

solve eq<sup>n</sup>

$$J = (Y^T - (X\theta)^T)(Y - X\theta)$$

$$= (Y^T - X^T\theta^T)(Y - X\theta)$$

$$= Y^T Y - Y^T X \theta - X^T \theta^T Y + X X^T \theta^T \theta$$

take derivative  
wrt  $\theta$

$$\frac{dJ}{d\theta} = 0 - (Y^T X)^T - (X^T Y) + 2 \underbrace{(X^T X)}_{\approx 0} \theta$$

$$- X^T Y^T - X^T Y + 2 X^T X \theta$$

$$0 = -2 X^T Y^T + 2 X^T X \theta$$

$$2 X^T X \theta = 2 X^T Y^T$$

$$\theta = (X^T X)^{-1} X^T Y^T$$

This is called closed-form solution since we know  $X$  &  $Y$  & close on the sol<sup>n</sup>

Summary of Closed-form Linear Regression:

Given:

- Training data matrix  $X$  of size  $N \times D$
- Training Target values  $Y$  of size  $N \times 1$

Add a Col<sup>n</sup> of ones to beginning  $X$

$$X \rightarrow [1 \ X]$$

Compute weights  $\theta = (X^T X)^{-1} X^T Y$