D Define $\forall k \in 1..8 \ A_k \ := \begin{cases} 1 & if \quad J_k\big(mod(3)\big) = 1 \\ 0 & if \quad J_k\big(mod(3)\big) = 0 \\ -1 & if \quad J_k\big(mod(3)\big) = 2 \end{cases}$

Show calculations on the facing side.

Hence

$A_1 =$ $A_2 =$ $A_3 =$ $A_4 =$

$A_5 =$ $A_6 =$ $A_7 =$ $A_8 =$

D Definitions for (1c), (2c), (3c):

(0) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right)$

(0a) $Unbdd\left(S\right) :\Leftrightarrow \quad \forall r \in [0.+\infty[\ \exists(P,Q) \in S^2\left(dst\left(P,Q\right) = r\right)$

(1) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \forall\left(P,Q\right) \in \left(S^2 \backslash \Delta_S\right)$

(1a) $RsVrRn\left(S,P,Q\right) := \left(\dfrac{\pi_2\left(Q\right)-\pi_2\left(P\right)}{\pi_1\left(Q\right)-\pi_1\left(P\right)}\right)$

(1b) $RnVrRs\left(S,P,Q\right) := \left(\dfrac{\pi_1\left(Q\right)-\pi_1\left(P\right)}{\pi_2\left(Q\right)-\pi_2\left(P\right)}\right)$

(2) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right)$

$$CnstIncln\left(S\right)$$

$$:\Longleftrightarrow \quad \forall\left(\left(A,B\right),\left(C,D\right)\right) \in \left(S^2\backslash\Delta_S\right)\times\left(S^2\backslash\Delta_S\right)$$

$$\left(\begin{array}{c}\left(RsVrRn\left(S,A,B\right) = RsVrRn\left(S,C,D\right)\right)\\ \vee \\ \left(RnVrRs\left(S,A,B\right) = RnVrRs\left(S,C,D\right)\right)\end{array}\right)$$

(3) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \qquad \forall P, Q \in \left(\mathbb{R}^2 \setminus \Delta_{\mathbb{R}}\right)$

$$StLn_1\left(S, P.Q\right)$$

$$:\Leftrightarrow \quad \left(P \in S\right) \wedge \left(Q \in S\right) \wedge \left(Unbdd\left(S\right)\right) \wedge \left(CnstIncln\left(S\right)\right)$$

$:\Leftrightarrow$

(4) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \qquad \forall P, Q \in \left(\mathbb{R}^2 \setminus \Delta_{\mathbb{R}}\right)$

$$StLn_2\left(S, P.Q\right)$$

$$:\Leftrightarrow \quad \left(P \in S\right) \wedge \left(Q \in S\right) \wedge \left(\begin{array}{c} \exists\left(p, q, r\right) \in \left(\mathbb{R}^3\right) \setminus \left\{(0.0.0)\right\} \\ \left(S := \left\{(x, y) \in \mathbb{R}^2 \middle| px + qy + r = 0\right\}\right) \end{array}\right)$$

R $StLn_1\left(S, P.Q\right)$ and $StLn_2\left(S, P.Q\right)$ are possible formalisations of the notion of a straight line through two distinct the points $P, Q \in \mathbb{R}$ in the senses of Philip and Jack respectively.