Math **270** Name (PLEASE PRINT) ____

Exam **1B Take Home** **2019 Fall**/Spring/Summer I /Summer II Score _________

Instructions

- All work must be done using a **MECHANICAL PENCIL following instructions given**.
- If you have any questions at all, please **ask me**. This includes the following possibilities:
  - something is **illegible,** or **ambiguous**, or **unclear** to you.
- **You may use your homework, notes, books, and calculators unless otherwise stated during the exam, as long all handwritten work is either done in the packages provided or on engineering-computation-pad paper.**
- **You will not get any partial credit**. You have to show **absolutely all your work** on all questions, **exactly as I do on the board in class, with no changes, variations, or omissions in form, unless otherwise noted whether orally or in writing**. If your answer is correct, but your procedure is incorrect or is not allowed by the instructions, you will get no credit all. All work must be displayed using **correct syntax**, and all **explanations**, written in **complete sentences**, must be complete.
- **You must write neatly, clearly and legibly. If you run out of room, you may not turn the page, but you must continue the work on the facing page (on the left side) labelling your work appropriately. You must draw every straight-line segment other than those in the list: ($=, \neq, \equiv, <, \leq, >, \geq, +, -, \times, \div$, $\neg, \vee, \wedge, \parallel, \perp, \backslash, \rightarrow, \leftrightarrow, \mapsto$) with your ruler. At least 1 point will be taken off for each instance of violation.**
- Remember that 'does not make sense' is a possible answer.
- I can grade you only on what you **actually** put on paper; I cannot read minds. You cannot say later that what you had written down is not what you had meant. **You need to learn to write exactly what you mean and mean exactly what you write**.
- **The answers for a problem should be noted on page *n***, as explained in class, and the **work must be shown with proper indices** on the back of page $(n-1)$ and in case the work cannot be completed thereon, the work may be continued with proper indexing on the back of page n, page $(n+1)$, back of page (n+1) .. back of page (n+k-1), and page $(n+k)$, as provided.
- After the exam is returned, all notes, if any, must be taken on the facing (blank) side with a ball-point with **blue ink**.

You may talk to anyone

Instructions:

Your J-number = ______________________________

D Define

$\forall k, l \in 1..8$

$$A_{kl} \quad \coloneqq \quad \begin{cases} 0 & if\ (k \neq l) \wedge (J_k = J_l) \\ 1 & otherwise \end{cases}$$

Show complete chart on the facing side for A_{kl} where $k, l \in 1..8$

D Define

$\forall k, l \in 1..8$

$$B_{kl} \coloneqq \begin{cases} -1 & if\ \Big((k < l) \wedge (J_k \leq J_l)\Big) \vee \Big((k > l) \wedge (J_k \geq J_l)\Big) \\ 0 & if\ (k = l) \\ 1 & if\ \Big((k < l) \wedge (J_k \geq J_l)\Big) \vee \Big((k > l) \wedge (J_k \leq J_l)\Big) \end{cases}$$

Show complete chart on the facing side for A_{kl} and B_{kl} where $k, l \in 1..8$

D Define

$\forall n \in \mathbb{N}\backslash\{0\}\ \forall k \in 1..8$

$$J_{7k} \quad \coloneqq \quad J_k (mod\ n)$$

Show calculations on the facing side for:

$J_{71} =$ $\qquad J_{72} =$ $\qquad J_{73} =$ $\qquad J_{74} =$

$J_{75} =$ $\qquad J_{76} =$ $\qquad J_{77} =$ $\qquad J_{78} =$

R **The above numbers will occur throughout the exam, so please make sure that they are correct. Otherwise all your answers will be wrong.**

D Define Y:= Yes N:=No

D Define Pf:= Proof W:=Witness

R **You will NOT get any points at all unless you check that your solutions are correct by substitution exactly as shown in class.**

R **Do not write anything down on any page if you cannot completely solve the problem or some part thereof that is indexed by a roman numeral.**

R **Any sloppiness, untidiness, and any departure from proper format (as indicated in class) will lead to a score of 0.**

R **On any problem whatsoever, if you do not check that your solution is correct exactly as shown in class you will get no points on this question.**

R If answers to different instances of some question on the exam contradict each other you will get a score of zero for every instance.

R **You will lose points for making any extraneous mark on the exam.**

R You may only submit complete answers to questions marked explicitly on the exam. You will lose points for unfinished work. **Do not write anything down on any page if you cannot completely solve the problem or some part thereof that is indexed by a roman numeral.**

R Separate pieces of work for any part of any problem recorded on a blank page intended for the purpose must be indexed by the index of the problem and also by numbers starting at 0 that record the order in which the pieces were completed; such parts must be separated from each other by straight lines parallel to the edges of the paper drawn with a ruler. You will lose points otherwise.

6 (1a) Solve the following system (S) of equations for:

$x := (x_1, x_2, x_3, x_4, x_5) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \in \mathbb{R}^5$, by row-reduction and check by substitution that your solution is correct. **Show the work for row-reduction and substitution on the facing side.**

$$\text{(S)} \quad \begin{array}{ll} (1) & B_{12}x_1 + B_{13}x_2 = B_{14} + B_{15} \\ (2) & B_{14}x_4 + B_{15}x_5 = B_{12} + B_{13} \\ (3) & B_{12}x_1 + B_{13}x_2 + B_{14}x_4 + B_{15}x_5 = B_{12} + B_{13} + B_{14} + B_{15} \end{array}$$

$$Flt\left(S \right) := \left\{ x \in \mathbb{R}^5 \,\middle|\, \begin{array}{lc} & (A_{12}x_1 + A_{13}x_2 = A_{14} + A_{15}) \\ \wedge & (A_{14}x_4 + A_{15}x_5 = A_{12} + A_{13}) \\ \wedge & (A_{12}x_1 + A_{13}x_2 + A_{14}x_4 + A_{15}x_5 = A_{12} + A_{13} + A_{14} + A_{15}) \end{array} \right\}$$

Rewrite the system using your values of e, f, g, and h and show all work on facing page and answer the following questions.

My problem:

(1) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 + (\quad)x_5 = (\quad) + (\quad) = (\quad)$

(2) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 + (\quad)x_5 = (\quad) + (\quad) = (\quad)$

(3) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 + (\quad)x_5 = (\quad) + (\quad) = (\quad) + (\quad) = (\quad)$

1 (i) Express $Flt\left(\quad\right)$ as the intersection of the least number of hyperplanes.

$$Flt\left(S \right) = \left\{ \quad \right.$$

You may talk to anyone

6 (1a) Continued

1 (ii) $dim\left(\mathbb{R}^5\right) = 5$ $\quad \nu\left(IndEqns\left(S\right)\right) =$ $\quad \nu\left(Prmtrs\left(S\right)\right) =$

Hence $Flt\left(S\right)$ represents a ___-flat in $\mathbb{R}^5$.

1 (iii) Vector-parametric form

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

1 (iv) Using parameters

$$Flt\left(S\right) = \left\{ \right.$$

1 (v) Flat form

1 (vi) Functional form $Flt(S) := Im(\Lambda)$

$\Lambda :=$

6 (1b) **Information as in (1a)**

2 (i) Find, if possible, a flat $F_1 \subseteq \mathbb{R}^5$ of largest possible dimension such that $prl\left(F_1, F\right) F$, and prove your assertion.

$F_1 =$ $dim\left(F_1\right) =$

2 (ii) Find, if possible, a flat $F_2 \subseteq \mathbb{R}^5$ of largest possible dimension such that $prp\left(F_2, F\right) F$, and prove your assertion.

$F_2 =$ $dim\left(F_2\right) =$

2 (iii) Find, if possible, a flat $F_{21} \subseteq \mathbb{R}^5$ of largest possible dimension such that $skw\left(F_2, F\right)$, and prove your assertion.

$F_1 =$ $dim\left(F_1\right) =$

5 (1c) 5 (3c) Solve the following system of equations for x, y, and z in $\mathbb{R}^3$, by row-reduction and check that your solution is correct. **Show the work for row-reduction on the facing side**. It is given that: $p, q, r \in \mathbb{R}$ and $p^3 + q^3 + r^3 - 3pqr \neq 0$. You may not divide by p, q, or r, as you do not know if any of these equals 0.

$$px + qy + rz = p + q + r$$

$$qx + ry + pz = p + q + r$$

$$rx + py + qz = p + q + r$$

Write the answer in:

1 (i) vector-parametric form

1 (ii) flat form

1 (iii) functional form

$\Lambda :=$

$F(S) :=$

1 (iv) On the basis of your answer, it follows that the system of equations in (1a) represents a

___-flat (also called a ________)in ___ .

1 (v) What happens to $F(S)$ from (1a) if: $p^3 + q^3 + r^3 - 3pqr = 0$

6 (2a) Find P and Q such that: $PAQ = CF(A)$, the canonical form of A.

Rewrite using your values of $e, f, g,$ and h. **Show work on the facing side. You will get 0 points unless you do part (v) to check that your answer is correct.**

$$A := \begin{bmatrix} A_{11} & A_{12} & 1 & 1 \\ 1 & A_{22} & A_{24} & 1 \\ 1 & i & A_{33} & A_{34} \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

1 (i) $P = \begin{bmatrix} \quad \end{bmatrix}$

1 (ii) $Q = \begin{bmatrix} \quad \end{bmatrix}$

1 (iii) $HF(A) = \begin{bmatrix} \quad \end{bmatrix}$

1 (iv) $CF(A) = \begin{bmatrix} \quad \end{bmatrix}$

1 (v) What is the rank of A? $Rnk(A) =$ ____________

1 (vi) Check by multiplying that:

$$PAQ = (HF(A))Q = CF(A).$$

6 (2b) Solve the following system (S) of equations for:

$x := (z_1, z_2) = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \in \left(\mathbb{C} \right)^2$, by row-reduction. Check by substitution that your solution is correct. **Show work on the facing side.**

(1) $(A_{11} + iA_{12})x_1 + (A_{13} + iA_{14})x_3 = (A_{15} + A_{16}) + i(A_{17} + A_{18})$

(S)

(2) $(A_{81} + iA_{82})x_1 + (A_{83} + iA_{84})x_3 = (A_{85} + A_{86}) + i(A_{87} + A_{88})$

Rewrite the system using your values of your coefficients from the chart found earlier and answer the following questions.

My system:

(1) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 = (\quad) + (\quad) + (\quad) + (\quad) = (\quad)$

(2) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 = (\quad) + (\quad) + (\quad) + (\quad) = (\quad)$

2 (i) $dim\left(\left(\mathbb{C}\right)^2\right) = 2 \qquad \nu\left(IndEqns\left(S\right)\right) = \quad \nu\left(Prmtrs\left(S\right)\right) =$

Hence $Flt\left(S\right)$ represents a __-flat in $\left(\mathbb{C}\right)^2$.

4 (ii) Write the answer in vector-parametric form:

$$x := \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} (\qquad) + i(\qquad) & (\qquad) + i(\qquad) \\ (\qquad) + i(\qquad) & (\qquad) + i(\qquad) \end{bmatrix}$$

You may talk to anyone

5 (2c) Solve the following system (S) of equations for:

$$x := (x_1, x_2, x_3, x_4) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \left(\mathbb{Z}_7\right)^4,$$ by row-reduction. Check by substitution that your solution is correct. **Show work on the facing side.**

$$\text{(S)} \quad \begin{array}{ll} (1) & J_{73}x_1 + J_{74}x_2 + J_{75}x_3 + J_{76}x_4 = J_{77} \\ (2) & J_{74}x_1 + J_{75}x_2 + J_{76}x_3 + J_{77}x_4 = J_{87} \end{array}$$

$$Flt\left(S\right) := \left\{ x \in \left(\mathbb{Z}_2\right)^4 \,\middle|\, \begin{array}{l} (J_{73}x_1 + J_{74}x_2 + J_{75}x_3 + J_{76}x_4 = J_{77}) \\ \wedge \ (J_{74}x_1 + J_{75}x_2 + J_{76}x_3 + J_{77}x_4 = J_{87}) \end{array} \right\}$$

Rewrite the system using your values of your coefficients from the chart found earlier and answer the following questions.

My system:

(1) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 = (\quad)$

(2) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 = (\quad)$

1 (i) Construct the addition-table for $\mathbb{Z}_7$ on the facing side.

1 (ii) Construct the multiplication-table for $\mathbb{Z}_7$ on the facing side.

1 (iii) $dim\left(\left(\mathbb{Z}_7\right)^4\right) = 4 \qquad \nu\left(IndEqns\left(S\right)\right) = \qquad \nu\left(Prmtrs\left(S\right)\right) =$

Hence $Flt\left(S\right)$ represents a ___-flat in $\left(\mathbb{Z}_7\right)^4$.

2 (2c) Continued.

1(iv) Vector-parametric form using only addition:

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

1 (v) $\nu\left(Flt\left(S\right)\right) =$

6 (3a) Solve the following system (S) of equations for:

$$x := (x_1, x_2, x_3, x_4) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \left(\mathbb{Z}_2\right)^4,$$ by generating, listing, computing, and counting. Check by substitution that your solution is correct. **Show work on the facing side.**

$$\text{(S)} \quad \begin{aligned} &(1) \quad A_{11}x_1 + A_{12}x_2 + A_{13}x_3 + A_{14}x_4 = A_{15} + A_{16} + A_{17} + A_{18} \\ &(2) \quad A_{81}x_1 + A_{82}x_2 + A_{83}x_3 + A_{84}x_4 = A_{85} + A_{86} + A_{87} + A_{88} \end{aligned}$$

$$Flt\left(S\right) := \left\{ x \in \left(\mathbb{Z}_2\right)^4 \;\middle|\; \begin{array}{l} (A_{11}x_1 + A_{12}x_2 + A_{13}x_3 + A_{14}x_4 = A_{15} + A_{16} + A_{17} + A_{18}) \\ \wedge \quad (A_{81}x_1 + A_{82}x_2 + A_{83}x_3 + A_{84}x_4 = A_{85} + A_{86} + A_{87} + A_{88}) \end{array} \right\}$$

Rewrite the system using your values of your coefficients from the chart found earlier and answer the following questions.

My system:

(1) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 = (\quad) + (\quad) + (\quad) + (\quad) = (\quad)$

(2) $(\quad)x_1 + (\quad)x_2 + (\quad)x_3 + (\quad)x_4 = (\quad) + (\quad) + (\quad) + (\quad) = (\quad)$

5 (i) Generate, list, and count the number of solutions:

$$Flt\left(S\right) = \left\{ \quad \right\}$$

1 (ii) $\nu\left(Flt\left(S\right)\right) =$

You may talk to anyone

1 (3b) Same information as in (3a)

Solve the following system (S) of equations for:

$x := (x_1, x_2, x_3, x_4) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \left(\mathbb{Z}_2\right)^4$, by row-reduction and check by substitution that your solution is correct. **Show the work for row-reduction and substitution on the facing side.**

My system:

(1) $(\ \ )x_1 + (\ \ )x_2 + (\ \ )x_3 + (\ \ )x_4 = (\ \ ) + (\ \ ) + (\ \ ) + (\ \ ) = (\ \ )$

(2) $(\ \ )x_1 + (\ \ )x_2 + (\ \ )x_3 + (\ \ )x_4 = (\ \ ) + (\ \ ) + (\ \ ) + (\ \ ) = (\ \ )$

1 (i) $dim\left(\left(\mathbb{Z}_2\right)^4\right) = 4 \qquad \nu\left(IndEqns\left(S\right)\right) = \quad \nu\left(Prmtrs\left(S\right)\right) =$

Hence $Flt\left(S\right)$ represents a ___-flat in $\left(\mathbb{Z}_2\right)^4$.

5 (3b) (3b) continued. Same information as in (3a). You get 0 on both (3a) and (3b) answer of (3a)(i) does not agree with the answer of (3b)(iii).

(A) Write the answer in:

1 (ii) Vector-parametric form using only addition:

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

4 (iii) as a finite set assigning all possible values to the parameters

$$Flt\left(S \right) = \left\{ \qquad \qquad \qquad \qquad \right\}$$

1 (iv) Functional form $Flt(S) := Im(\Lambda)$

$\Lambda :=$

5 (3c) Generate, list, and count the 0-flats, 1-flats, 2-flats, 3-flats, and 4-flats of $\left(\mathbb{Z}_2\right)^4$

1(i) $$0\text{-}Flt\left(S\left(\mathbb{Z}_2\right)^4\right) = \left\{ \qquad \right\}$$

$$\nu\left(0 - Flt\left(S\left(\mathbb{Z}_2\right)^4\right)\right) =$$

1(ii) $$1\text{-}Flt\left(S\left(\mathbb{Z}_2\right)^4\right) = \left\{ \qquad \right\}$$

$$\nu\left(1 - Flt\left(S\left(\mathbb{Z}_2\right)^4\right)\right) =$$

1(iii) $$2\text{-}Flt\left(S\left(\mathbb{Z}_2\right)^4\right) = \left\{ \qquad \right\}$$

$$\nu\left(2 - Flt\left(S\left(\mathbb{Z}_2\right)^4\right)\right) =$$

1(iv) $$3\text{-}Flt\left(S\left(\mathbb{Z}_2\right)^4\right) = \left\{ \qquad \right\}$$

$$\nu\left(3 - Flt\left(S\left(\mathbb{Z}_2\right)^4\right)\right) =$$

1(v) $$4\text{-}Flt\left(S\left(\mathbb{Z}_2\right)^4\right) = \left\{ \qquad \right\}$$

$$\nu\left(4 - Flt\left(S\left(\mathbb{Z}_2\right)^4\right)\right) =$$

You may talk to anyone