MatrixForm

In[11]:=

$$A = \begin{pmatrix} 0 & 1 \\ 1 & -\nu \end{pmatrix};$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix};$$

$$Q = \begin{pmatrix} q11 & 0 \\ 0 & q22 \end{pmatrix};$$

$$R = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix};$$

In[15]:=

$$P = \begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix};$$

In[16]:=

```
eqRiccati = P.A + Transpose[A].P + Q - P.B. Inverse[R].Transpose[B].P;
eqRiccati // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 2\ p12 + q11 - \frac{p12^2}{r} & p11 + p22 - \frac{p12\ p22}{r} - p12\ \nu \\ p11 + p22 - \frac{p12\ p22}{r} - p12\ \nu & 2\ p12 + q22 - \frac{p22^2}{r} - 2\ p22\ \nu \end{pmatrix}$$

In[18]:=

```
sols = Solve[
  {eqRiccati[[1, 2]] == 0, eqRiccati[[1, 1]] == 0, eqRiccati[[2, 2]] == 0}, {p11, p12, p22}]
```

Out[18]=

$$\Big\{\Big\{p11 \to \frac{r^2\ \nu + \sqrt{q11\ r + r^2}\ \sqrt{q22\ r + 2\ r^2 - 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}}{r},$$

$$p12 \to r - \sqrt{q11\ r + r^2},\ p22 \to -r\ \nu - \sqrt{q22\ r + 2\ r^2 - 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}\Big\},$$

$$\Big\{p11 \to \frac{r^2\ \nu - \sqrt{q11\ r + r^2}\ \sqrt{q22\ r + 2\ r^2 - 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}}{r},$$

$$p12 \to r - \sqrt{q11\ r + r^2},\ p22 \to -r\ \nu + \sqrt{q22\ r + 2\ r^2 - 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}\Big\},$$

$$\Big\{p11 \to \frac{r^2\ \nu - \sqrt{q11\ r + r^2}\ \sqrt{q22\ r + 2\ r^2 + 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}}{r},$$

$$p12 \to r + \sqrt{q11\ r + r^2},\ p22 \to -r\ \nu - \sqrt{q22\ r + 2\ r^2 + 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}\Big\},$$

$$\Big\{p11 \to \frac{r^2\ \nu + \sqrt{q11\ r + r^2}\ \sqrt{q22\ r + 2\ r^2 + 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}}{r},$$

$$p12 \to r + \sqrt{q11\ r + r^2},\ p22 \to -r\ \nu + \sqrt{q22\ r + 2\ r^2 + 2\ r\ \sqrt{q11\ r + r^2} + r^2\ \nu^2}\Big\}\Big\}$$

In[19]:= `sols[[4]] /. {q11 → q₁₁, q22 → q₂₂}`
`TeXForm[%]`

Out[19]=

$$\left\{p11\to \frac{r^2 \nu + \sqrt{r^2 + r\, q_{11}}\ \sqrt{2\, r^2 + r^2\, \nu^2 + 2\, r\, \sqrt{r^2 + r\, q_{11}} + r\, q_{22}}}{r},\right.$$
$$\left. p12 \to r + \sqrt{r^2 + r\, q_{11}},\ p22 \to -r\, \nu + \sqrt{2\, r^2 + r^2\, \nu^2 + 2\, r\, \sqrt{r^2 + r\, q_{11}} + r\, q_{22}}\right\}$$

Out[20]//TeXForm=

```
\left\{\text{p11}\to \frac{\sqrt{q_{11} r+r^2} \sqrt{2 r \sqrt{q_{11} r+r^2}+q_{22} r+\n
   r+r^2}+r,\text{p22}\to \sqrt{2 r \sqrt{q_{11} r+r^2}+q_{22} r+\nu ^2 r^2+2 r^2}-\nu
```

In[∘]:= `P.B.Inverse[R].Transpose[B].P /. r → 1 // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} p12^2 & p12\, p22 \\ p12\, p22 & p22^2 \end{pmatrix}$$

In[23]:= `params = {r22 → 1, q22 → 10, q11 → 0.1, r → 1};`
`Table[P /. sols[[i]] /. params , {i, 1, 4}];`
`Table[Eigenvalues[%[[i]]], {i, 1, 4}]`

Out[25]=

$$\left\{\left\{\frac{1}{2}\left(0.0488088\ \sqrt{9.90238 + \nu^2} - \sqrt{41.5759 + 8.19762\, \nu^2 + 8.19524\, \nu\ \sqrt{9.90238 + \nu^2}}\right),\right.\right.$$
$$\left.\frac{1}{2}\left(0.0488088\ \sqrt{9.90238 + \nu^2} + \sqrt{41.5759 + 8.19762\, \nu^2 + 8.19524\, \nu\ \sqrt{9.90238 + \nu^2}}\right)\right\},$$
$$\left\{\frac{1}{2}\left(-0.0488088\ \sqrt{9.90238 + \nu^2} - \sqrt{41.5759 + 8.19762\, \nu^2 - 8.19524\, \nu\ \sqrt{9.90238 + \nu^2}}\right),\right.$$
$$\left.\frac{1}{2}\left(-0.0488088\ \sqrt{9.90238 + \nu^2} + \sqrt{41.5759 + 8.19762\, \nu^2 - 8.19524\, \nu\ \sqrt{9.90238 + \nu^2}}\right)\right\},$$
$$\left\{\frac{1}{2}\left(-2.04881\ \sqrt{14.0976 + \nu^2} - \sqrt{16.8241 + 4.00238\, \nu^2 - 0.195235\, \nu\ \sqrt{14.0976 + \nu^2}}\right),\right.$$
$$\left.\frac{1}{2}\left(-2.04881\ \sqrt{14.0976 + \nu^2} + \sqrt{16.8241 + 4.00238\, \nu^2 - 0.195235\, \nu\ \sqrt{14.0976 + \nu^2}}\right)\right\},$$
$$\left\{\frac{1}{2}\left(2.04881\ \sqrt{14.0976 + \nu^2} - \sqrt{16.8241 + 4.00238\, \nu^2 + 0.195235\, \nu\ \sqrt{14.0976 + \nu^2}}\right),\right.$$
$$\left.\left.\frac{1}{2}\left(2.04881\ \sqrt{14.0976 + \nu^2} + \sqrt{16.8241 + 4.00238\, \nu^2 + 0.195235\, \nu\ \sqrt{14.0976 + \nu^2}}\right)\right\}\right\}$$

In[26]:= `K = -Inverse[R].Transpose[B].P /. sols`

Out[26]=

$$\left\{\left\{\{0, 0\}, \left\{-\frac{r - \sqrt{q11\, r + r^2}}{r}, -\frac{-r\, \nu - \sqrt{q22\, r + 2\, r^2 - 2\, r\, \sqrt{q11\, r + r^2} + r^2\, \nu^2}}{r}\right\}\right\},\right.$$
$$\left\{\{0, 0\}, \left\{-\frac{r - \sqrt{q11\, r + r^2}}{r}, -\frac{-r\, \nu + \sqrt{q22\, r + 2\, r^2 - 2\, r\, \sqrt{q11\, r + r^2} + r^2\, \nu^2}}{r}\right\}\right\},$$
$$\left\{\{0, 0\}, \left\{-\frac{r + \sqrt{q11\, r + r^2}}{r}, -\frac{-r\, \nu - \sqrt{q22\, r + 2\, r^2 + 2\, r\, \sqrt{q11\, r + r^2} + r^2\, \nu^2}}{r}\right\}\right\},$$
$$\left.\left\{\{0, 0\}, \left\{-\frac{r + \sqrt{q11\, r + r^2}}{r}, -\frac{-r\, \nu + \sqrt{q22\, r + 2\, r^2 + 2\, r\, \sqrt{q11\, r + r^2} + r^2\, \nu^2}}{r}\right\}\right\}\right\}$$

In[27]:=
```
K⟦4⟧ /. {q11 → q11, q22 → q22} // MatrixForm
TeXForm[%]
```

Out[27]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ -\frac{r+\sqrt{r^2+r\,q_{11}}}{r} & -\frac{-r\,\nu+\sqrt{2\,r^2+r^2\,\nu^2+2\,r\,\sqrt{r^2+r\,q_{11}}+r\,q_{22}}}{r} \end{pmatrix}$$

Out[28]//TeXForm=
```
\left(
\begin{array}{cc}
 0 & 0 \\
 -\frac{\sqrt{q_{11} r+r^2}+r}{r} & -\frac{\sqrt{2 r \sqrt{q_{11} r+r^2}+q_{22} r+\nu ^2
\end{array}
\right)
```

In[ ]:=
```
params = {r22 → 1, q22 → 1.0, q11 → 0., r → 1};
A + B.K⟦4⟧ /. params;
Eigenvalues[%]
```

Out[ ]=
```
{-1.61803, -0.618034}
```

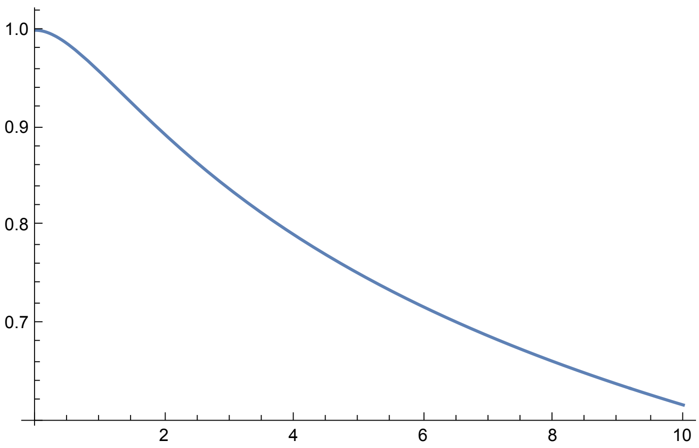
In[ ]:=
```
x = {α[t], α'[t]}
```

Out[ ]=
```
{α[t], α′[t]}
```

In[ ]:=
```
∂t x - (A.x + B.K⟦4⟧.x) + Sin[α[t] + π] /. params
αs = α /. NDSolve[{%⟦2⟧ == 0, α[0] == 1, α'[0] == 0}, α, {t, 0, 10}]⟦1⟧;
Plot[αs[t], {t, 0, 10}, PlotRange → All]
```

Out[ ]=
```
{-Sin[α[t]], -Sin[α[t]] + 1. α[t] + 2.02485 α′[t] + α″[t]}
```

Out[ ]=



Scalar

Variational form

Total LQR