

Functions

We recall from our discussion of relations the following section.

Properties of Relations

D A relation $R: S \rightarrow T$ is said to be functional and we write

$$Fncl(R) :\Leftrightarrow \forall s \in S \exists ! t \in T \text{ such that } sRt ,$$

that is, if and only if every source-element is assigned exactly one target-element as an output.

R Note that although every source-element serves as an input for a functional relation, not every target-element need serve as an output.

D Given sets S, T we define the set of all functions from S to T as follows:

$$Fnc(S, T) := \{R \in \mathcal{P}(S \times T) | Fncl(R)\}$$

R Since functions are relations of a certain type it is possible to specify functions as subsets, graphs, arrow-diagrams, digraphs, or matrices under those conditions under which relations may be so specified. It is, however, more convenient to specify functions using the following notational scheme.

S A function is *specified* by listing *five items*, separated by *specific symbols* as shown below:

$$\begin{pmatrix} \text{Name of} \\ \text{of} \\ \text{function} \end{pmatrix} := \begin{pmatrix} \text{Name of} \\ \text{generic} \\ \text{input} \end{pmatrix} \mapsto \begin{pmatrix} \text{Output at} \\ \text{generic} \\ \text{input} \end{pmatrix} : \begin{pmatrix} \text{Source} \\ \text{of} \\ \text{Function} \end{pmatrix} \rightarrow \begin{pmatrix} \text{Target} \\ \text{of} \\ \text{Function} \end{pmatrix}$$

We use lowercase greek letters (such as α , as in the example below) or certain defined strings of latin letters (such as \sin) to denote functions. We denote the generic input by lowercase latin letters; it is x in the example below. $\alpha(x)$, read alpha of x and not alpha times x , denotes the output at the generic input below. The set $Src(\alpha)$ denotes the source of the function; its elements are called source-elements. Every source-element serves as an input for a function. The set $Tgt(\alpha)$ denotes the target of the function; its elements are called target-elements. Not every target-element arises as an output, that is, as a value of the function at an input.

D The *image* of the set: $Src(\alpha)$ under the function

$$\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha),$$

denoted by $Im(Src(\alpha), \alpha)$, or in a more abbreviated form by $Im(\alpha)$, if there is no danger of confusion, is defined as follows:

$$Im(\alpha) := \{y \in Tgt(\alpha) | \exists x \in Src(\alpha) \text{ such that } y = \alpha(x)\}$$

R Alternative terminology

Source, Target, and Image are alternatively called Domain, Codomain, and Range respectively. Domain and Range for Source and Target respectively were also in use in the past.

D The graph of the function

$$\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha),$$

denoted by $Gr(\alpha)$ is defined as follows:

$$Gr(\alpha) := \{(x, \alpha(x)) \in Src(\alpha) \times Tgt(\alpha) | x \in Src(\alpha)\}$$

R Note that:

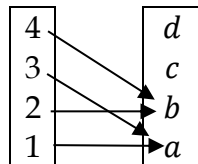
$$Gr(\alpha) \subseteq Src(\alpha) \times Tgt(\alpha)$$

E
$$\varphi := \begin{bmatrix} 4 \mapsto b \\ 3 \mapsto a \\ 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2, 3, 4\} \rightarrow \{a, b, c, d\}$$

R Because of software limitations I cannot produce the proper notation. When

you present a function on an exam, please replace $\begin{bmatrix} 4 \mapsto b \\ 3 \mapsto a \\ 2 \mapsto b \\ 1 \mapsto a \end{bmatrix}$ with the table

appearing below. Note that every input must be assigned exactly one output.



Note that $Src(\varphi) = \{1, 2, 3, 4\}$, $Tgt(\varphi) = \{a, b, c, d\}$, and $Im(\varphi) = \{a, b\}$

Special functions

R The following functions arise sufficiently often to deserve special names and symbols.

D Given a set S , we define the identity function of the set S as follows:

$$\iota_S := x \mapsto x: S \rightarrow S$$

D Given a set S and a set T such that $S \subseteq T$, we define the inclusion function of the set S into the set T as follows:

$$\iota_{S \subseteq T} := x \mapsto x: S \rightarrow T$$

D Given a set S and a set T and $t \in T$, we define the constant function from the set S into the set T with value t as follows:

$$\tilde{t} := t_{S \rightarrow T} := x \mapsto t: S \rightarrow T$$

Given a family of sets $(S_k | k \in 1..n)$ we define $\forall k \in 1..n$

D the k^{th} projection

$$\pi_k := (s_k | k \in 1..n) \mapsto s_k: \prod_{k \in 1..n} S_k \rightarrow S_k$$

D and the k^{th} insertion

$$\iota_k := s \mapsto (s, k): S_k \rightarrow \coprod_{k \in 1..n} S_k$$

Operations on Functions

D Composition of Functions

Given two functions:

$$\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha), \text{ and}$$

$$\beta := x \mapsto \beta(x): Src(\beta) \rightarrow Tgt(\beta)$$

$$\text{where} \quad Tgt(\alpha) = Src(\beta)$$

$$\text{that is:} \quad Src(\alpha) \xrightarrow{\alpha} Tgt(\alpha) = Src(\beta) \xrightarrow{\beta} Tgt(\beta)$$

we define the composite $\beta \circ \alpha$ (read β circle α) as follows:

$$\beta \circ \alpha := x \mapsto \beta(\alpha(x)): Src(\alpha) \rightarrow Tgt(\beta)$$

$$\text{Note that:} \quad \forall x \in Src(\alpha), (\beta \circ \alpha)(x) = \beta(\alpha(x))$$

R Note that function composition agrees relational composition. If functions were composed as relations, the relation so obtained would agree with the result of composing them as functions.

D Given a function: $\alpha \in Fnc(S)$, we may compose α with itself; the n-fold composite of α with itself, denoted by $R^{\circ n}$ is defined recursively as follows:

$$\text{BC} \quad \alpha^{\circ 0} := \iota$$

$$\text{RcS} \quad \forall n \in \mathbb{N} \quad \alpha^{\circ(n+1)} := (\alpha^{\circ(n)}) \circ \alpha$$

R The notion of composition may be modified to include situations in which $Tgt(\alpha) \neq Src(\beta)$.

Products and Multiples

Given two functions:

$$\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha), \text{ and}$$

$$\beta := x \mapsto \beta(x): Src(\beta) \rightarrow Tgt(\beta)$$

D we define the product $\alpha \times \beta$ as follows:

$$\alpha \times \beta := (x, y) \mapsto (\alpha(x), \beta(y)): (Src(\alpha)) \times (Src(\beta)) \rightarrow (Tgt(\alpha)) \times (Tgt(\beta))$$

and if: $Src(\alpha) = Src(\beta) = S$

D we define the pair (α, β) as follows:

$$(\alpha, \beta) := s \mapsto (\alpha(s), \beta(s)): S \rightarrow (Tgt(\alpha)) \times (Tgt(\beta))$$

Restriction of Source, Extension of Target, and Adjustment of Target to Image

R In some situations, one encounters functions that may be made bijective by first restricting the source, and then by adjusting the target to the image.

Given a function $\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha)$,

a set S , such that $S \subseteq Src(\alpha)$, and

a set T , such that $Tgt(\alpha) \subseteq T$, we define:

D the restriction of α ,

$$\alpha \Big|_S := x \mapsto \alpha(x): S \rightarrow Tgt(\alpha)$$

D the extension of α ,

$$\alpha \Big|_T := x \mapsto \alpha(x): Src(\alpha) \rightarrow T$$

D the adjustment of α ,

$$\alpha \Big|_{Im}, := x \mapsto \alpha(x): Src(\alpha) \rightarrow Im(\alpha)$$

E To illustrate these ideas we consider the following example:

E

$$\varphi := \begin{bmatrix} 3 \mapsto a \\ 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2, 3\} \rightarrow \{a, b, c, d\}$$

$$Src(\varphi) = \{1, 2, 3\} \quad Tgt(\varphi) = \{a, b, c, d\} \quad Im(\varphi) = \{a, b\}$$

$Src(\varphi) = \{1, 2, 3\}$ has $2^3 = 8$ subsets which are

$$\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}.$$

We may restrict φ to any of these 8 subsets and adjust them to image. We exhibit some of them.

$$1 \quad \varphi \Big|_{\{\}} := [\] : \{\} \rightarrow \{a, b, c, d\}$$

$$\varphi \Big|_{\{\}}^{Im} := [\] : \{\} \rightarrow \{\} \text{ which is a bijective restriction. Note}$$

that this is precisely $\iota_{\{\} \subseteq \{a, b, c, d\}}$, the inclusion of the empty set $\{\}$ into the set $\{a, b, c, d\}$.

$$2 \quad \varphi \Big|_{\{2\}} := [2 \mapsto b] : \{2\} \rightarrow \{a, b, c, d\}$$

$$\varphi \Big|_{\{2\}}^{Im} := [2 \mapsto b] : \{2\} \rightarrow \{b\} \text{ which is a bijective restriction. It}$$

is not a maximal bijective restriction as the φ may be bijectively extended to a larger source, for instance, $\{1, 2\}$, or $\{2, 3\}$.

$$3 \quad \varphi \Big|_{\{1, 3\}} := \begin{bmatrix} 3 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1, 3\} \rightarrow \{a, b, c, d\}$$

$$\varphi \Big|_{\{1, 3\}}^{Im} := \begin{bmatrix} 3 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1, 3\} \rightarrow \{a\} \text{ which is not a bijective}$$

restriction as $\varphi(1) = a = \varphi(3)$.

$$4 \quad \varphi \Big|_{\{2,3\}} := \begin{bmatrix} 3 \mapsto a \\ 2 \mapsto b \end{bmatrix} : \{2,3\} \rightarrow \{a,b\}$$

$$\varphi \Big|_{\{2,3\}}^{Im} := \begin{bmatrix} 3 \mapsto a \\ 2 \mapsto b \end{bmatrix} : \{2,3\} \rightarrow \{a,b\} \quad \text{which is a bijective}$$

restriction. It is a maximal bijective restriction because it cannot be bijectively extended any further.

Functions induced by $\alpha \in \text{Fnc}(\text{Src}(\alpha), \text{Tgt}(\alpha))$

D Given $\alpha \in \text{Fnc}(\text{Src}(\alpha), \text{Tgt}(\alpha))$, and $S \in \mathcal{P}(\text{Src}(\alpha))$, $T \in \mathcal{P}(\text{Tgt}(\alpha))$, we define:

$\alpha_>(A)$ the image of A under α and

$\alpha^<(B)$, the pre-image of B under α as follows:

D
$$\alpha_>(S) := \{t \in \text{Tgt}(\alpha) \mid \exists s \in S (t = \alpha(s))\} = \{\alpha(s) \mid s \in S\}$$

and

D
$$\alpha^<(T) := \{s \in \text{Src}(\alpha) \mid \exists t \in T (\alpha(s) = t)\} = \{s \in \text{Src}(\alpha) \mid \alpha(s) \in T\}$$

D Given $\alpha \in \text{Fnc}(\text{Src}(\alpha), \text{Tgt}(\alpha))$, we define:

the image function induced by α :

$$\alpha_>: \mathcal{P}(\text{Src}(\alpha)) \rightarrow \mathcal{P}(\text{Tgt}(\alpha)) \text{ and}$$

the pre-image function induced by α :

$$\alpha^<: \mathcal{P}(\text{Tgt}(\alpha)) \rightarrow \mathcal{P}(\text{Src}(\alpha))$$

as follows:

D
$$\alpha_> := S \mapsto \alpha_>(S): \mathcal{P}(\text{Src}(\alpha)) \rightarrow \mathcal{P}(\text{Tgt}(\alpha))$$

and

D
$$\alpha^< := S \mapsto \alpha^<(S): \mathcal{P}(\text{Tgt}(\alpha)) \rightarrow \mathcal{P}(\text{Src}(\alpha))$$

Properties of Functions

D Equality

Two functions: $\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha)$ and

$$\beta := x \mapsto \beta(x): Src(\beta) \rightarrow Tgt(\beta)$$

are said to be *equal*, if the following holds:

$$(\alpha = \beta) :\Leftrightarrow \left(\begin{array}{l} (1) Src(\alpha) = Src(\beta) \\ \wedge (2) Tgt(\alpha) = Tgt(\beta) \\ \wedge (3) \forall x \in Src(\alpha) (\alpha(x) = \beta(x)) \end{array} \right)$$

D Injectivity

The function $\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha)$ is said to be *injective*, denoted by the predicate *Inj*, if the following holds:

$$Inj(\alpha) :\Leftrightarrow \forall x, y \in Src(\alpha), \left(\begin{array}{l} (1) (x \neq y) \Rightarrow (\alpha(x) \neq \alpha(y)) \\ \text{or equivalently,} \\ (2) (\alpha(x) = \alpha(y)) \Rightarrow (x = y) \end{array} \right)$$

R An injective function preserves the distinctness of inputs, that is, an injective function does not assign the same output to more than one input.

A To prove that a function is injective, if $Src(\alpha)$ and $Tgt(\alpha)$ are both finite sets, use a display.

If $Src(\alpha)$ is not a finite set, see if: $\frac{\alpha(x)=\alpha(y)}{x=y}$ is valid.

Otherwise, to establish non-injectivity, find a counterexample, that is $x, y \in Src(\alpha)$, such that $x \neq y$ but $\alpha(x) = \alpha(y)$.

E $\theta := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b, c\}$ which is injective by inspection.

E $\varphi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b\}$ which is not injective as $\varphi(1) = a = \varphi(2)$.

E $\psi := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b\}$ which is injective by inspection.

D Surjectivity

The function $\alpha := x \mapsto \alpha(x): \text{Src}(\alpha) \rightarrow \text{Tgt}(\alpha)$ is said to be *surjective*, denoted by the predicate Srl , if the following holds:

$$\text{Srl}(\alpha) : \Leftrightarrow \forall y \in \text{Tgt}(\alpha), \exists x \in \text{Src}(\alpha), \text{ such that: } y = \alpha(x).$$

R A surjective function hits every target-element, that is, every target element is realized as an output; that is if $\text{Im}(\alpha) = \text{Tgt}(\alpha)$.

A To prove that a function is surjective, if $\text{Src}(\alpha)$ and $\text{Tgt}(\alpha)$ are both finite sets, use a display.

If $\text{Src}(\alpha)$ is not a finite set, see if: $\frac{\alpha(x)=y}{x=\beta(y)}$ is valid, that is, see if you can solve the equation $\alpha(x) = y$ for x in terms of y .

Otherwise to establish non-surjectivity, find a counterexample, that is, some element $b \in \text{Tgt}(\alpha)$ that is not hit.

E $\theta := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b, c\}$ which is not surjective because no input has been assigned the target element c as output. 'c is not hit'.

E $\varphi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b\}$ which is not surjective $\text{Im}(\varphi) = \{a\} \neq \text{Tgt}(\varphi)$

E $\psi := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b\}$ which is surjective because $\text{Im}(\psi) = \{a, b\} = \text{Tgt}(\psi)$, that is, every target element is hit.

Q Prove or disprove:

The set of injective functions $\text{InjFncs}\left(\{1, 2\}, \{1\}\right)$ from $\{1, 2\}$ to $\{1\}$ is empty, that is: $\text{InjFncs}\left(\{1, 2\}, \{1\}\right) = \{\}$.

The set of surjective functions $SurjFncs\left(\{1,2\},\{1\}\right)$ from $\{1,2\}$ to $\{1\}$ is empty,
that is: $SurjFncs\left(\{1,2\},\{1\}\right) = \{ \}$.

R Note that

$$E \quad \theta := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1,2\} \rightarrow \{a,b,c\} \quad \left(Inj(\theta) \right) \wedge \left(\cancel{Surj}(\theta) \right).$$

$$E \quad \varphi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1,2\} \rightarrow \{a,b\} \quad \left(\cancel{Inj}(\varphi) \right) \wedge \left(\cancel{Surj}(\varphi) \right)$$

$$E \quad \psi := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1,2\} \rightarrow \{a,b\} \quad \left(Inj(\psi) \right) \wedge \left(Surj(\psi) \right)$$

$$E \quad \chi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1,2\} \rightarrow \{a\} \quad \left(\cancel{Inj}(\chi) \right) \wedge \left(Surj(\chi) \right)$$

D Bijectivity

The function $\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha)$ is said to be *bijective*, denoted by the predicate *Bij*, if the following holds:

$$Bij(\alpha): \Leftrightarrow Inj(\alpha) \wedge Srj(\alpha)$$

E $\theta := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix}: \{1,2\} \rightarrow \{a,b,c\}$ which is not bijective because ~~*Srj*~~(θ).

E $\varphi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix}: \{1,2\} \rightarrow \{a,b\}$ which is not bijective because ~~*Inj*~~(φ).

E $\psi := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix}: \{1,2\} \rightarrow \{a,b\}$ which is bijective *Inj*(ψ) \wedge *Srj*(ψ).

E $\chi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix}: \{1,2\} \rightarrow \{a\}$ which is not bijective because ~~*Inj*~~(χ).

D Invertibility

The function $\alpha := x \mapsto \alpha(x): Src(\alpha) \rightarrow Tgt(\alpha)$ is said to be *invertible*, denoted by the predicate *Inv*, if the following holds:

$$Inv(\alpha): \Leftrightarrow \exists \beta: Tgt(\alpha) (= Src(\beta)) \rightarrow Src(\alpha), \left(\begin{array}{c} (1) (\beta \circ \alpha = \iota_{Src(\alpha)}) \\ \text{and} \\ (2) (\alpha \circ \beta = \iota_{Src(\beta)}) \end{array} \right)$$

Under these conditions, β is called *the inverse of α* , and α is called *the inverse of β* .

N Note that α and β appear symmetrically in the above definition and, therefore, it makes no sense to talk of 'inverse' functions, as it is impossible to distinguish between the 'direct' and the 'inverse' function. It may be the case that one of the functions came to be considered earlier, leading to a higher degree of familiarity with it, thereby earning the one that came to be considered later the qualifier 'inverse'. This event, however, is a historical accident. Two functions that satisfy the conditions of the above definition are called inverses of each other.

N We denote the *inverse of α* , by the symbol α^{\leftarrow} , that is:

$$\beta = \alpha^{\leftarrow},$$

$$\alpha = \beta^{\leftarrow}, \text{ and}$$

$$(\alpha^{\leftarrow})^{\leftarrow} = \alpha$$

R Note that in the definition of invertibility, the conditions (1) and (2) are respectively equivalent to the conditions (1)' and (2)' that appear below:

$$(1)' \quad \forall x \in \text{Src}(\alpha), (\beta \circ \alpha)(x) = \beta(\alpha(x)) = \iota_{\text{Src}(\alpha)}(x) = x \text{ and}$$

$$(2)' \quad \forall y \in \text{Tgt}(\alpha), (\alpha \circ \beta)(x) = \alpha(\beta(x)) = \iota_{\text{Tgt}(\alpha)}(y) = y.$$

R The function α is invertible, if and only if, there is a function β going backwards that undoes the effect of α , and in such a way, that α likewise undoes the effect of β .

$$\mathbf{T} \quad \text{Bij}(\alpha) \Leftrightarrow \text{Inv}(\alpha)$$

Pf Omitted.

R If the function: $\alpha := x \mapsto \alpha(x): \text{Src}(\alpha) \rightarrow \text{Tgt}(\alpha)$ is invertible, to find the inverse α^{\leftarrow} of α , we first solve the equation $\alpha(x) = y$ for x in terms of y , as shown below:

$$\frac{\alpha(x)=y}{x=\beta(y)},$$

and then set: $\alpha^{\leftarrow} := y \mapsto \beta(y): \text{Tgt}(\alpha) \rightarrow \text{Src}(\alpha).$

$$\mathbf{E} \quad \theta := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b, c\} \quad \text{Bij}(\theta), \text{ therefore } \text{Inv}(\theta).$$

$$\mathbf{E} \quad \varphi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b\} \quad \text{Bij}(\theta), \text{ therefore } \text{Inv}(\theta).$$

$$\mathbf{E} \quad \chi := \begin{bmatrix} 2 \mapsto a \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a\} \quad \text{Bij}(\theta), \text{ therefore } \text{Inv}(\theta).$$

$$\mathbf{E} \quad \psi := \begin{bmatrix} 2 \mapsto b \\ 1 \mapsto a \end{bmatrix} : \{1, 2\} \rightarrow \{a, b\} \quad \text{Bij}(\theta), \text{ therefore } \text{Inv}(\theta)$$

$$\psi^{\leftarrow} := \begin{bmatrix} b \mapsto 2 \\ a \mapsto 1 \end{bmatrix} : \{a, b\} \rightarrow \{1, 2\} \quad \text{Bij}(\theta), \text{ therefore } \text{Inv}(\theta)$$

Home Work

It is given that the relations:

$$\begin{aligned} & Gr(\alpha), Gr(\beta), Gr(\gamma) \\ \in & \left\{ \begin{array}{lll} \{(a, a), (b, a), (c, a)\}, & \{(a, a), (b, b), (c, a)\}, & \{(a, a), (b, b), (c, c)\}, \\ \{(a, b), (b, a), (c, c)\}, & \{(a, b), (b, c), (c, a)\}, & \{(a, c), (b, a), (c, b)\}, \\ \{(a, b), (b, c), (c, b)\}, & \{(a, a), (b, c), (c, b)\}, & \{(a, c), (b, c), (c, a)\} \end{array} \right\} \\ \subseteq & \mathcal{P}\left(\{a, b, c\} \times \{a, b, c\}\right) \end{aligned}$$

are **graphs of functions**, that is: $\alpha, \beta, \gamma \in Fnc(\{a, b, c\})$, and that

$$S \in \mathcal{P}(\{a, b, c\}), T \in \{\{a, b, c, d\}, \{a, b, c, d, e\}\} \quad \odot, \otimes, \odot \in \left\{ \circ, \left|_S, \left|_T, \left|_{Im} \right. \right\}$$

R Count how many variants there are of each problem. Do as many of these variants as you need to in order to understand what is going on.

0 Find for every α : its specification as a function.

1 Determine for every α , if:

$$Inj(\alpha), Srj(\alpha), Bij(\alpha)$$

and if $Bij(\alpha)$, find

$$\alpha^{\leftarrow}.$$

2 Compute the following expressions using particular choices for functions: α, β , and γ , for sets: S , and T , and operations: \odot, \otimes , and \odot from appropriate sets in the above.

$$20 \quad \odot (R)$$

$$21 \quad R \odot S$$

$$22 \quad (R \odot S) \otimes T$$

- 3 For each of your answers α in 2 determine if:

$$Inj(\alpha), Surj(\alpha), Bij(\alpha)$$

and if $Bij(\alpha)$, find

$$\alpha^{-1}.$$

- 4 Determine for every α , how many bijective restrictions there are and find their specifications.
- 5 Determine for every α , $\alpha_{>}$ and $\alpha_{<}$.