

# logistic Regression Cost function:

→ new stuff from ml course

→ to train the parameters  $w$  &  $b$  of logistic regression model, you need to define a cost function.

$$\hat{y} = \sigma(w^T x + b) \quad \text{where } \sigma(z) = \frac{1}{1 + e^{-z}} \quad z^{(i)} = w^T x^{(i)} + b$$

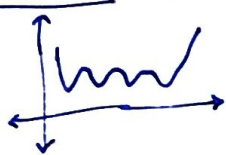
given  $\{(x^{(1)}, y^{(1)}) \dots (x^{(m)}, y^{(m)})\}$  we want  $\hat{y}^{(i)} \approx y^{(i)}$

## \* Loss (error function):

1) You could do  $L(\hat{y}, y) = \frac{1}{2} (\hat{y} - y)^2$  → half square error

→ this basically measures how well our algo is doing

→ If you do this however, the optimization problem (later) becomes non-convex → end up with optimization problem with lots of local optima



↓  
gradient descent might not find local optima

2) another way

$$L(\hat{y}, y) = - \left( \{y \log \hat{y}\} + \{(1-y) \log (1-\hat{y})\} \right)$$

Rationale: if  $y=1$   $L(\hat{y}, 1) = -\log \hat{y}$  (← want as small as possible)  
 $\hat{y} = \text{large} \rightarrow \text{sigmoid can't be } > 1$   
if  $y=0$   $L(\hat{y}, 0) = -\log (1-\hat{y})$  want  $\log 1-\hat{y}$  large want  $\hat{y}$  small

## \* Cost function

→ loss  $f^n$  was defined wrt single training example

→ Cost  $f^n$  determines how well you're doing on entire training set

$$J(w, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) = \frac{1}{m} \sum_{i=1}^m \left( y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log (1-\hat{y}^{(i)}) \right)$$

basically gives average of  $L(\hat{y}, y)$

→ In training log. reg. model, we're going to find params  $w$  &  $b$  that minimize overall cost  $f^n J$ .