DFS with the graph formulation. Each state can only move 1 step to the left or right direction. Once DFS has been completed, use the closed set position of each state to get the heuristic value, i.e., if the final closed set after DFS is S,A,C,B,D, then h(S)=5-1. The cost of moving left or right is 1. Now solve this problem using A* search. **Draw "only" the last tree and "other values" for both methods. Donot miss out nodes that cannot be expanded. Also, write down the final solution for DFS and A*.** Note for this question: In cases where multiple children of a node have to be inserted into the fringe simultaneously and they have the same overall cost depending on the algorithm, insert them in such a way that when removing them from the fringe, the removal is automatically in alphabetical order.

1	2	3	4	5
B	A	S	C	D

———————END———————