Part 1: Written Problems

Q.1) Number of possible permutations = $\frac{8!}{4! \times 4!}$

Number of permutations when no two adjcent parakeets are the same color = 2

$p(\text{given Condition}) = \frac{2}{\frac{8!}{4!4!}} = \frac{1}{35}$

$= 0.0285714$

Q.2) a) $(0.7)^8(0.3)^0\,{}^8C_8 = (0.7)^8$

~~= 0.0057~~

$= 0.05764$

b)

Using formula = $(p)^r(1-p)^{n-r}\,{}^nC_{n-r}$

P → Probability of core working correctly

r → Number of cores running correctly.

r ↓		Number of CPUs = probability X1000
0	$0.7^0\,0.3^8\,{}^8C_8$	0.06561 ≈ 0
1	$0.7^1\,0.3^7\,{}^8C_7$	1.22472 ≈ 1
2	$0.7^2\,0.3^6\,{}^8C_6$	10.00188 ≈ 10
3	$0.7^3\,0.3^5\,{}^8C_5$	46.67544 ≈ 47

4	$0.7^4\ 0.3^4\ {}^8C_4$	136.1367	$\approx$ 136
5	$0.7^5\ 0.3^3\ {}^8C_3$	254.12	$\approx$ 254
6	$0.7^6\ 0.3^2\ {}^8C_2$	296.47	$\approx$ 296
7	$0.7^7\ 0.3^1\ {}^8C_1$	197.65	$\approx$ 198
8	$0.7^8\ 0.3^0\ {}^8C_0$	57.64	$\approx$ 58
			1000

No. of Great = 58
$n \to 1$ to 3

No. of Advanced = 884
$n \to 4$ to 7

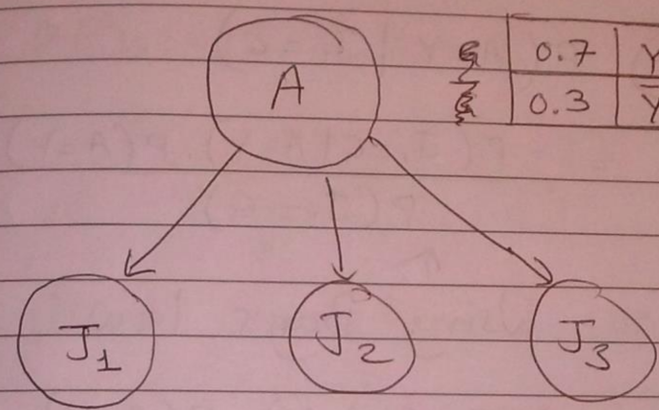
No. of Extreme = 58
$n \to 8$

c) Expected revenue -

58×50	2900
884×100	88400
58×1000	+ 58000
	149,300

$\therefore$ Expected Revenue - 149,300 \$

Q. 3)



G	0.7	Y
$\bar{G}$	0.3	$\bar{Y}$

J_1	A(Y)	A($\bar{Y}$)
G	0.7	0.32
$\bar{G}$	0.3	0.8

J_2	A(Y)	A($\bar{Y}$)
G	0.7	0.2
$\bar{G}$	0.3	0.8

J_3	A(Y)	A($\bar{Y}$)
G	0.7	0.2
$\bar{G}$	0.3	0.8

- Ussing
  A → to donote actually guilty or not

  Y → Actually guilty
  $\bar{Y}$ → Actually Not guilty

J_1, J_2, J_3 → to denote judge's decision

G - Guilty
$\bar{G}$ → Not Guilty.

a) $P(A=Y \mid J_1=G)$

$$= \frac{P(J_1=G \mid A=Y) \cdot P(A=Y)}{P(J_1=G)}$$

↑
Using Bay's law

$$= \frac{P(J_1=G \mid A=Y) \cdot P(A=Y)}{P(J_1=G \mid A=Y) \cdot P(A=Y) + P(J_1=G \mid A=\bar{Y}) \cdot P(A=\bar{Y})}$$

$$= \frac{0.7 \times 0.7}{0.7 \times 0.7 + 0.2 \times 0.3}$$

$= 0.8909$

b) $P(A=Y \mid J_1=G, J_2=G, J_3=G)$

$$= \frac{P(J_1=G, \cancel{P}J_2=G, J_3=G \mid A=Y) \cdot P(A=Y)}{P(J_1 \cancel{GC}=G \mid A=Y) P(J_2=G \mid A=Y) \cdot P(J_3=G \mid A=Y) \cdot P(A=Y) + P(J_1=G \mid A=\bar{Y}) \cdot P(J_2=G \mid A=\bar{Y}) \cdot P(J_3=G \mid A=\bar{Y}) P(A=\bar{Y})}$$

$$= \frac{(0.7)^4}{(0.7)^4 + (0.2)^3\, 0.3}$$

$= 0.9901$

c) $P(J_3 = G \mid J_1 = \bar{G}, J_2 = \bar{G})$

$$= \frac{P(J_3 = G, J_1 = \bar{G}, J_2 = \bar{G})}{P(J_1 = \bar{G}, J_2 = \bar{G})} \begin{array}{l} \leftarrow (1) \\ \leftarrow (2) \end{array}$$

Using conditional probability formula

(1) $P(J_3 = G, J_1 = \bar{G}, J_2 = \bar{G})$ (by $\sum_A$)

$= P(J_3 = G \mid A = Y).P(J_2 = \bar{G} \mid A = Y).P(J_3 = \bar{G} \mid A = Y).P(A = Y) + P(J_3 = G \mid A = \bar{Y}).P(J_2 = \bar{G} \mid A = \bar{Y}).P(J_1 = \bar{G} \mid A = \bar{Y}).P(A = \bar{Y})$

$= 0.7 \times 0.3 \times 0.3 \times 0.7 + 0.2 \times 0.8 \times 0.8 \times 0.3$

$= 0.0441 + 0.0384$

$= 0.0825$ ← Value for (1)

(2) $P(J_1 = \bar{G}, J_2 = \bar{G})$

$$= \sum_A \sum_{J_3} P(J_1 = \bar{G} \mid A) P(J_2 = \bar{G} \mid A) P(J_3 \mid A) P(A)$$

$$= \sum_A P(J_1 = \bar{G} \mid A).P(J_2 = \bar{G} \mid A) P(A) \underbrace{\sum_{J_3} P(J_3 \mid A)} \leftarrow (3)$$

$\sum_{J_3} P(J_3 \mid A)$

$=$ $A = Y$

$P(J_3 = G \mid A = Y) + P(J_3 = \bar{G} \mid A = Y) = 1$

$A = \bar{Y}$

$P(J_3 = G \mid A = \bar{Y}) + P(J_3 = \bar{G} \mid A = \bar{Y}) = 1$

(6)

classmate

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$$\sum_{J_3} P(J_3 | A) = t(A) \Rightarrow \begin{matrix} Y = 1 \\ \bar{Y} = 1 \end{matrix}$$

$\therefore$ Neglekting $t(A)$ as it gives value 1 for both Y & $\bar{Y}$

by (3) & (4)

$$\Rightarrow = \sum_{A} P(J_1 = \bar{G} | A) \cdot P(J_2 = \bar{G} | A) \cdot P(A)$$

$$= P(J_1 = \bar{G} | A = Y) \cdot P(J_2 = \bar{G} | A = Y) \cdot P(A = Y)$$
$$+ P(J_1 = \bar{G} | A = \bar{Y}) \cdot P(J_2 = \bar{G} | A = \bar{Y}) \cdot P(A = \bar{Y})$$

$$= 0.3 \times 0.3 \times 0.7 + 0.8 \times 0.8 \times 0.3$$

$= 0.255 \leftarrow$ value for (2)

$\therefore P(J_3 = G | J_1 = \bar{G}, J_2 = \bar{G}) \Rightarrow$ $\frac{0.0825}{0.255}$

value for (1)

value for (2)

$= \cancel{0.3667}$

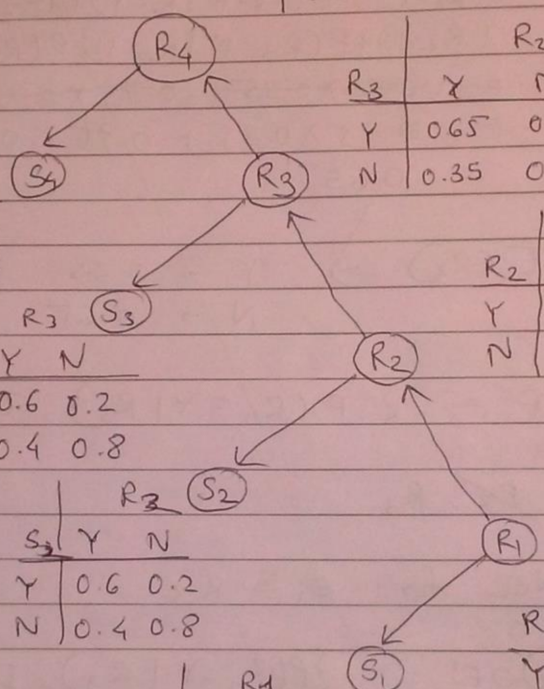
$= 0.3235$

$$\therefore P(J_3 = G | J_1 = \bar{G}, J_2 = \bar{G})$$

$= \cancel{0.3667}$

$= 0.3235$

Q. 4) a)



R_4 \ R_3	Y	N
Y	0.65	0.25
N	0.35	0.75

R_3 \ R_2	Y	N
Y	065	0.25
N	0.35	0.75

R_2 \ R_1	Y	N
Y	0.65	0.25
N	0.35	0.75

S_4 \ R_4	Y	N
Y	0.6	0.2
N	0.4	0.8

S_3 \ R_3	Y	N
Y	0.6	0.2
N	0.4	0.8

S_2 \ R_2	Y	N
Y	0.6	0.2
N	0.4	0.8

S_1 \ R_1	Y	N
Y	0.6	0.2
N	0.4	0.8

R_1	
Y	x
N	$1-x$

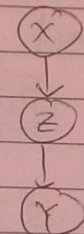
Value of x or $1-x$ not given for part (a)

$S_i \rightarrow$ NOT DATA SENT BY SENOR.
its data received at computer.

$S_i \rightarrow$ Data Received on i^{th} day
$R_i \rightarrow$ If it rain or not on i^{th} day

~~4 obserbed~~
4 observed variables = S_1, S_2, S_3, S_4
4 unobserved variables = R_1, R_2, R_3, R_4

Using



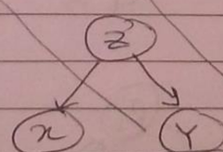
$x \perp Y | Z$ = Yes
$x \perp Y$ = No

Hence

Ex: $R_4 \leftarrow R_3 \leftarrow R_2 \leftarrow R_1$

1. $R_i \perp R_j$ = No, where $i = j$
and $4 \geq i, j \geq 1$

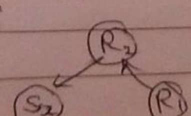
~~Using~~



$\cancel{x \perp Y | Z = \text{Yes}}$
$\cancel{X \perp Y = \text{NO}}$

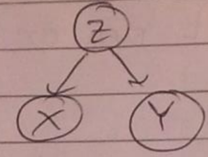
2. $S_i \perp R_j = \text{No}$ where
~~i⇒~~ $i \geq j$
and $4 \geq i, j \geq 1$

Ex:



Using $X \perp Y | Z$ = Yes

$X \perp Y$ = No

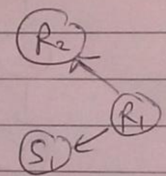


3. $S_i \perp R_j$ = No where

$j > i$

and $4 \geq i,j \geq 1$

Ex:



~~4 $S_i \perp S_j$ = No~~

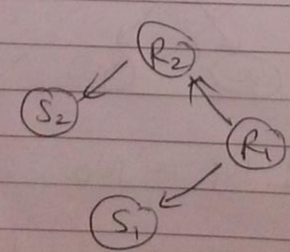
4.

$S_i \perp S_j$ = No where

$i \neq j$

$4 \geq i,j \geq 1$

Ex:



∴ Four rules -

1. $R_i \perp R_j = \text{No}$, where $i \neq j$ and $4 \geq i, j \geq 1$
2. $S_i \perp R_j = \text{No}$, where $i \geq j$ and $4 \geq i, j \geq 1$
3. $S_i \perp R_j = \text{No}$, where $j > i$ and $4 \geq i, j \geq 1$
4. $S_i \perp S_j = \text{No}$, where $i \neq j$ $4 \geq i, j \geq 1$

Now making conditional independance rules by distrubing trails :-

1. $R_i \perp R_j | R_k = \text{True}$, where $i > k > j$ and $4 \geq i, j, k \geq 1$
2. $S_i \perp R_j | R_k = \text{True}$, where $i \geq k > j$ $4 \geq i, j, k \geq 1$
3. $S_i \perp R_j | R_k = \text{True}$, where $j > k \geq i$ $4 \geq i, j, k \geq 1$
4. $S_i \perp S_j | R_k = \text{True}$, where $i \geq k$, $k \geq j$, $i > j$, $4 \geq i, j, k \geq 1$

b) $P(R_4 = Y \mid R_1 = N)$

here we have given that it did not rain on first day.
Hence,

$$P(R_1 = N) = 1$$

$$P(R_4 = Y \mid R_1 = N) = \frac{P(R_4 = Y, R_1 = N)}{P(R_1 = N)}$$

$$= P(R_4 = Y, R_1 = N)$$

$$P(R_4 = Y,\ R_1 = N)$$

$$= \sum_{R_3} \sum_{R_2} \sum_{S_1} \sum_{S_2} \sum_{S_3} \sum_{S_4} P(R_4 = Y \mid R_3) .$$

$$P(R_3 \mid R_2) .\ P(R_2 \mid R_1 = N) .\ P(R_1 = N)$$

$$P(S_4 \mid R_4) .\ P(S_3 \mid R_3)$$

$$.\ P(S_2 \mid R_2) .\ P(S_1 \mid R_1 = N)$$

$P(R_1 = N) = 1$

hence neglecting

Variable elimination Algorithm

1) Sort Non-query elements

2) Initialize factors F

3) for each $i = 1 \ldots n$

   a) Identify F'
   b) Take product of factors in F'
   c) Sum ~~all~~ over all values of z_i, producing new factor f parameterized by $V - \{z_i\}$
   d) Remove elements in F' from F, then add f to F

Alogrithm in action:-

$n =$ 1 2 3 4 5 6

1) Sorting $\rightarrow S_1, S_2, S_3, S_4, R_2, R_3$

2) $F = \{ \cancel{P(R_3 | R} \; P(R_4 = r | R_3),$
   $P(R_3 | R_2), P(R_2 | R_1 = N),$
   $P(S_4 | R_4), P(S_3 | R_3),$
   $P(S_2 | R_2), P(S_1 | R_1 = N) \}$

3) for $n = 1 \rightarrow S_1$

$$\sum_{S_1} P(S_1 | R_1 = N)$$

$= P(S_1 = Y | R_1 = N) +$
$P(S_1 = N | R_1 = N)$

$= 0.2 + 0.8$

$= 1$

it's equal to 1, hence not adding in F, but removing $P(S_1 | R_1 = N)$ from F

for $n = 2 \rightarrow S_2$

$$\sum_{S_2} P(S_2 | R_2)$$

$\rightarrow R_2 = Y$

$P(S_2 = Y | R_2 = Y) + P(S_2 = N | R_2 = Y)$

$R_2 = N$

$P(S_2 = Y | R_2 = N) + P(S_2 = N | R_2 = N)$

$R_2 = Y \rightarrow 0.6 + 0.4 = 1$

$R_2 = N \rightarrow 0.2 + 0.8 = 1$

$\therefore \tau(R_2) \rightarrow$

R_2	
Y	1
N	1

Hence, neglecting $\tau(R_2)$ as it will always give value = 1 which will not affect equations but removing $P(S_2 | R_2)$ from F.

for $n=3 \longrightarrow S_3$

$$\sum_{S_3} P(S_3 | R_3)$$

Similar to $n=2$

$\therefore$ Removing $P(S_3 | R_3)$ from F

for $n=4 \longrightarrow S_4$

$$\sum_{S_4} P(S_4 | R_3)$$

Similar to $n=2$

$\therefore$ Removing $P(S_4 | R_4)$ from F

Now $F = \{ P(R_4 = Y | R_3),\ P(R_3 | R_2)\ P(R_2 | R_1 = N) \}$

for $n=5 \longrightarrow R_2$

$$\sum_{R_2} P(R_3 | R_2)(R_2 | R_1 = N)$$

$\rightarrow$ $R_3 = N$

$$P(R_3 = N | R_2 = Y) . P(R_2 = Y | R_1 = N)$$
$$+ P(R_3 = N | R_2 = N) . P(R_2 = N | R_1 = N)$$
$$= 0.35 \times 0.25 + 0.75 \times 0.75$$
$$= 0.65$$

$R_3 = Y$

$$P(R_3 = Y | R_2 = Y) . P(R_2 = Y | R_1 = N)$$
$$+ P(R_3 = Y | R_2 = N) . P(R_2 = N | R_1 = N)$$
$$= 0.65 \times 0.25 + 0.25 \times 0.75$$
$$= 0.35$$

$\therefore \tau(R_3) \Rightarrow$

R_3	
Y	0.35
N	0.65

Now $F = \{ P(R_4 = Y | R_3), \tau(R_3) \}$

for $n = 6 \rightarrow R_3$

$$\sum_{R_3} P(R_4 = Y | R_3) . \tau(R_3)$$

$$= P(R_4 = Y | R_3 = Y) . \tau(R_3 = Y)$$
$$+ P(R_4 = Y | R_3 = N) . \tau(R_3 = N)$$
$$= 0.65 \times 0.35 + 0.25 \times 0.65$$

$= 0.39$

$\therefore \tau(R_4 = Y) = 0.39$

~~$\therefore P(R_4 = Y)$~~

$\therefore P(R_4 = Y \mid R_1 = N)$

$= P(R_4 = Y, R_1 = N)$

$= \tau(R_4 = Y)$

$= 0.39$

$\therefore P(R_4 = Y \mid R_1 = N) = 0.39$