Math **270** Name (PLEASE PRINT) ____

Exam **1A Take-Home** **2019 Fall**/Spring/Summer I /Summer II Score___________

Instructions

- All work must be done using a **MECHANICAL PENCIL following instructions given**.
- If you have any questions at all, please **ask me**. This includes the following possibilities:
    - something is **illegible,** or **ambiguous**, or **unclear** to you.
- **You may use your homework, notes, books, and calculators unless otherwise stated during the exam, as long all handwritten work is either done in the packages provided or on engineering-computation-pad paper.**
- **You will not get any partial credit**. You have to show **absolutely all your work** on all questions, **exactly as I do on the board in class, with no changes, variations, or omissions in form, unless otherwise noted whether orally or in writing**. If your answer is correct, but your procedure is incorrect or is not allowed by the instructions, you will get no credit all. All work must be displayed using **correct syntax**, and all **explanations**, written in **complete sentences**, must be complete.
- **You must write neatly, clearly and legibly. If you run out of room, you may not turn the page, but you must continue the work on the facing page (on the left side) labelling your work appropriately. You must draw every straight-line segment other than those in the list: ($=, \neq, \equiv, <, \leq, >, \geq, +, -, \times, \div$, $\neg, \vee, \wedge, \parallel, \perp, \backslash, \rightarrow, \leftrightarrow, \mapsto$) with your ruler. At least 1 point will be taken off for each instance of violation.**
- Remember that 'does not make sense' is a possible answer.
- I can grade you only on what you **actually** put on paper; I cannot read minds. You cannot say later that what you had written down is not what you had meant. **You need to learn to write exactly what you mean and mean exactly what you write**.
- **The answers for a problem should be noted on page *n***, as explained in class, and the **work must be shown with proper indices** on the back of page $(n-1)$ and in case the work cannot be completed thereon, the work may be continued with proper indexing on the back of page n, page $(n+1)$, back of page (n+1) .. back of page (n+k-1), and page $(n+k)$, as provided. Work for separate parts must be separated from each other by lines drawn by a ruler parallel to the edges of the paper.
- After the exam is returned, all notes, if any, must be taken on the facing (blank) side with a ball-point with **blue ink**.

You may talk to anyone

R The distinct numbers $a, b, c,$ and $d,$ as defined below, will occur throughout the exam, so please make sure that they are correct. Otherwise all your answers will be wrong.

D Your J-number $J :=$ ____________ $:= (J_k | k \in 1..8)$

Hence

$J_1 =$ $J_2 =$ $J_3 =$ $J_4 =$

$J_5 =$ $J_6 =$ $J_7 =$ $J_8 =$

D $$a := mnm\left\{ J_k \middle| \left(k \in 1..8 \right) \wedge \left(J_k \neq 0 \right) \right\} =$$

D $$b := mxm\left\{ J_k \middle| k \in 1..8 \right\} =$$

D $$c := \left\lfloor \left(\frac{b+a}{2} \right) \right. = = = =$$

D $$d := mxm_{k \in \mathbb{N}}\left\{ 2k+1 \middle| 2k+1 < c \right\} =$$

R If $c := \left\lfloor \left(\frac{b+a}{2} \right) \right. = 1$ then take $d = 1$

D Define $\forall k \in 1..8 \ \ A_k \quad := \quad \begin{cases} 1 & if \quad J_k\big(mod(3)\big) = 1 \\ 0 & if \quad J_k\big(mod(3)\big) = 0 \\ -1 & if \quad J_k\big(mod(3)\big) = 2 \end{cases}$

Show calculations on the facing side.

Hence

$$A_1 = \qquad A_2 = \qquad A_3 = \qquad A_4 =$$

$$A_5 = \qquad A_6 = \qquad A_7 = \qquad A_8 =$$

D Definitions for (1c), (2c), (3c):

(0) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right)$

(0a) $Unbdd\left(S\right) \quad :\Leftrightarrow \quad \forall r \in \mathbb{R}\ \exists (P,Q) \in S^2 \left(dst\left(P,Q\right) = r \right)$

(0) $CnstIncln\left(S\right) \quad :\Leftrightarrow \quad \forall \left(\left(A,B\right), \left(C,D\right) \right) \in \left(S^2 \backslash \Delta_S\right) \times \left(S^2 \backslash \Delta_S\right)$

$$\left(\begin{array}{c} \left(RsVrRn\left(S,A,B\right) = RsVrRn\left(S,C,D\right) \right) \\ \vee \\ \left(RnVrRs\left(S,A,B\right) = RsVrRn\left(S,C,D\right) \right) \end{array} \right)$$

(1) $$\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \qquad \forall \left(P, Q\right) \in \left(S^2 \setminus \Delta_S\right)$$

(1a) $$RsVrRn\left(S, P, Q\right) := \left(\frac{\pi_2\left(Q\right) - \pi_2\left(P\right)}{\pi_1\left(Q\right) - \pi_1\left(P\right)}\right)$$

(1b) $$RnVrRs\left(S, P, Q\right) := \left(\frac{\pi_1\left(Q\right) - \pi_1\left(P\right)}{\pi_2\left(Q\right) - \pi_2\left(P\right)}\right)$$

(2) $$\forall S \in \mathcal{P}\left(\mathbb{R}^2\right)$$

$$CnstIncln\left(S\right)$$

$$:\Leftrightarrow \quad \forall \left(\left(A, B\right), \left(C, D\right)\right) \in \left(S^2 \setminus \Delta_S\right) \times \left(S^2 \setminus \Delta_S\right)$$

$$\left(\begin{array}{c}\left(RsVrRn\left(S, A, B\right) = RsVrRn\left(S, C, D\right)\right) \\ \vee \\ \left(RnVrRs\left(S, A, B\right) = RsVrRn\left(S, C, D\right)\right)\end{array}\right)$$

(3) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \qquad \forall P, Q \in \left(\mathbb{R}^2 \backslash \Delta_{\mathbb{R}}\right)$

$$StLn_1\left(S, P.Q\right)$$

$$:\Leftrightarrow \quad \left(P \in S\right) \wedge \left(Q \in S\right) \wedge \left(Unbdd\left(S\right)\right) \wedge \left(CnstIncln\left(S\right)\right)$$

$$:\Leftrightarrow$$

(4) $\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \qquad \forall P, Q \in \left(\mathbb{R}^2 \backslash \Delta_{\mathbb{R}}\right)$

$$StLn_2\left(S, P.Q\right)$$

$$:\Leftrightarrow \quad \left(P \in S\right) \wedge \left(Q \in S\right) \wedge \left(\begin{array}{c} \exists \left(p, q, r\right) \in \left(\mathbb{R}^3\right) \backslash \left\{(0.0.0)\right\} \\ \forall \left(x, y\right) \in \left(S\right) \\ \left(px + qy + r = 0\right) \end{array}\right)$$

R $\quad StLn_1\left(S, P.Q\right)$ and $StLn_2\left(S, P.Q\right)$ are possible formalisations of the notion of a straight line through two distinct the points $P, Q \in \mathbb{R}$ in the senses of Philip and Jack respectively.

R Do all work on the facing side and in the additional space provided and record ONLY your answers on the right side as indicated.

R **Do not make any extraneous marks on the exam; one point will be deducted for each extraneous mark.**

R You may **only** submit **complete** answers to questions marked explicitly on the exam. You will lose points for unfinished work.

R **On any problem whatsoever, if you do not check that your solution is correct exactly as shown in class you will get no points on this question.**

R If answers to different instances of some question on the exam contradict each other you will get a score of zero for every instance.

R **You will lose points for making any extraneous mark on the exam.**

R You may only submit complete answers to questions marked explicitly on the exam. You will lose points for unfinished work. **Do not write anything down on any page if you cannot completely solve the problem or some part thereof that is indexed by a roman numeral.**

R Separate pieces of work for any part of any problem recorded on a blank page intended for the purpose must be indexed by the index of the problem and also by numbers starting at 0 that record the order in which the pieces were completed; such parts must be separated from each other by straight lines parallel to the edges of the paper drawn with a ruler. You will lose points otherwise.

R **Any sloppiness, untidiness, and any departure from proper format (as indicated in class) will lead to a score of 0.**

6 (1a) Compute the following:

1 (i) $A \quad := \begin{bmatrix} a & 0 & d \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \quad = \quad [\qquad\qquad]$

1 (ii) $A^2 \quad = \left([\qquad] \right)^2 \quad = \quad [\qquad\qquad]$

1 (ii) $A^3 \quad = \left([\qquad] \right)^3 \quad = \quad [\qquad\qquad]$

3 (iv) $A^3 - (a+b+c)A^2 + \ (ab+bc+ca)A - \ (abc)I_3$

$= \quad [\qquad\qquad]$

6 (1b) Define $\forall u, v \in \mathbb{R}^4$:

$$(0) \quad u.v \quad := \quad au_1v_1 + bu_2v_2 + cu_3v_3 + du_4v_4$$

$$(1) \quad \|u\|^2 := \quad u.u$$

$$(2) \quad \Big(prp(u,v)\Big) \quad :\Leftrightarrow \Big(u.v = 0\Big)$$

My $\quad u.v \quad := \quad (\quad)u_1v_1 + (\quad)u_2v_2 + (\quad)u_3v_3 + (\quad)u_4v_4$

My $\quad \|u\|^2 := \quad (\quad)(u_1)^2 + (\quad)(u_2)^2 + (\quad)(u_3)^2 + (\quad)(u_4)^2$

Circle correct choices from among Y, N, Proof, and Witness and provide showing work on the facing side a proof or witness as the case may be:

Define $\quad u := \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ My $u := \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$

1 (i) $\|u\|^2 \quad :=$

2 (ii) $\exists\, v \in \left(\mathbb{R}^4\right) \backslash \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\} \left(prp\left(u, v\right)\right)$ $\quad Y \quad N \quad Pf \quad W$

3 (iii) $\forall u \in \left(\mathbb{R}^4\right) \left(\dfrac{\|u\|^2 = 0}{u = 0}\right)$ $\quad Y \quad N \quad Pf \quad W$

You may talk to anyone

5 (1c) Define: $S := \left\{(x,y) \in \mathbb{R}^2 \middle| ax + by + c = 0\right\}$

My $S := \left\{(x,y) \in \mathbb{R}^2 \middle| \left(\quad\right)x + \left(\quad\right)y + \left(\quad\right) = 0\right\}$

Find $\left(P,Q\right) \in \left(S^2 \backslash \Delta_S\right)$ and prove your assertion.

My $\left(P,Q\right) := \left(\left(\quad,\quad\right),\left(\quad,\quad\right)\right) \in \left(S^2 \backslash \Delta_S\right)$

1 (i) Prove that: $\left(P,Q\right) \in \left(S^2 \backslash \Delta_S\right)$

4 (ii) Prove that: $StLn_1\left(S,P,Q\right)$

6 (2a) **You will not get any points on this page unless you can do part (v) and part (vi) completely and exhibit exact calculations with all details as explained in class**. Do all your work on the facing side and only fill in the blanks with real numbers to express the answers in the forms indicated:

Note that: $\boldsymbol{k, l, m, n, p, q, r, s} \in \mathbb{R}$

1 (i) $u := \frac{a+ib}{c+id} \quad = \quad p + iq \quad = \quad \Big(\qquad \Big) + i \Big(\qquad \Big)$

1 (ii) $u := \frac{a+ib}{c+id} \quad = \quad ke^{il} \quad = \quad \Big(\qquad \Big) e^{i\left(\qquad \right)}$

1 (iii) $v := \frac{c+id}{a+ib} \quad = \quad r + is \quad = \quad \Big(\qquad \Big) + i \Big(\qquad \Big)$

1 (iv) $v := \frac{c+id}{a+ib} \quad = \quad me^{in} \quad = \quad \Big(\qquad \Big) e^{i\left(\qquad \right)}$

1 (v) $\left(p + iq\right)\left(r + is\right) \quad = \quad 1$ $\qquad Y \quad N \quad Pf \quad W$

1 (vi) $\left(ke^{il}\right)\left(me^{in}\right) \quad = \quad 1$ $\qquad Y \quad N \quad Pf \quad W$

6 (2b) Solve the following system of equations for x and y, in $\mathbb{R}^2$, **by row-reduction**. **Show the work for row-reduction on the facing side** and **check that your solution is correct exactly as shown in class; otherwise you will get 0 on this question.**

4 (i)

$$ax + cy = a + c$$
$$bx + dy = b + d$$

$$(\quad)x + (\quad)y = (\quad) + (\quad) = (\quad)$$

$$(\quad)x + (\quad)y = (\quad) + (\quad) = (\quad)$$

My problem: ______________________________

$x=$

$y=$

1(ii) The solution requires ________ parameters, and therefore represents a ____ - flat, also called a _________.

1 (iii) Check, exactly as shown in class, that your solution works; on the facing side and other blank sheets provided for the purpose. **Otherwise you will not get any points on (2b).**

5 (2c) Consider:

$S \in \mathcal{P}\left(\mathbb{R}^2\right)$ such that:

$$P := (a, b) \in S \text{ and } Q := (b, a) \in S \text{ and } StLn_1\left(S, P. Q\right)$$

My $P := \left(\qquad \right)$ and my $Q := \left(\qquad \right)$

Prove that: $StLn_2\left(S, P. Q\right)$

6 (3a) Find the Hermite form $HF(A)$of the matrix:

$$A \quad := \quad \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \\ A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \end{bmatrix}$$

$$= \quad \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Use the row reductions on:

$$\left[\begin{array}{cccccc|ccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & 1 & 0 & 0 \\ A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & 0 & 1 & 0 \\ A_3 & A_4 & A_5 & A_6 & A_7 & A_8 & 0 & 0 & 1 \end{array}\right]$$

and obtain the matrix P such that $PA = HF(A)$ **Start the work for row-reduction on the facing side** and **check that your solution is correct exactly as shown in class by computing PA; otherwise you will get no points on this question.**

$$HF(A) \quad = \quad \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$P \quad = \quad \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$PA \quad = \quad \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

6 (3b) Write down a system of equations S whose augmented matrix is of the form:

$$AM(S) = \left[\begin{matrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \\ A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \end{matrix} \left| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right. \right]$$

1 (i) Use unknowns of the form x_1, x_2 etc. using as many unknowns as you need.

1 (ii) Using your answer from (3a) and as many parameters: p_1, p_2, etc. as you need, write below a solution of the system. **On the facing side check that your solution is correct exactly as shown in class; otherwise you will get 0 on this question.**

1 (iii) Rewrite the solution in vector-parametric form.

1(iv) Rewrite the solution in functional form.

2(v) The solution is a _____ in ________.

5 (3c) Prove that:

$$\forall S \in \mathcal{P}\left(\mathbb{R}^2\right) \qquad \forall P, Q \in \left(\mathbb{R}^2 \backslash \Delta_{\mathbb{R}}\right)$$

$$\left(\left(StLn_1\left(S, P, Q\right)\right) \iff \left(StLn_2\left(S, P, Q\right)\right)\right)$$