Artificial Intelligence

Lec 10: Adversarial Search

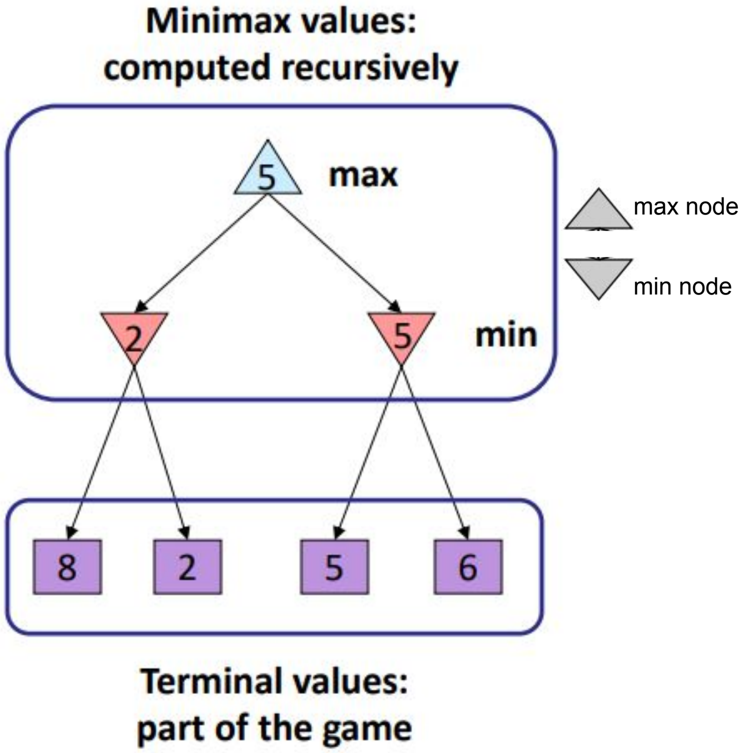
Pratik Mazumder

Content taken from various sources including
CS188 Intro to AI at UC Berkeley

Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value:
    - the best achievable utility **against a rational (optimal) adversary**


Minimax values: computed recursively
5
max
2
5
min
8
2
5
6
Terminal values: part of the game
max node
min node

Adversarial Search (Minimax)

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    for each successor of state:
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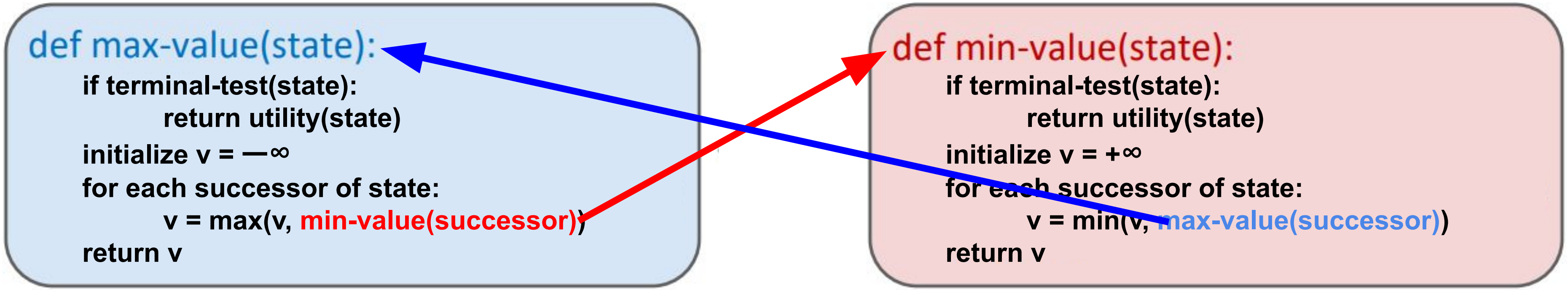

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Minimax Terminology

- move: a move by both players

- ply: a half move, i.e., action by one player

- backed-up value
  - Of a max position: the value of its largest successor
  - Of a min position: the value of its smallest successor

- Minimax procedure:
  - Search down several levels.
  - At the bottom level, apply the utility function.
  - Back-up values all the way up to the root node
  - Select a move starting from the root node [if you perform the first move of the game].

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▲ max node

▼ min node

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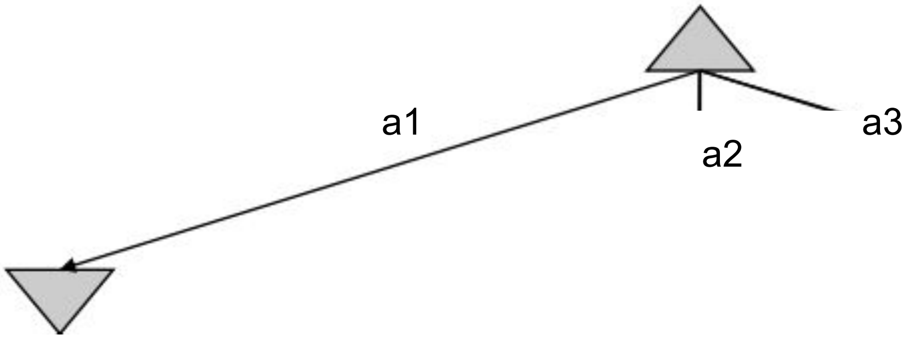

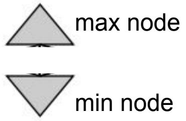

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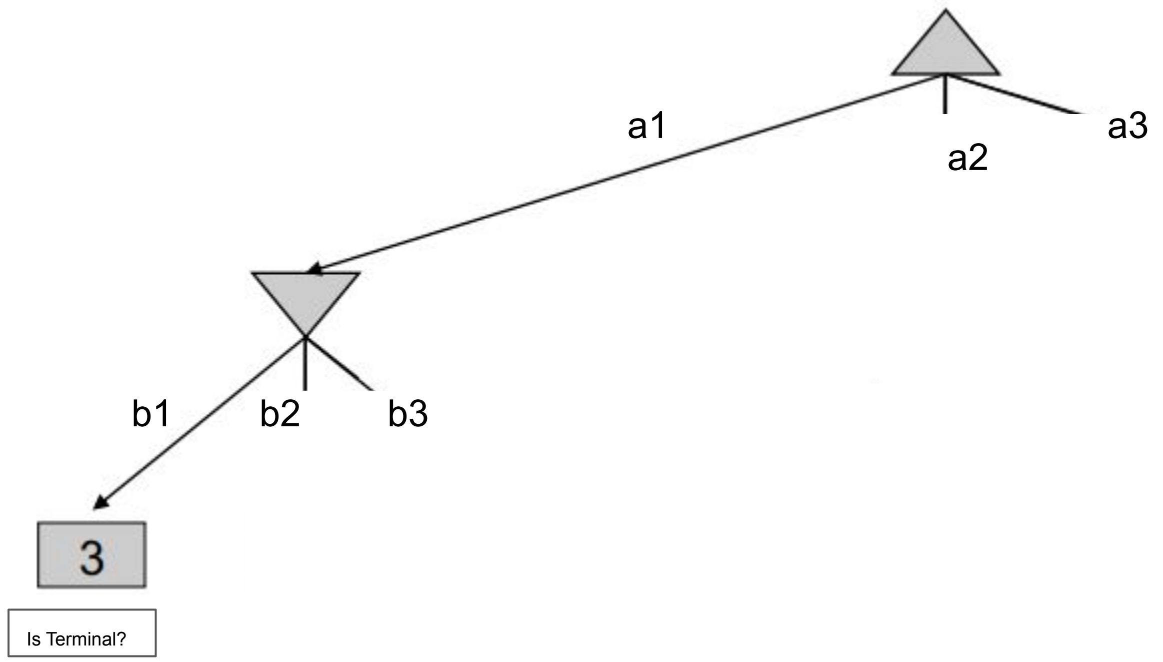

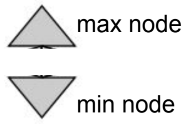

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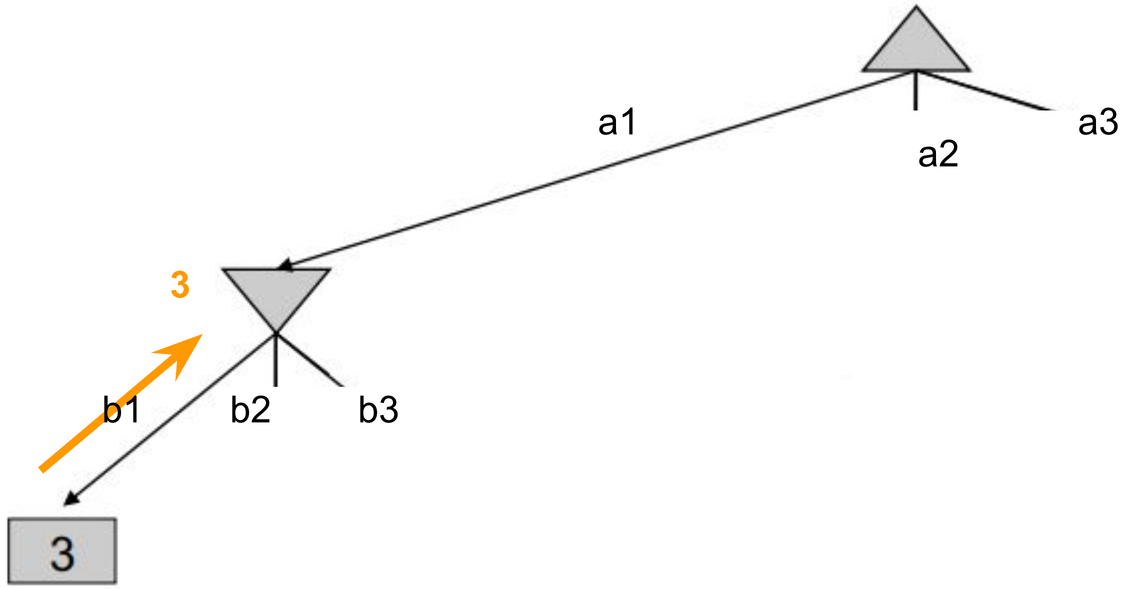

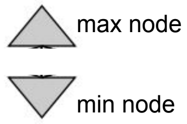

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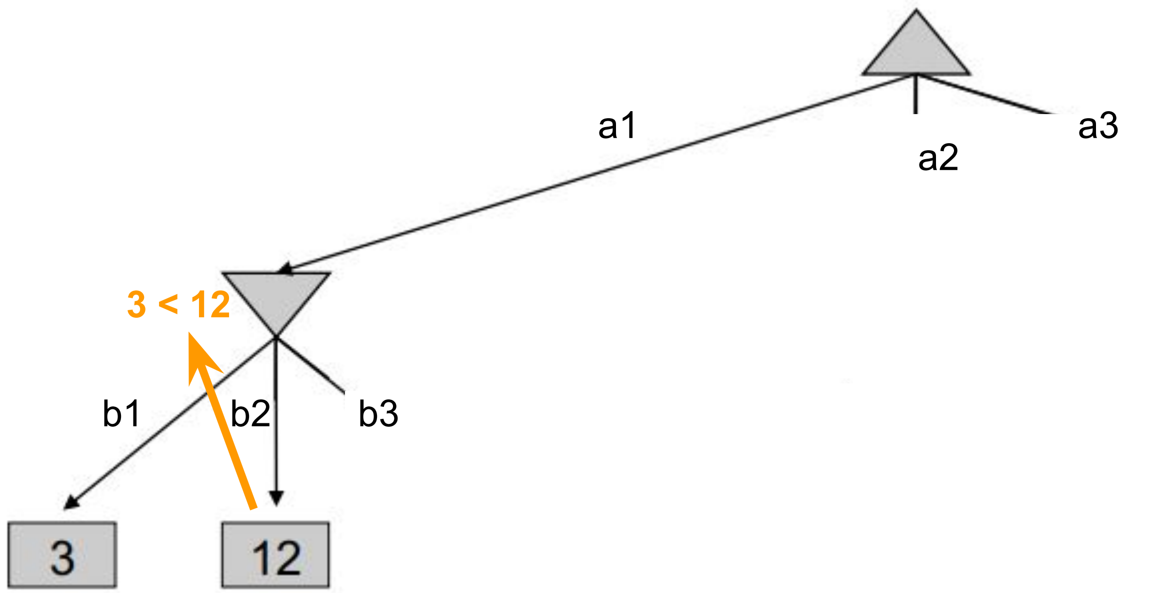

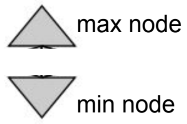

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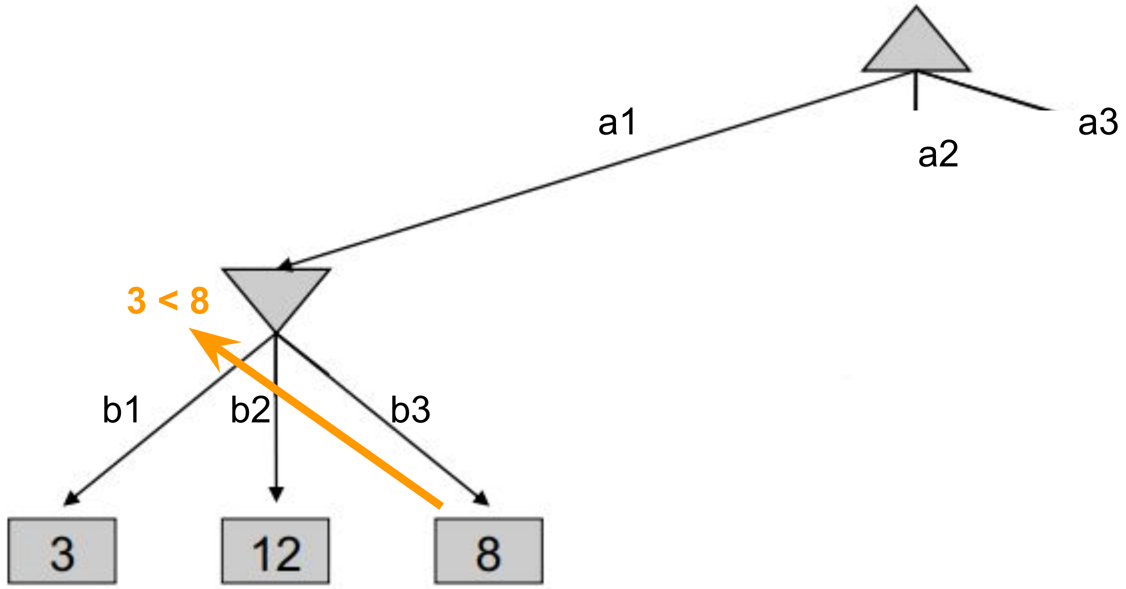

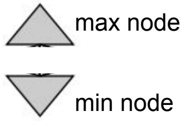

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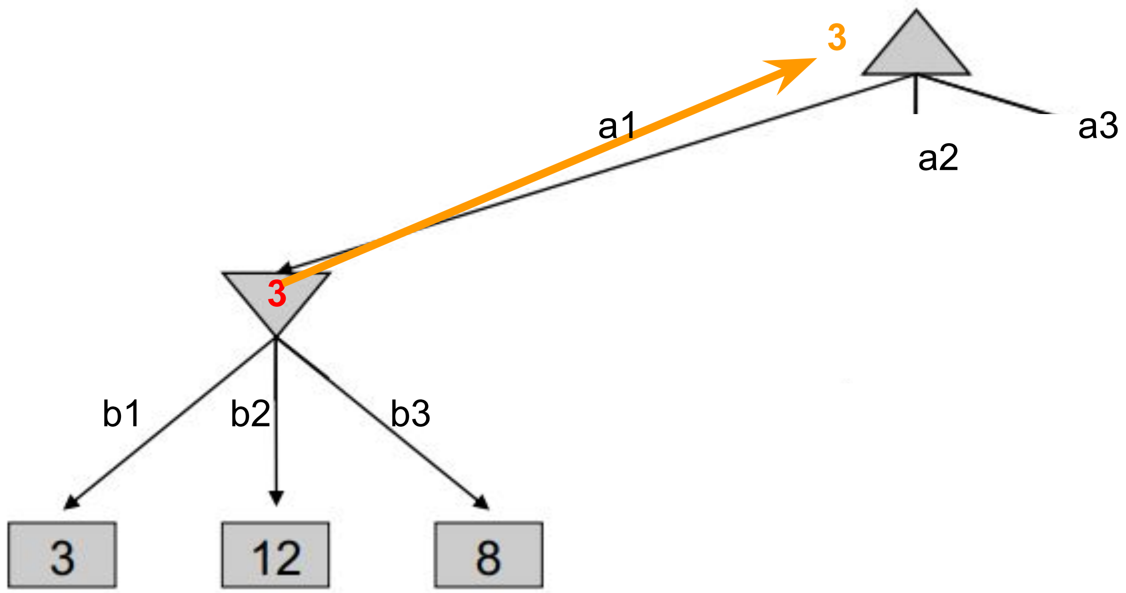

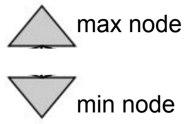

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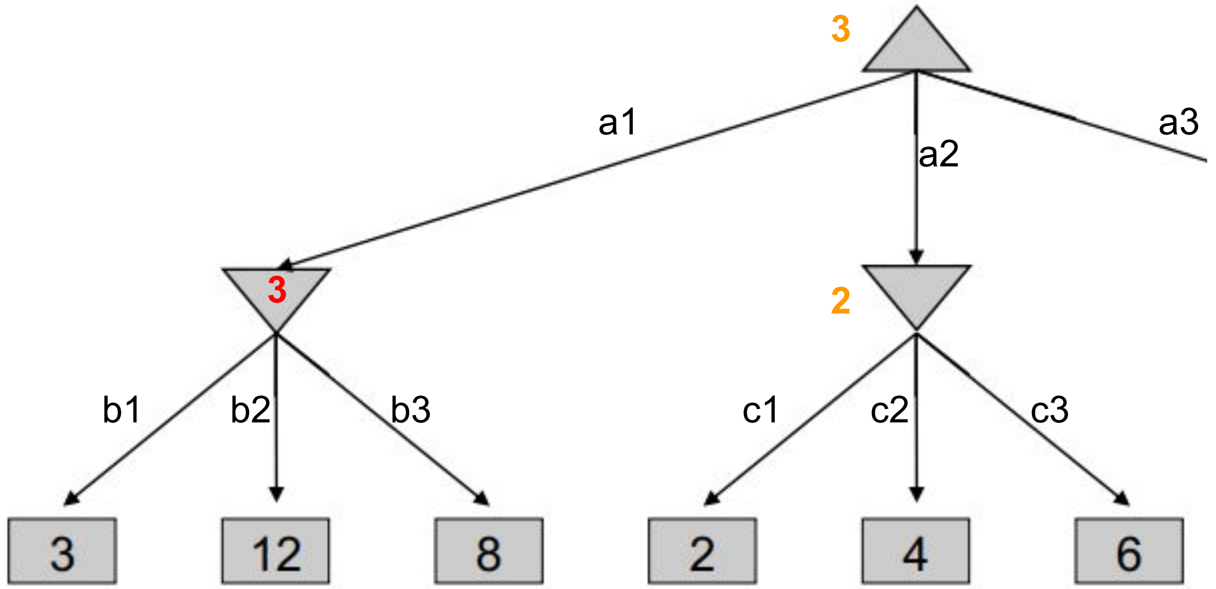

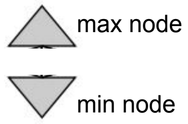

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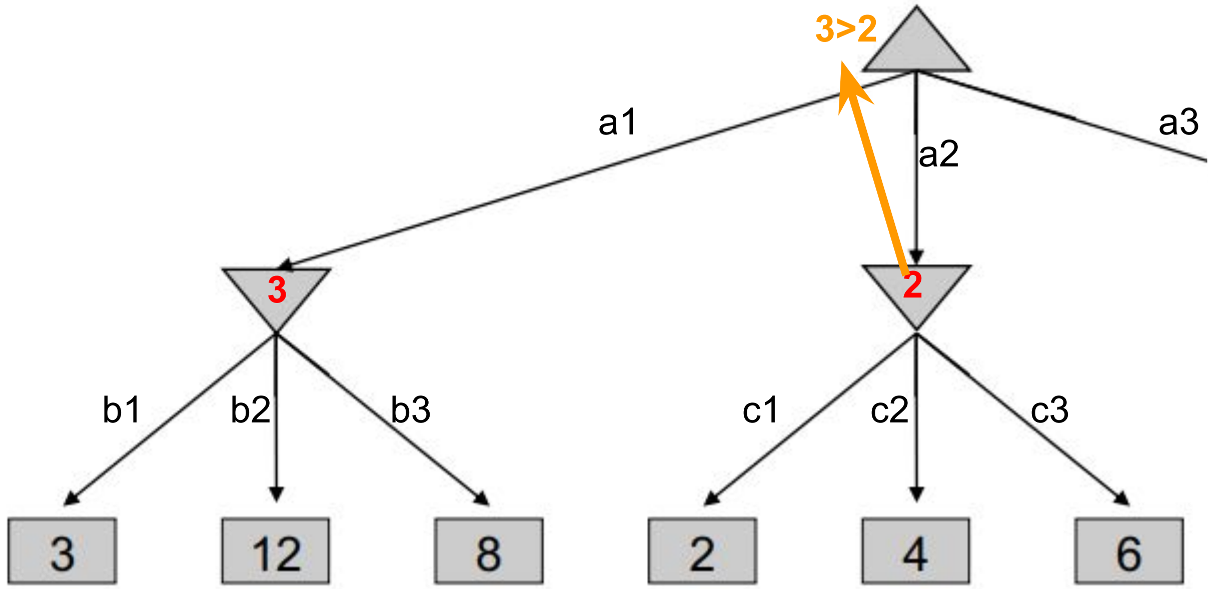

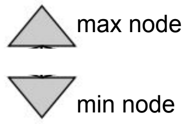

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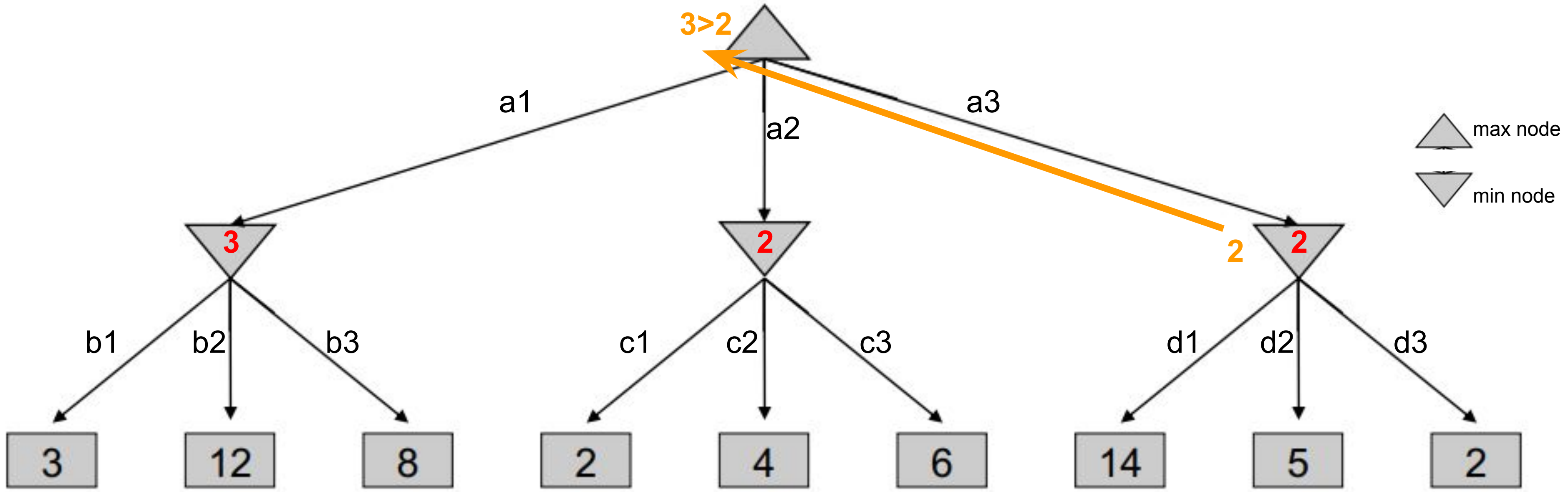

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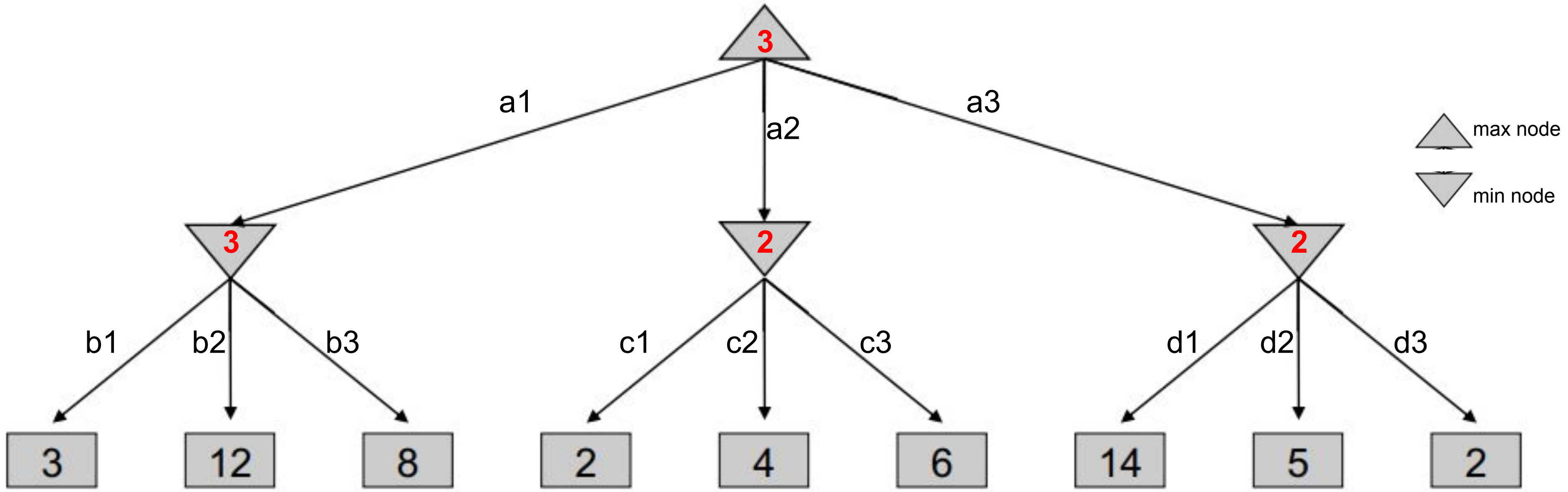

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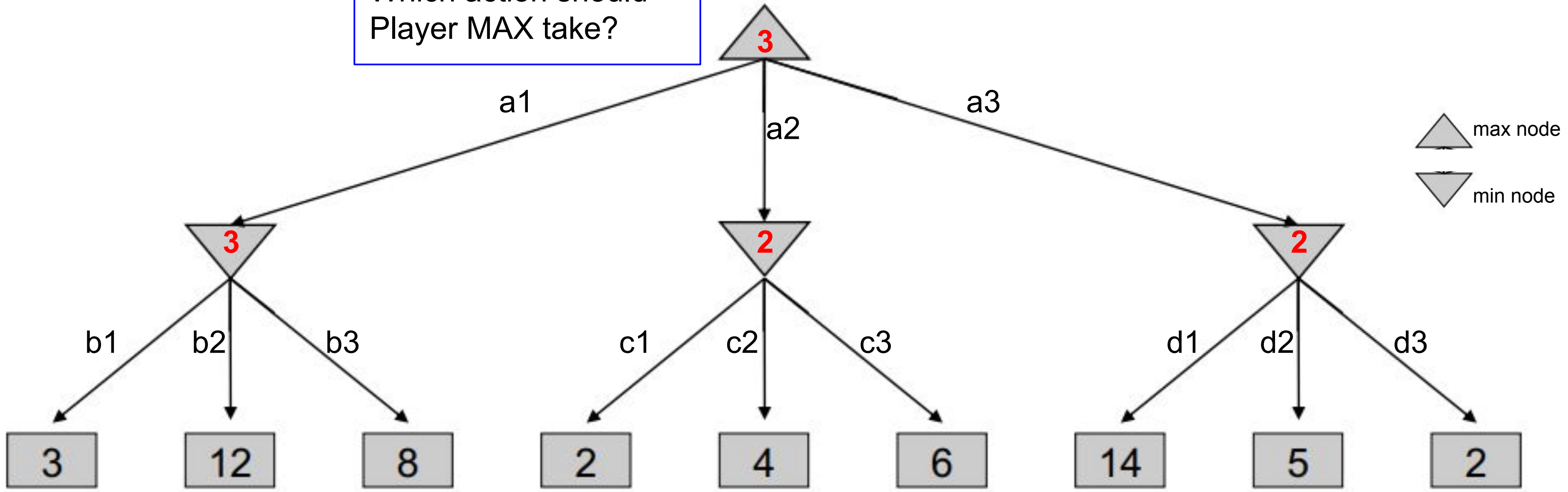

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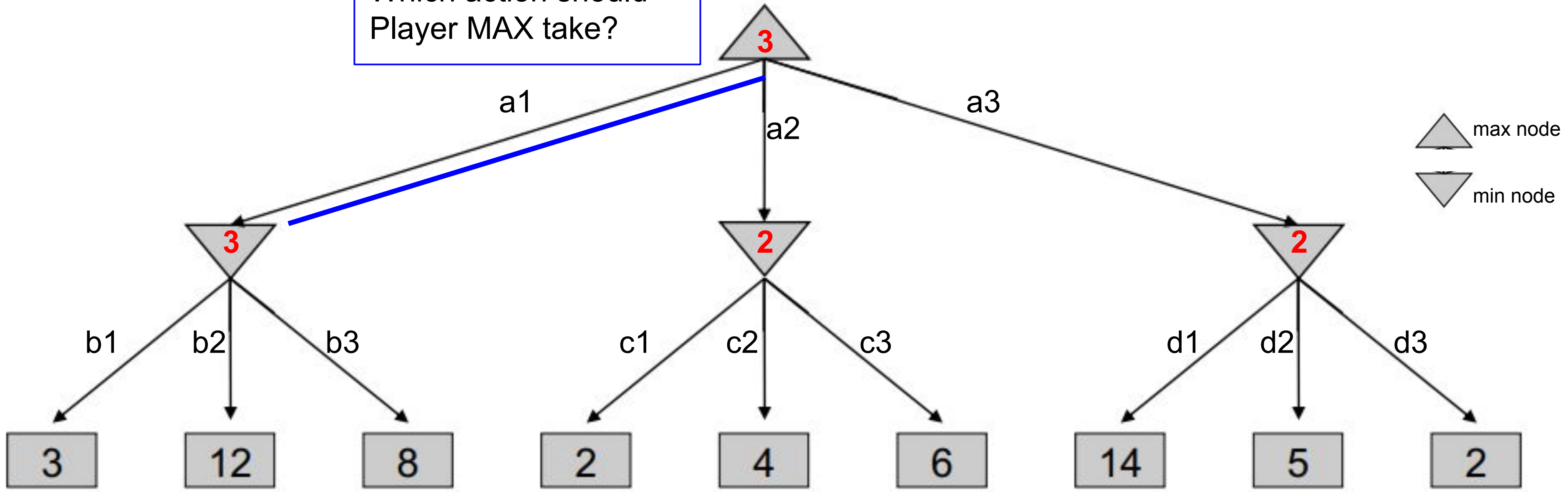

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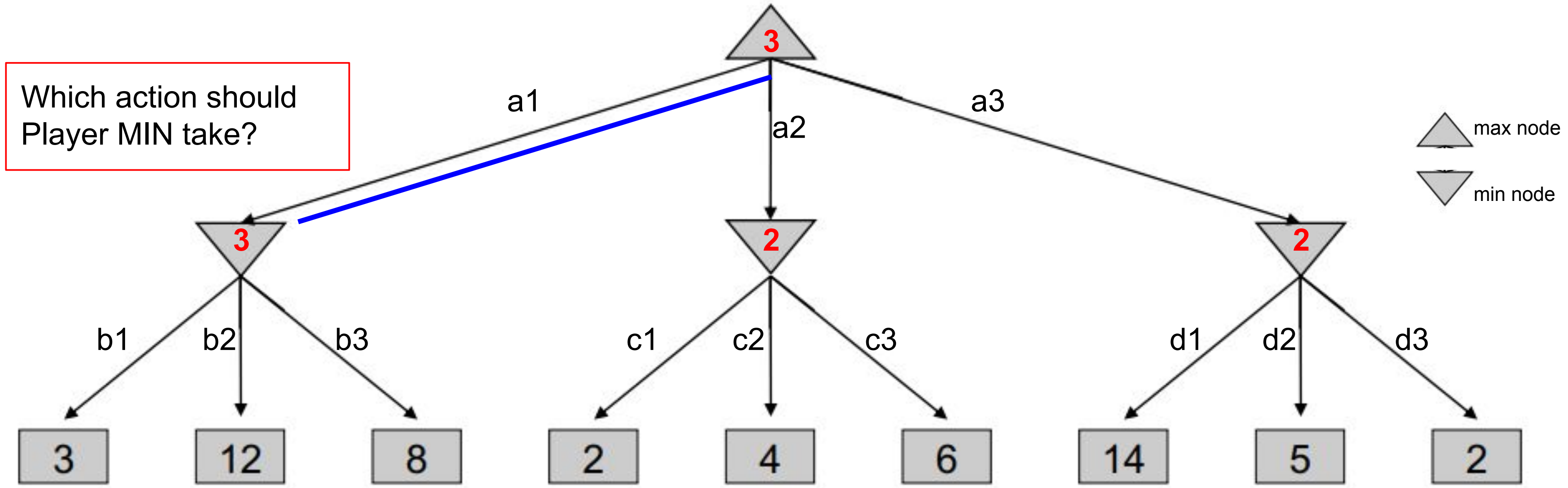
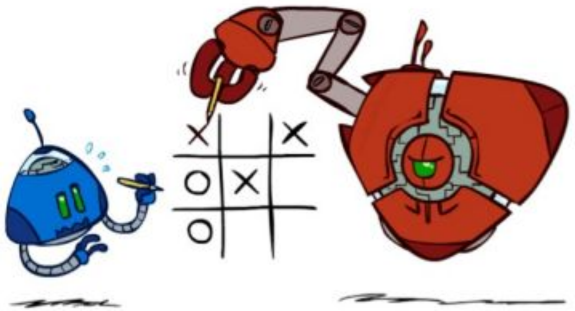
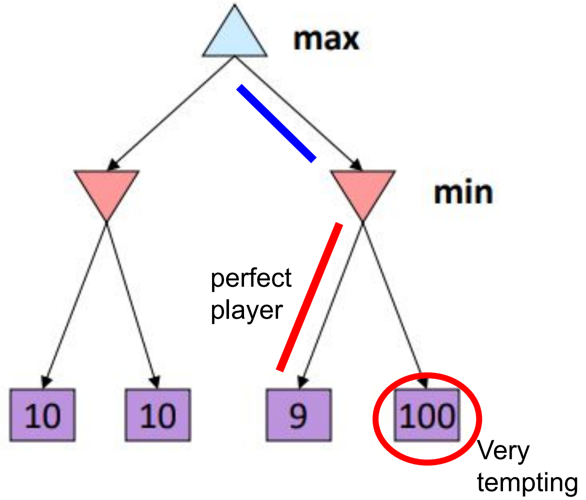


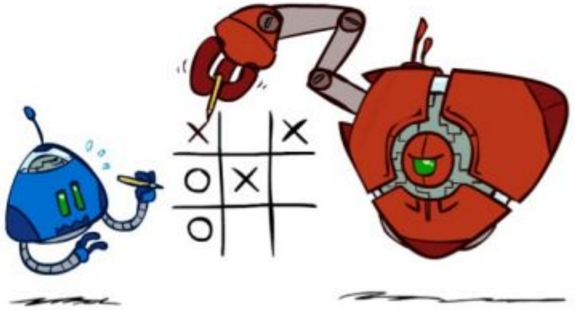
Minimax Properties

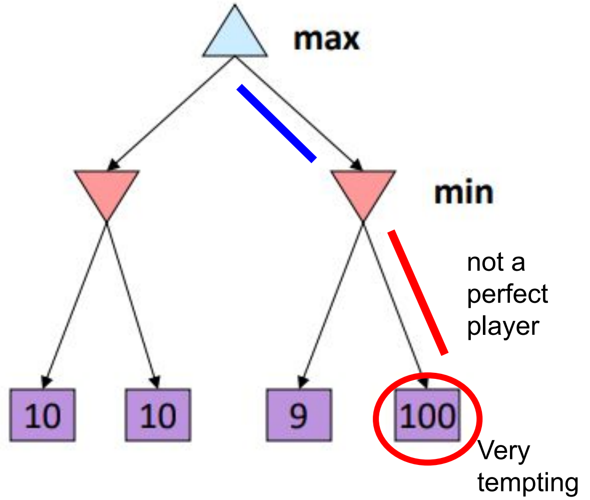


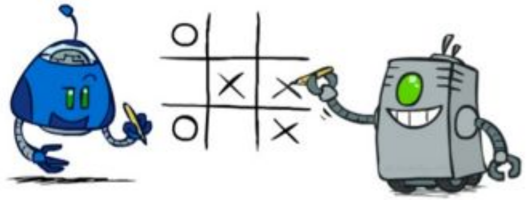


Optimal against a perfect player.

Minimax Properties



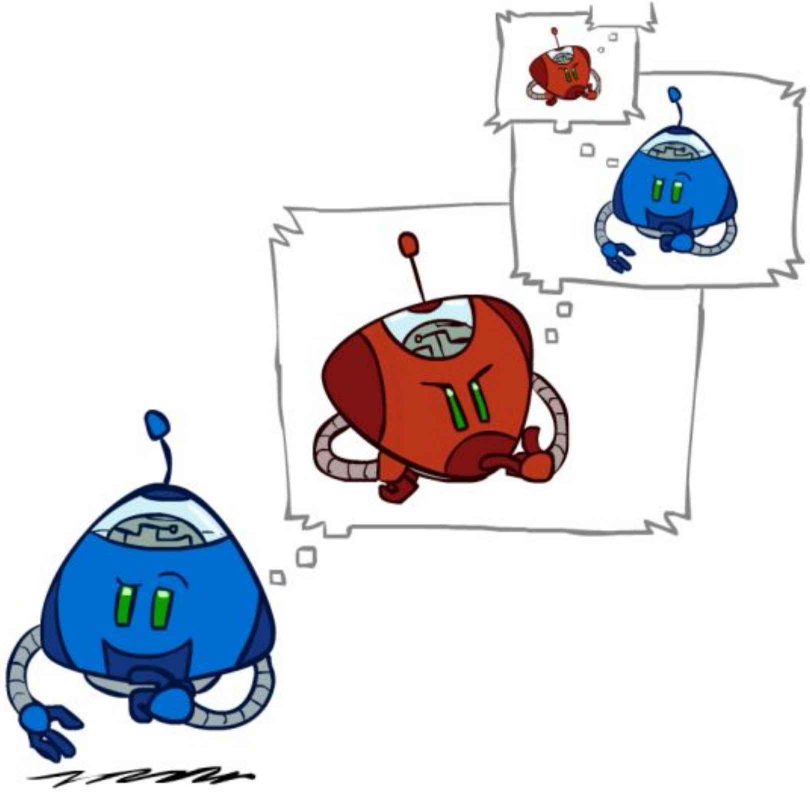




Optimal against a perfect player. Otherwise?

Minimax Efficiency

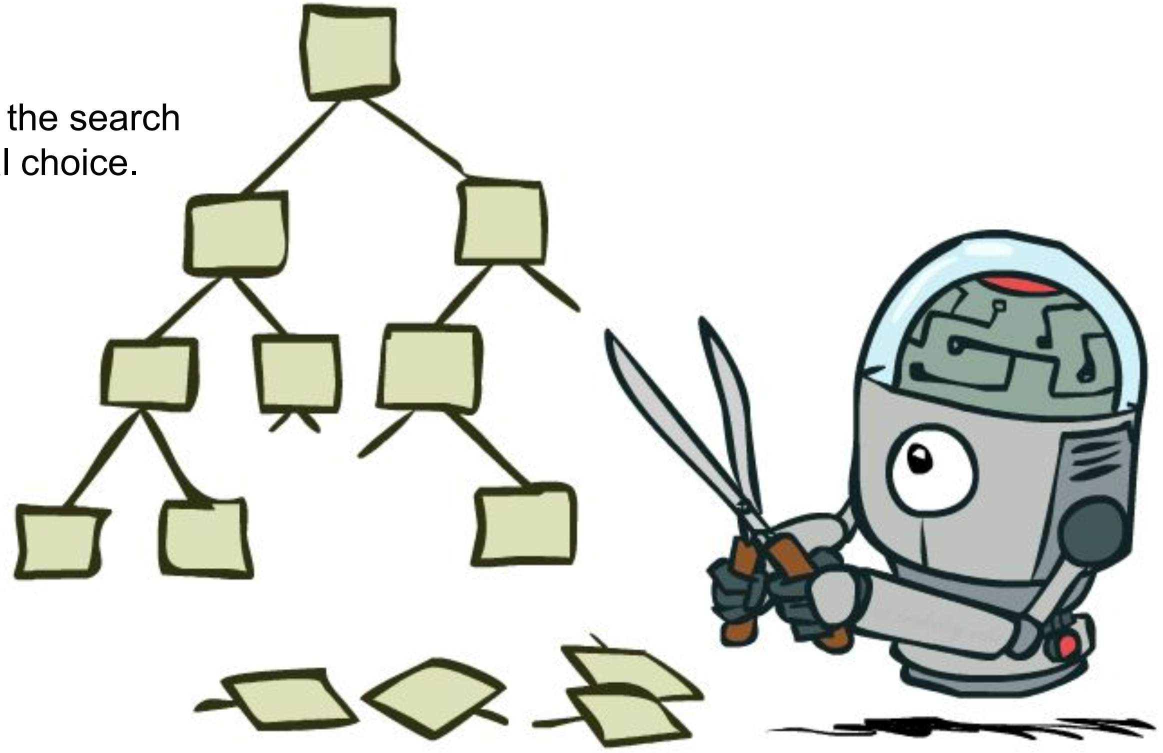
- How efficient is minimax?
  - Just like (exhaustive) DFS
- Example: For chess, branching factor b ≈ 35, solution depth/how many turns the game lasts typically m ≈ 100
  - Search space $b^m = 35^{100} \approx 10^{154}$
- Interesting Analogy: Universe
  - Number of atoms ≈ 10^{78}
  - Age ≈ 10^{18} seconds
  - Avg no of chemical reactions: 10^8/sec
  - 10^8 moves/sec x 10^{78} x 10^{18} = 10^{104}
- For Go, b ≈ 250-300, m ≈ 150
- Exact solution is not very feasible.
- But, do we need to explore the whole tree?



Overcoming Resource Limits

Overcoming Resource Limits: Game Tree Pruning

Pruning allows us to ignore portions of the search tree that make no difference to the final choice.



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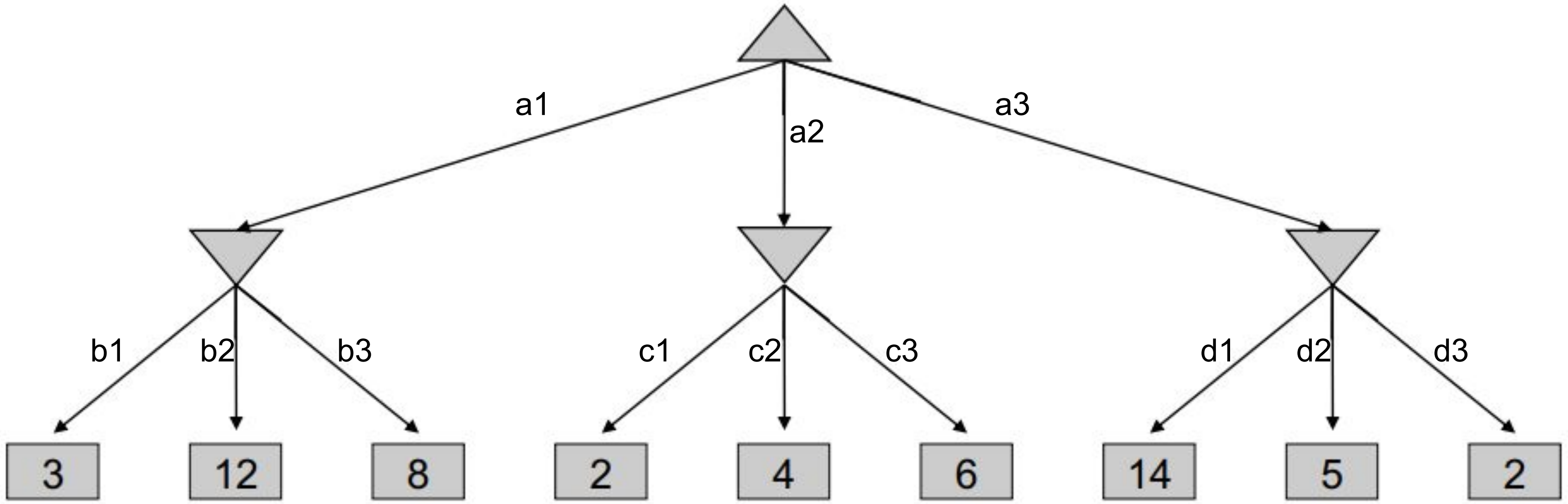

Pruning

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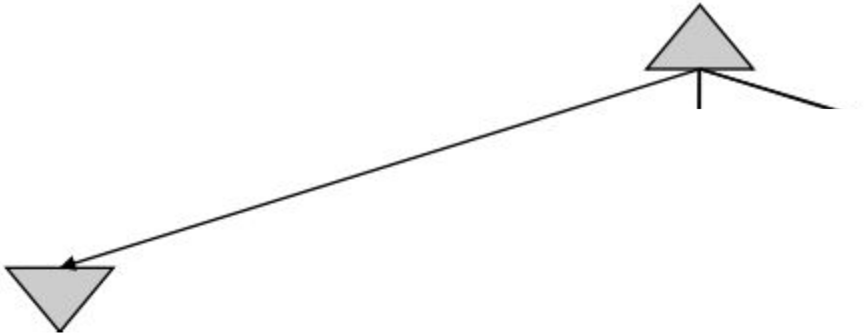

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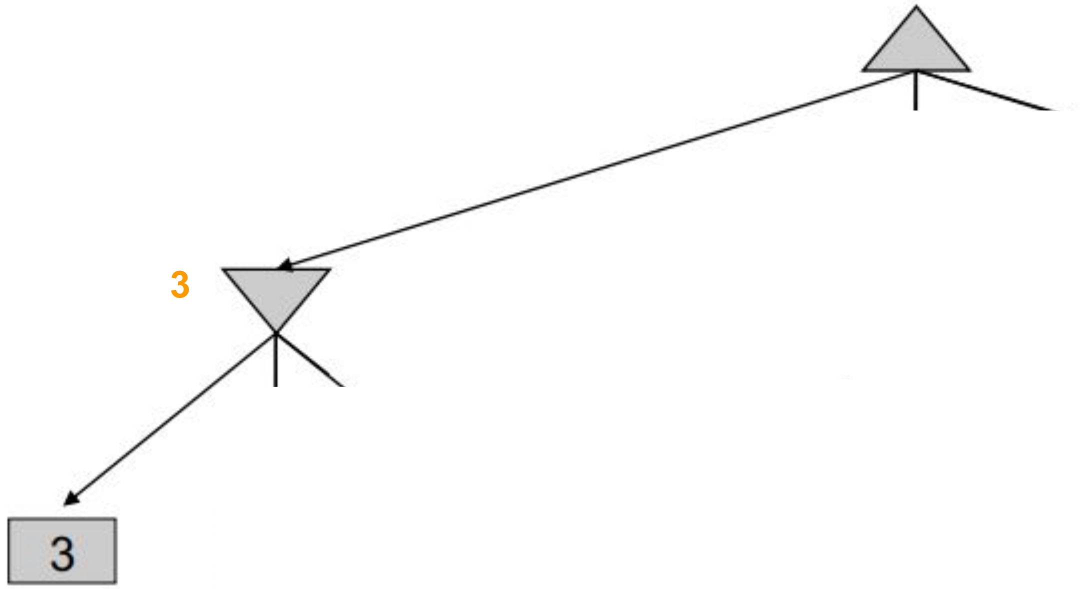

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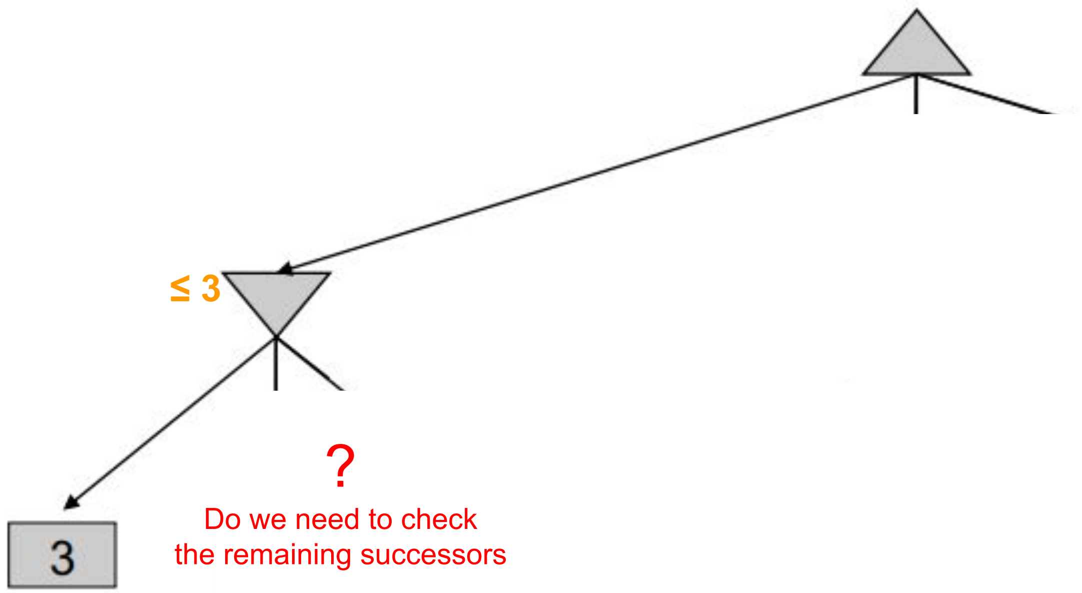

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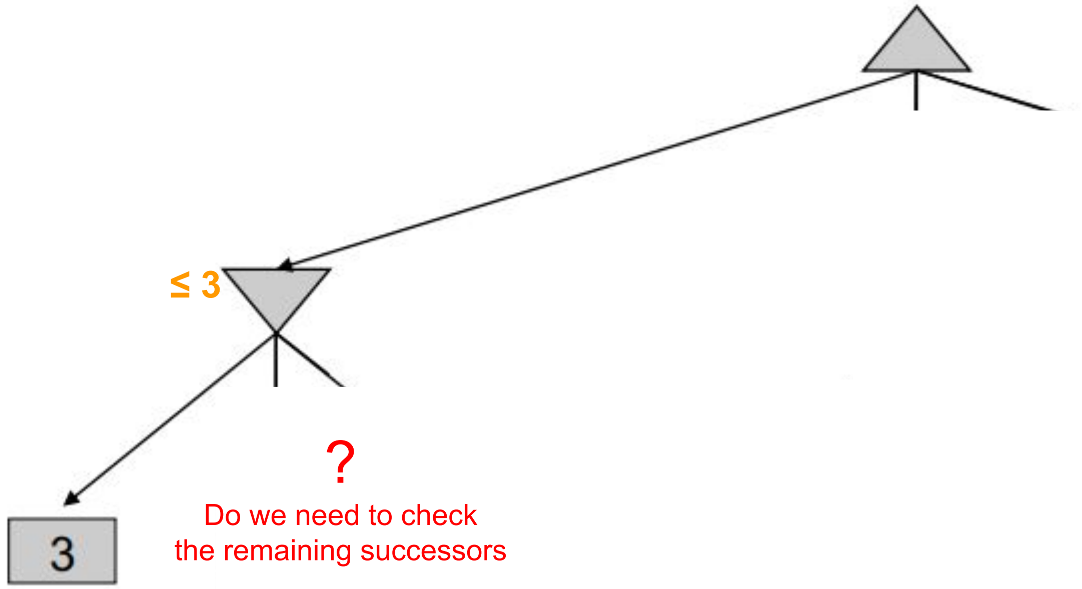
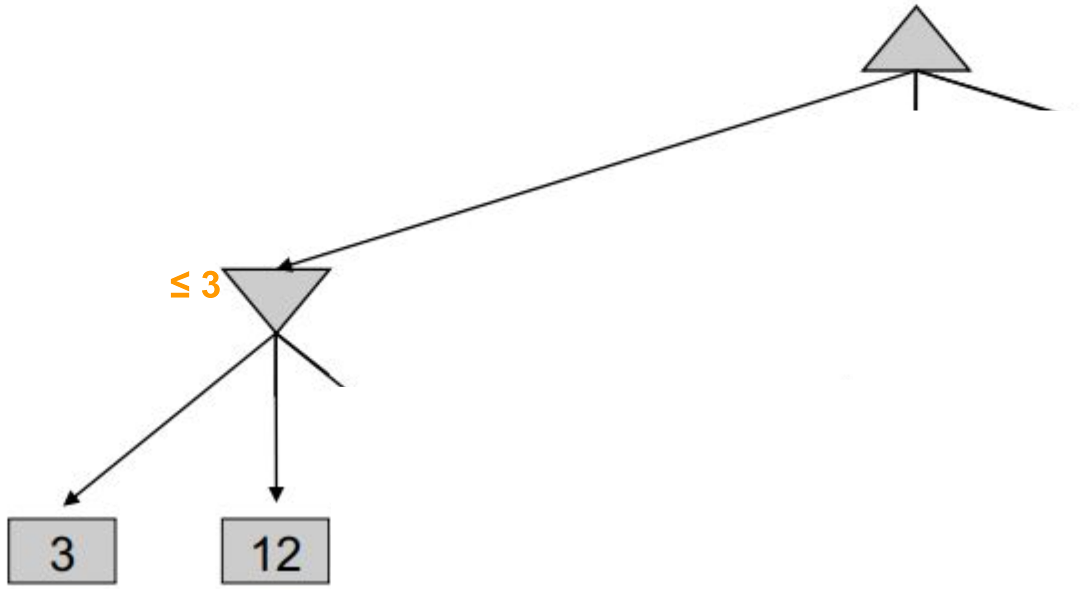


Do we need to check
the remaining successors

Pruning



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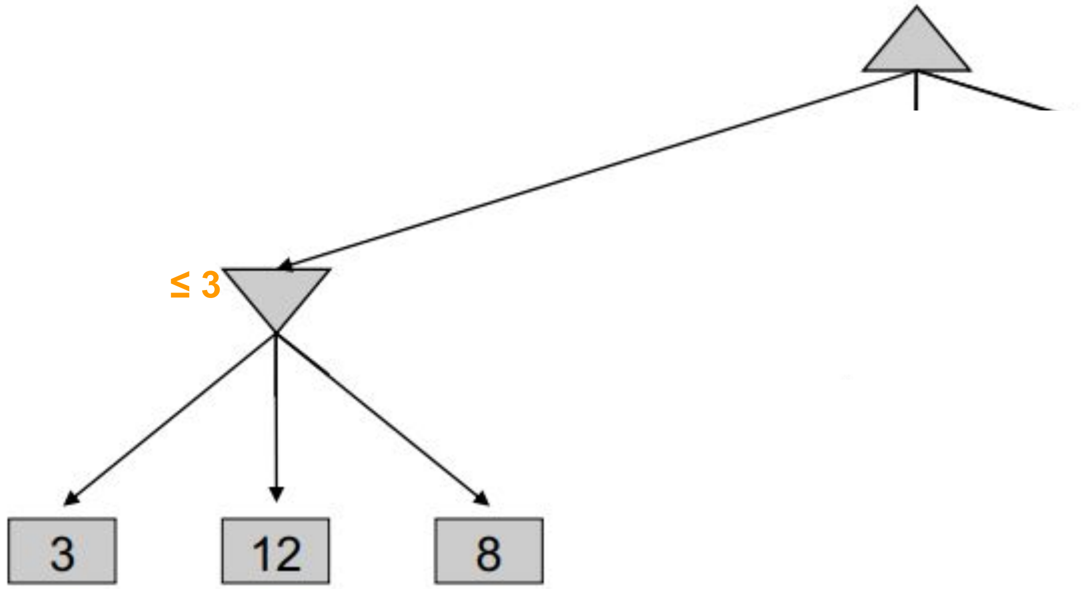

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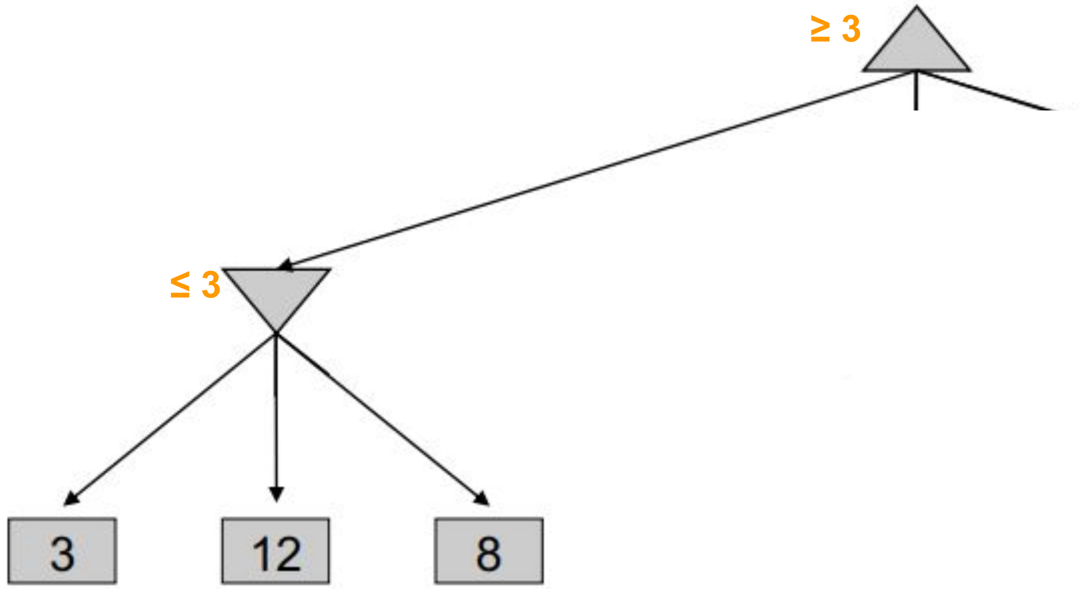
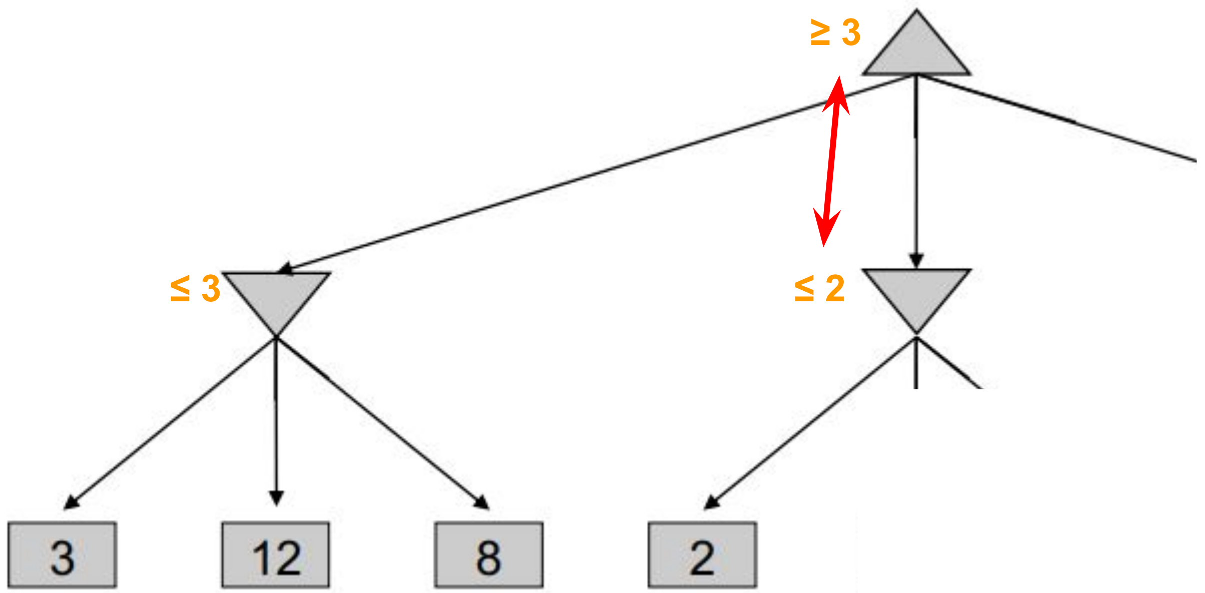
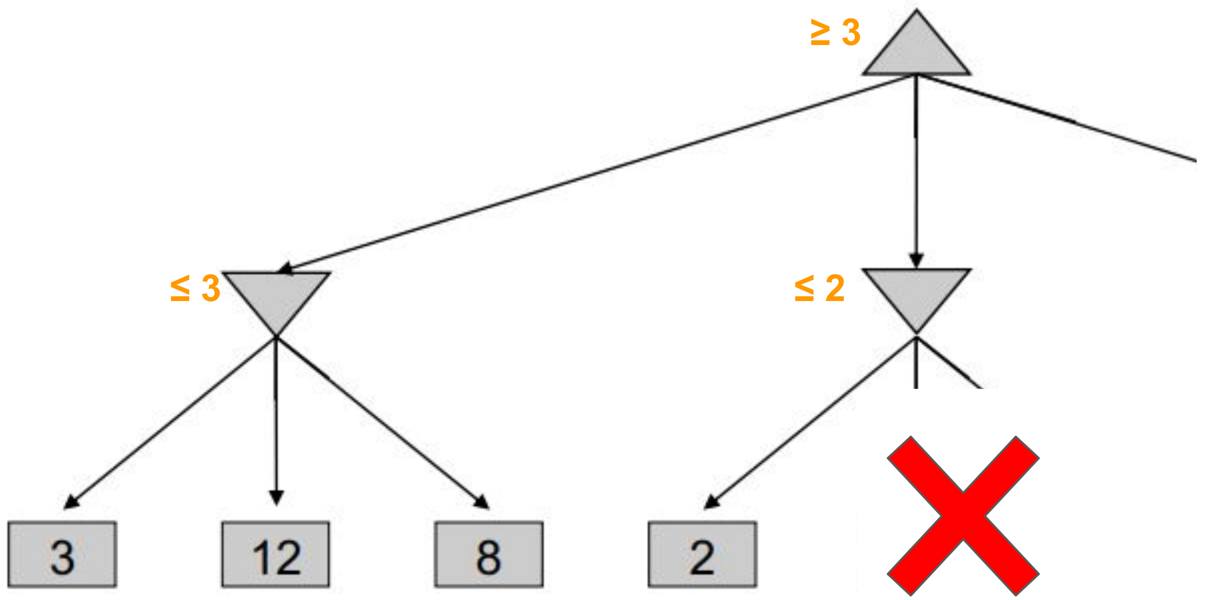


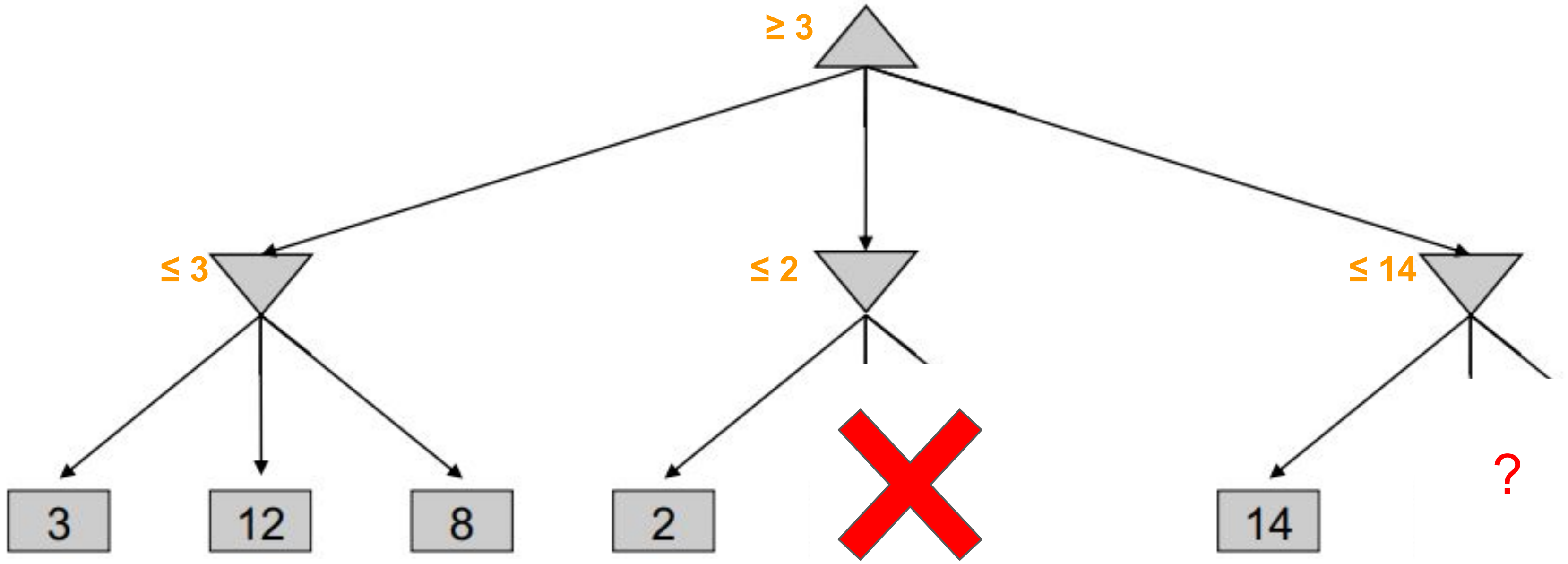
Pruning



Pruning



Pruning



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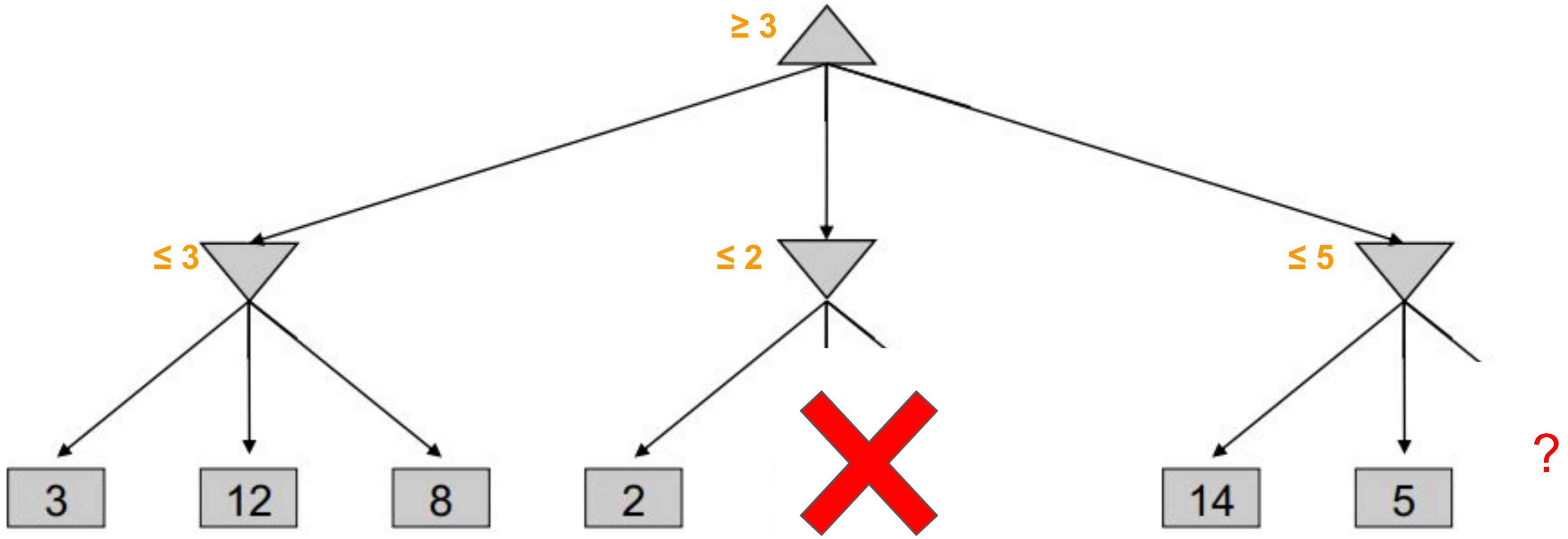
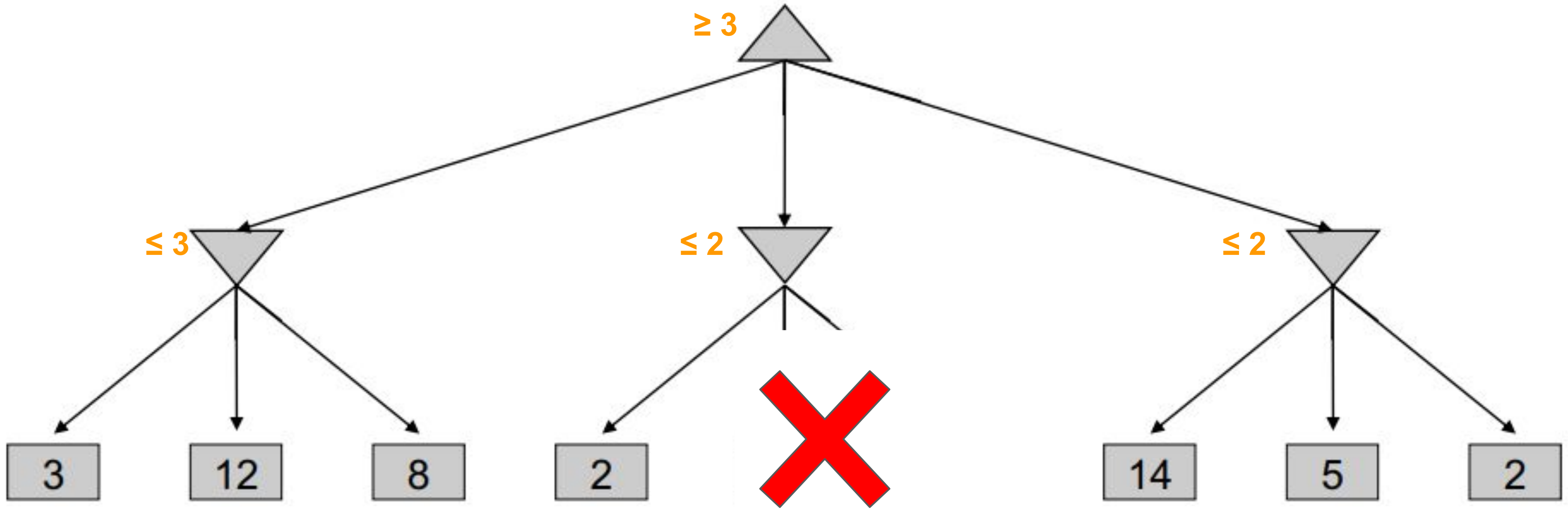
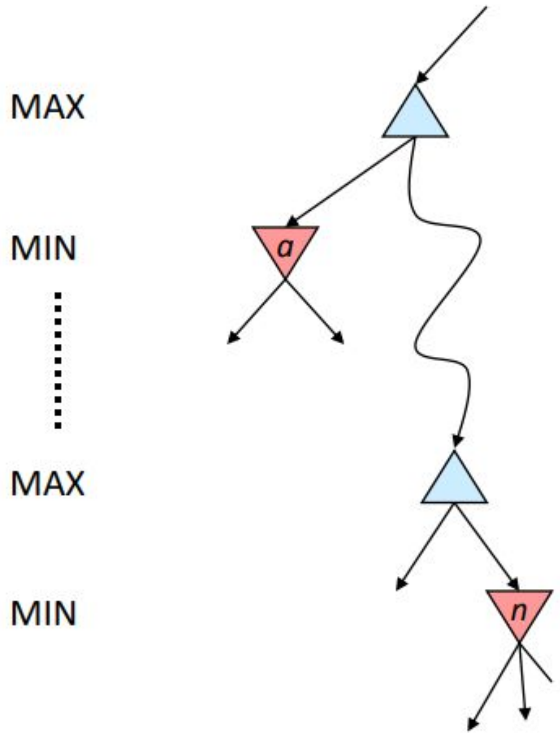


Pruning



Alpha-Beta Pruning

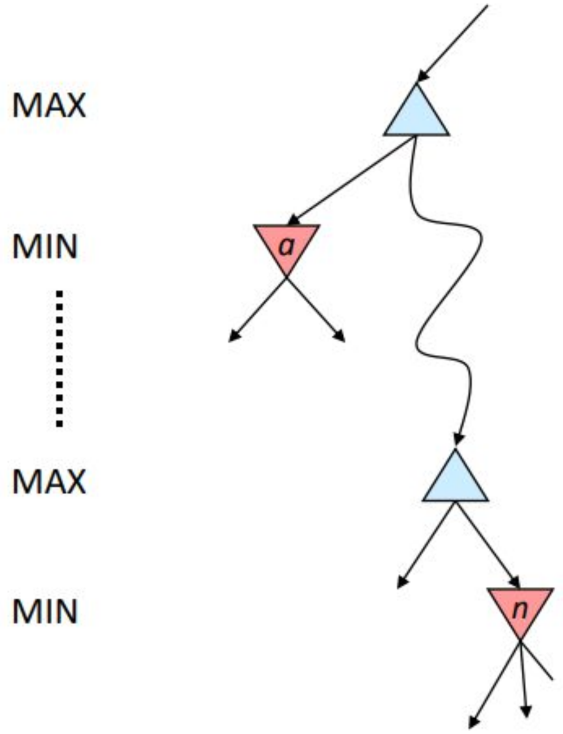
- The **problem with minimax** search is that the **number of game states** it has to examine is **exponential** in the **depth** of the tree.
- We can effectively **cut** the **search** and compute the correct minimax decision **without looking at every node** in the game tree.
- **Alpha–beta pruning** can be applied to trees of any depth, and it is often possible to **prune entire subtrees rather than just leaves**.
- Alpha–beta pruning gets its name from the following two parameters that describe **bounds on the backed-up values** that appear anywhere along the path:
  - **α** = the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX .
  - **β** = the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN .



Alpha-Beta Pruning

- Alpha–beta search **updates the values of α and β** as it goes along.

- It **prunes** the **remaining branches at a node** (i.e., terminates the recursive call) as soon **as the value** of the current node is known to be **worse than the current α or β** value depending on whether the current node is MAX or MIN.



Alpha-Beta Implementation

```
def max-value(state):
    if terminal-test(state):
            return utility(state)
    initialize v = —∞
    for each successor of state:
            v = max(v, min-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

```
def min-value(state):
    if terminal-test(state):
            return utility(state)
    initialize v = +∞
    for each successor of state:
            v = min(v, max-value(successor))
    return v
```

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

α: MAX's best option on path to root

β: MIN's best option on path to root

```
def max-value(state, α, β):
    if terminal-test(state):
            return utility(state)
    initialize v = —∞
    for each successor of state:
            v = max(v, min-value(successor, α, □))
            if v ≥ □ return v
            α = max(α,v)
    return v
```

```
def min-value(state , α, β):
    if terminal-test(state):
            return utility(state)
    initialize v = +∞
    for each successor of state:
            v = min(v, max-value(successor, α, □))
            if v ≤ α return v
            □ = min(□,v)
    return v
```

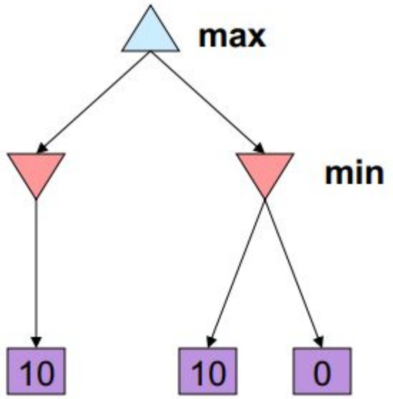
Alpha-Beta Pruning Properties

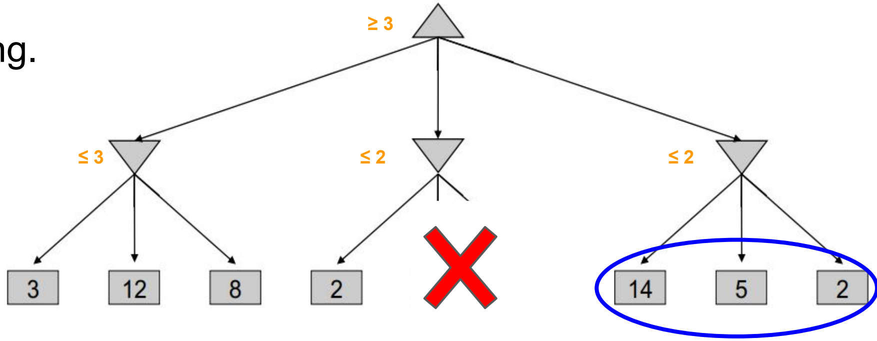
- This pruning has **no effect on the minimax value computed for the root**!

- **Values of intermediate nodes** might be **wrong**.
  - Important: children of the root may have the wrong value.
  - So the most naive version won't let you do action selection.

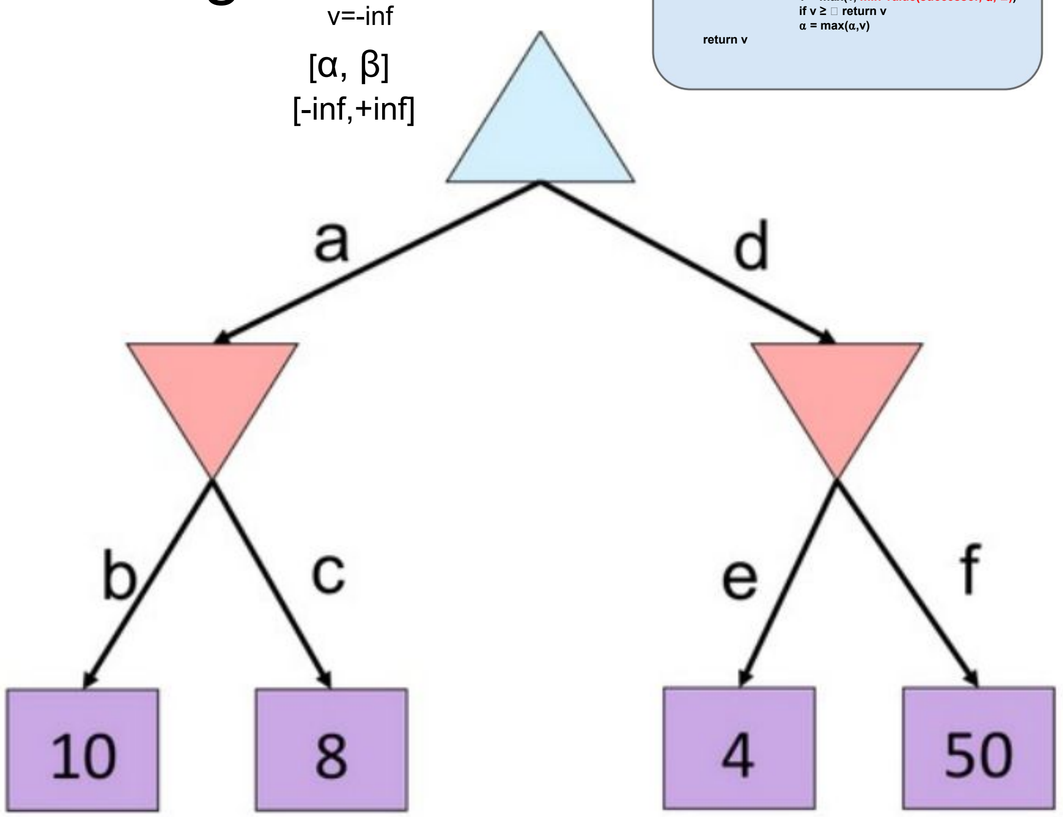
- Good child ordering improves the effectiveness of pruning.

- With "perfect ordering":
  - Time for search goes down.
  - Doubles solvable depth!
  - Full search of, e.g., chess, is still hopeless…





Alpha-Beta Pruning



```
def max-value(state, α, β):

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    return v
```

```
def min-value(state , α, β):

    if terminal-test(state):
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    initialize v = +∞
    for each successor of state:
                v = min(v, max-value(successor, α, □))
                if v ≤ α return v
                □ = min(□,v)
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```

Alpha-Beta Pruning

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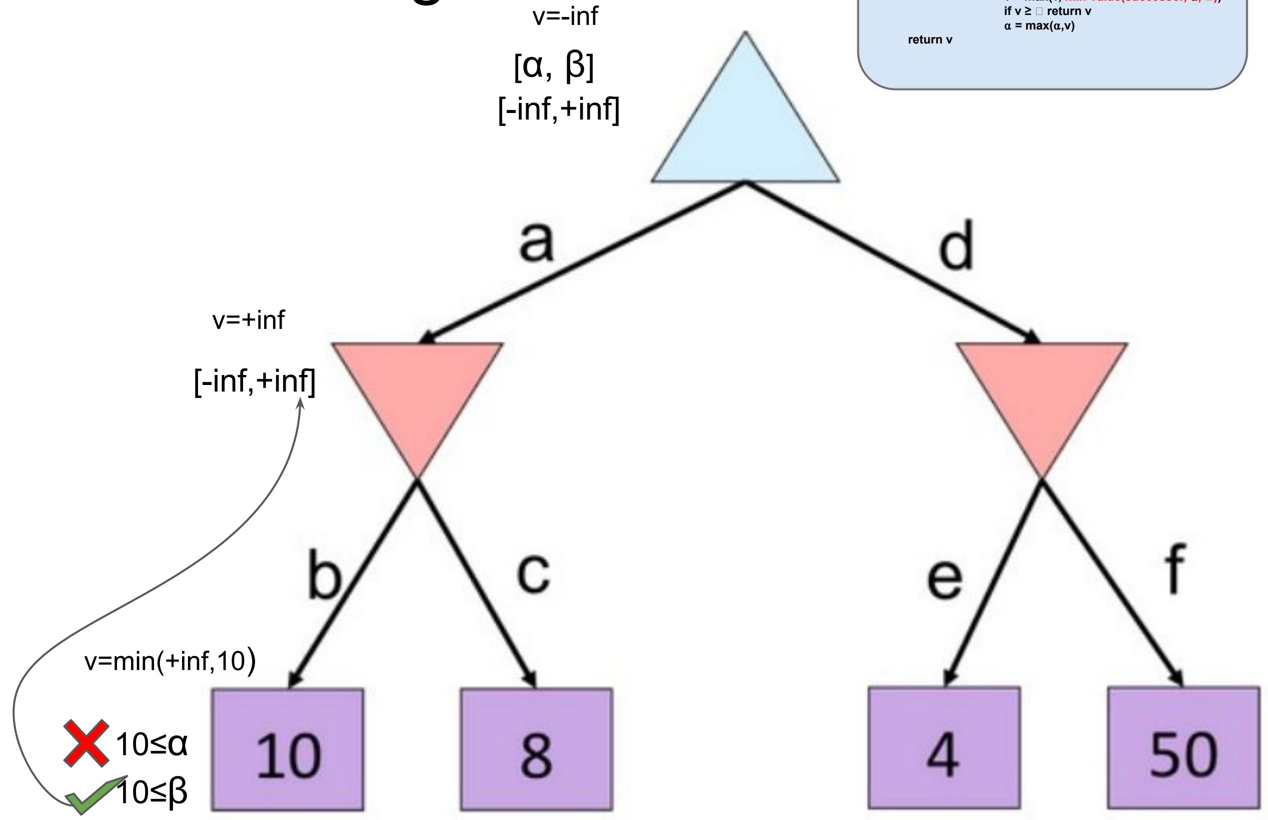

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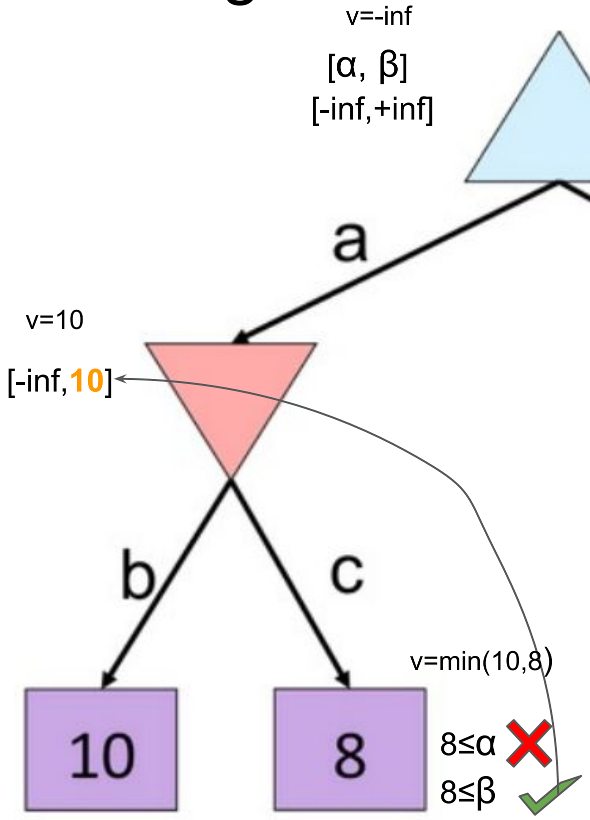

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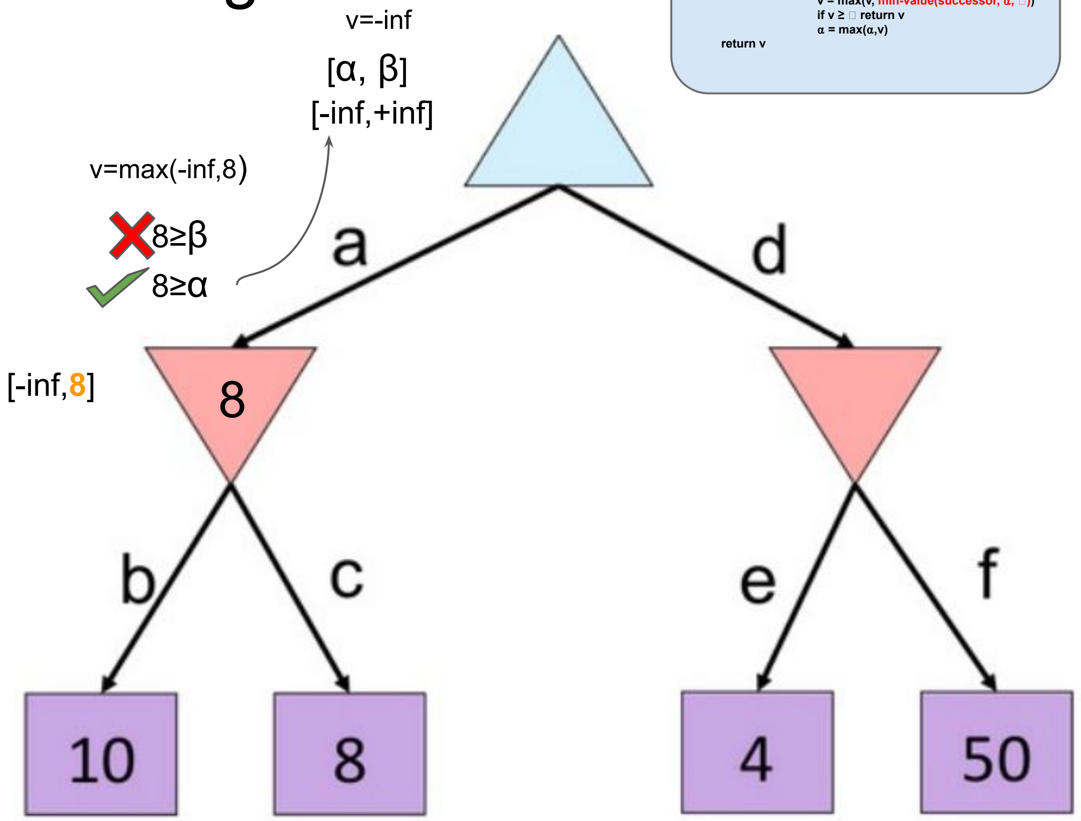

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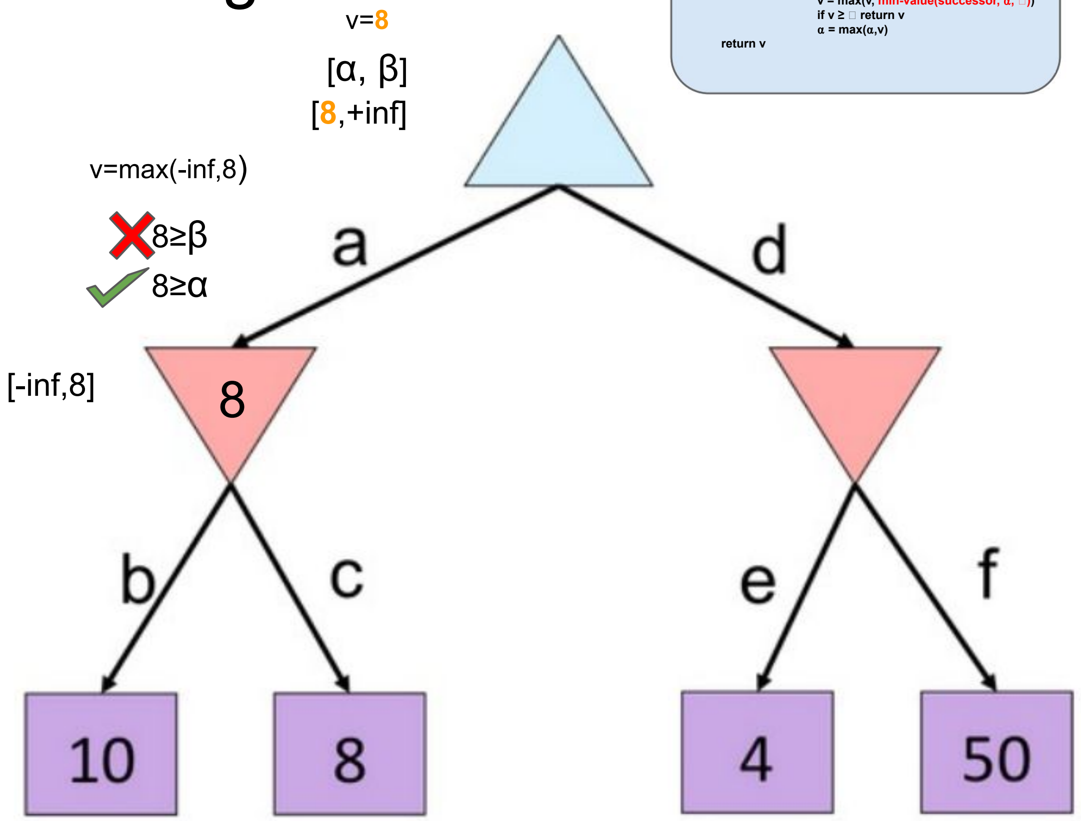

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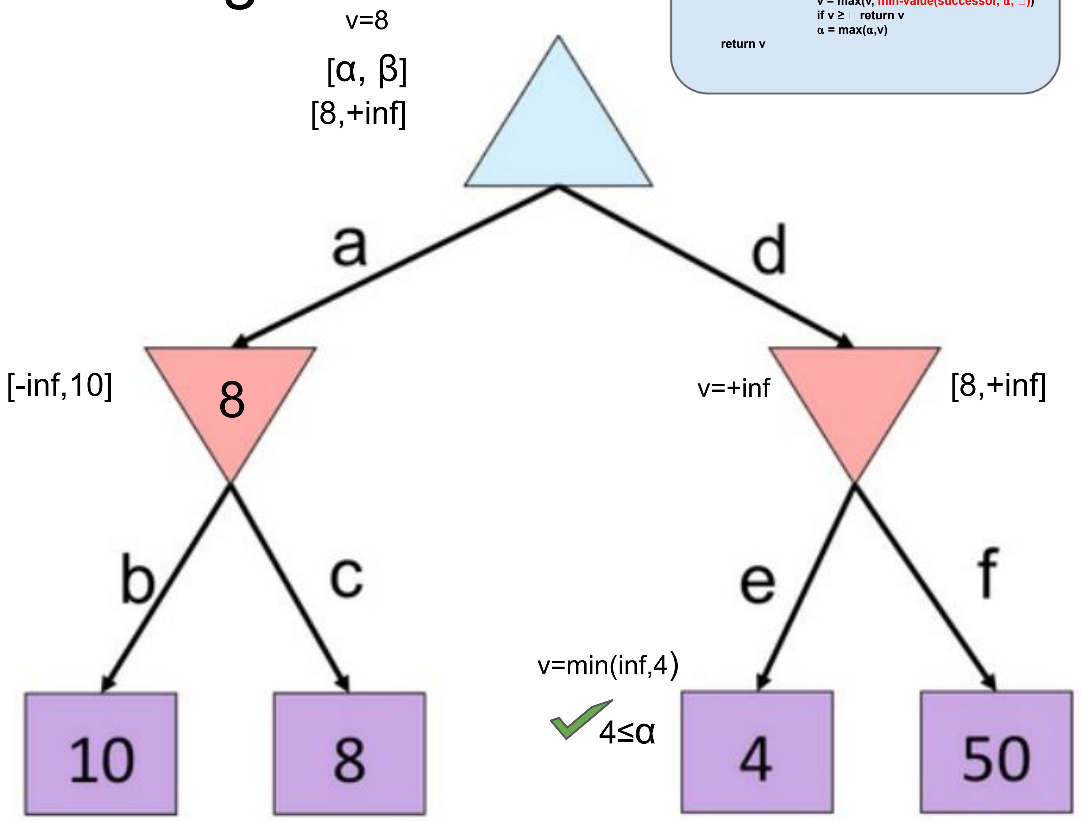

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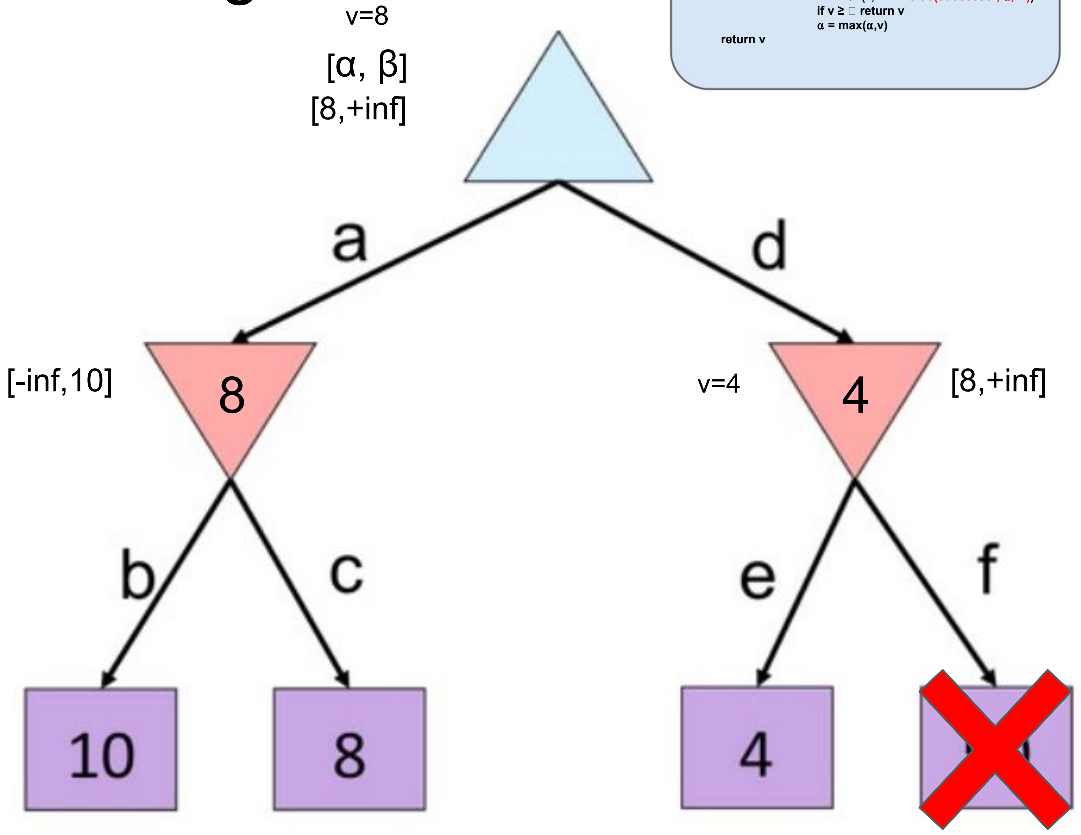

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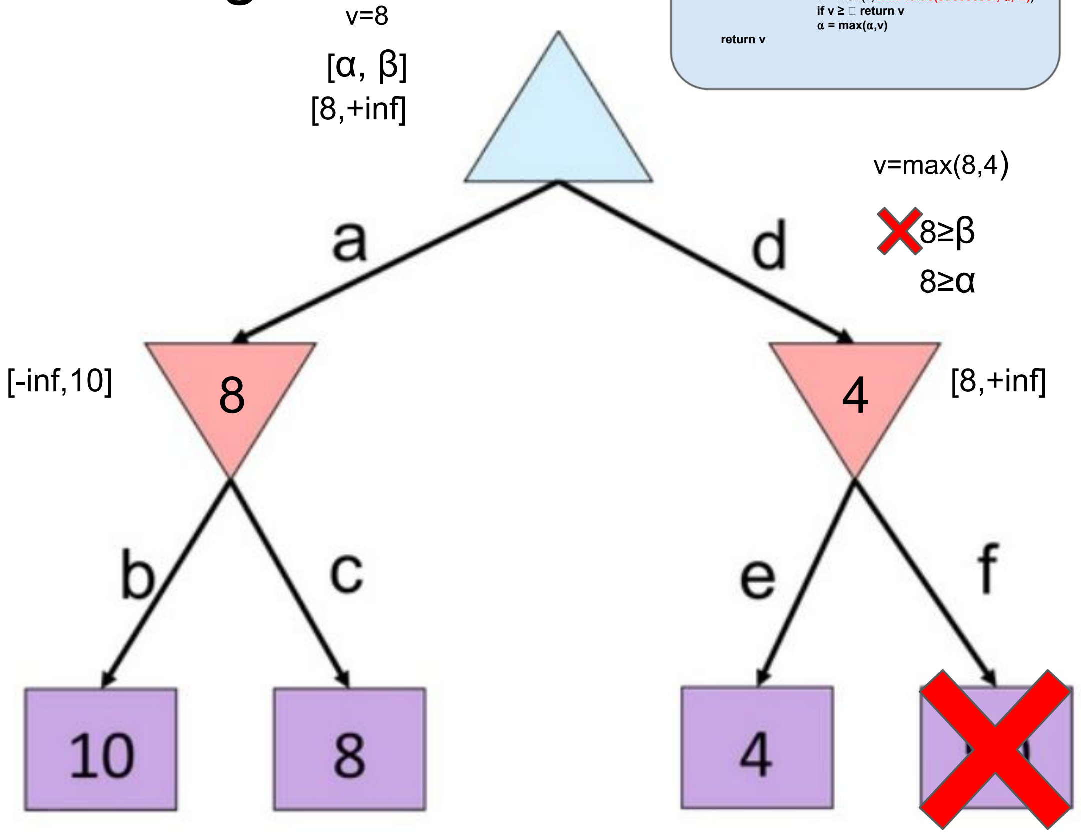

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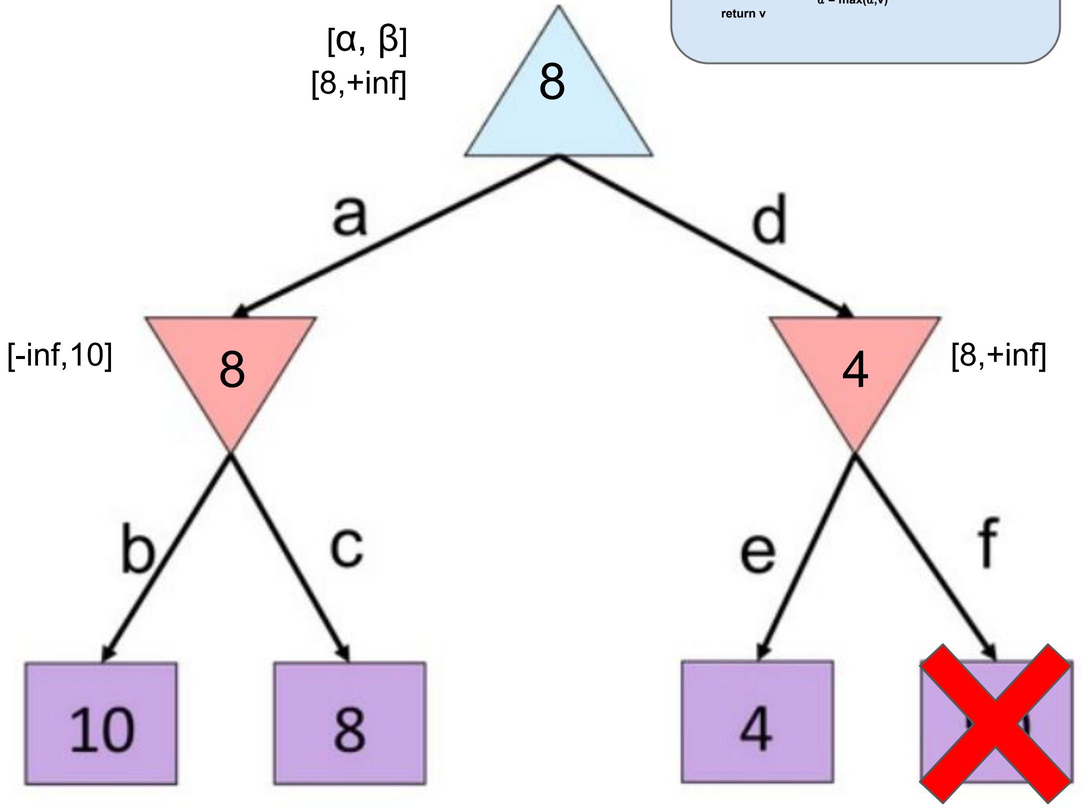

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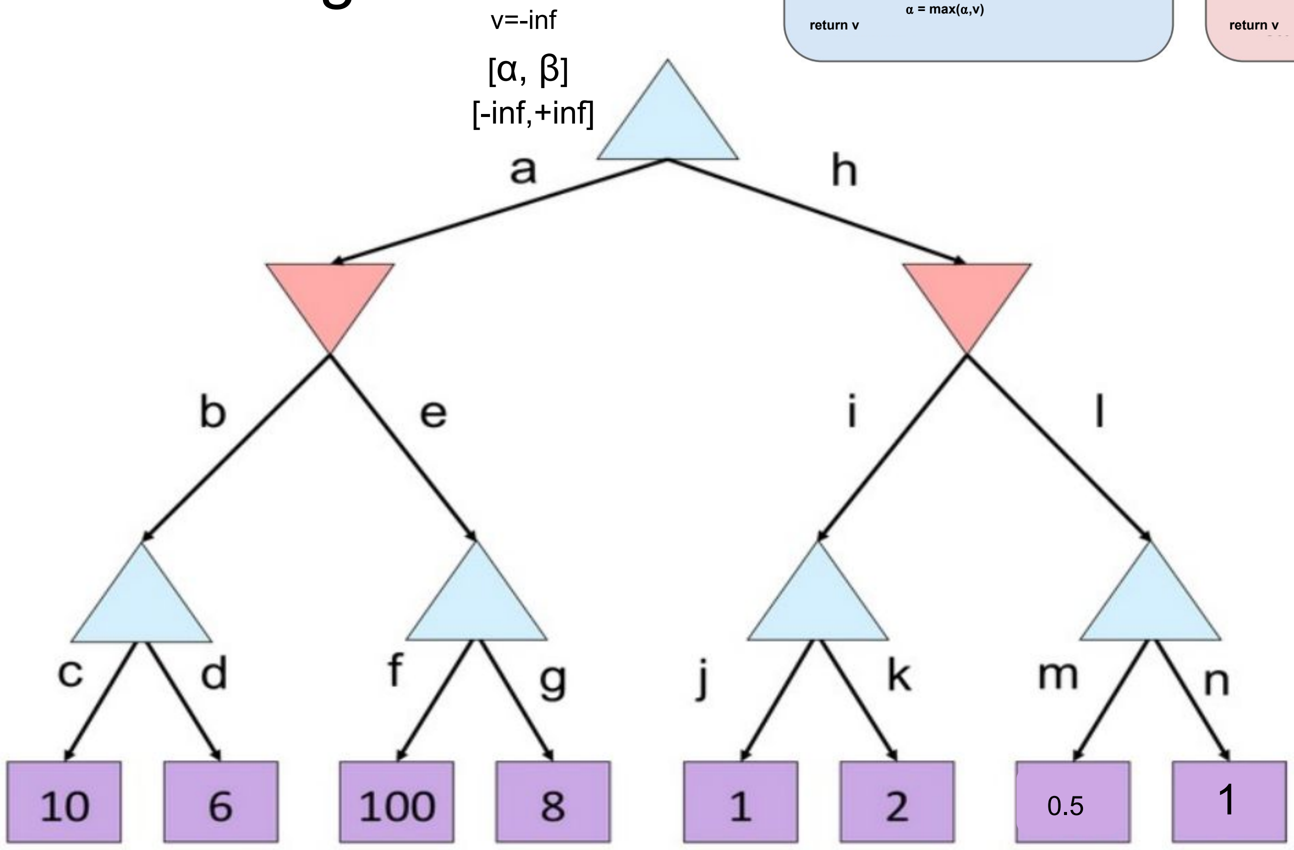

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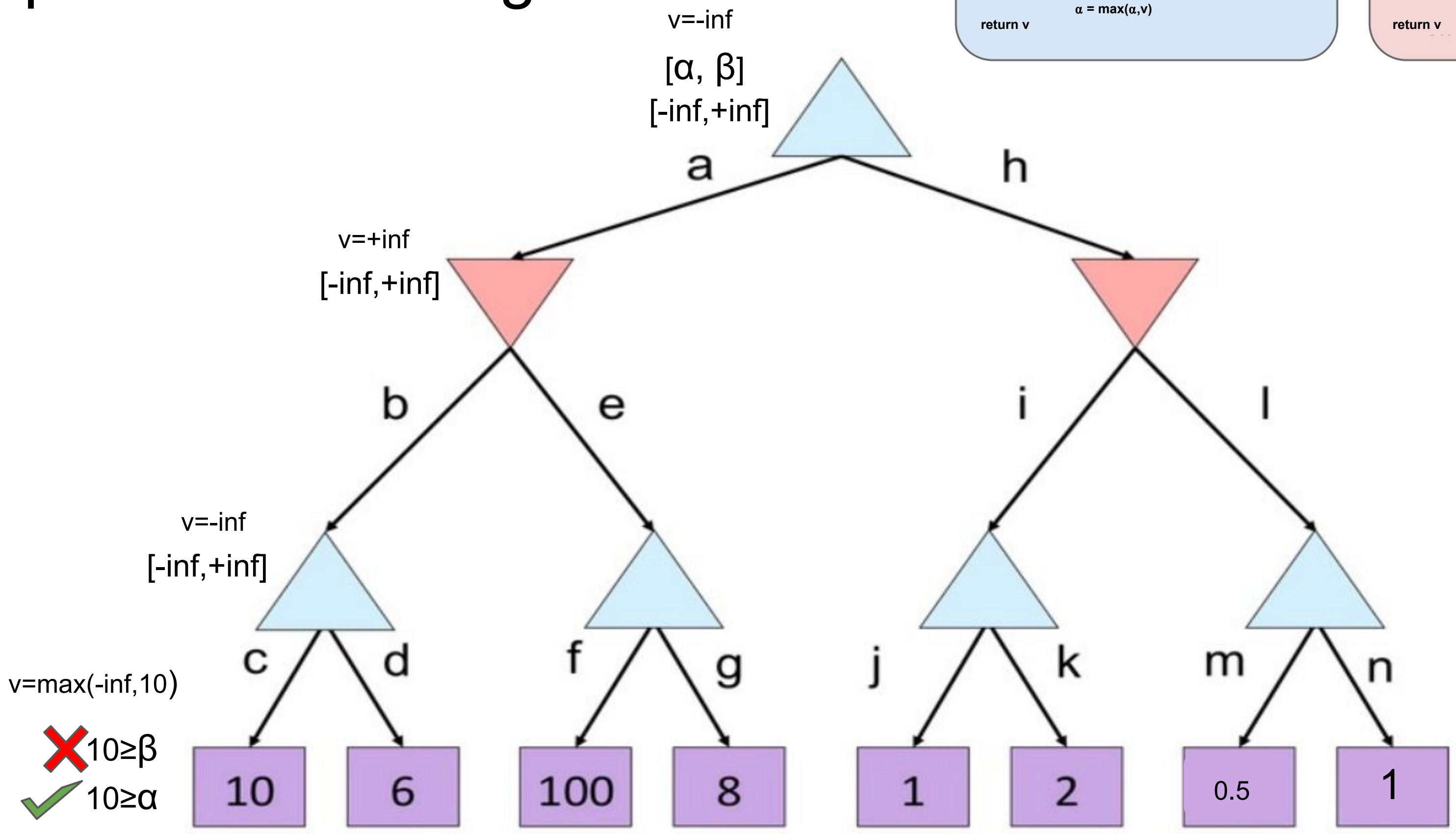

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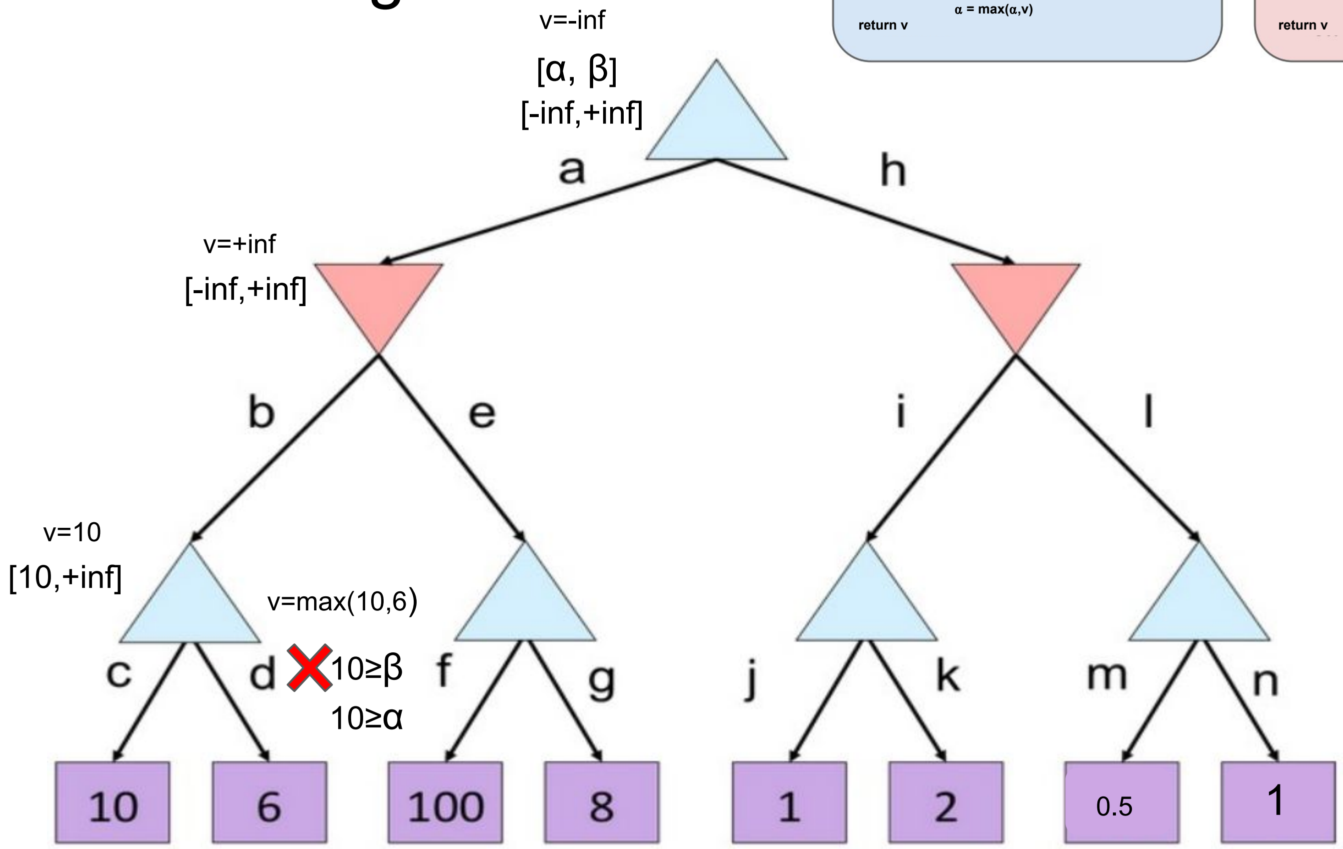

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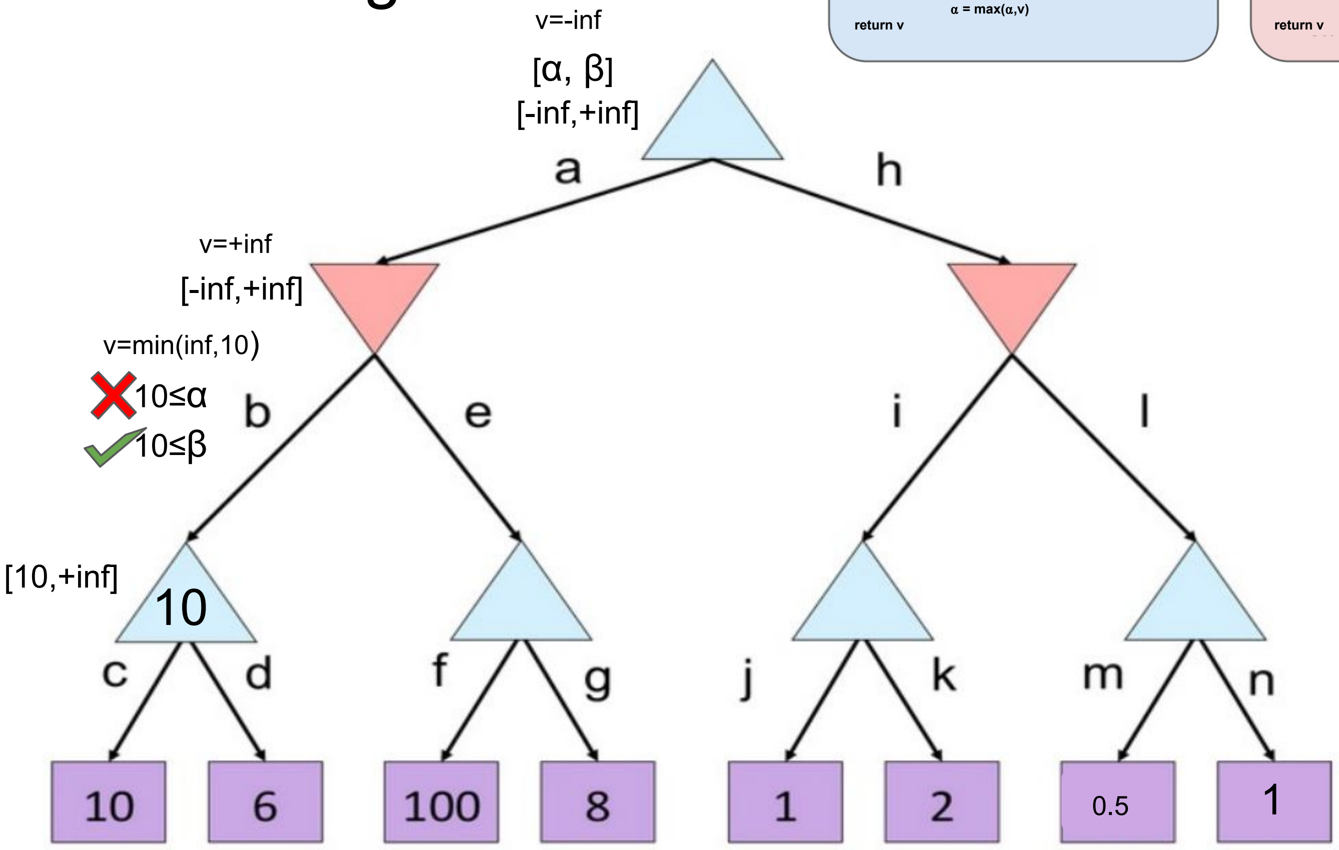

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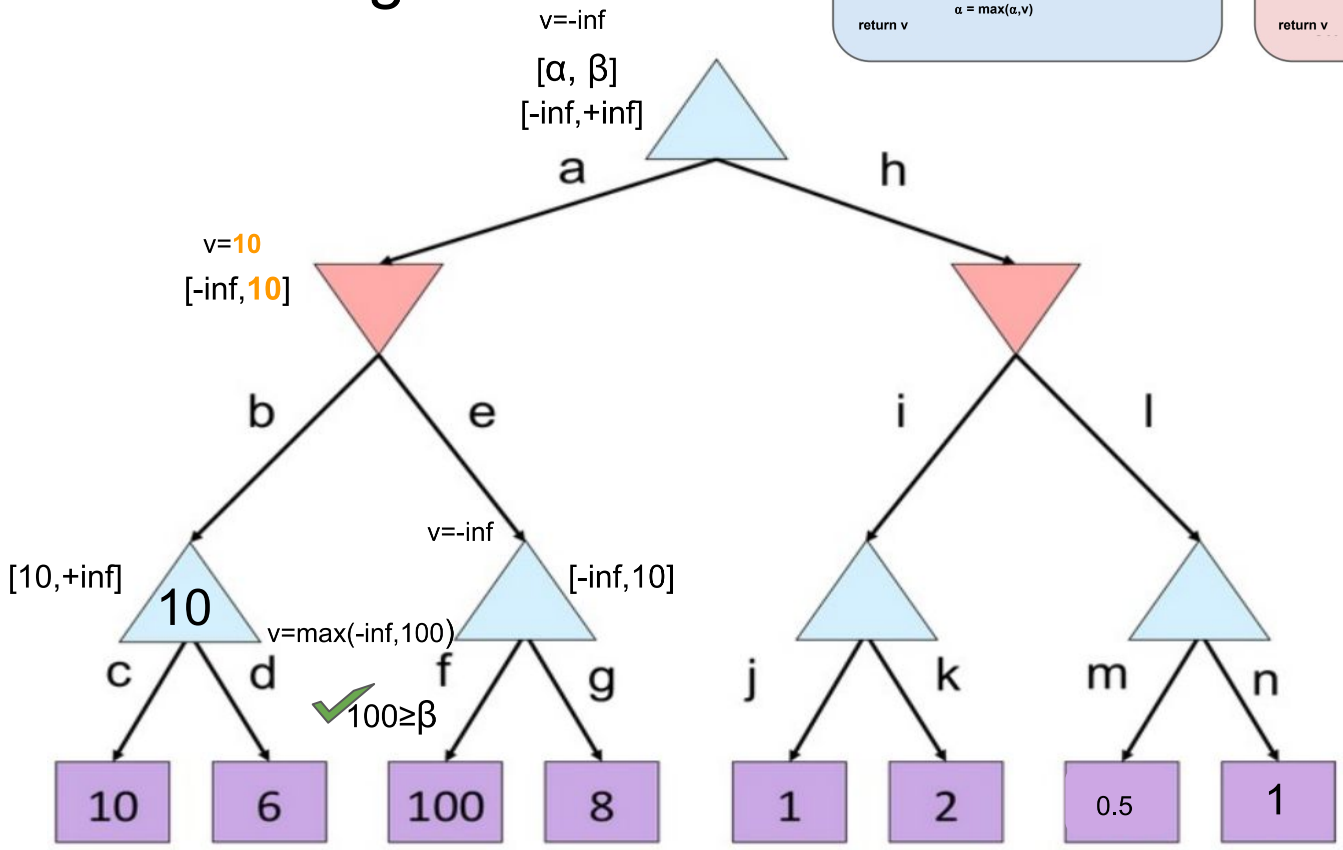

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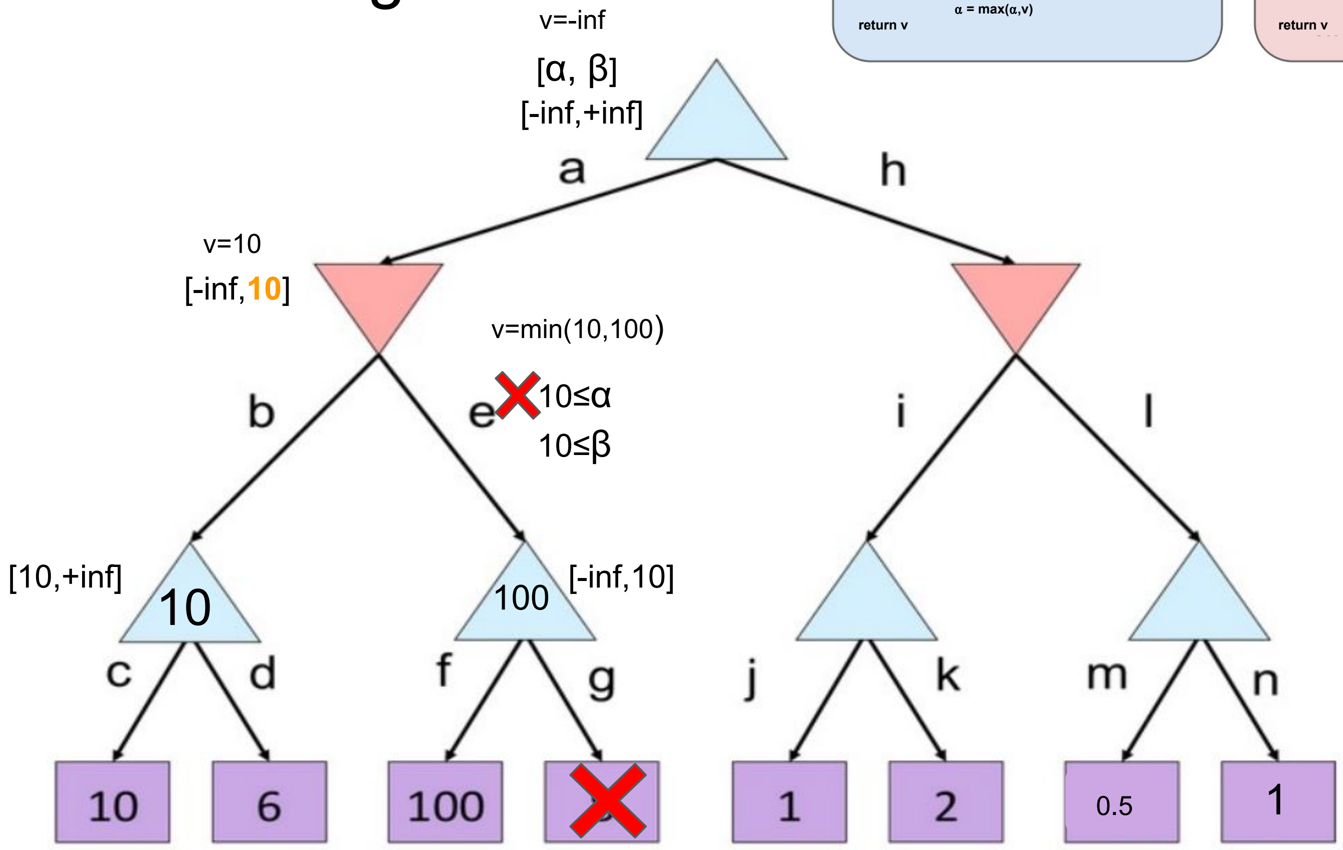

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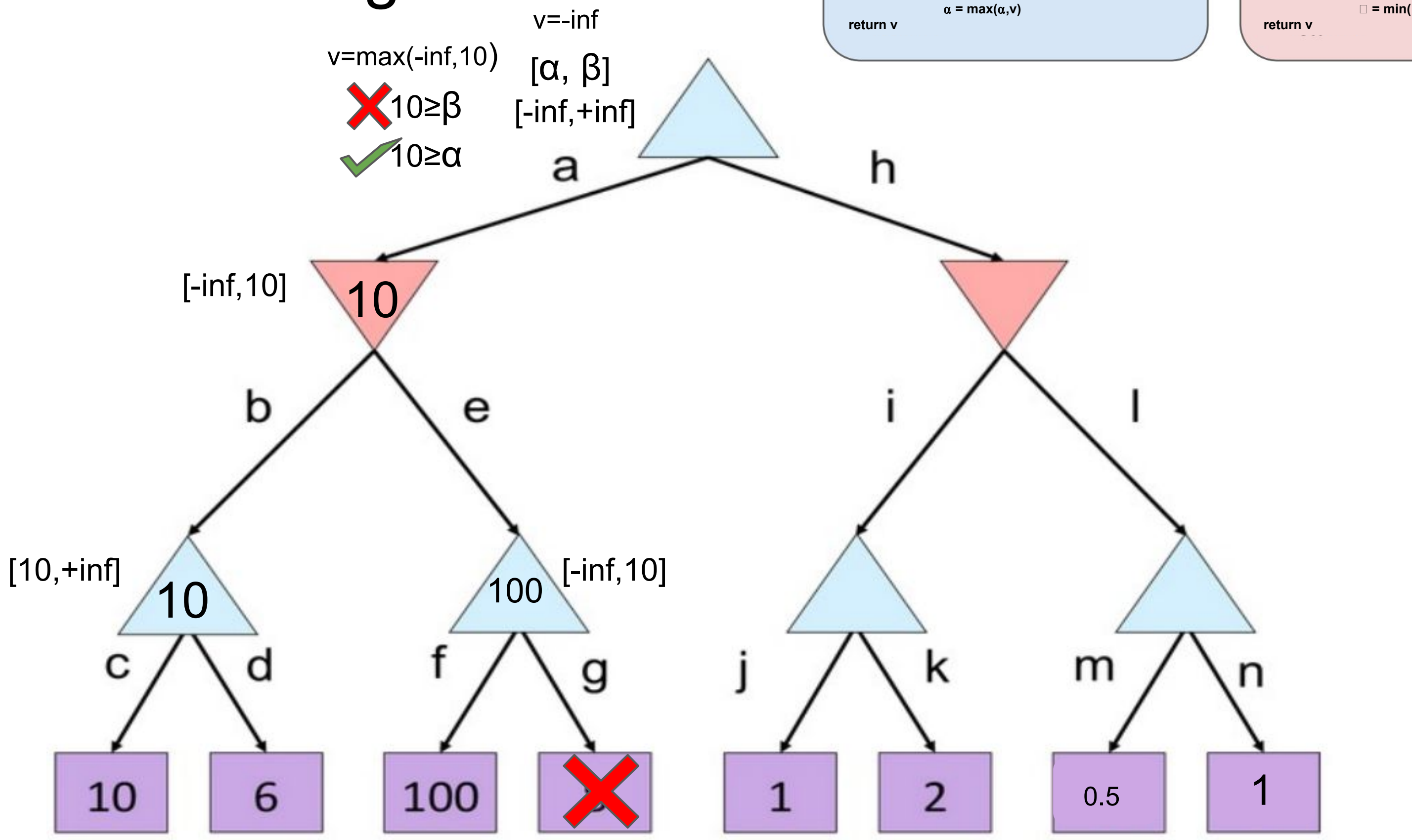

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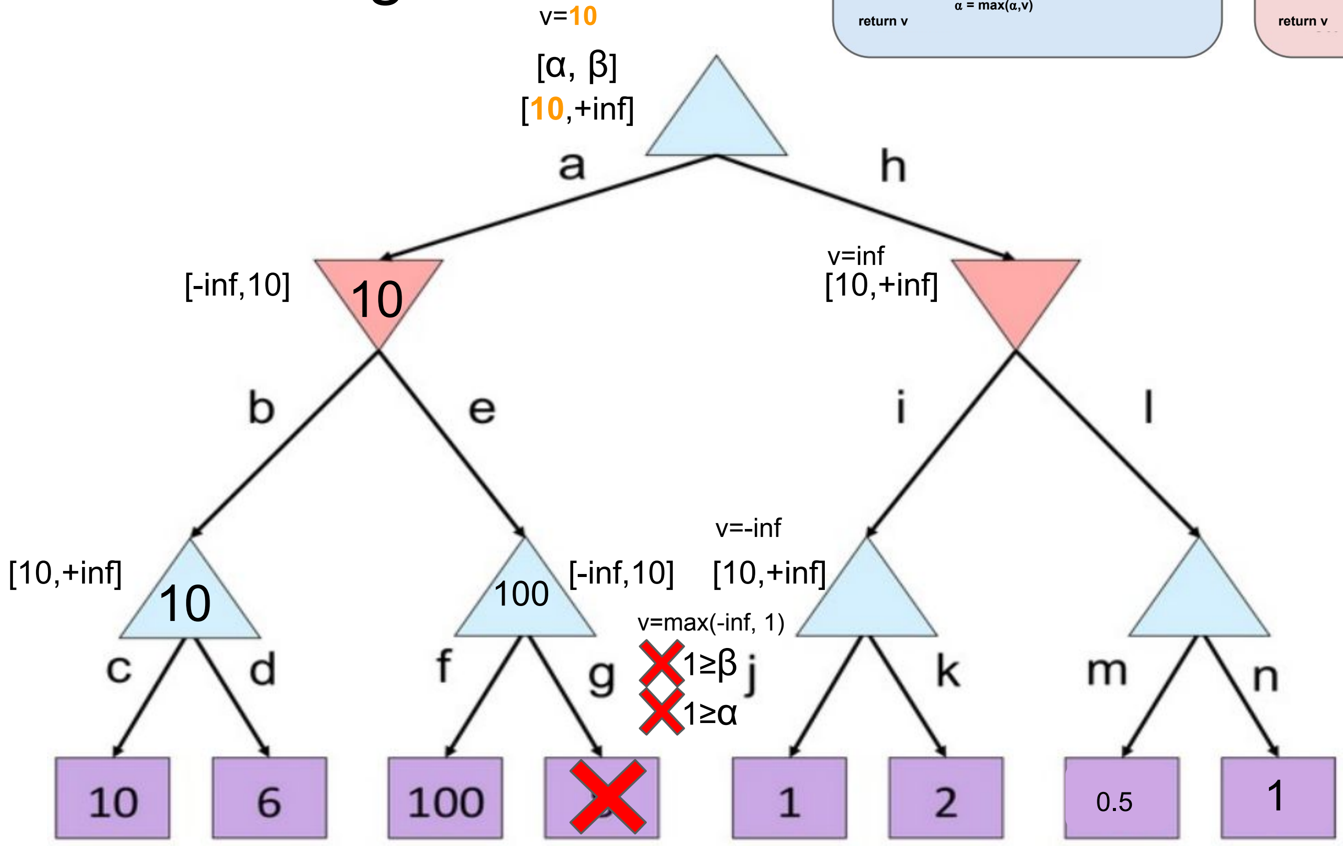

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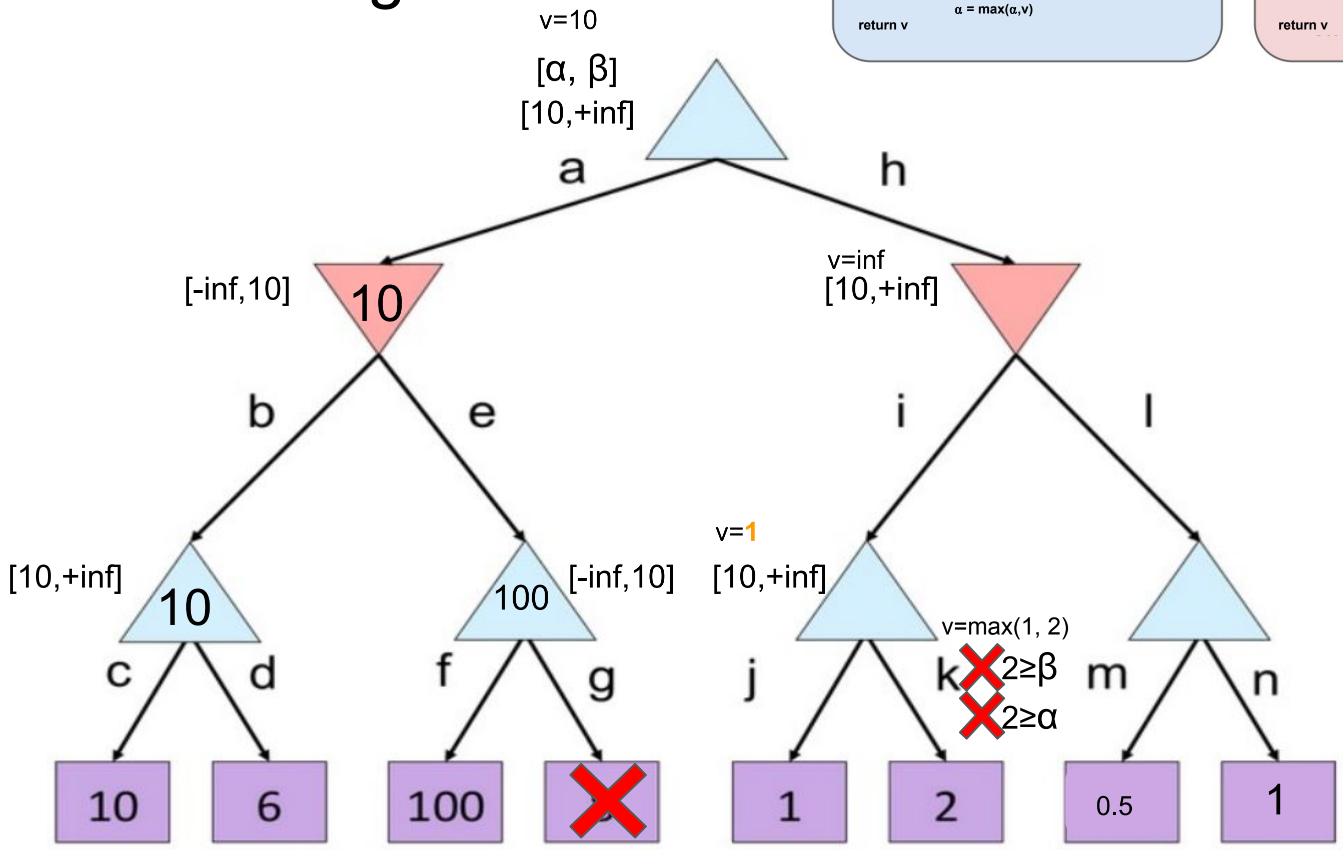

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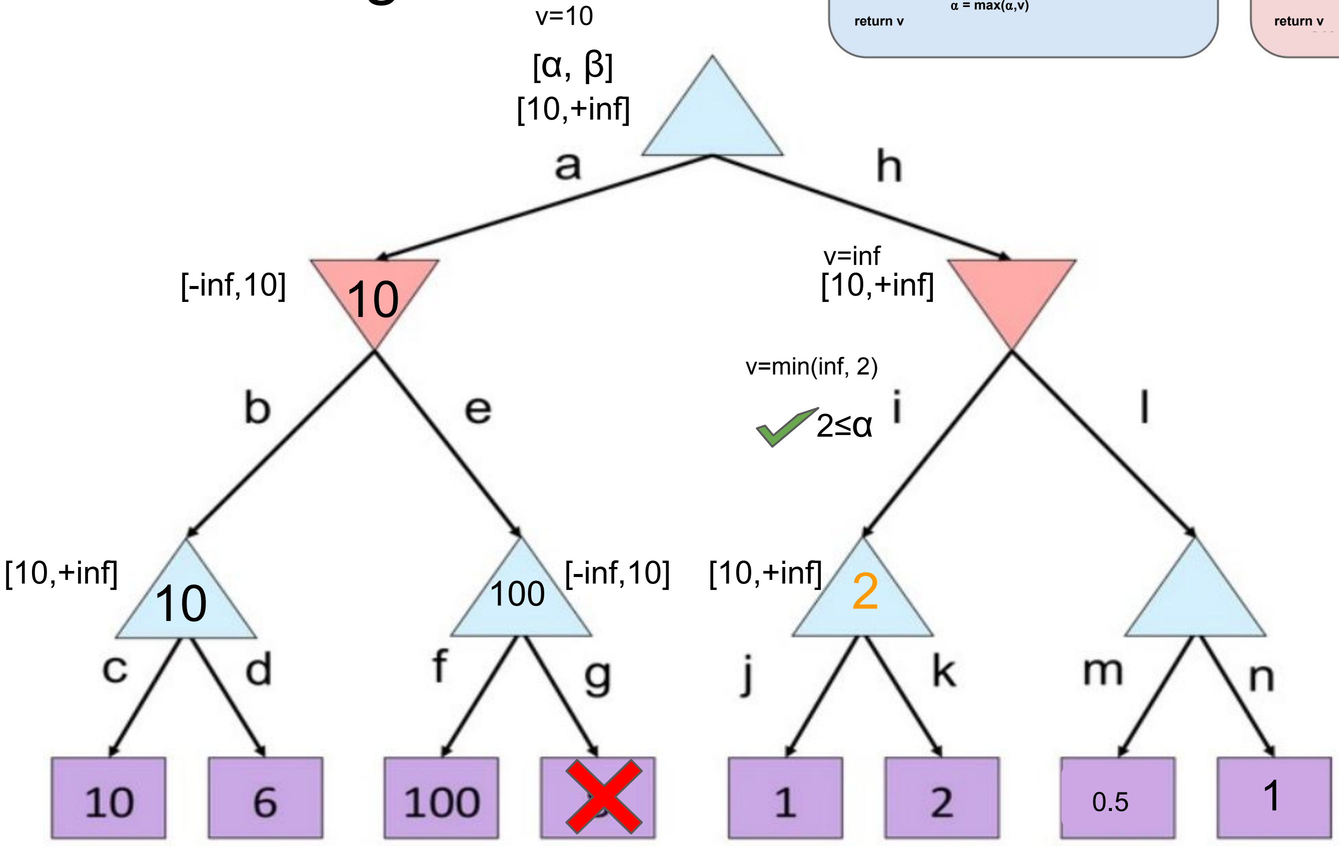

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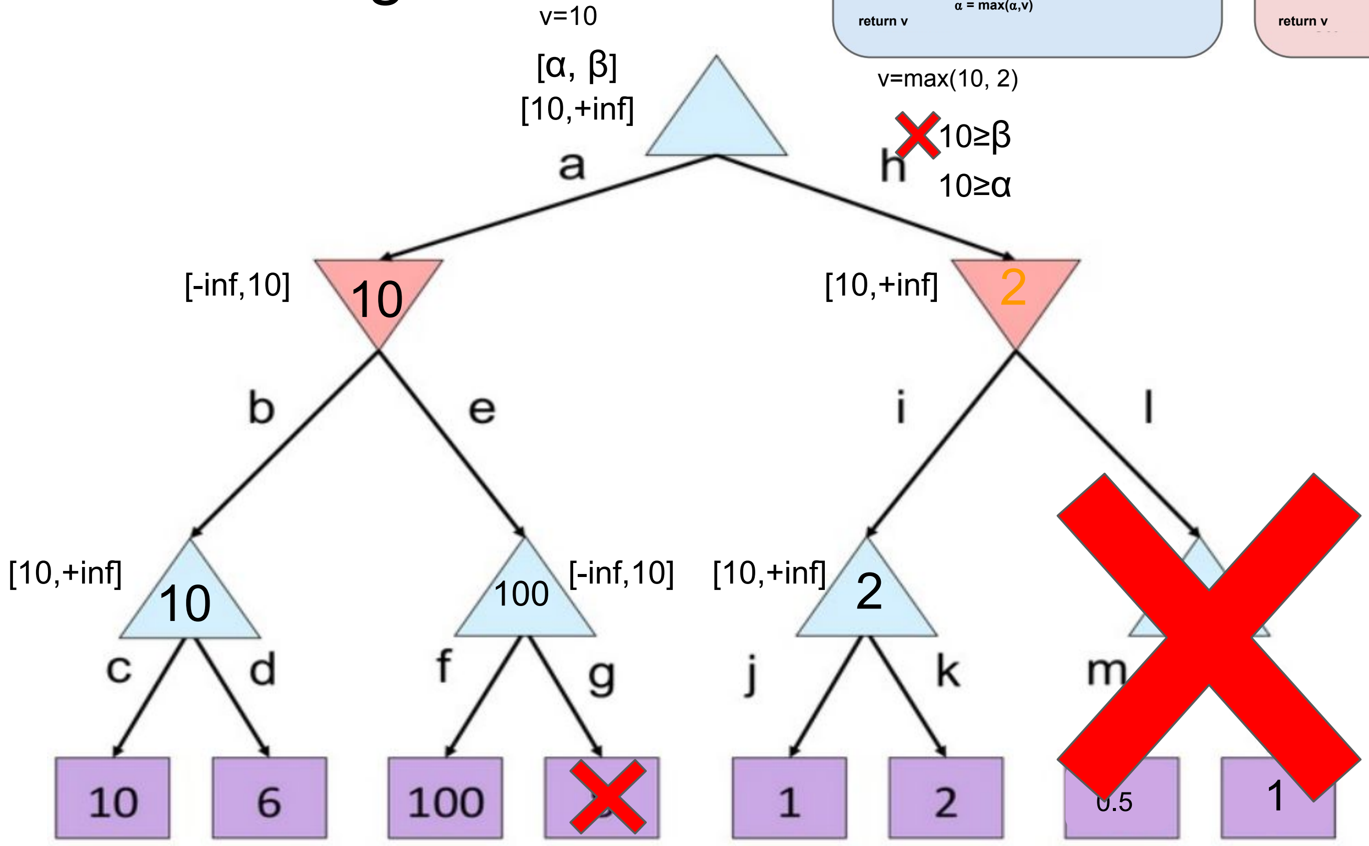
    return v
```



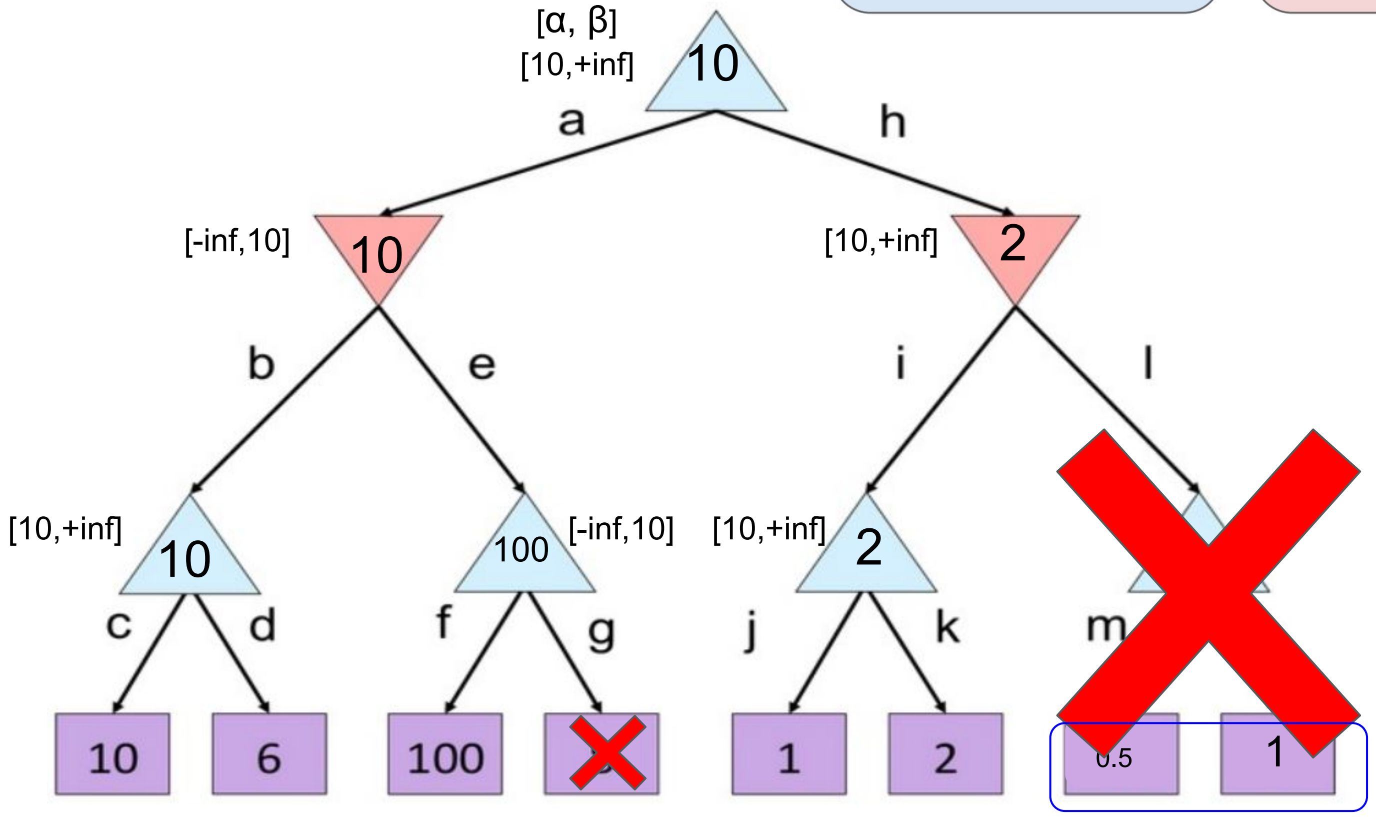
Alpha-Beta Pruning

```
def max-value(state, α, β):

    if terminal-test(state):
            return utility(state)
    initialize v = —∞
    for each successor of state:
            v = max(v, min-value(successor, α, □))
            if v ≥ □ return v
            α = max(α,v)
    return v
```

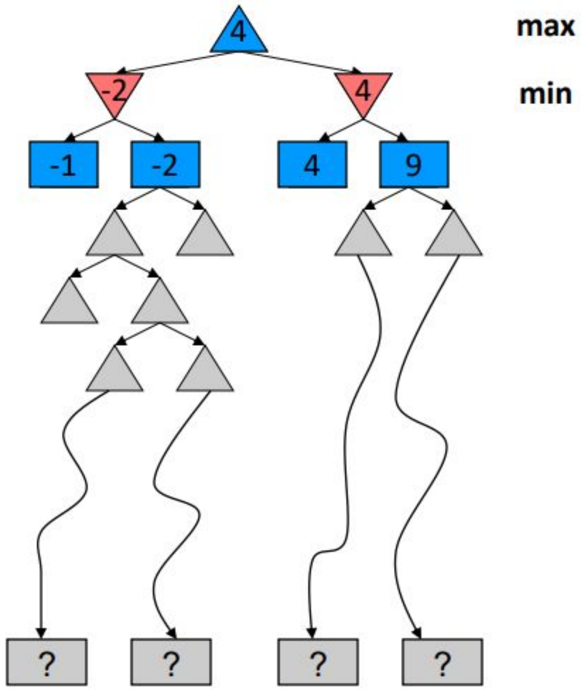
```
def min-value(state , α, β):

    if terminal-test(state):
            return utility(state)
    initialize v = +∞
    for each successor of state:
            v = min(v, max-value(successor, α, □))
            if v ≤ α return v
            □ = min(□,v)
    return v
```

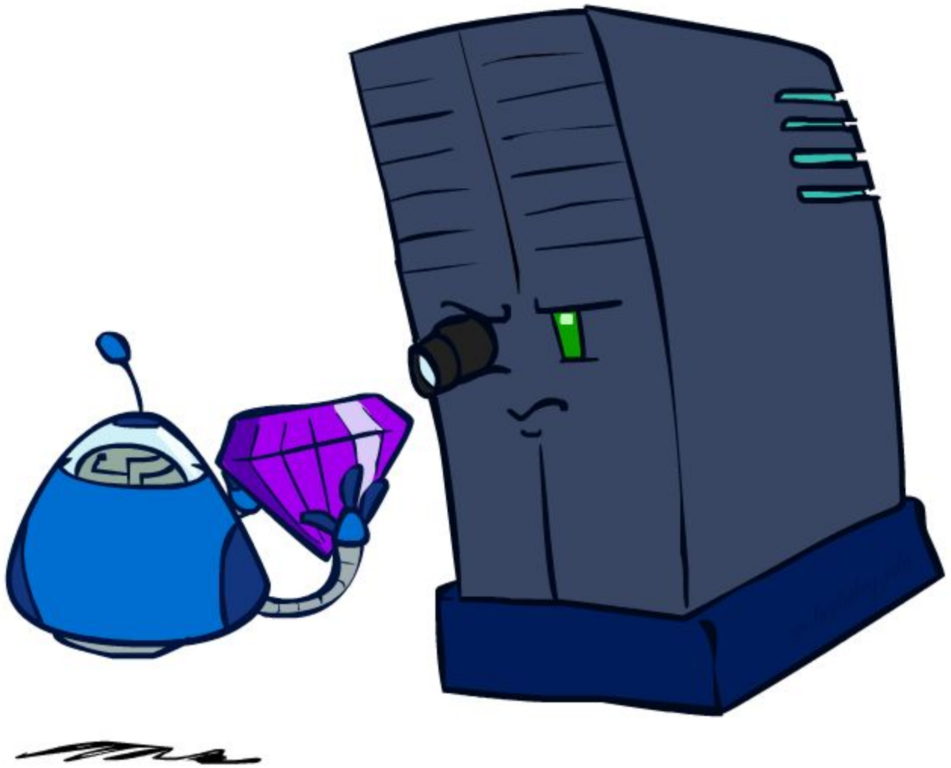


Overcoming Resource Limits: Limiting Depth

- Problem: In realistic games, you cannot search upto leaves!
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Use an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds for a move, and can explore 10K nodes per second.
  - So can check 1M nodes per move
- Guarantee of optimal play is gone.
- More plies/moves makes a BIG difference.
- Use iterative deepening.

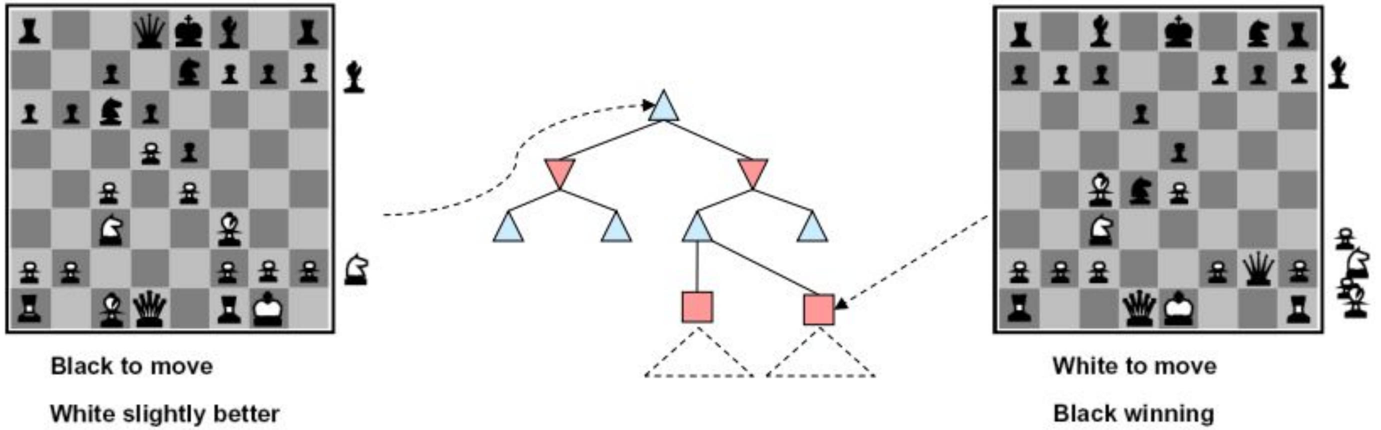


Evaluation Function



Evaluation Function

- Evaluation functions score non-terminals in depth-limited search



- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. f1(s) = (num white queens – num black queens), etc.

Evaluation Function

- Evaluation functions are always imperfect.

- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters.

- Tradeoff between complexity of features and complexity of computation