# STAT 5120 Final Project

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# 1 Introduction

This report is to build and select a model for the response y, using the generated data set "dat.cvs" given by the course instructor. Most of the tools studied in course STAT 5120 (Linear Regression) will be used to facilitate model selection, model estimation, model validation, and model diagnosis procedures. This report is written by Shaoran Sun individually.

#### 2 Materials and methods

Data is given by the instructor. Data contains one response variable, y, and 10 predictors,  $x_1, x_2, \ldots, x_{10}$ . There are 100 intances of data, in total of 100 rows. See the data on Appendix I

The analysis method used are:

- Linear Transformation
- Detecting Outliers in Predictors with leverages
- Detecting Infuential Observations with DFFITS and Cook's Distance
- Automated Forward Selection Procedure
- Diagnosing Multicollinearity with Variance Inflation Factor (VIF)
- General Linear F Test

The criteria for statistical significance is P < 0.05.

### 3 Results

With the given data set "dat.cvs",

- y is log-transformed
- Entry 10 is reomoved due to large influence to the overall data
- Predictors x1, x6, x7, x8, x9, and x10 are removed due to insignificance.
- Predictors x2, x3, x4, and x5 are kept, and combined as a first order model.

The final model is:

```
y = 2.00 \cdot x_2 + 0.91 \cdot x_3 + 0.40 \cdot x_4 + 3.35 \cdot x_5 + 1.51
```

```
Call:
lm(formula = y ~ x2 + x3 + x4 + x5)
Residuals:
                 1Q Median
-2.29129 -0.46534 0.00512 0.62212 1.87873
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.50163
                           0.08829 22.605 < 2e-16 ***
               1.99583
                                      8.999 2.46e-14 ***
x3
               0.91006
                            0.10112
               0.40358
3.34805
                           0.08986 4.491 2.01e-05 ***
0.09792 34.191 < 2e-16 ***
х5
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 0.9338 on 94 degrees of freedom Multiple R-squared: 0.955, Adjusted R-squared: 0.9
                                     Adjusted R-squared: 0.9531
F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16
```

The intercept, x2, and x5 all have p-values of (< 2e - 16), x3 has p-value of 2.46e - 14, and x4 has p-value of 2.01e - 05. The overall p-value is (< 2.2e - 16). These p-values are all less than 0.05 and significant.

The  $R^2$  is 0.955, which means the model represents 95.5% of the data.

# 4 Detailed Procedures

#### 4.1 Read in data and fit the originial data to a model

I first plot in all 10 variables in responsing to y. All the p-values to the variables are greater than 0.05, which indicates that none of the variable is significant.

The 5 assumptions about linear regression are:

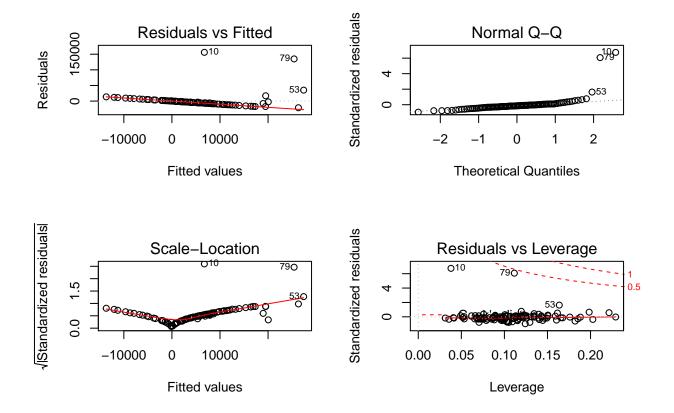
- 1. There exists a linear relation between the response and predictor variable(s).
- 2. The error terms have the constant variance
- 3. The error terms are independent, have mean 0.
- 4. Model fits all observations well (no outliers).
- 5. The errors follow a Normal distribution.

The Residuals vs Fitted graph suggested that residuals do not fall in a horizontal band around 0, and they have an apparent pattern. Assumption 1 and 3 are not met.

The residuals also do not have similar vertical variation across fits. Assumption 2 is not met.

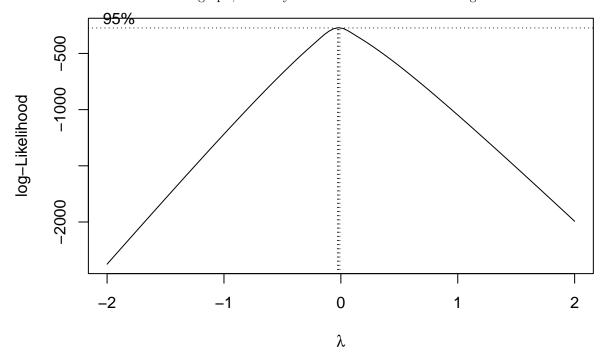
Hence, we consider using a boxcox transformation to test for log-likelihood.

```
##
## Call:
## lm(formula = dat$y ~ ., data = dat)
##
## Residuals:
##
      Min
              1Q Median
                             3Q
                                   Max
## -21213 -7359
                 -3119
                          1056 156003
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                                      1.902
## (Intercept)
                 4856.9
                             2554.0
                                              0.0604
## x1
                18253.4
                            50397.7
                                      0.362
                                              0.7181
               -16220.4
                            50336.8
                                     -0.322
                                              0.7480
## x2
## x3
                 -723.3
                             2565.5
                                     -0.282
                                              0.7786
                  520.9
                             2317.8
                                      0.225
                                              0.8227
## x4
## x5
                 4563.1
                             2562.1
                                      1.781
                                              0.0783
                -2807.6
                             2457.2
                                     -1.143
                                              0.2563
## x6
                                     -0.902
## x7
                -2535.7
                             2810.4
                                              0.3694
## x8
                 2839.7
                             2348.2
                                      1.209
                                              0.2297
## x9
                -3654.9
                             2392.7
                                     -1.527
                                              0.1302
                 -577.3
                             2607.0 -0.221
## x10
                                              0.8253
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 23590 on 89 degrees of freedom
## Multiple R-squared: 0.1113, Adjusted R-squared:
## F-statistic: 1.115 on 10 and 89 DF, p-value: 0.36
```



# 4.2 Boxcox transformation

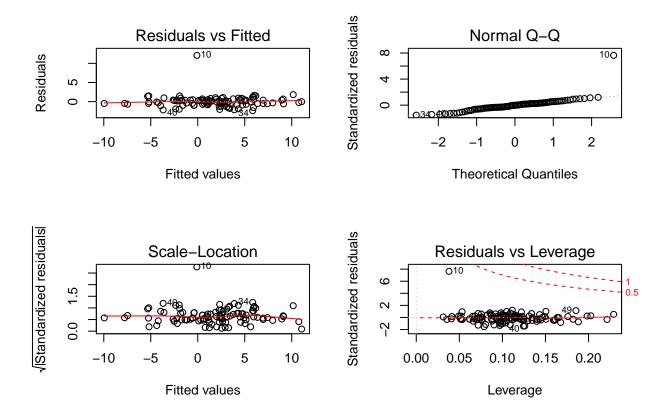
As we can see from the boxcox graph,  $\lambda$  is very close to 0. So we consider a log-transformation.



#### 4.3 Fit the log-transformed y

After the transformation, some of the predictors are starting to become significant, namely, x3, x5, and intercept. The overall p-value also becomes very significant, comparing to before the transformation, where p-value = 0.36.

```
##
## Call:
## lm(formula = dat$y ~ ., data = dat)
##
## Residuals:
##
       Min
                1Q Median
                                 3Q
                                        Max
## -2.3787 -0.6130 -0.0369 0.4978 12.0928
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                1.610750
                            0.174805
                                       9.215 1.35e-14 ***
               -1.323510
                            3.449464
                                      -0.384
                                               0.7021
## x1
## x2
                3.262825
                            3.445294
                                       0.947
                                               0.3462
## x3
                0.945029
                            0.175598
                                       5.382 5.92e-07 ***
                0.349093
                                       2.201
                                               0.0304 *
## x4
                            0.158641
                3.261169
                            0.175364
                                     18.597
                                              < 2e-16 ***
## x5
## x6
               -0.104953
                            0.168182
                                      -0.624
                                               0.5342
                                       0.007
## x7
                0.001392
                            0.192358
                                               0.9942
## x8
                0.086216
                            0.160720
                                       0.536
                                               0.5930
               -0.066157
## x9
                            0.163769
                                      -0.404
                                               0.6872
## x10
                0.080254
                            0.178437
                                       0.450
                                               0.6540
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.614 on 89 degrees of freedom
## Multiple R-squared: 0.8795, Adjusted R-squared: 0.866
## F-statistic: 64.97 on 10 and 89 DF, p-value: < 2.2e-16
```



#### 4.4 Detect and remove outliers

Next, I will detect outliers.

From the above graphs, we can see that outliers definitely exist, possibly, data 10.

Obtain leverages and two measures that can be used to identify influential points, DFFITS (difference in fts) and Cook's Distance.

From the output of influence, DFFITS (difference in fts), and Cook's Distance, we can see that observation 10 is indeed an influential outlier. Hence we remove it.

After removing observation 10, the model fits the data better than before, as we can see in the following output table. Next, we will be choosing which predictors are actually significant.

```
##
## Call:
## lm(formula = no10$y ~ ., data = no10)
##
## Residuals:
##
       Min
                 1Q
                     Median
                                          Max
   -2.3706 -0.4279
                     0.0241
                              0.5760
                                      1.8382
##
##
##
  Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
                                                < 2e-16 ***
## (Intercept)
                 1.482703
                             0.103688
                                       14.300
##
                 0.692642
                             2.042699
                                        0.339
                                                  0.735
  x1
##
  x2
                 1.295552
                             2.039959
                                        0.635
                                                  0.527
  xЗ
                 0.909189
                             0.103719
                                        8.766 1.25e-13 ***
##
                             0.093778
                                        4.344 3.73e-05 ***
## x4
                 0.407380
```

```
## x5
                3.322817
                           0.103653
                                     32.057
                                             < 2e-16 ***
                                     -0.080
  x6
                                                0.937
##
               -0.007948
                           0.099586
##
  x7
                0.130916
                           0.114019
                                       1.148
                                                0.254
                0.070550
                                       0.743
                                                0.459
## x8
                           0.094905
##
  x9
                0.027593
                           0.096969
                                       0.285
                                                0.777
## x10
                0.052156
                           0.105381
                                       0.495
                                                0.622
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9533 on 88 degrees of freedom
## Multiple R-squared: 0.9561, Adjusted R-squared:
## F-statistic: 191.6 on 10 and 88 DF, p-value: < 2.2e-16
```

#### 4.5 Check for multicolinearity issue

```
##
            у
                  x1
                         x2
                                xЗ
                                        x4
                                               x5
                                                      x6
                                                             x7
                                                                     8x
                                                                            x9
## y
        1.000
               0.554
                      0.555
                             0.324
                                    0.042
                                            0.808 -0.015
                                                          0.141
                                                                 0.205 - 0.096
                                                   0.001 -0.003
## x1
        0.554
               1.000
                      0.999
                             0.060 - 0.020
                                            0.063
                                                                 0.077 - 0.085
                                                          0.008
## x2
        0.555
               0.999
                      1.000
                             0.060 -0.020
                                                   0.003
                                            0.064
                                                                 0.079 - 0.088
##
        0.324
               0.060
                      0.060
                             1.000 -0.003
                                            0.126
                                                   0.028
                                                          0.063
                                                                 0.017 - 0.007
##
        0.042 -0.020 -0.020 -0.003
                                    1.000 -0.061
                                                   0.000
                                                          0.071 - 0.144
                                                                        0.085
  x4
##
  x5
        0.808
               0.063
                      0.064
                             0.126 -0.061
                                            1.000 -0.030
                                                          0.125
                                                                 0.211 -0.087
       -0.015
               0.001
                      0.003
                             0.028
                                    0.000 -0.030
                                                   1.000
##
                                                          0.034
                                                                 0.109 - 0.166
  x6
        0.141 -0.003
                      0.008
                             0.063
                                    0.071
                                            0.125
                                                   0.034
                                                          1.000
                                                                 0.019 -0.068
##
  x7
                             0.017 - 0.144
        0.205
               0.077
                      0.079
                                            0.211
                                                   0.109
                                                          0.019
                                                                 1.000
##
  x8
                                                                        0.044
       -0.096 -0.085 -0.088 -0.007
                                    0.085 -0.087 -0.166 -0.068
  x9
                                                                 0.044
               ##
  x10
       0.033
                                                                 0.078 - 0.058
##
          x10
## y
        0.033
## x1
        0.158
## x2
        0.160
## x3
       -0.069
## x4
       -0.112
## x5
       -0.042
##
  x6
        0.100
## x7
       -0.051
## x8
        0.078
## x9
       -0.058
## x10 1.000
##
                                                        x5
                                                                   x6
           x1
                      x2
                                 x3
                                             x4
## 515.180302 515.882996
                           1.028513
                                       1.049128
                                                  1.099961
                                                             1.060420
##
           x7
                      x8
                                 x9
                                            x10
     1.095493
                1.101787
                           1.069866
                                       1.068766
```

We can see that x1 and x2 are highly correlated with 0.999 correlation, and 515 VIFs, which is way greater than 10, the threshold. We will next apply automated search to search for significant predictors. If x1 and x2 are both in the result, we will remove one of them in the final predictors.

#### 4.6 Use automated search

Now I use automated search, after removing data 10 from the data set.

```
##
## Call:
## lm(formula = y \sim x5 + x2 + x3 + x4, data = data)
##
## Residuals:
##
       Min
                  1Q
                      Median
                                    3Q
                                            Max
   -2.29129 -0.46534 0.00512 0.62212 1.87873
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept)
               1.50163
                           0.09635
                                    15.584
                                           < 2e-16 ***
                                            < 2e-16 ***
## x5
                3.34805
                           0.09792
                                    34.191
                1.99583
## x2
                           0.08829
                                    22,605
                                           < 2e-16 ***
## x3
                0.91006
                           0.10112
                                     8.999 2.46e-14 ***
                0.40358
                           0.08986
                                     4.491 2.01e-05 ***
## x4
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9338 on 94 degrees of freedom
## Multiple R-squared: 0.955, Adjusted R-squared:
## F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16
```

Using automated forward search, we get x2, x3, x4, and x5 are chosen, among which all are significant with p-values way less than 0.05. The over all fitting of the 4 predictors results in a p-value of < 2.2e-16. This proves that after log-transformation, removing data 10 and automated forward selection, the model fits better. x1 and x2 are not both in the result, so there is no multicolinearity issue.

### 4.7 Model Comparison and Selection

Next, we will check use techniques in Model Comparison and Selection, by comparing R2.adj, PRESS, AIC, BIC, and Cp. We will be using data after removing outlier and log-transformation.

After comparing all permutation's R2.adj, PRESS, AIC, BIC, and Cp. We have the following results:

Permutation Number	Criteria	Result Predictors	p-value	R-squared
489	R2.adj	x2+x3+x4+x5+x7	< 2.2e-16	0.9556
481	PRESS, AIC, BIC, Cp	x2+x3+x4+x5	< 2.2e-16	0.9550

#### 4.8 General F-test

Model 481 is a reduced model of 489, with an extra x7. We apply a general F-test to test if x7 is significant.

```
## Analysis of Variance Table
##
## Model 1: y ~ x2 + x3 + x4 + x5
## Model 2: y ~ x2 + x3 + x4 + x5 + x7
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 94 81.960
## 2 93 80.929 1 1.0317 1.1856 0.279
```

p-value is greater than 0.05. Hence, x7 is not significant, we only need x2, x3, x4, and x5.

Finally, fit our final model again, and test for goodness of fit.

```
##
## Call:
   lm(formula = y \sim x2 + x3 + x4 + x5)
##
##
   Residuals:
##
         Min
                          Median
                                         3Q
                    1Q
                                                  Max
   -2.29129 -0.46534
                        0.00512 0.62212
##
##
   Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
##
   (Intercept)
                  1.50163
                               0.09635
                                         15.584 < 2e-16 ***
                  1.99583
                               0.08829
                                         22.605 < 2e-16 ***
##
                  0.91006
                               0.10112
                                          8.999 2.46e-14 ***
##
   xЗ
                                          4.491 2.01e-05 ***
## x4
                  0.40358
                               0.08986
## x5
                  3.34805
                               0.09792 34.191 < 2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9338 on 94 degrees of freedom
## Multiple R-squared: 0.955, Adjusted R-squared: 0.9531
## F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16
                                                    Standardized residuals
                 Residuals vs Fitted
                                                                        Normal Q-Q
     ^{\circ}
                                                                                        2000000
                                          0
                                                         ^{\circ}
Residuals
     0
                                                         0
                                         0
     7
                                                         7
                                                                                            2
          -10
                   -5
                                   5
                                          10
                                                                  -2
                                                                               0
                                                                                     1
                           0
                      Fitted values
                                                                     Theoretical Quantiles
(Standardized residuals)
                                                    Standardized residuals
                   Scale-Location
                                                                   Residuals vs Leverage
                                                                          ¯
Ω,0730
                                                         \alpha
     1.0
                                                         0
                   %
                                            0
                                                                            Splistance
     0.0
                                                             0.00
          -10
                   -5
                                   5
                                          10
                                                                       0.05
                                                                                0.10
                                                                                          0.15
                           0
                      Fitted values
                                                                           Leverage
```

# 5 Conclusion

With the given data set "dat.cvs",

- y is log-transformed
- Entry 10 is reomoved due to large influence to the overall data
- Predictors x1, x6, x7, x8, x9, and x10 are removed due to insignificance.
- Predictors x2, x3, x4, and x5 are kept, and combined as a first order model.

The final model is:

$$y = 2.00 \cdot x_2 + 0.91 \cdot x_3 + 0.40 \cdot x_4 + 3.35 \cdot x_5 + 1.51$$

```
##
## Call:
## lm(formula = y \sim x2 + x3 + x4 + x5)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
                                            Max
## -2.29129 -0.46534 0.00512 0.62212
                                       1.87873
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
               1.50163
                           0.09635
                                   15.584 < 2e-16 ***
## x2
                1.99583
                           0.08829
                                    22.605 < 2e-16 ***
                                     8.999 2.46e-14 ***
## x3
                0.91006
                           0.10112
## x4
                0.40358
                           0.08986
                                     4.491 2.01e-05 ***
## x5
                3.34805
                           0.09792
                                   34.191 < 2e-16 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9338 on 94 degrees of freedom
## Multiple R-squared: 0.955, Adjusted R-squared: 0.9531
## F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16
```

Predictor	P-value
intercept	(< 2e - 16)
x2	(< 2e - 16)
x3	2.46e - 14
x4	2.01e - 05
x5	(< 2e - 16)
overall	(< 2.2e - 16)

The intercept, x2, and x5 all have p-values of (< 2e - 16), x3 has p-value of 2.46e - 14, and x4 has p-value of 2.01e - 05. The overall p-value is (< 2.2e - 16). These p-values are all less than 0.05 and significant.

The  $R^2$  is 0.955, which means the model represents 95.5% of the data. The model fits the data very well.

# 6 Appendix

# 6.1 Data

```
##
                                        x2
                                                    xЗ
                                                               x4
                           x1
                 y
## 1
      2.436641e+02
                    1.08580359
                               1.114076637
                                            0.45462375
                                                       1.69147892
##
      1.348317e+00
                    1.30488896
                               1.311393972 -2.01623853 -0.14386029
##
  3
      2.785373e+00
                    1.03064839
                               1.032495758
                                           1.02785284 -0.30580807
##
      4.870431e+01
                    0.15783723
                               0.115275934 -0.12345028 1.71466970
##
  5
      5.107077e+03
                    1.32553541
                               1.270779866 -0.47953329 -1.26886600
##
  6
      2.461621e+01
                    0.17907553
                               0.169041211
                                           1.10933527 -1.54807478
## 7
      4.053033e-01 -1.15115545 -1.158052862 -0.26611181 -1.04691094
## 8
                    0.34749784
                               0.413766183 -1.14806766
                                                       2.07995485
      1.273015e+00
                               1.068349232 -0.31490694
## 9
      1.750351e+04
                    1.09237146
                                                       0.78322136
##
      1.627548e+05 -0.12377383 -0.101591919
  10
                                            0.14495762 -0.80827415
                                            0.08461995 -2.67794463
##
      1.142597e+04
                    3.62769114
                               3.608260193
  11
                    1.30989523
                               1.307003554
##
  12
      3.286586e+00
                                            0.14383078 -0.93232006
##
  13
      1.317835e+01
                   0.75371880
                               0.861374750 -0.47295516 0.72538498
## 14
      7.994837e+01
                   1.08365898
                               1.057540228
                                            0.40165760 -2.39403767
      1.254888e+01
                   0.36798425
                               0.355010226 -0.91801520 -1.46354593
## 15
##
  16
      6.475994e+02 0.56864183
                              0.640659644
                                            0.54047899 0.18522984
      1.883914e+02 -1.28581387 -1.212272920
##
  17
                                            1.48905504 -0.41666222
      1.435408e-02 -1.26743305 -1.234753509
                                            0.97972598 -0.19470807
  18
##
      2.152715e+03 -1.06712497 -1.084466308
                                            0.53220005
                                                       0.22177933
##
  20
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                                            2.09600223 -0.67593870
##
  21
      4.269482e+00 1.54605968 1.601931488 -0.23083356
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##
  22
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##
      1.362112e+00 -1.32641841 -1.315556541 -0.46236821 -0.07518034
##
      1.557144e+03 0.35885165 0.341494903
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                                                       1.20814059
##
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## 30
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##
  34
                                                       0.91527663
##
  35
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##
  36
      0.28060109
  37
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                                            1.91593170
                                                       1.52952326
      9.732534e-01 -0.72132166 -0.721717588
##
  38
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                                                       1.66975546
##
      8.182734e-01 0.25261865 0.283697303 -0.61464193
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##
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      1.257483e+01 -0.67510456 -0.666402448 0.52190114 -0.77282839
##
  41
##
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##
      7.227816e-02 0.93858479 0.886642035 -1.51878437
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  43
                    0.20579738 0.240324984
                                           1.10446829 -0.85297338
      2.300922e+00
      1.840516e+01
                   ##
  45
                    0.07543418 -0.009001722
                                            0.23625602 -0.15096018
      5.036520e+00
##
      1.587850e+02 -1.11552973 -1.183864277 0.71806593 -0.10832554
                              0.197831749 -0.10367126  0.81746507
      2.266285e+02 0.12797868
```

```
1.132980e+01 1.03329881 0.945368691 -0.99770994 -0.80953017
      3.306079e-03 -0.75883165 -0.698295514 -0.26168555 -1.01764340
## 51
      1.071461e+01 0.50535547 0.562841300 -0.15480350 -0.08928677
      6.252071e+04 1.97540640 2.022707360 -1.06790809 -1.07014097
##
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##
  55
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##
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##
  61
##
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  62
##
  63
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      ##
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##
##
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      6.441906e-01 0.30558641
                               0.290689350 -0.65171173 0.87539299
##
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##
  68
##
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##
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##
  72
##
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##
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      1.937389e-02 -1.07592262 -1.101434275 -1.86585366 -1.70593077
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      1.140421e+03 0.52876397
                               0.588710603
##
  76
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##
  77
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                   1.84080141
                               1.823142945 -0.85451309 -1.18813479
##
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  79
      1.604147e+05
                    1.14786183
                               1.076712312
                                            0.16096038 0.56421133
## 80
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                                2.011348457
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##
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##
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##
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##
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##
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##
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## 95
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                    1.31951096
                                1.343514903 0.70519380
##
  100 4.444400e+01
                                                        0.51040363
##
                            x6
                                        x7
                                                    8x
## 1
                    1.90546080 -0.64313318 0.83133738 0.069397226
       0.106603041
## 2
       0.048113406 0.13304981 0.77112214 -1.00337110 -0.637381125
```

```
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                                          1.17447004 -0.444843952
       0.111867633 0.72595169 1.42055461
## 6
                                          0.27589284 -0.815191465
## 7
      -0.234039291
                   0.24481605 0.97600319
                                          1.83334807 0.185934914
## 8
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                                          0.16025309 -2.035447258
## 9
       1.674133143 0.23688817 -0.47114442 0.20716043 -0.798898709
## 10
      -0.469902806 -0.58441930 -0.68181262 -0.06050827 -0.629779662
##
       0.271191966 - 1.28401940 - 0.55178324 0.14962121 - 0.624638810
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##
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      -0.417091415 0.28162045 0.71172742 0.45612887 0.713363388
       0.846814894 -0.67379802 0.40452412 -0.22245711 0.940187027
##
  14
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##
       0.606506960 -0.04757949 -0.34626245
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## 16
       1.028743441 0.04754917 -0.16413534 0.21921100 0.364608977
       1.107018328 -0.56406240 0.12190554 -0.36376932 -0.241138967
## 17
## 18
      -0.760967134 -0.80492656 -0.86978750 0.39825746 1.158414868
       1.829845819 -0.73342870 -0.94666949 0.19451624 -0.905648810
## 19
##
       20
      -1.103237249 -0.43830930 2.71029359 0.20913206 -1.170123589
##
  21
##
  22
      -0.595501385 -0.33002193 -1.93515195 -1.67960528
                                                      0.841716695
##
  23
       0.660742855 - 0.62337290 \quad 0.79310289 \quad 0.22332240 - 0.069931890
       1.277794077 -0.19302307 0.36741309 0.54070993
## 24
## 25
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                                                      1.106197464
## 26
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## 27
       0.230299672 - 0.07183219 - 0.69532621 - 1.35099094 0.665157761
  28
       0.051483369 -1.01026450 -1.20622548 0.69385552 -0.226950504
      -1.639354604 -1.71285686 -0.68068843 -0.38012039
##
  29
                                                      1.646191506
##
  30
       0.227475120 1.27095593 0.54184668 -0.78465842
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      -0.962824638 -0.82055062 -0.50390831 -3.71666465
##
  31
                                                      0.147991891
##
  32
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##
  33
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##
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##
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                   ##
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                    0.81315420 -0.66453940 0.14655152
##
  37
                                                      0.265973462
      -0.515142922   0.40762951   -1.09066780   -0.55211951
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  38
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##
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##
  44
                                                      0.399725069
##
  45
      -0.178258312 0.02239690 -0.38183419 1.32645517
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       0.511254635 -0.16785673 -0.65089906 -0.89565732
##
  46
                                                     0.000981539
## 47
       1.394295955 -0.63117192 0.39522009 1.76262402
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## 48
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## 50
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## 51
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## 52
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## 53
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                                          0.85675322 -2.434304298
## 54
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      -1.953302876 -3.32045561 -0.75547571 -1.08434120 1.079579708
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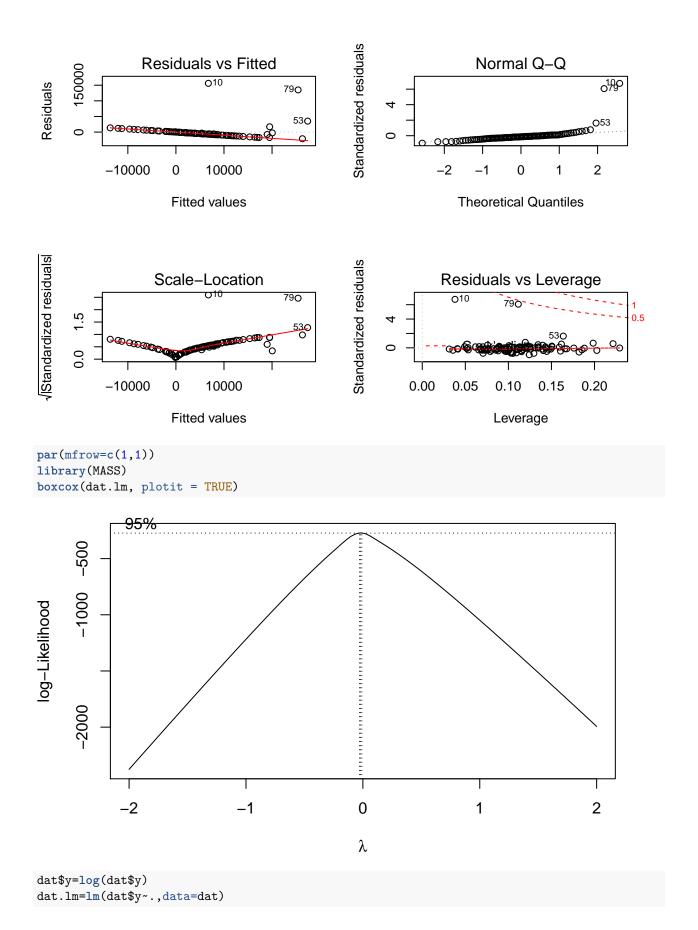
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      -1.113642189 0.90204810 -0.43588234 -0.29226662 -1.412875319
##
       0.099917732 1.33957013 -1.00504540 -1.27988134
                                                      0.010248035
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##
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                                                      0.903785763
      -0.328917315 -1.03580965 -0.19824551 -1.56607538 -0.083228044
##
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       2.176541490 1.25439732 0.01221935 0.53989788 -1.176444197
## 64
##
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       0.359155361 -0.38137468 -0.21197593 1.06951079 -1.437608440
##
  66
       0.175534351 -0.65950463 0.30733107 -0.41385941 0.241848292
##
  67
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                              0.30618828 0.64407819
                                                      0.294383024
                              0.24405404 -0.29996224
##
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##
      -1.125241257 1.59653610 1.63733360 -1.94479562
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  69
                                         1.15353352
##
  70
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                                                      0.804810292
      -0.828135290 -0.08516574 -0.69509037 0.09517886
                                                      0.985063420
##
  71
##
      -0.749955503 -0.22971663 -0.36249241
                                          0.25308145 -0.001017382
      -0.851768270 -1.23800605 -0.92535162
##
  73
                                         0.52885085
                                                      0.803418511
##
      -0.731668258 1.35741196
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                                         0.80694433 -0.126094011
  74
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##
  75
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##
  76
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##
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                                                     1.194503979
      -1.089315254 -1.26292043 -0.38436811 -0.62779261 -0.363265560
##
  78
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##
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## 81
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                  1.25912009 0.63157173
                                          0.72074879 -1.970260344
##
  83
      -0.058817748
##
  84
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                                         0.14306334 0.777691117
       0.873171495 -0.07761400 -0.29553356 1.26574014 1.681854681
##
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##
  86
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## 87
      -0.625515250 -1.64171035 -0.40230057 -0.66624567 -0.787110302
##
  88
      -0.986388177
                   ##
  89
      -1.770361614
                   1.71259812 -0.18706843 -1.22297350 -1.986949883
                   ##
      -0.846689080
  90
      -0.481208263
                   0.92316435 0.25664424 1.72950423
                                                      0.830516979
##
  91
      ##
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## 94
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##
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  97
       0.074184141 -1.19916492  0.80995952 -1.23494372 -0.885696554
##
  98
##
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##
  100 -0.749197139
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##
              x10
       1.13955430
## 1
## 2
       0.36182605
## 3
       0.50456596
## 4
      -0.53134563
## 5
       0.47327357
## 6
       2.17941996
## 7
       1.06412371
## 8
       1.41167025
## 9
      -0.46691668
```

- ## 10 0.55953961
- ## 11 1.82517774
- ## 12 0.18046497
- ... 12 0.10010101
- ## 15 0.54561368
- ## 16 1.21556816
- ## 17 0.72169162
- ## 18 -0.30776948
- ## 19 -0.18771025
- "" 10 0:10171020
- ## 20 1.45771451
- ## 21 1.28044712
- ## 22 0.23462303
- ## 23 1.41575945
- ## 24 0.56858929
- ## 25 -0.81765305
- ## 26 0.40475753
- ## 27 1.85524370
- ## 28 0.65924981
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- ## 30 -0.89032164
- ## 31 0.62618347
- ## 32 -1.11311773
- ## 33 0.77693533
- ## 34 0.98145266
- ## 04 0.3014020C
- ## 35 0.65527363
- ## 36 0.02572670
- ## 37 0.11347937
- ## 38 -1.15593094 ## 39 -0.53636143
- ## 39 -0.55050143
- ## 40 -2.16853334 ## 41 -0.44425598
- ## 42 -0.68087290
- ## 43 -0.65405770
- ## 44 0.85179036
- ## 45 0.42393059
- ## 46 1.69118235
- ## 47 -0.80354661 ## 48 -0.30648334
- ## 49 -2.42375948
- ## 50 0.73293923
- ## 51 -0.63928612
- ## 52 0.41350240
- ## 53 0.24463690
- ## 54 0.08462778
- ## 55 -0.69888375
- ## 56 0.99111492
- ## 57 0.55902480 ## 58 1.05353168
- ## 58 1.05353168 ## 59 -0.90060787
- ## 60 -0.24721684
- ## 61 0.23984043
- ## 62 1.06366407
- ## 63 0.41491551

```
## 64
       -0.89119739
## 65
       -0.52471453
## 66
        1.09703071
## 67
        0.12833406
## 68
        2.47980070
## 69
        1.91084613
## 70
        1.21482615
## 71
        1.18321540
## 72
        0.72596536
## 73
        0.16631868
## 74
        0.45776133
## 75
        0.50387033
## 76
       -1.24822048
## 77
        0.96434421
## 78
       -0.57090721
## 79
        0.24812163
## 80
        1.49124498
## 81
        0.51753355
## 82
       -0.80164123
## 83
       -0.46179780
## 84
        0.39187139
## 85
        1.16138340
        0.20073253
## 86
## 87
       -0.79295204
## 88
        2.43258380
## 89
        0.01108954
## 90
        0.65262878
## 91
       -0.52023995
## 92
        0.82471275
## 93
        0.20835946
## 94
       -0.58733443
## 95
       -1.58502046
## 96
        1.13572107
## 97
        0.12638203
## 98
       -0.33588327
## 99
       -0.93919173
## 100 0.19932810
```

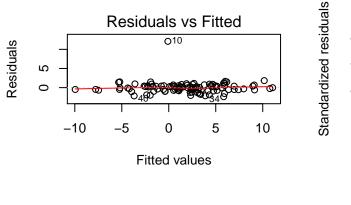
#### 6.2 R Code

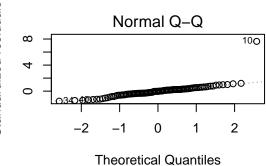
```
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
## The following objects are masked from dat (pos = 7):
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
dat.lm=lm(dat$y~.,data=dat)
summary(dat.lm)
##
## Call:
## lm(formula = dat$y ~ ., data = dat)
##
## Residuals:
##
     Min
              1Q Median
                            3Q
                                 Max
## -21213 -7359 -3119
                        1056 156003
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
               4856.9
                           2554.0
## (Intercept)
                                    1.902
                                            0.0604 .
## x1
               18253.4
                          50397.7
                                   0.362
                                            0.7181
              -16220.4
                          50336.8 -0.322
## x2
                                            0.7480
                -723.3
                            2565.5 -0.282
## x3
                                            0.7786
## x4
                 520.9
                           2317.8
                                    0.225
                                            0.8227
## x5
                4563.1
                           2562.1
                                    1.781
                                            0.0783 .
               -2807.6
                           2457.2 -1.143
                                            0.2563
## x6
## x7
               -2535.7
                            2810.4 -0.902
                                            0.3694
## x8
                2839.7
                            2348.2
                                   1.209
                                            0.2297
## x9
               -3654.9
                            2392.7 -1.527
                                             0.1302
                -577.3
                            2607.0 -0.221
                                            0.8253
## x10
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 23590 on 89 degrees of freedom
## Multiple R-squared: 0.1113, Adjusted R-squared: 0.01149
## F-statistic: 1.115 on 10 and 89 DF, p-value: 0.36
par(mfrow=c(2,2))
plot(dat.lm)
```

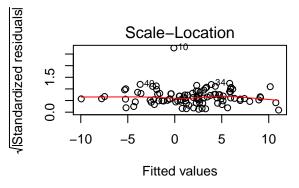


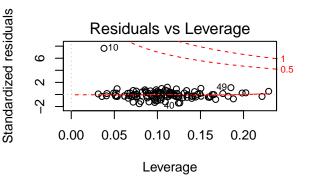
#### summary(dat.lm)

```
##
## Call:
## lm(formula = dat$y ~ ., data = dat)
##
## Residuals:
##
      Min
              1Q Median
                             3Q
                                    Max
## -2.3787 -0.6130 -0.0369 0.4978 12.0928
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.610750 0.174805 9.215 1.35e-14 ***
             -1.323510 3.449464 -0.384
                                          0.7021
## x1
              3.262825 3.445294 0.947
## x2
                                          0.3462
## x3
             0.349093 0.158641 2.201
## x4
                                         0.0304 *
                       0.175364 18.597 < 2e-16 ***
## x5
              3.261169
## x6
             -0.104953
                        0.168182 -0.624 0.5342
                                 0.007
## x7
              0.001392
                        0.192358
                                          0.9942
## x8
              0.086216
                        0.160720
                                 0.536
                                          0.5930
## x9
             -0.066157
                        0.163769 -0.404
                                          0.6872
              0.080254
                                 0.450 0.6540
## x10
                        0.178437
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1.614 on 89 degrees of freedom
## Multiple R-squared: 0.8795, Adjusted R-squared: 0.866
## F-statistic: 64.97 on 10 and 89 DF, p-value: < 2.2e-16
par(mfrow=c(2,2))
plot(dat.lm)
```









#### attach(dat)

```
## The following object is masked _by_ .GlobalEnv:
##
##
       у
##
   The following objects are masked from dat (pos = 3):
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
  The following objects are masked from dat (pos = 4):
##
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
  The following objects are masked from dat (pos = 6):
##
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
  The following objects are masked from dat (pos = 8):
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
##
```

#### dat.lm\$residuals

```
##
                                  9 10
                                                         11
   0.91445506 - 0.42747145 \ 0.67615956 \ 12.09277675 \ 0.39684465 \ 0.70673482
##
                      14
                                  15
                                              16
                                                         17
   0.61301290 -1.54270869 -0.57145897 -0.49268510
                                                 0.90761831 -1.89567053
##
##
                       20
                                  21
                                              22
   1.50472567 0.03107907 0.21320996 0.73273280 -0.66043868 0.39177543
##
           25
                       26
                                  27
                                                         29
##
   -0.97302267 -2.12985599 0.39241405 -0.96315529
                                                  0.36131924 0.52319298
##
           31
                       32
                                  33
                                              34
                                                         35
   -0.47421161 -1.21115573 0.28165607 -2.37873048
                                                  0.36915979 -0.05378739
                       38
                                  39
                                              40
                                                         41
   0.24585338 0.43959228
##
##
           43
                      44
                                  45
                                              46
                                                         47
   ##
           49
                      50
                                             52
                                                         53
##
                                  51
##
   1.64630937 -1.89003794 -0.47072953 -0.26611134 -0.01188162 -0.82421181
##
           55
                      56
                                  57
                                              58
                                                         59
   -1.06336897 -0.88784144 -0.48258014 0.84599502 -0.66984518 -0.51079971
##
           61
                      62
                                  63
                                              64
                                                         65
##
   0.22063896 -0.96786752 -0.63538811 -0.24984842 -1.38626689 0.14034692
##
           67
                      68
                                  69
                                             70
                                                         71
    1.48881825 -0.53317116 0.34388223 -0.04063531 -0.29729509 -0.36282110
##
           73
                      74
                                                         77
##
                                  75
                                              76
   -0.09116264 1.38740278 -0.32815101 1.19629503 0.57524205 -0.71165091
##
##
                       80
                                  81
                                              82
                                                         83
   1.82581743 -0.41373586 -0.03320768 -0.36846335 -1.62344667 -0.53836283
##
           85
                      86
                                  87
                                             88
                                                         89
   0.86190740 \quad 0.16400262 \quad 0.82308678 \quad -0.46164896 \quad 1.50089022 \quad -1.98097576
##
##
                      92
                                  93
                                             94
                                                         95
           91
   0.40999962 0.34303544 0.28850747 0.16727313 -0.05365871 -0.60549986
##
           97
                       98
                                  99
                                             100
## -0.52297153 1.18638495 0.02364513 1.10739083
tmp<-lm.influence(dat.lm)</pre>
tmp$hat[tmp$hat>2*10/100]
                            31
##
                   22
         11
## 0.2026707 0.2291824 0.2214943
DFFITS<-dffits(dat.lm)</pre>
DFFITS[DFFITS>1]
##
       10
## 2.57098
dat[DFFITS>1,]
                                    xЗ
                x1
                          x2
                                               x4
                                                         x5
## 10 12 -0.1237738 -0.1015919 0.1449576 -0.8082742 -0.4699028 -0.5844193
                                   x9
             x7
                        8x
                                            x10
## 10 -0.6818126 -0.06050827 -0.6297797 0.5595396
```

```
COOKS<-cooks.distance(dat.lm)
round(COOKS,3)
                                                   4
                                                               5
                                                                            6
                                                                                        7
                                                                                                    8
                                                                                                                 9
                                                                                                                           10
                                                                                                                                                    12
##
              1
                                       3
                                                                                                                                        11
## 0.001 0.017 0.007 0.003 0.002 0.000 0.003 0.002 0.002 0.209 0.002 0.003
##
            13
                                    15
                                                 16
                                                             17
                                                                          18
                                                                                      19
                                                                                                  20
                                                                                                               21
                                                                                                                           22
                                                                                                                                        23
                        14
## 0.002 0.010 0.001 0.001 0.004 0.014 0.010 0.000 0.000 0.007 0.001 0.000
                                    27
                                                 28
                                                             29
##
            25
                        26
                                                                         30
                                                                                      31
                                                                                                  32
                                                                                                               33
                                                                                                                           34
                                                                                                                                        35
## 0.003 0.026 0.001 0.004 0.001 0.001 0.003 0.012 0.000 0.018 0.000 0.000
            37
                        38
                                    39
                                                 40
                                                             41
                                                                         42
                                                                                      43
                                                                                                  44
                                                                                                               45
                                                                                                                           46
                                                                                                                                        47
## 0.000 0.001 0.003 0.026 0.000 0.001 0.006 0.000 0.003 0.011 0.002 0.001
                        50
                                    51
                                                 52
                                                             53
                                                                         54
                                                                                      55
                                                                                                  56
                                                                                                              57
                                                                                                                           58
                                                                                                                                       59
## 0.026 0.017 0.001 0.000 0.000 0.004 0.011 0.003 0.002 0.003 0.004 0.001
##
            61
                        62
                                    63
                                                 64
                                                             65
                                                                         66
                                                                                      67
                                                                                                  68
                                                                                                               69
                                                                                                                           70
                                                                                                                                       71
## 0.000 0.006 0.003 0.000 0.009 0.000 0.005 0.002 0.001 0.000 0.000 0.000
            73
                        74
                                    75
                                                 76
                                                             77
                                                                         78
                                                                                      79
                                                                                                  80
                                                                                                              81
                                                                                                                           82
                                                                                                                                        83
## 0.000 0.011 0.000 0.006 0.001 0.004 0.016 0.001 0.000 0.000 0.011 0.001
                                    87
                                                 88
                                                             89
                                                                         90
                                                                                      91
                                                                                                  92
                                                                                                              93
                                                                                                                           94
                        86
## 0.002 0.000 0.003 0.001 0.016 0.011 0.001 0.000 0.000 0.000 0.000 0.002
            97
                        98
                                    99
                                               100
## 0.000 0.005 0.000 0.006
COOKS>qf(0.5,10,90)
                                                                                        7
##
                                       3
                                                   4
                                                               5
                                                                            6
                                                                                                    8
                                                                                                                 9
                                                                                                                           10
                                                                                                                                        11
                                                                                                                                                    12
## FALSE FALSE
                                                             17
                                                                         18
                                                                                      19
                                                                                                   20
                                                                                                               21
                        14
                                    15
                                                 16
                                                                                                                           22
## FALSE FALSE
            25
                        26
                                    27
                                                 28
                                                             29
                                                                          30
                                                                                      31
                                                                                                   32
                                                                                                               33
                                                                                                                           34
                                                                                                                                        35
## FALSE FAL
            37
                        38
                                    39
                                                 40
                                                             41
                                                                          42
                                                                                      43
                                                                                                   44
                                                                                                               45
                                                                                                                           46
## FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##
            49
                        50
                                    51
                                                 52
                                                             53
                                                                          54
                                                                                      55
                                                                                                   56
                                                                                                               57
                                                                                                                           58
                                                                                                                                        59
                                                                                                                                                    60
## FALSE FALSE
            61
                                    63
                                                 64
                                                             65
                                                                         66
                                                                                      67
                                                                                                  68
                                                                                                               69
                                                                                                                           70
                                                                                                                                        71
                        62
## FALSE FALSE
                        74
                                    75
                                                 76
                                                             77
                                                                         78
                                                                                      79
                                                                                                  80
##
            73
                                                                                                               81
                                                                                                                           82
                                                                                                                                        83
## FALSE FALSE
                                    87
                                                             89
                                                                         90
                                                                                                   92
##
            85
                        86
                                                 88
                                                                                      91
                                                                                                               93
                                                                                                                           94
                                                                                                                                        95
## FALSE FALSE
                        98
                                               100
##
            97
                                    99
## FALSE FALSE FALSE
no10<- dat[-10, ]
no10.lm=lm(no10$y~.,data=no10)
summary(no10.lm)
##
## Call:
## lm(formula = no10$y ~ ., data = no10)
```

## Residuals:

```
10 Median
       Min
                                 3Q
## -2.3706 -0.4279 0.0241 0.5760 1.8382
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                           0.103688 14.300 < 2e-16 ***
## (Intercept) 1.482703
                                      0.339
## x1
                0.692642
                            2.042699
                                                0.735
## x2
                1.295552
                           2.039959
                                       0.635
                                                0.527
## x3
                0.909189
                           0.103719
                                      8.766 1.25e-13 ***
## x4
                0.407380
                           0.093778
                                      4.344 3.73e-05 ***
## x5
                3.322817
                           0.103653 32.057
                                              < 2e-16 ***
                           0.099586 -0.080
## x6
               -0.007948
                                                0.937
## x7
                0.130916
                           0.114019
                                      1.148
                                                0.254
## x8
                0.070550
                           0.094905
                                     0.743
                                                0.459
## x9
                0.027593
                            0.096969
                                       0.285
                                                0.777
## x10
                0.052156
                           0.105381
                                       0.495
                                                0.622
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9533 on 88 degrees of freedom
## Multiple R-squared: 0.9561, Adjusted R-squared: 0.9511
## F-statistic: 191.6 on 10 and 88 DF, p-value: < 2.2e-16
round(cor(no10),3)
                                               x5
                                                                     x8
##
                  x1
                         x2
                                 xЗ
                                        x4
                                                      x6
                                                              x7
            У
## y
        1.000 0.554 0.555 0.324 0.042 0.808 -0.015 0.141 0.205 -0.096
## x1
        0.554 1.000 0.999
                             0.060 -0.020 0.063 0.001 -0.003
                                                                  0.077 - 0.085
## x2
        0.555 0.999 1.000
                             0.060 -0.020 0.064 0.003
                                                          0.008 0.079 -0.088
## x3
        0.324 \quad 0.060 \quad 0.060 \quad 1.000 \quad -0.003 \quad 0.126 \quad 0.028 \quad 0.063 \quad 0.017 \quad -0.007
        0.042 \; -0.020 \; -0.020 \; -0.003 \quad 1.000 \; -0.061 \quad 0.000 \quad 0.071 \; -0.144 \quad 0.085
## x4
## x5
        0.808   0.063   0.064   0.126   -0.061   1.000   -0.030   0.125
                                                                  0.211 -0.087
      -0.015 0.001 0.003 0.028 0.000 -0.030 1.000 0.034
## x6
                                                                  0.109 - 0.166
## x7
        0.141 -0.003 0.008 0.063 0.071 0.125 0.034 1.000
                                                                  0.019 -0.068
## x8
        0.205 0.077 0.079 0.017 -0.144 0.211 0.109 0.019
                                                                  1.000 0.044
      -0.096 -0.085 -0.088 -0.007 0.085 -0.087 -0.166 -0.068
## x9
                                                                  0.044 1.000
               0.158   0.160   -0.069   -0.112   -0.042   0.100   -0.051
## x10 0.033
                                                                  0.078 -0.058
##
          x10
## y
        0.033
        0.158
## x1
## x2
        0.160
## x3 -0.069
## x4 -0.112
## x5 -0.042
## x6
        0.100
## x7 -0.051
## x8
        0.078
## x9 -0.058
## x10 1.000
library(faraway)
vif(no10.lm)
##
                      x2
                                  x3
                                             x4
                                                         x5
                                                                    x6
           x1
```

```
## 515.180302 515.882996
                           1.028513
                                      1.049128
                                                 1.099961
                                                             1.060420
##
           <sub>x</sub>7
                      x8
                                 x9
                                           x10
##
     1.095493
              1.101787
                           1.069866
                                      1.068766
data=no10
start<-lm(y~1, data=data)
end<-lm(y~.,data=data)</pre>
result<-step(start, scope=list(lower=start, upper=end), direction="forward", trace=FALSE)
summary(result)
##
## Call:
## lm(formula = y ~ x5 + x2 + x3 + x4, data = data)
##
## Residuals:
##
       Min
                  1Q
                     Median
                                    30
                                            Max
## -2.29129 -0.46534 0.00512 0.62212 1.87873
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
                           0.09635 15.584 < 2e-16 ***
## (Intercept) 1.50163
## x5
                3.34805
                           0.09792 34.191 < 2e-16 ***
## x2
               1.99583
                           0.08829 22.605 < 2e-16 ***
               0.91006
                                    8.999 2.46e-14 ***
## x3
                           0.10112
                           0.08986
                                    4.491 2.01e-05 ***
               0.40358
## x4
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9338 on 94 degrees of freedom
## Multiple R-squared: 0.955, Adjusted R-squared: 0.9531
## F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16
dat=data
attach(dat)
## The following object is masked by .GlobalEnv:
##
##
       у
## The following objects are masked from dat (pos = 3):
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
## The following objects are masked from dat (pos = 4):
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
## The following objects are masked from dat (pos = 5):
##
##
       x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
## The following objects are masked from dat (pos = 7):
##
##
      x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
```

```
## The following objects are masked from dat (pos = 9):
##
     x1, x10, x2, x3, x4, x5, x6, x7, x8, x9, y
##
##STEP 1: create design matrix for maximal model##
x < -cbind(Const=1, x1, x2, x3, x4, x5, x6, x7, x8, x9, x10)
у<-у
##STEP 2: logical matrix to indicate for each possible model, which predictors are in/out##
models<-matrix(F, 2^10, 10) ##10 predictors, change accordingly
dimnames(models)<-list(NULL, c("x1", "x2", "x3", "x4", "x5", "x6", "x7", "x8", "x9", "x10"))
row<-0
for (a in c(F,T)) { ##loop has 10 for statements, one for each predictor
 for (b in c(F,T)) {
  for (c in c(F,T)) {
    for (d in c(F,T)) {
     for (e in c(F,T)) {
       for (f in c(F,T)) {
        for (g in c(F,T)) {
          for (h in c(F,T)) {
           for (i in c(F,T)) {
             for (j in c(F,T)) {
row<-row+1
models[row,] < -c(a,b,c,d,e,f,g,h,i,j)
}}}}}}
##STEP 3: Matrix to store results from criteria##
results<-matrix(NA, 2^10, 7) ##10 predictors, store 7 statistics
dimnames(results)<-list(NULL, c("p","R2","R2.adj","PRESS","AIC","BIC","Cp"))</pre>
##STEP 4: MSE for maximal model##
tmp<-lsfit(x,y,intercept=F)</pre>
n<-nrow(dat)</pre>
p < -ncol(x)
MSE.max<-sum(tmp$res^2)/(n-p)
#####################################
##STEP 5: Fit all possible models##
```

```
time1<-Sys.time()</pre>
for (i in 1:(2^10)){ ##10 predictors
  which <-c(T, models[i,]) ##pull out the row of model and append an intercept
  tmp<-lsfit(x[,which], y, intercept=F) ##fit the model and compute the criteria
  p<-sum(which) ##number of parameters for chosen model</pre>
  SSTo <-(n-1)*var(y)
  MSTo<-var(y)
  SSE<-sum(tmp$res^2)</pre>
  MSE < -SSE/(n-p)
  R2<-1-(SSE/SSTo)
  R2.adj<-1-(MSE/MSTo)
  hi<-ls.diag(tmp)$hat ##leverages
  res.PRESS<-tmp$res/(1-hi)
  PRESS<-sum(res.PRESS^2)</pre>
  AIC < -n * log(SSE/n) + 2 * p
  BIC < -n * log(SSE/n) + p * log(n)
  Cp < -(SSE/MSE.max) - n + 2 * p
  ##store the results
  results[i,1]<-p
  results[i,2]<-R2
  results[i,3]<-R2.adj
  results[i,4]<-PRESS
  results[i,5]<-AIC
  results[i,6]<-BIC
  results[i,7]<-Cp
  ##have R print out the iteration we are at
  print(paste("run",i))}
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## [1] "run 1014"
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## [1] "run 1017"
## [1] "run 1018"
## [1] "run 1019"
## [1] "run 1020"
## [1] "run 1021"
## [1] "run 1022"
## [1] "run 1023"
## [1] "run 1024"
Sys.time()-time1 ##see how long it takes!
```

### ## Time difference of 0.6356552 secs

```
################
##Save results##
#################
save(x,y,results,models,file="results.RData")
#######################
##Give results names##
#######################
p<-results[,1]
R2<-results[,2]
R2.adj<-results[,3]
PRESS<-results[,4]
AIC<-results[,5]
BIC<-results[,6]
Cp<-results[,7]</pre>
###################################
##Find model with best R2.adj##
```

```
i < -R2.adj == max(R2.adj) ##what index has the maximum R2.adj
(1:2<sup>10</sup>)[i] ##2<sup>10</sup> because 10 potential predictors
## [1] 489
round(results[i,],3)
            R2 R2.adj PRESS
                              AIC
## 6.000 0.956 0.953 92.026 -7.954 7.617 2.057
models[i,]
     x1
          x2
                xЗ
                     x4
                           x5
                                x6
                                      x7
                                           8x
                                                 x9
                                                     x10
## FALSE TRUE TRUE TRUE TRUE FALSE TRUE FALSE FALSE
##Find model with best PRESS, AIC, BIC, Cp##
i2<-PRESS == min(PRESS) ##what index has the min PRESS
(1:2<sup>10</sup>)[i2] ##2<sup>10</sup> because 10 potential predictors
## [1] 481
i3 < -AIC == min(AIC) ##what index has the min AIC
(1:2^10)[i3] ##2^10 because 10 potential predictors
## [1] 481
i4<-BIC == min(BIC) ##what index has the min BIC
(1:2^10)[i4] ##2^10 because 10 potential predictors
## [1] 481
i5 < -Cp == min(Cp) ##what index has the min Cp
(1:2^10)[i5] ##2^10 because 10 potential predictors
## [1] 481
models[i2,]
##
                           x5
                                x6
                                      x7
                                           8x
     x1
          x2
                xЗ
                     x4
```

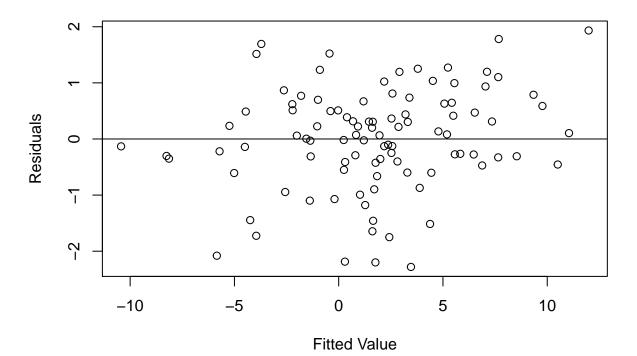
## FALSE TRUE TRUE TRUE FALSE FALSE FALSE FALSE

```
models[i3,]
     x1
           x2
                xЗ
                     x4
                           x5
                                x6
                                      x7
                                           8x
                                                x9
## FALSE TRUE TRUE TRUE FALSE FALSE FALSE FALSE
models[i4,]
          x2
                xЗ
                     x4
                           x5
                                x6
                                      x7
                                           8x
## FALSE TRUE TRUE TRUE FALSE FALSE FALSE FALSE
models[i5,]
     x1
          x2
                x3
                     x4
                           x5
                                x6
                                      <sub>x</sub>7
                                           x8
                                                x9
## FALSE TRUE TRUE TRUE TRUE FALSE FALSE FALSE FALSE
##plot the residual against fitted values##
dat489.lm=lm(y \sim x2+x3+x4+x5+x7)
summary(dat489.lm)
##
## Call:
## lm(formula = y \sim x2 + x3 + x4 + x5 + x7)
##
## Residuals:
       Min
                1Q
                   Median
## -2.28020 -0.41751 0.06503 0.60385 1.93313
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.49739 0.09634 15.543 < 2e-16 ***
## x2
              1.99599
                        0.08820 22.629 < 2e-16 ***
              0.90480
                        0.10114
                                8.946 3.47e-14 ***
## x3
## x4
              0.39580
                        0.09006
                                4.395 2.94e-05 ***
## x5
              3.33487
                        0.09857 33.832 < 2e-16 ***
              0.11749
                        0.10791
                                1.089
## x7
                                         0.279
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.9328 on 93 degrees of freedom
## Multiple R-squared: 0.9556, Adjusted R-squared: 0.9532
## F-statistic: 399.9 on 5 and 93 DF, p-value: < 2.2e-16
dat481.lm=lm(y \sim x2+x3+x4+x5)
summary(dat481.lm)
##
```

## Call:

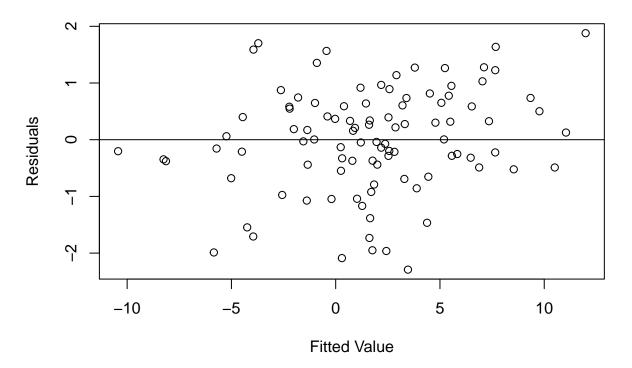
```
## lm(formula = y \sim x2 + x3 + x4 + x5)
##
## Residuals:
##
       Min
                      Median
                                    ЗQ
                  1Q
                                            Max
##
   -2.29129 -0.46534 0.00512 0.62212
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 1.50163
                           0.09635
                                   15.584 < 2e-16 ***
                           0.08829
                                    22.605 < 2e-16 ***
## x2
                1.99583
## x3
                0.91006
                           0.10112
                                     8.999 2.46e-14 ***
                                     4.491 2.01e-05 ***
## x4
                0.40358
                           0.08986
                3.34805
                           0.09792 34.191 < 2e-16 ***
## x5
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9338 on 94 degrees of freedom
## Multiple R-squared: 0.955, Adjusted R-squared: 0.9531
## F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16
dat489.res=resid(dat489.lm)
par(mfrow=c(1,1))
plot(y, dat489.res, ylab="Residuals", xlab="Fitted Value", main="Residual Plot")
abline(0, 0)
                              # the horizon
```

## **Residual Plot**



```
dat481.res=resid(dat481.lm)
par(mfrow=c(1,1))
plot(y, dat481.res, ylab="Residuals", xlab="Fitted Value", main="Residual Plot")
abline(0, 0)  # the horizon
```

# **Residual Plot**



#### anova(dat481.lm,dat489.lm)

```
## Analysis of Variance Table
##
## Model 1: y ~ x2 + x3 + x4 + x5
## Model 2: y ~ x2 + x3 + x4 + x5 + x7
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 94 81.960
## 2 93 80.929 1 1.0317 1.1856 0.279

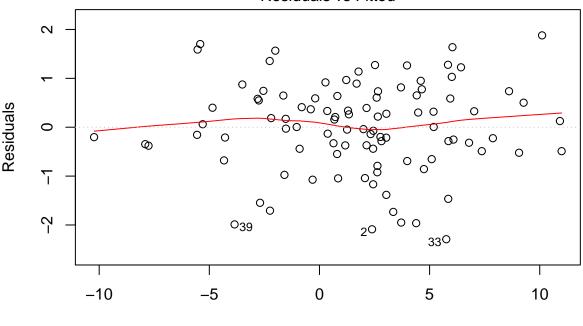
result.lm=lm(y ~ x2+x3+x4+x5)
summary(result.lm)
```

```
##
## Call:
## lm(formula = y \sim x2 + x3 + x4 + x5)
##
## Residuals:
                  1Q
                      Median
## -2.29129 -0.46534 0.00512 0.62212 1.87873
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.50163
                           0.09635 15.584 < 2e-16 ***
## x2
                1.99583
                           0.08829
                                    22.605 < 2e-16 ***
                                    8.999 2.46e-14 ***
## x3
                0.91006
                           0.10112
## x4
                0.40358
                           0.08986
                                    4.491 2.01e-05 ***
                           0.09792 34.191 < 2e-16 ***
                3.34805
## x5
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9338 on 94 degrees of freedom
## Multiple R-squared: 0.955, Adjusted R-squared: 0.9531
## F-statistic: 498.6 on 4 and 94 DF, p-value: < 2.2e-16</pre>
```

## plot(result.lm)

# Residuals vs Fitted



Fitted values  $Im(y \sim x2 + x3 + x4 + x5)$ 

