Artificial Intelligence

Lec 16: Knowledge Representation and Logic

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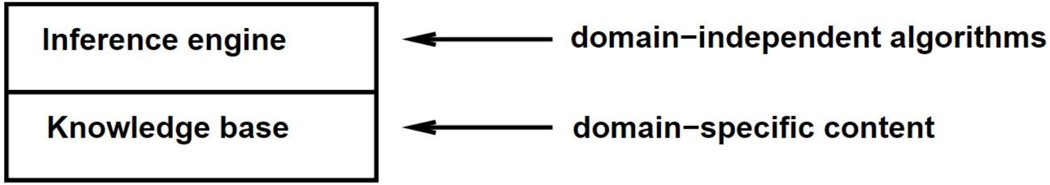
Content taken from various sources

Last Class: Knowledge Representation

- Represent knowledge about the world in a manner that facilitates inferencing (i.e., drawing conclusions) from knowledge.

- Knowledge base (KB) = set of sentences in a formal language

- Declarative approach to building an agent:
  - Tell it what it needs to know (**KB**).
  - Then it can **Ask itself** what to do.
  - Answers are consequences of the KB.



- Example: Medical treatment agent - KB: Symptom 1 and Symptom 2 → Treatment, etc.
  - Given an observation of Symptom X and Symptom Y, infer the treatment.

Last Class: Components of Knowledge Representation

- Syntax: What is a correctly formed sentence?
- Semantics: What is the meaning of the sentence?
- Inference Procedure (reasoning, entailment): what sentence logically follows from the given knowledge?
  - Algorithm
- Knowledge Base
- Propositional Logic is used as a KR technique with
  - Syntax
  - Semantics
  - Inference Procedure

Propositional Logic (PL): Syntactical Elements

- PL vocabulary
  - A set of **propositional symbols** (P,Q,R etc.), each of which can be True or False.
  - Set of **Logical Operators**
    - $\land$ (AND), $\lor$ (OR), ¬ or ! (NOT), ⇒ (implies), ⇔ (if and only if or biconditional).
    - Often use parentheses () for grouping.
  - There are two **special symbols**.
    - TRUE (T) and FALSE (F) - logical constants.

Propositional Logic (PL): How to form logical sentences

- Each symbol (a proposition or constant) is a sentence.

- A **sentence** is a statement that can be assigned a **truth value**.

- **Sentences** are also called well-formed formulae (WFF).

- A **well-formed formula** (WFF) is a formula that is constructed **according to the syntax** rules of the language.

- A WFF is made up of **propositional variables, logical connectives, and parentheses**, and it must be constructed in such a way that it can be assigned a truth value.

Propositional Logic (PL): How to form logical sentences

- Are these WFF?
  - $(P \land Q) \lor \neg R$
    - is a WFF
  - $(P \land) Q \lor \neg R$
    - is not, because it violates the syntax rules by not having a complete expression after the AND operator.
  - TRUE is a WFF
  - (P) is a WFF
  - $P \land Q$ is a WFF
  - $P \lor Q$ is a WFF
  - $\neg P$ is a WFF
  - $P \Rightarrow Q$ is a WFF
  - $P \Leftrightarrow Q$ is a WFF
  - $(PvQ) \Rightarrow R$ is a WFF

Propositional Logic (PL)

$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} & \rightarrow & \textit{True} \mid \textit{False} \mid P \mid Q \mid R \mid \ldots \\
\textit{ComplexSentence} & \rightarrow & (\ \textit{Sentence}\ ) \mid [\ \textit{Sentence}\ ] \\
 & \mid & \neg\ \textit{Sentence} \\
 & \mid & \textit{Sentence} \land \textit{Sentence} \\
 & \mid & \textit{Sentence} \lor \textit{Sentence} \\
 & \mid & \textit{Sentence} \Rightarrow \textit{Sentence} \\
 & \mid & \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
 & & \\
\text{OPERATOR PRECEDENCE} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

- Syntax

- Say, P is a proposition “Animesh is intelligent”
- Say, Q is a proposition “Animesh is foolish”
- ¬P ?
- P v Q ?
- P ∧ Q ?

These are Complex/Compound Proposition/Sentence

- Conjunctive Normal Form (clause ∧ clause) - clause is a disjunction of literals, e.g., (P v Q) ∧ R ∧ S
- Disjunctive Normal Form (disjunct v disjunct) - disjunct is a conjunction of literals, e.g., (P∧Q) v R v S

Implication $\Rightarrow$

- $P \Rightarrow Q$
- If P is True then Q is True.
- P is a sufficient but not necessary condition for Q to be True.
- If $\underbrace{\text{it rains}}_{P}$ then the $\underbrace{\text{roads are wet}}_{Q}$
- If the roads are wet, then it rains ??
- Q being True is not a necessary condition for P to be True.
    - Road can be wet even without rain.

BiConditional ⇔

- $P \Leftrightarrow Q$
- If P is True then Q is True and if Q is True then P is True
- Two sides of a triangle are equal **if and only if** two base angles are equal.
- If the <u>two sides of the triangle are equal</u> then <u>two base angles of the triangle are equal</u>
  P Q
  And
  If the <u>two base angles of the triangle are equal</u> then <u>two sides of the triangle are equal</u>
  Q P
- Equivalence can be expressed as
  $(P \Rightarrow Q) \land (Q \Rightarrow P)$

Operator Precedence

- What is the correct interpretation of the following formula:

  $P \lor Q \land R \Leftrightarrow Q \Rightarrow \neg R$

  - $((P \lor (Q \land R)) \Leftrightarrow Q) \Rightarrow (\neg R)$
  - $(((P \lor Q) \land R) \Leftrightarrow Q) \Rightarrow (\neg R)$
  - $(P \lor (Q \land R)) \Leftrightarrow (Q \Rightarrow \neg R)$
  - $(P \lor ((Q \land R) \Leftrightarrow Q)) \Rightarrow (\neg R)$
- Precedence: $\neg$, $\land$, $\lor$, $\Rightarrow$, $\Leftrightarrow$

Operator Precedence

- Precedence: $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

- When an operand is surrounded by two $\Rightarrow$ operators or by two $\Leftrightarrow$ operators, the operand associates to the right.

  $P \Rightarrow Q \Rightarrow R$ $\quad$ $(P \Rightarrow (Q \Rightarrow R))$
  $P \Leftrightarrow Q \Leftrightarrow R$ $\quad$ $(P \Leftrightarrow (Q \Leftrightarrow R))$

- What about $P \wedge Q \wedge R$?
- What about P v Q v R?

PL

If P is True and Q is True, then find the truth value of the following

- $P \Rightarrow Q$
- $(\neg P \vee Q) \Rightarrow Q$
- $(\neg P \vee Q) \Rightarrow P$
- $P \vee \neg P \Rightarrow T$

Ans.

- T
- T
- T
- T

Semantics: Interpretation

- An **interpretation** is a **mapping** of **propositional symbols** to **truth values** (either true or false).

- An interpretation specifies **which propositions** are **True**, and which are **False**, and can be thought of as a way of **assigning meaning to the symbols** in a formula.

- For example, for the formula $P \wedge Q$, an interpretation might assign P = True, and Q = False, in which case the formula would be False.

- Let P be a proposition, “The child knows about Newton’s laws of motion”
  - Suppose the world/environment is the KG class.
    - If we interpret P in this world, then P is False.
  - Suppose the world/environment is the 10th class.
    - If we interpret P in this world, then P is True.

- For a compound sentence,
  - Each atomic proposition in it has to be interpreted in the same world and assigned a truth value.
  - Finally compute the truth value of the compound sentence.

Validity of a Sentence

- If a propositional sentence is **true under all possible interpretations,** then it is a **valid** sentence.
- **Tautology**: is a **formula that is always true**, regardless of the truth values assigned to its propositional variables.

  $P \vee \neg P$ is always true.
- If P is True then the above will be True.
- If P is False then the above will again be True since $\neg P$ is True.

PL

Express the following English statements in PL

- It is snowing.
- The bus is faulty.
- If the coal keeps burning and the room is not ventilated then one will suffer from carbon monoxide poisoning.
- Proposition S - snowing.
- Proposition F - bus is faulty.
- If $\underbrace{\text{the coal keeps burning}}_{C}$ and the room is not $\underbrace{\text{ventilated}}_{V}$ then $\underbrace{\text{one will suffer from carbon monoxide poisoning}}_{P}$
- $C \land \neg V \Rightarrow P$
- $(C \land \neg V) \Rightarrow P$

PL

If P is True and Q is True, then find the truth value of the following:

- $P \Rightarrow Q$
- $(\neg P \vee Q) \Rightarrow Q$
- $(\neg P \vee Q) \Rightarrow P$
- $P \vee \neg P \Rightarrow T$

Ans.

- T
- T
- T
- T

Procedure to derive the Truth value for compound sentences

Truth Table

- Shows the truth value of a propositional formula for all possible values of its constituent atomic propositions.
- Can be used to derive the correctness or validity of any propositional statement/sentence.

P	Q	¬P	P∧Q	P∨Q
T	T	F	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

Procedure to derive the Truth value for compound sentences

IMPORTANT: Process to be followed while assigning Truth Values to Propositional Symbols, e.g., P, Q, R, etc

- The first set of columns should be dedicated to all the propositional symbols needed for the compound sentence.
- If there are n propositional symbols, the truth table will have 2^n rows.
- Alternately assign $2^n/2^c$ number of True and False in each column, where c is the column number (starting from 1).

P	Q	¬P	P∧Q	P∨Q
T	T	F	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

Procedure to derive the Truth value for compound sentences

- if there are n propositional symbols, the truth table will have 2^n rows.
- Alternately assign $2^n/2^c$ number of True and False, where c is the column (starting from 1)

<u>This process **MUST** be followed while assigning Truth Values to Propositional Symbols</u>

Otherwise No Considerations

P	Q	R	P⋁Q⋁R

Procedure to derive the Truth value for compound sentences

- if there are n propositional symbols, the truth table will have 2^n rows.
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This process **MUST** be followed while assigning Truth Values to Propositional Symbols

Otherwise No Considerations

P	**Q**	**R**	**P∨Q∨R**
T			
T			
T			
T			
F			
F			
F			
F			

Procedure to derive the Truth value for compound sentences

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P	**Q**	**R**	**P⋁Q⋁R**
T	T		
T	T		
T	F		
T	F		
F	T		
F	T		
F	F		
F	F		

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Otherwise No Considerations

P	**Q**	**R**	**P∨Q∨R**
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Procedure to derive the Truth value for compound sentences

- if there are n propositional symbols, the truth table will have 2^n rows.
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<u>This process **MUST** be followed while assigning Truth Values to Propositional Symbols</u>

Otherwise No Considerations

P	**Q**	**R**	**P⋁Q⋁R**
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

Procedure to derive the Truth value

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

$P \Rightarrow Q$ can also be written as $\neg P \vee Q$

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Procedure to derive the Truth value

P⇒Q

P	Q	P⇒Q
T	T	T
T	F	F
F	T	T
F	F	T

Q⇒P

P	Q	Q⇒P
T	T	T
T	F	T
F	T	F
F	F	T

P⇔Q can also be written as (P⇒Q)∧(Q⇒P)

P	Q	P⇔Q
T	T	T
T	F	F
F	T	F
F	F	T

Procedure to derive the Truth value

$\neg P \lor Q \Rightarrow P \land Q$

P	Q	$\neg P$	$P \land Q$	$\neg P \lor Q$	$\neg P \lor Q \Rightarrow P \land Q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	F	T	F
F	F	T	F	T	F

Equivalence

- Two Formulas F_1 and F_2 are equivalent if they have the same truth value for every interpretation.
  - e.g., $P \vee P$ and P

- $F_1 \equiv F_2$

- There are some **standard known equivalences** that can be used to simplify/reduce a formula or prove that two formulas are equivalent. **But the equivalence name has to be mentioned.**

Equivalences

The **name** of the equivalence used **MUST** be mentioned.

Otherwise No Considerations

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Equivalences

The **name** of the equivalence used **MUST** be mentioned.

Otherwise No Considerations

- Law of double negation: $\neg\neg\phi \equiv \phi$
- Identity Laws: $\phi \wedge T \equiv \phi \qquad \phi \vee F \equiv \phi$
- Domination Laws: $\phi \vee T \equiv T \qquad \phi \wedge F \equiv F$
- Idempotent Laws: $\phi \vee \phi \equiv \phi \qquad \phi \wedge \phi \equiv \phi$
- Negation Laws: $\phi \wedge \neg\phi \equiv F \qquad \phi \vee \neg\phi \equiv T$
- Absorption Laws: $\phi_1 \wedge (\phi_1 \vee \phi_2) \equiv \phi_1 \qquad \phi_1 \vee (\phi_1 \wedge \phi_2) = \phi_1$