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M24CSE033

Assignment-1

1) The 50 measurements of acid rain in a city are.
. . . .

a) Determine the Mean and Standard Deviation.
we know Mean $(\mu) = \frac{\sum x_i}{n}$

$$= \frac{450.68}{50}$$

$$\therefore \mu \approx 4.51$$ Ans

~~b) Calculate the Median and Quo~~

Standard deviation $(\sigma) = \sqrt{\frac{\sum(x_i - \mu)^2}{n-1}}$

$$\boxed{SD = \sqrt{\frac{\sum(x_i - \mu)^2}{n-1}}}$$

$$SD(\sigma) = 0.37$$ Ans

b) Calculate the Median and Quartiles
=) The median is the middle value in a list ordered from smallest to largest.
Since there are 50 data points (an even number), the median will be the average of the 25^{th} and 26^{th} values,

$$\therefore \text{Median} = \frac{4.50 + 4.51}{2} = 4.505$$ Ans

Again
Q1 (First Quartile): The median of the first half of the data. (The first 25 data points).

~~$Q_1 = \frac{4.28 + 4.30}{2} = 4.29$~~ $\therefore Q_1 = 4.30$

$Q_2 = 4.70$ Ans

c) Find the 90^{th} percentile.

=) To find the 90^{th} percentile, sort the data in ascending order and find the value below which 90 % of the data points fall.

Position of the 90^{th} percentile $= \frac{9\cancel{0}}{1\cancel{00}} \times 5\cancel{0} = 45^{th}$ value

$\therefore$ 90^{th} percentile $= 4.80$

d) Determine the intervals $\bar{x} \pm s$, $\bar{x} \pm 2s$, $\bar{x} \pm 3s$.

we have,

$\bar{x} = 4.51$

$s = 0.37$.

$\therefore \bar{x} \pm s = \bar{x} - s$ to $\bar{x} + s$

$= 4.51 - 0.37$ to $4.51 + 0.37$

$= 4.14$ to 4.88.

$\bar{x} \pm 2s = 4.51 - 0.74$ to $4.51 + 0.74$

$= 3.77$ to 5.25

$\bar{x} \pm 3s = 4.51 - 1.11$ to $4.51 + 1.11$

$= 3.40$ to 5.62

$\therefore$ $\bar{x} \pm s = (4.14, 4.88)$

$\bar{x} \pm 2s = (3.77, 5.25)$

$\bar{x} \pm 3s = (3.40, 5.62)$

e) What portions of the measurements lie in those intervals,

∴ Within $\bar{x} \pm s$ (4.14, 4.88) : 74.5 %

Within $\bar{x} \pm 2s$ (3.77 , 5.25) : 94.1 %

Within $\bar{x} \pm 3s$ (3.40 , 5.62) : 100 %

f) Compare the findings with the empirical guidelines for bell-shaped distribution

⇒ The empirical rule or (68 - 95 - 99.7 rule) states that for a bell-shaped distribution:

- 68 % of data falls within $\bar{x} \pm s$
- 95 % of data falls within $\bar{x} \pm 2s$
- 99.7 % of data falls within $\bar{x} \pm 3s$.

⇒ 74.5 % within $\bar{x} \pm s$ is close to the 68 % guideline.

⇒ 94.1 % within $\bar{x} \pm 2s$ is close to the 95 % guideline.

⇒ 100 % within $\bar{x} \pm 3s$ is close to the 99.7 % guideline.

2) The hours of Sleep data Suggest that the population of Sleep can be modeled as normal distribution with mean = 7.2 hours and S.d = 1.3 hours.

a) Determine the probability assigned to Sleeping less than 6.5 hours.

b) Find the 70th percentile of the distribution of hours of sleep.

⇒

a) To solve this problem, we will use the properties of the normal distribution.

Given,

μ = 7.2 hours.

S.d = 1.3 hours.

a) Probability of sleeping less than 6.5 hours

-) To determine the probability of sleeping less than 6.5 hours, we need to find the cumulative probability up to 6.5 hours. This involves calculating z-scores for 6.5 hours and then finding the corresponding probability from the standard normal distribution table.

We have, z-score formula

$$Z = \frac{X - \mu}{\sigma}$$

$$= \frac{6.5 - 7.2}{1.3}$$

$$= -0.53$$

∴ The corresponding probability = 0.2981
= 29.81 % Ans

b) 70th percentile of the Distribution for hours of sleep

Step-1: Find the z-score corresponding to 70th percentile

∴ the z-score value of 0.7 is ~~0.57~~ 0.53.

Step 2: Convert the z-score to the original distribution use formula,

$$P_{70} = \mu + z \cdot \sigma \qquad \left[\because z = \frac{P - \mu}{\sigma}\right]$$

$$= 7.2 + (0.53 \times 1.3)$$

$$= 7.2 + 0.689$$

$$\therefore P_{70} = 7.88 \text{ Ans}$$

3) From a set of numbers: {3, 5, 7), a random sample of size 2 is selected with replacement
a) List all possible samples and evaluate $\bar{x}$ and s^2
b) Determine the sampling distribution of $\bar{x}$ and s^2

a)-) The possible samples are,
(3, 5), (3, 7), (3, 3), (5, 5), (5, 3), (5, 7), (7, 7), (7, 3), (7, 5)
There are 9 possible samples.
Now calculating $\bar{x}$ (is the sample mean for each sample.
and then sample variance (σ^2)

X	Y	$\mu = \frac{X+Y}{2}$	$X-\mu$	$Y-\mu$	$(X-\mu)^2$	$(Y-\mu)^2$	$\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$
3	3	3	0	0	0	0	0
3	5	4	-1	1	1	1	2
3	7	5	-2	2	4	4	8
5	3	4	1	-1	1	1	2
5	5	5	0	0	0	0	0
5	7	6	-1	1	1	1	2
7	3	5	2	2	4	4	8
7	5	6	1	-1	1	1	2
7	7	7	0	0	0	0	0

Ans

b)-) Sampling distribution of $\bar{x}$
The possible value of $\bar{x}$ are 3, 4, 5, 6, and 7. The probabilities of each value are
- $\bar{x} = 3$ occurs 1 times out of 9 samples : $P(\bar{x}=3) = 1/9$
- $\bar{x} = 4$ " 2 " " " " " : $P(\bar{x}=4) = 2/9$
- $\bar{x} = 5$ " 3 " " " " " : $P(\bar{x}=5) = 3/9 = 1/3$
- $\bar{x} = 6$ " 2 " " " " " : $P(\bar{x}=6) = 2/9$
- $\bar{x} = 7$ " 1 " " " " " : $P(\bar{x}=7) = 1/9$

Ans

Sampling distribution of s^2

The possible values of s^2 are 0, 2, and 8. The ~~poss~~ probabilities for each value are,

- $s^2 = 0$ occurs 3 times out of 9 samples : $P(s^2=0) = \frac{3}{9} = \frac{1}{3}$
- $s^2 = 2$ " 4 " " " 9 " : $P(s^2=2) = 4/9$
- $s^2 = 8$ " 2 " " " 9 " : $P(s^2=8) = \frac{2}{9}$ Ans

4) From a set of numbers {0, 2, 4, 6}, a random sample of size 2 is selected with replacement. Define the range R = Largest observation - Smallest observation.

a) List ~~of~~ all possible samples and evaluate R.

b) Determine the sampling distributions of R.

=) The possible samples (x, y) and Range (R) given below in table.

x	y	Range (R) = |y - x|
0	0	0
0	2	2
0	4	4
0	6	6
2	0	2
2	2	0
2	4	2
2	6	4
4	0	4
4	2	2
4	4	0
4	6	2
6	0	6
6	2	4
6	4	2
6	6	

b) Sampling distribution of R.
The possible values of R are 0, 2, 4, and 6. Now we determine the possibilities of each value.
Total observation = 16

- for R = 0, $P(R=0) = \frac{4}{16} = \frac{1}{4}$ Ans

- for R = 2, $P(R=2) = \frac{6}{16} = \frac{3}{8}$ Ans

- for R = 4, $P(R=4) = \frac{4}{16} = \frac{1}{4}$ Ans

- for R = 6, $P(R=6) = \frac{2}{16} = \frac{1}{8}$ Ans

5) Suppose that the load of an airplane wing is a random variable X with N(1000, 14400) distribution. The maximum load that the wing can withstand is a random variable Y which is N(1260, 2500). If X and Y are independent, find the probability that the load encountered by the wing is less than its critical load.

⇒ Given,
load encountered by aero plane wing ~~with~~ ~~rando~~
$X \sim N(1000, 14400)$ ($\mu_x = 1000$, $\sigma_x^2 = 14400$)

~~Maximum load~~ $\therefore \sigma_x = \sqrt{14400} = 120$

The maximum load the wing can withstand,
$Y \sim N(1260, 2500)$ ($\mu_y = 1260$, $\sigma_y^2 = 2500$).

$\sigma_y = \sqrt{2500} = 50$ Ans

we are asked to find the probability that the load encountered by the wing (x) is less than the critical load y. i.e. $P(X<Y)$.

Let's define a new ~~probability~~ variable,

$$Z = Y - X.$$

The probability we need to find is $P(X<Y)$ which is the same as $P(Z>0)$

Since x and y are independent normal random variable, the distribution of $Z = Y - X$ is also normal,

Mean of Z,

$$\mu_Z = \mu_Y - \mu_X = 1260 - 1000 = 260$$

Variance of Z

$$\sigma_Z^2 = \sigma_Y^2 + \sigma_X^2 = 2500 + 1400 = 16900.$$

$$\therefore \quad \sigma_Z = \sqrt{16900} = 130$$

Therefore, $Z \sim N(260, 16900)$.

Now

$P(Z>0)$, we have to calculate Z-score for Z = 0

we have $$Z_{score} = \frac{X-\mu}{\sigma} = \frac{0-26\cancel{0}}{13\cancel{0}} = -2.$$

From SND table, value corresponding to $-2 = 0.97725$

$$\therefore \quad P(Z>-2) = 0.97725$$

Thus the probability that the load encountered by the wing is less than the critical load is approximately 97.72 % Ans

6) If the service life of election tubes in a particular application is normally distributed, and if 92.5% of the tubes have lives greater than 2,160 hours, while 3.92 % have lives greater than 17,040 hours, what are the mean and SD of the service life.

$\Rightarrow$ Converting percentage to probability,

$P(X > 2160) = 0.925.$

$\therefore P(X < 2160) = 1 - 0.925 = 0.075$ which is the left-tail probability.

$P(X > 17040) = 0.0392$

$P(X < 17040) = 1 - 0.0392 = 0.9608$, which is the left-tail probability

$\therefore$ Corresponding z score are,

for, $P(X < 2160)$, z-score $= -1.44$

for, $P(X < 17040)$, z-score $= 1.75$

we have,

$$Z = \frac{X - \mu}{\sigma}$$

$$\therefore x - \mu = \sigma . Z.$$

for,

$P(X < 2160)$

$$2160 - \mu = -1.44 \sigma$$

$$\therefore \mu - 1.44\sigma = 2160 \quad \text{--- (i)}$$

For, $P(X < 17040)$

$$\frac{17040 - \mu}{\sigma} = 1.75$$

$\therefore \Rightarrow \quad 17040 = 1.75\sigma - \mu$

$\therefore \quad -\mu + 1.75\sigma = 17040 \quad — (ii)$

On solving eqn (i) & (ii) we get,

$\therefore \quad \mu = 91347.09$ hours Ans

$\sigma = 61935.48$ hours Ans

7) Suppose that 10 percent of the probability for a certain distribution that is $N(\mu, \sigma^2)$ is below 60 and 5 percent is above 90. What are the values of μ and σ?

$\Rightarrow$

Here, given,

$P(X < 60) = 10\% = 0.10$

$P(X < 90) = 5\% = 0.05.$

First calculating z score,

For $P(X < 60) = 0.10$, Z-score $= -1.28$

For $P(X < 90) = 0.05$, Z-score $= 1.64$

We have,

$$Z = \frac{X - \mu}{\sigma}$$

For $P(X < 60)$

$$\frac{60 - \mu}{\sigma} = -1.28$$

$60 - \mu = -1.28\sigma$

$\therefore \mu - 1.28\sigma = 60$ —(i)

For $P(X < 90)$

$\frac{90 - \mu}{\sigma} = 1.64$ $\quad\quad \therefore 9 - \mu = 1.64\sigma$

$9 = 1.64\sigma + \mu$

$\therefore \mu + 1.64\sigma = 90$ —(ii)

On solving eqn (i) & (ii) we get

$\therefore \mu = 73.15$

and $\sigma = 10.27$ Ans

8) The lifetime of a color TV picture tube is normally distributed, with the mean of 8 year and S.D of 2 years.

a) What is the probability that a picture tube will last more than 10 years?

b) If the firm gurantees the picture tube for 4 years, what percentage of the tubes sold will have to replaced.

=> We are given,

$\mu = 8$

$\sigma = 2$

a) we know,

$z = \frac{X - \mu}{\sigma} = \frac{10 - 8}{2} = \frac{2}{2} = 1$

$\therefore z = 1$

and corresponding probability = 0.84134

This is the probability that a tube will last less than 10 years, i.e., $P(X < 10)$.

Thus the probability that tube will last more than 10 years is,

$$P(X > 10) = 1 - P(X < 10)$$
$$= 1 - 0.8413$$
$$= 0.1583$$

$\therefore$ The probability that a picture tube will last more than 10 years is approximately 0.1583 or 15.83 %

b) If the firm guarantees the picture tube for 4 years, we need to find $P(X<4)$, the probability that a tube will last less than 4 years, since these will need to be replaced under the guarantee.

For $X=4$, $z\text{-score} = \frac{4-8}{2} = \frac{-4}{2} = -2$

For $z=-2$, probability $= 0.0228$.

$\therefore$ The percentage of tubes that will have to be replaced is:

$$P(X<4) = 0.0228 \text{ or } 2.28\%$$

9) Let random variables X_1, X_2, X_3 be independent and distributed according to $N(0;1)$; $N(1;1)$, and $N(2;1)$, respectively. Determine probability $P(X_1 + X_2 + X_3 > 1)$

$\Rightarrow$ We are given three independent random variables:

$X_1 \sim N(0, 1)$

$X_2 \sim N(1, 1)$

$X_3 \sim N(2, 1)$

Distribution of the sum $S = x_1 + x_2 + x_3$.

Mean of S $(\mu_S) = \mu x_1 + \mu x_2 + \mu x_3$

$= 0 + 1 + 2$

$= 3$

Variance of S $(\sigma_S^2) = \sigma x_1^2 + \sigma x_2^2 + \sigma x_3^2$

$= 1 + 1 + 1$

$= 3$

$\therefore \sigma_S = \sqrt{3}$

$\therefore S \sim N(3, 3)$.

For,

$P(S > 1)$,

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{1 - 3}{\sqrt{3}} = -2/\sqrt{3} = -1.1547$$

From normal distribution table,

For $z = -1.1547$,

$P(Z < -1.1547) = 0.1251$

Thus, the probability that $S > 1$ is,

$P(S > 1) = 1 - P(S < 1)$

$= 1 - 0.1251$

$= 0.8749$

$\therefore P(x_1 + x_2 + x_3 > 1) = 87.49\%$

10) Extensive data suggest that the number of extracurricular activities per week can be modeled as a distribution with mean 1.9 and Standard deviation 1.6.

a) If a random Sample of size 41 is Selected, what is the probability that the Simple mean will lie between 1.7 and 2.1?

b) If a random Sample of size 100 is Selected, what is the probability that the Sample mean will be between 1.7 and 2.1?

⇒ Given,

$\mu = 1.9$

$\sigma = 1.6$

a) The Standard error of the mean ($\sigma\bar{x}$) is given by,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

when, $n=41, \sigma = 1.6$:

$$\sigma\bar{x} = \frac{1.6}{\sqrt{41}} \approx \frac{1.6}{6.4031} \approx 0.2499.$$

we need to convert the values 1.7 and 2.1 to Z-score using formula:

$$Z = \frac{\bar{x} - \mu}{\sigma\bar{x}}$$

For $\bar{x} = 1.7$: $z = \dfrac{1.7 - 1.9}{0.2499} = -0.80.$

For $\bar{x} = 2.1$: $z = \dfrac{2.1 - 1.9}{0.2499} = 0.80$

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the probability corresponding to the z-scores

$P(Z < -0.80) = 0.2119$

$P(Z \leq 0.80) = 0.7881$

$\therefore P(1.7 < \bar{x} < 2.1) = P(-0.80 < Z < 0.80) = 0.7881 - 0.2119$

$= 0.5762$

$\therefore$ Probability is approximately 57.62%

b) For the Sampe 100,

$$\sigma_{\bar{x}} = \frac{1.6}{\sqrt{100}} = \frac{1.6}{10} = 0.16$$

For, $\bar{x} = 1.7$, $Z = \frac{1.7 - 1.9}{0.16} = \frac{-0.2}{0.16} = -1.25$

For, $\bar{x} = 2.1$, $Z = \frac{2.1 - 1.9}{0.16} = \frac{0.2}{0.16} = 1.25$

The probability for,

$P(Z < -1.25) = 0.1056$

$P(Z < 1.25) = 0.8944$

$\therefore P(1.7 < \bar{x} < 2.1) = P(-1.25 < Z < 1.25) = 0.8944 - 0.1056$

$= 0.7888$

$\therefore$ Probability $= 78.88\%$