(1)

Ssable

Assignment 4

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Q. 1) $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Functions :-

Sum (x,y) : Returns $x+y$

Mult (x,y) : Return $x * y$

Predicates

Equal (x,y) : if $x == y$:
Return true
else :
Return False

Z(c) : if $c \in Z$:
Return true
else :
Return False

(a) $\forall_x \forall_y \; Z(x) \wedge Z(y) \rightarrow$
Equal (Sum (x,y), Sum (y,x)) $\wedge$
Equal (Mult (x,y), Mult (y,x))

(b) $\forall_x \forall_y \; Z(x) \; Z(y) \rightarrow$
Z(Sum (x,y)) $\wedge$ Z(Mult (x,y))

(c) $\forall_x \forall_y \forall_z \; Z(x) \wedge Z(y) \wedge Z(z)$
$\rightarrow$ Equal (Mult (x, Sum (y, z)),
Add (Mult (x, y), Mult (x, z)))

(d) $\forall_x \forall_y \forall_z \; Z(x) \wedge Z(y) \wedge Z(z) \rightarrow$
$\text{Equal}(\text{Sum}(x, \text{Sum}(y,z)),$
$\text{Sum}(\text{Sum}(x,y), z)) \;\wedge$
$\text{Equal}(\text{Mult}(x, \text{Mult}(y,z)),$
$\text{Mult}(\text{Mult}(x,y), z))$

(e) $\forall_x \; Z(x) \rightarrow \text{Equal}(\text{Sum}(x,0), x)$
$\wedge \; \text{Equal}(\text{Mult}(x,1), x)$

Q. 2) $\alpha = \forall_x (P(x) \vee Q(x))$
$\beta = \forall_x P(x) \vee \forall_x Q(x)$

$\alpha \models \beta$ is equivalent to $\alpha \rightarrow \beta$

$\forall_x (P(x) \vee Q(x)) \rightarrow \forall_x P(x) \vee \forall_x Q(x)$
$= \neg \forall_x (P(x) \vee Q(x)) \vee \forall_x P(x) \vee \forall_x Q(x)$
$= \exists_x \neg (P(x) \vee Q(x)) \vee \forall_x P(x) \vee \forall_x Q(x)$
$= \exists_x \neg P(x) \wedge \neg Q(x) \vee \forall_x P(x) \vee \forall_x Q(x)$

By skolemization

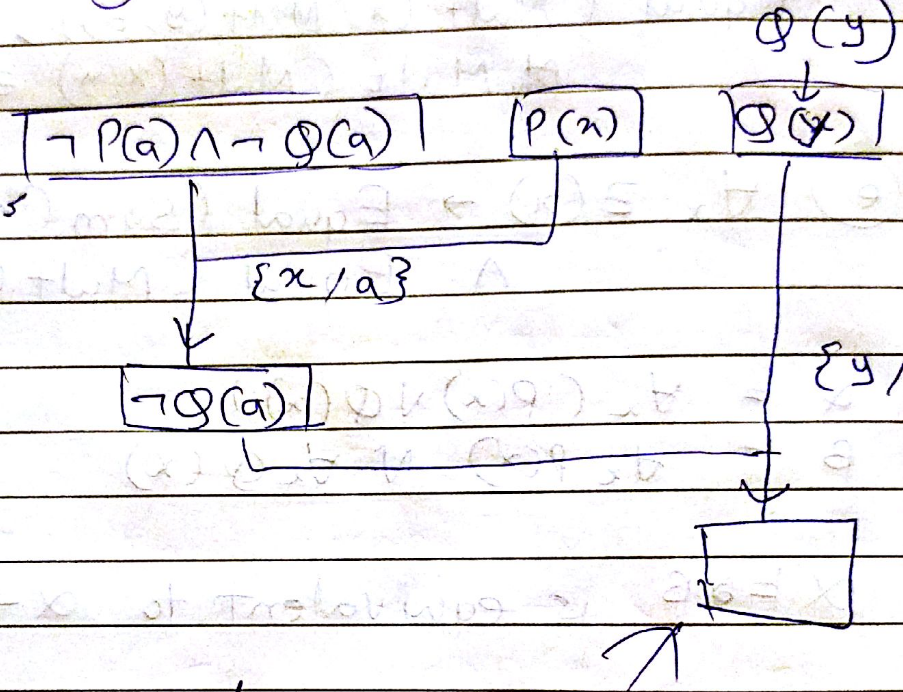
$\cancel{= \neg P(a) \wedge \neg P(b) \vee P(x) \vee Q(x)}$

~~Using Resolution~~

$\cancel{\boxed{\neg P(a) \wedge \neg Q(b)}} \quad \boxed{P(x)} \quad \boxed{Q(x)}$

$= \neg P(a) \wedge \neg Q(a) \vee P(x) \vee Q(y)$

Using Resolution :-



hence $\alpha \to \beta$ is not valid

hence $\alpha \models \beta$ is not valid

Using Example

let S be the set of x balls
$x > 10$ and x is even

Balls have only two colors -
Yellow or Red

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Ball is either totally yellow or totally red.

$P(x) \rightarrow$ ball is yellow
$Q(x) \rightarrow$ ball is red

$\forall_x (P(x) \vee Q(x)) = \text{True}$
$\forall_x P(x) \vee \forall_x Q(x) = \text{False}$

$\therefore \alpha \models \beta$ is not valid as we have found a model in which α is true but β is not.

Q. 3) Converting to first order logic

OwnFalcon (x) : X owns millennium falcon.
Unhappy (x) : X is unhappy
Visitobi (x) : X visits Obi-Van-Kenoby.
Wise (x) : X is wise
Teach (x) : X is ~~ta~~ being taught how to use light staber by obi van Kenoby.
Rebel (x) : X joins Rebel allians
Declares (x, y) : X declares love for Y
~~Frd~~ Frnd (x) : X is friend with chewbacca.
Loves (x, y) : X loves Y

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Using the given data we can write

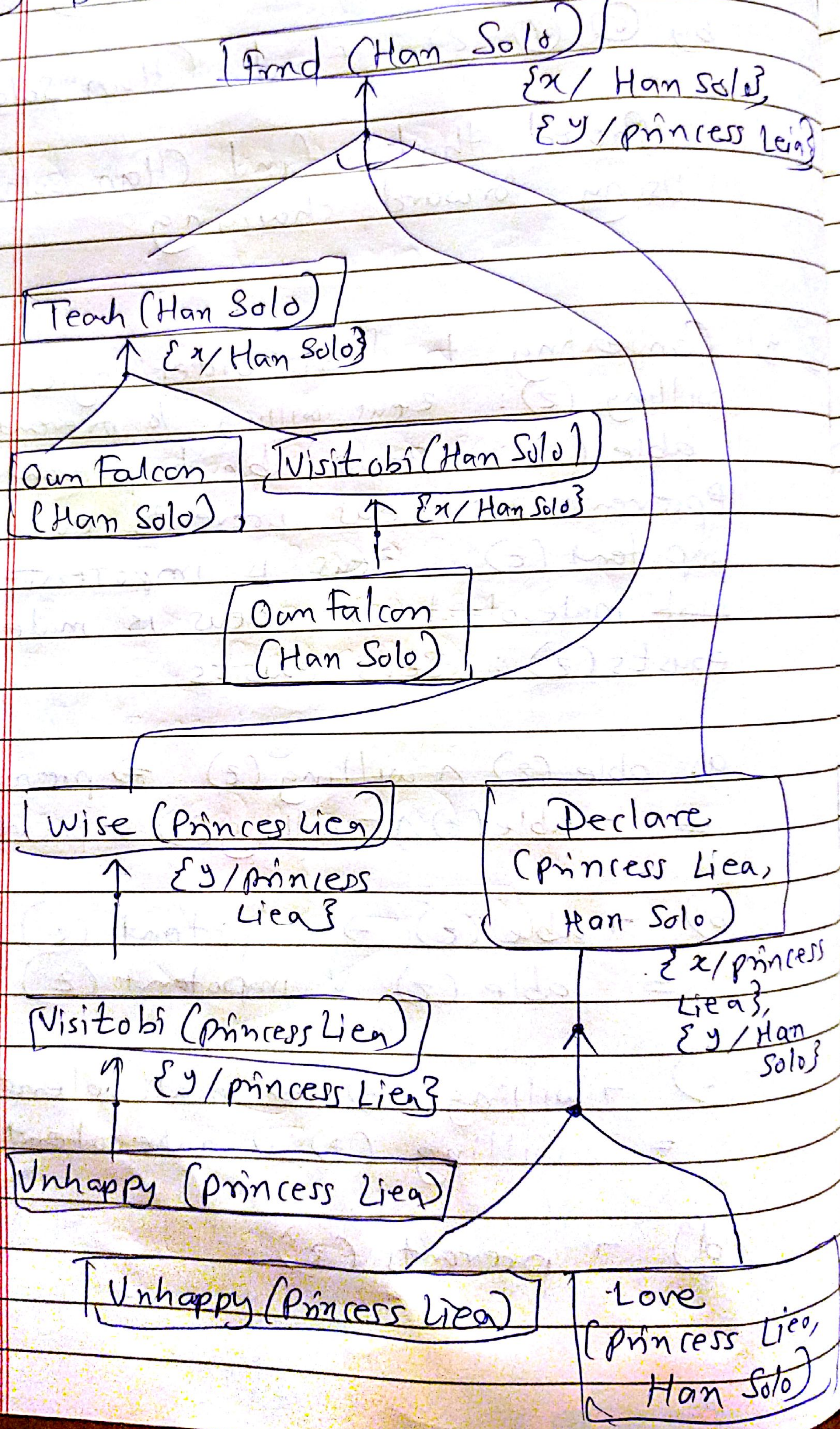
a. OwnFalcon (Han Solo)
b. Unhappy (Princess Leia)
c. Loves (Princess Leia, Han Solo)
d. $\forall x$ ((OwnFalcon (x) $\vee$ Unhappy(x)) $\rightarrow$ Visitobi (x))
e. $\forall x$ Visitobi (x) $\rightarrow$ wise(x)
f. $\forall x$ OwnFalcon (x) $\wedge$ Visitobi (x) $\rightarrow$ Teach (x)
g. $\forall x$ Unhappy (x) $\vee$ OwnFalcon (x) $\wedge$ Teach(x) $\rightarrow$ Rebel (x)
h. $\forall x \forall y$ Unhappy (x) $\wedge$ loves (x,y) $\rightarrow$ Declare (x,y)
i. $\forall x \forall y$ Teach (x) $\wedge$ Declare (y,x) $\wedge$ wise (y) $\rightarrow$ frnd (x)

We need to prove:-

~~Friend (Han Solo)~~

Frnd (Han Solo)

i) Backward chaining :-



ii) Forward chaining:-

~~Forward chaining:~~

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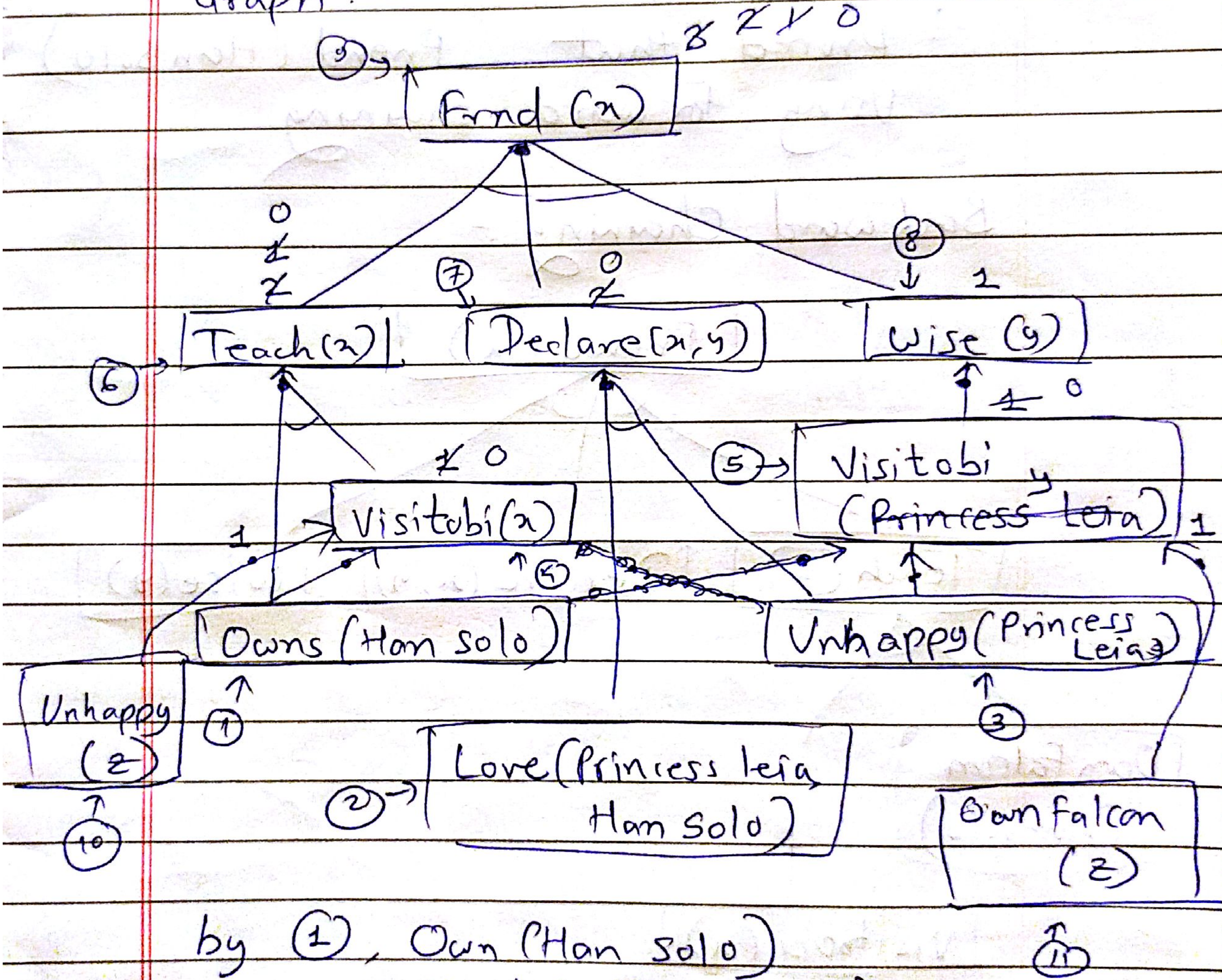
~~OwnFalcon (Han Solo)~~ ~~Unhappy (Princess Leia)~~

Graph:-



by ①, Own (Han solo)
visitobi (Han Solo)

by ① & ④ Teach (Han Solo)

by ② & ③ Declare (Princess Leia, Han Solo)

by ③ Visitobi (princess Leia)

by (5) Wise(princess Leia)

by (6), (7), (8) frnd (Han Solo)

∴ Proved that frnd (Han solo)
Using forward chaining

Q. 4) Converting to first order logic

willing (z) : Zeus willing to prevent evil

able (z) : Zeus able to prevent evil

prevents (z) : Zeus prevents evil

impotent (z) : Zeus is impotent

~~mal~~ malevolent (z) : Zeus is malevolent

exists (z) : Zeus exists

a) $able(z) \wedge willing(z) \Rightarrow prevent(z)$

$\equiv \neg able(z) \wedge \neg willing(z) \vee prevent(z)$

b) $\neg able(z) \Rightarrow impotent(z)$

$= able(z) \vee impotent(z)$

c) $\neg willing(z) \Rightarrow malevolent(z)$

$= willing(z) \vee malevolent(z)$

d) $\neg prevents(z)$

e) $exists(z) \rightarrow \neg\, impotent(z) \wedge \neg\, malevolent(z)$

$= \neg\, exists(z) \vee \neg\, impotent(z) \wedge \neg\, exists(z) \vee malevolent(z)$

for simplicity using short forms.

$\boxed{\neg A(z) \vee \neg W(z) \vee \neg P(z)}$ $\boxed{A(z) \vee I(z)}$

$\downarrow$

$\boxed{\neg W(z) \vee \neg P(z) \vee I(z)}$ $\boxed{W(z) \vee M(z)}$

$\boxed{\neg E(z) \vee \neg M(z)}$

$\downarrow$

$\boxed{W(z) \vee \neg E(z)}$

$\downarrow$

$\boxed{\neg P(z) \vee I(z) \vee E(z)}$ $\boxed{\neg E(z) \vee \neg I(z)}$

$\downarrow$

$\boxed{\neg P(z) \vee \neg E(z)}$ $\boxed{\neg P(z)}$

$\downarrow$

$\boxed{\neg E(z)}$ $\boxed{E(z)}$

$\downarrow$

empty $\boxed{}$

Hence Using Resolution proved that zeus does not exist.

Q.5) a) $\forall_x (P(x) \Rightarrow P(x))$

Using Resolution, ~~for~~

taking negation

$\neg \cancel{A} \forall_x P(x) \Rightarrow P(x))$

$\equiv \exists_x \neg (P(x) \Rightarrow P(x))$

$\equiv \exists_x \neg (\neg P(x) \vee P(x))$

$\equiv \exists_x P(x) \wedge \neg P(x)$

Skolemizing

$= P(a) \wedge \neg P(a)$

$\boxed{P(a)}$ $\boxed{\neg P(a)}$

$\boxed{}$

Empty

Hence proved.

b) $\neg \exists_x P(x) \Rightarrow \forall_x \neg P(x)$

Using Resolution

taking negation

$\neg (\neg \exists_x P(x) \Rightarrow \forall_x \neg P(x))$

$\cancel{\neg \exists_x P(x)}$

$= \neg (\exists_x P(x) \vee \forall_x \neg P(x))$

$= \neg \exists_x P(x) \wedge \neg \forall_x \neg P(x)$

$= A_x \neg P(x) \wedge \exists_x P(x)$

Skolemizing

$= \neg P(x) \wedge P(a)$

$\boxed{\neg P(x)}$ $\boxed{P(a)}$

$\{x/a\}$

$\boxed{}$

Empty

Hence proved.

c) $\forall_x (P(x) \vee Q(x)) \rightarrow \forall_x P(x) \vee \exists_x Q(x)$

taking negation

$\neg (\forall_x (P(x) \vee Q(x)) \rightarrow \forall_x P(x) \vee \exists_x Q(x))$

$\equiv \neg (\neg \forall_x (P(x) \vee Q(x)) \vee \forall_x P(x) \vee \exists_x Q(x))$

$\equiv \forall_x (P(x) \vee Q(x)) \wedge \neg \forall_x P(x) \wedge \neg \exists_x Q(x)$

$\equiv \forall_x (P(x) \vee Q(x)) \wedge \exists_x \neg P(x) \wedge \forall_x \neg Q(x)$

Skolemizing.

$\equiv (P(x) \vee Q(x)) \wedge \neg P(a) \wedge \neg Q(y)$

$\equiv (P(x) \wedge \neg P(a) \wedge \neg Q(y)) \vee$
$(Q(x) \wedge \neg P(a) \wedge \neg Q(y))$

(a) (b)

$\Rightarrow$ in above expressing a

$a = P(x) \wedge \neg P(a) \wedge \neg Q(y)$
for $\{x/a\}$ $a = F$

in expression b
$Q(x) \wedge \neg P(a) \wedge \neg Q(y)$
for $\{x/a\}, \{y/a\}$ $b = F$
$\therefore$ Expression is False
$\therefore$ proved by contradiction