ELECTRICAL ENGINEERING

FE Review Course
Outline

1. Electrostatics
    a. Charge
    b. Voltage
    c. Current
    d. Resistance

2. Circuit Analysis Basics
    a. Resistor simplification
        i. Parallel
        ii. Series
    b. Source Equivalents
        i. Thevenin
        ii. Norton
    c. Node Analysis
    d. Loop Analysis

3. Transient Circuits
    a. RC Circuits
    b. RL Circuits

4. AC Circuits
        i. RMS
        ii. Phasor Transforms
        iii. AC impedance
        iv. AC Steady State analysis

5. Power
    a. DC Power
        i. Power supplied
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    b. AC Power
        i. Complex power
        ii. power factor

6. Transformers
    a. Current and Voltage in an Ideal transformers
    b. Impedance seen at the input of an ideal transformer

7. Operational Amplifiers (OP-AMPS)
    a. Ideal OP-AMPS
    b. solving OP-AMP Circuits

8. Resonant Circuits
    a. Series Resonance
    b. Parallel Resonance
    c. Quality Factor
    d. Bandwidth

What you need to know:

1. Electrostatics
   a. Charge
      i. Units: Coulombs (C) , 1 C is defined as the charge of $6.24\text{x}10^{16}$ electrons. The charge of an electron is $1.6\text{x}10^{-19}$ C.
      ii. The force of one charge on another charge: $\overrightarrow{\mathbf{F_{12}}} = \frac{Q_1 Q_2}{4\pi\epsilon r^2}\overrightarrow{a_{12}}$;

         Where:
         1. Q_i = the i the point charge
         2. $\overrightarrow{\mathbf{F_{12}}}$ = the force on charge 2 due to charge 1
         3. r = the distance between the two charges
         4. $\overrightarrow{a_{12}}$ = a unit vector directed from 1 to 2
         5. ϵ = the permittivity of the medium (how capable is the medium in which the charges exist of allowing these forces to exist)

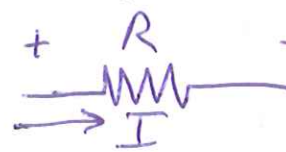
   b. Voltage – The potential difference between two points, is the work done per unit charge required to move the charge between two points.

   c. Current – Rate of charge passing across a surface:

$$I = i(t) = \frac{dQ}{dt}$$

   d. Resistance –Measure of the ability of charge to move from one point to another for a given potential difference (voltage).

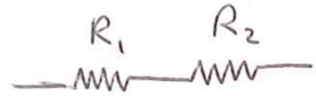
$$R = \frac{\rho L}{A}$$



$V_R = IR$

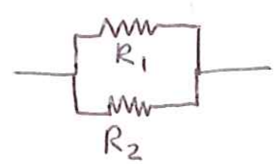
   e. Ohm's Law V=IR or $v(t) = i(t)R$

2. Circuit Analysis
   a. Resistor Simplification



      i. Two resistors in series: $R_{TOT} = R_1 + R_2$

      ii. Two resistors in parallel: $R_{TOT} = \frac{R_1 R_2}{R_1 + R_2}$



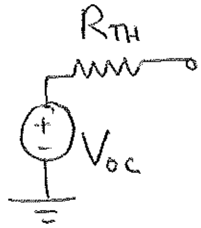
   b. Node Analysis – Using Kirchhoff's Current Law (KCL): ($\sum I = 0$) at any node.

      And: $I_{AB} = \frac{V_A - V_B}{R_{AB}}$

Write a system of equations to solve for unknown node voltages.

   c. Loop Analysis – Using Kirchhoff's Voltage Law (KVL): ($\sum V = 0$) around any closed path.
      Write a system of equations to solve for the unknown currents in each branch.

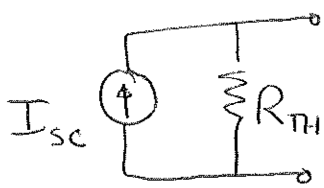
   d. Source equivalents – At any port a linear circuit can be simplified into an ideal source and a resistance
      i. Thevenin Equivalent Circuit – circuit is simplified into a series combination of an ideal voltage source and a resistance.
         1. Find the open circuit voltage (V_{oc}) at the port of interest



2. Find the equivalent resistance (R_{TH})at the port of interest
Or Find the short circuit current (I_{sc}) at the port of interest

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$

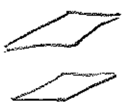
ii. Norton Equivalent Circuit – circuit is simplified into a parallel combination of an ideal current source and a resistance



1. Find the short circuit current (I_{sc}) at the port of interest
2. Find the equivalent resistance (R_{TH})at the port of interest
Or Find the open circuit voltage (V_{oc}) at the port if interest

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$

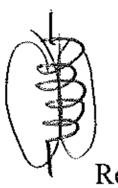
3. Transient Circuits – Circuits that contain capacitors and/or inductors as well as resistors. Voltages or Current sources are switched on or off at t = 0. Response is analyzed



a. Capacitors: $C = \frac{\varepsilon A}{d}$; capacitance for a parallel plate capacitor

$$i(t) = C\frac{dv_c(t)}{dt}$$

open ckt to DC



b. Inductors: $L = \frac{N\phi}{i(t)}$; L SHORT TO DC

inductance for a coil with N turns and magnetic flux ϕ enclosed in the coil

$$v(t) = L\frac{di_L(t)}{dt}$$

Requires solving first order (C or L only) or second order (L and C) differential eqns.

4. AC Circuits – Steady state analysis of circuits with (periodic) sinusoidal voltage and current sources. $\omega = 2\pi f$
   a. Capacitors replaced with frequency dependent impedance $Z_C = \frac{1}{j\omega C}$
   b. Inductors replaced with frequency dependent impedance $Z_L = j\omega L$
   c. Circuit Analysis is simplified from DIFFEQ's to (complex) algebraic techniques used in resistive circuit analysis

5. Power
   a. DC (Resistive) Circuits
      i. Real power P supplied by a source; P = VI
      ii. Power absorbed by a resistor; $P = \frac{V^2}{R} = I^2R$
      iii. Circuits with multiple sources may have some sources ABSORBING power. (Battery charging)
   b. AC (Complex) Circuits

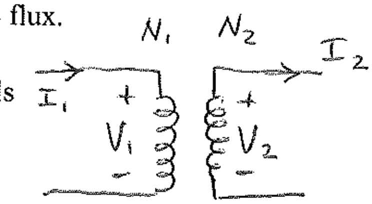
      Complex power; S = P + jQ

      Real Power $P = (1/2)V_{max}I_{max}cos\theta$
      $= V_{rms}I_{rms}cos\theta$

      Complex Power Q = $(1/2)V_{max}I_{max}sin\theta$

θ is the angle measured from V to I.

6. Transformers: Two coils in proximity sharing magnetic flux.

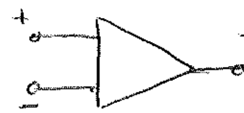


$n = \frac{N_1}{N_2}$; the ratio of the number of turns in the two coils

$$n = \frac{|V_1|}{|V_2|} = \frac{|I_2|}{|I_1|}$$

Beware of the Dots!

7. Operational Amplifiers



$\frac{V_{OUT}}{V_{IN}} = \text{GAIN}$

TRANSFER { FCN / CHARACTERISTIC

For Ideal Op-Amps:

- No current flows into the input terminals
- The input terminals have the same voltage

8. Resonant Circuits – Parallel and Series LC circuits have a bandpass frequency response. This response is located at a center frequency (ω_o) and has a bandwidth (BW)

Resonant Frequency: $\omega_o = \frac{1}{\sqrt{LC}}$

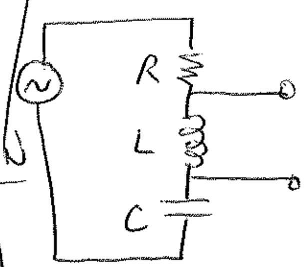
$\omega_o = \frac{1}{\sqrt{LC}}$

Impedance at Resonance: Z = R

Series Resonance: $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$

$Z_L + Z_c = 0$

$BW = \frac{\omega_o}{Q}$

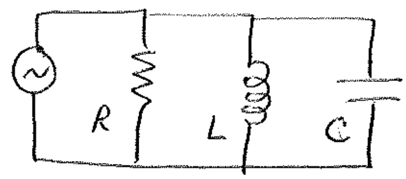


Parallel Resonance: $Q = \omega_o RC = \frac{R}{\omega_o L}$

$\infty = Z_L || Z_c$

Quality Factor

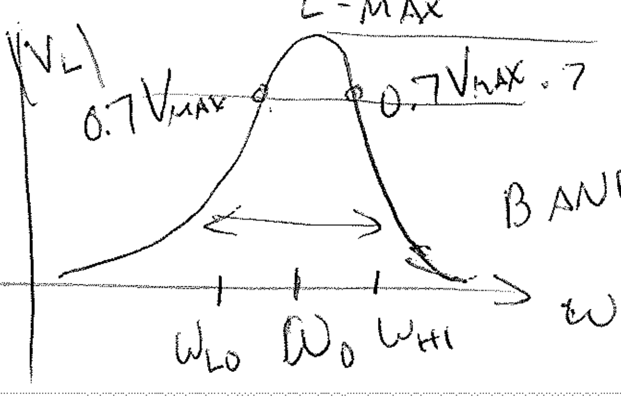
V_L as a fcn of ω



V_L @ DC

$V_L = I Z_L$

$V_L(\omega)\big|_{w=0} = I j\omega L = 0$



ELECTRIC CIRCUITS

UNITS

The basic electrical units are coulombs for charge, volts for voltage, amperes for current, and ohms for resistance and impedance.

ELECTROSTATICS

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}\mathbf{a}_{r12}, \text{ where}$$

$\mathbf{F}_2$ = the force on charge 2 due to charge 1,

Q_i = the ith point charge,

r = the distance between charges 1 and 2,

$\mathbf{a}_{r12}$ = a unit vector directed from 1 to 2, and

ε = the permittivity of the medium.

For free space or air:

$$\varepsilon = \varepsilon_o = 8.85 \times 10^{-12} \text{ Farads/meter}$$

Electrostatic Fields

Electric field intensity **E** (volts/meter) at point 2 due to a point charge Q_1 at point 1 is

$$\mathbf{E} = \frac{Q_1}{4\pi\varepsilon r^2}\mathbf{a}_{r12}$$

For a line charge of density ρ_L C/m on the z-axis, the radial electric field is

$$\mathbf{E}_L = \frac{\rho_L}{2\pi\varepsilon r}\mathbf{a}_r$$

For a sheet charge of density ρ_s C/m^2 in the x-y plane:

$$\mathbf{E}_s = \frac{\rho_s}{2\varepsilon}\mathbf{a}_z,\ z > 0$$

Gauss' law states that the integral of the electric flux density $\mathbf{D} = \varepsilon\mathbf{E}$ over a closed surface is equal to the charge enclosed or

$$Q_{encl} = \oiint_{S} \varepsilon\mathbf{E}\cdot d\mathbf{S}$$

The force on a point charge Q in an electric field with intensity **E** is $\mathbf{F} = Q\mathbf{E}$.

The work done by an external agent in moving a charge Q in an electric field from point p_1 to point p_2 is

$$W = -Q\int_{p_1}^{p_2} \mathbf{E}\cdot d\mathbf{l}$$

The energy stored W_E in an electric field **E** is

$$W_E = (1/2)\iiint_V \varepsilon\,|\mathbf{E}|^2\,dv$$

Voltage

The potential difference V between two points is the work per unit charge required to move the charge between the points.

For two parallel plates with potential difference V, separated by distance d, the strength of the E field between the plates is

$$E = \frac{V}{d}$$

directed from the + plate to the – plate.

Current

Electric current $i(t)$ through a surface is defined as the rate of charge transport through that surface or

$i(t) = dq(t)/dt$, which is a function of time t

since $q(t)$ denotes instantaneous charge.

A constant current $i(t)$ is written as I, and the vector current density in amperes/m^2 is defined as **J**.

Magnetic Fields

For a current carrying wire on the z-axis

$$\mathbf{H} = \frac{\mathbf{B}}{\mu} = \frac{I\mathbf{a}_\phi}{2\pi r}, \text{ where}$$

H = the magnetic field strength (amperes/meter),

B = the magnetic flux density (tesla),

$\mathbf{a}_\phi$ = the unit vector in positive ϕ direction in cylindrical coordinates,

I = the current, and

μ = the permeability of the medium.

For air: $\mu = \mu_o = 4\pi \times 10^{-7}$ H/m

Force on a current carrying conductor in a uniform magnetic field is

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}, \text{ where}$$

L = the length vector of a conductor.

The energy stored W_H in a magnetic field **H** is

$$W_H = (1/2)\iiint_V \mu\,|\mathbf{H}|^2\,dv$$

Induced Voltage

Faraday's Law; For a coil of N turns enclosing flux ϕ:

$$v = -N\,d\phi/dt, \text{ where}$$

v = the induced voltage, and

ϕ = the flux (webers) enclosed by the N conductor turns, and

ϕ = $\int_S \mathbf{B}\cdot d\mathbf{S}$

Resistivity

For a conductor of length L, electrical resistivity ρ, and area A, the resistance is

$$R = \frac{\rho L}{A}$$

For metallic conductors, the resistivity and resistance vary linearly with changes in temperature according to the following relationships:

$$\rho = \rho_o\,[1 + \alpha\,(T - T_o)], \text{ and}$$

$$R = R_o\,[1 + \alpha\,(T - T_o)], \text{ where}$$

ρ_o is resistivity at T_o, R_o is the resistance at T_o, and

α is the temperature coefficient.

Ohm's Law: $V = IR$; $v(t) = i(t)\,R$

Resistors in Series and Parallel

For series connections, the current in all resistors is the same and the equivalent resistance for n resistors in series is

$$R_T = R_1 + R_2 + \ldots + R_n$$

For parallel connections of resistors, the voltage drop across each resistor is the same and the resistance for n resistors in parallel is

$$R_T = 1/(1/R_1 + 1/R_2 + \ldots + 1/R_n)$$

For two resistors R_1 and R_2 in parallel

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Power in a Resistive Element

$$P = VI = \frac{V^2}{R} = I^2 R$$

Kirchhoff's Laws

Kirchhoff's voltage law for a closed path is expressed by

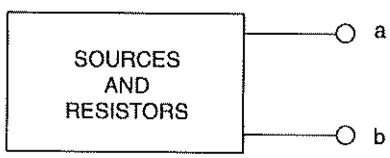
$$\Sigma\, V_{\text{rises}} = \Sigma\, V_{\text{drops}}$$

Kirchhoff's current law for a closed surface is

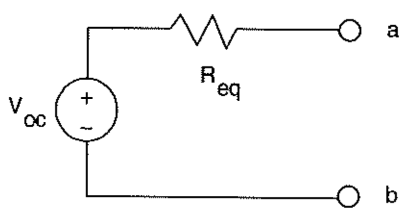
$$\Sigma\, I_{\text{in}} = \Sigma\, I_{\text{out}}$$

SOURCE EQUIVALENTS

For an arbitrary circuit



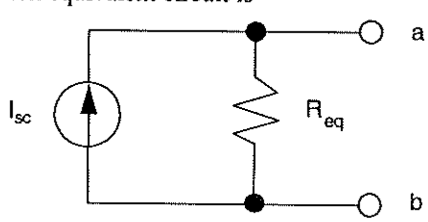
The Thévenin equivalent is



$$R_{eq} = \frac{V_{oc}}{I_{sc}}$$

The open circuit voltage V_{oc} is $V_a - V_b$, and the short circuit current is I_{sc} from a to b.

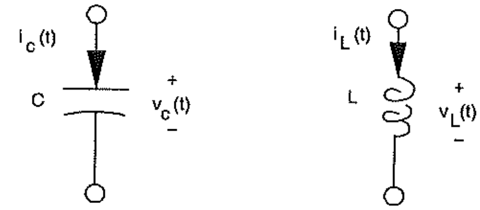
The Norton equivalent circuit is



where I_{sc} and R_{eq} are defined above.

A load resistor R_L connected across terminals a and b will draw maximum power when $R_L = R_{eq}$.

CAPACITORS AND INDUCTORS



The charge $q_C(t)$ and voltage $v_C(t)$ relationship for a capacitor C in farads is

$$C = q_C(t)/v_C(t) \qquad \text{or} \qquad q_C(t) = Cv_C(t)$$

A parallel plate capacitor of area A with plates separated a distance d by an insulator with a permittivity ε has a capacitance

$$C = \frac{\varepsilon A}{d}$$

The current-voltage relationships for a capacitor are

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(\tau)d\tau$$

and $\quad i_C(t) = C\,(dv_C/dt)$

The energy stored in a capacitor is expressed in joules and given by

$$\text{Energy} = Cv_C^2/2 = q_C^2/2C = q_C v_C/2$$

The inductance L of a coil is

$$L = N\phi/i_L$$

and using Faraday's law, the voltage-current relations for an inductor are

$$v_L(t) = L\,(di_L/dt)$$

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)d\tau, \text{ where}$$

v_L = inductor voltage,
L = inductance (henrys), and
i = current (amperes).

The energy stored in an inductor is expressed in joules and given by

$$\text{Energy} = Li_L^2/2$$

Capacitors and Inductors in Parallel and Series

Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Capacitors in Series

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + \ldots + 1/C_n}$$

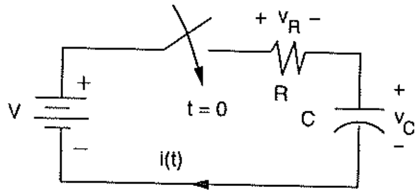
Inductors in Parallel

$$L_{eq} = \frac{1}{1/L_1 + 1/L_2 + \ldots + 1/L_n}$$

Inductors in Series

$$L_{eq} = L_1 + L_2 + \ldots + L_n$$

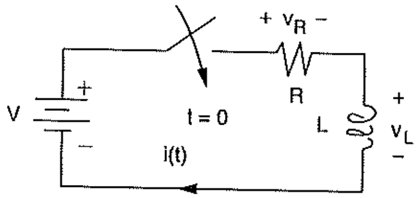
RC AND RL TRANSIENTS



$$t \ge 0;\ v_C(t) = v_C(0)e^{-t/RC} + V(1 - e^{-t/RC})$$

$$i(t) = \{[V - v_C(0)]/R\}e^{-t/RC}$$

$$v_R(t) = i(t)\,R = [V - v_C(0)]e^{-t/RC}$$



$$t \ge 0;\ i(t) = i(0)e^{-Rt/L} + \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

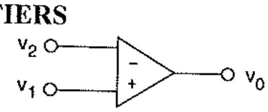
$$v_R(t) = i(t)\,R = i(0)\,Re^{-Rt/L} + V(1 - e^{-Rt/L})$$

$$v_L(t) = L\,(di/dt) = -\,i(0)\,Re^{-Rt/L} + Ve^{-Rt/L}$$

where $v(0)$ and $i(0)$ denote the initial conditions and the parameters RC and L/R are termed the respective circuit time constants.

OPERATIONAL AMPLIFIERS

$v_o = A(v_1 - v_2)$



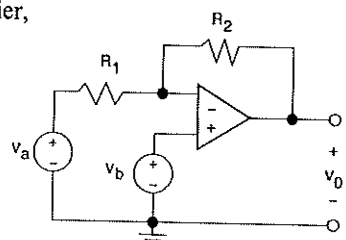
where

A is large ($> 10^4$), and

$v_1 - v_2$ is small enough so as not to saturate the amplifier.

For the ideal operational amplifier, assume that the input currents are zero and that the gain A is infinite so when operating linearly $v_2 - v_1 = 0$.

For the two-source configuration with an ideal operational amplifier,



$$v_o = -\frac{R_2}{R_1}v_a + \left(1 + \frac{R_2}{R_1}\right)v_b$$

If $v_a = 0$, we have a non-inverting amplifier with

$$v_o = \left(1 + \frac{R_2}{R_1}\right)v_b$$

If $v_b = 0$, we have an inverting amplifier with

$$v_o = -\frac{R_2}{R_1}v_a$$

AC CIRCUITS

For a sinusoidal voltage or current of frequency f (Hz) and period T (seconds),

$f = 1/T = \omega/(2\pi)$, where

ω = the angular frequency in radians/s.

Average Value

For a periodic waveform (either voltage or current) with period T,

$$X_{ave} = (1/T)\int_0^T x(t)dt$$

The average value of a full-wave rectified sine wave is

$$X_{ave} = (2X_{max})/\pi$$

and half this for a half-wave rectification, where

X_{max} = the peak amplitude of the waveform.

Effective or RMS Values

For a periodic waveform with period T, the rms or effective value is

$$X_{rms} = \left[(1/T)\int_0^T x^2(t)dt\right]^{1/2}$$

For a sinusoidal waveform and full-wave rectified sine wave,

$$X_{rms} = X_{max}/\sqrt{2}$$

For a half-wave rectified sine wave,

$$X_{rms} = X_{max}/2$$

Sine-Cosine Relations

$\cos(\omega t) = \sin(\omega t + \pi/2) = -\sin(\omega t - \pi/2)$

$\sin(\omega t) = \cos(\omega t - \pi/2) = -\cos(\omega t + \pi/2)$

Phasor Transforms of Sinusoids

$P[V_{max}\cos(\omega t + \phi)] = V_{rms} \angle \phi = \boldsymbol{V}$

$P[I_{max}\cos(\omega t + \theta)] = I_{rms} \angle \theta = \boldsymbol{I}$

For a circuit element, the impedance is defined as the ratio of phasor voltage to phasor current.

$$Z = \frac{\boldsymbol{V}}{\boldsymbol{I}}$$

For a Resistor,

$$Z_R = R$$

For a Capacitor,

$$Z_C = \frac{1}{j\omega C} = jX_C$$

For an Inductor,

$Z_L = j\omega L = jX_L$, where

X_C and X_L are the capacitive and inductive reactances respectively defined as

$$X_C = -\frac{1}{\omega C} \quad \text{and} \quad X_L = \omega L$$

Impedances in series combine additively while those in parallel combine according to the reciprocal rule just as in the case of resistors.

Complex Power

Real power P (watts) is defined by

$$P = (\tfrac{1}{2})V_{max}I_{max}\cos\theta$$
$$= V_{rms}I_{rms}\cos\theta$$

where θ is the angle measured from V to I. If I leads (lags) V, then the power factor ($p.f.$),

$$p.f. = \cos\theta$$

is said to be a leading (lagging) $p.f.$

Reactive power Q (vars) is defined by

$$Q = (\tfrac{1}{2})V_{max}I_{max}\sin\theta$$
$$= V_{rms}I_{rms}\sin\theta$$

Complex power $\boldsymbol{S}$ (volt-amperes) is defined by

$$\boldsymbol{S} = \boldsymbol{V}\boldsymbol{I}^* = \mathrm{P} + \mathrm{j}Q,$$

where $\boldsymbol{I}^*$ is the complex conjugate of the phasor current.

For resistors, θ = 0, so the real power is

$$P = V_{rms}I_{rms} = V_{rms}^2/R = I_{rms}^2 R$$

RESONANCE

The radian resonant frequency for both parallel and series resonance situations is

$$\omega_o = \frac{1}{\sqrt{LC}} = 2\pi f_o \text{ (rad/s)}$$

Series Resonance

$$\omega_o L = \frac{1}{\omega_o C}$$

$Z = R$ at resonance.

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$BW = \omega_o/Q \text{ (rad/s)}$$

Parallel Resonance

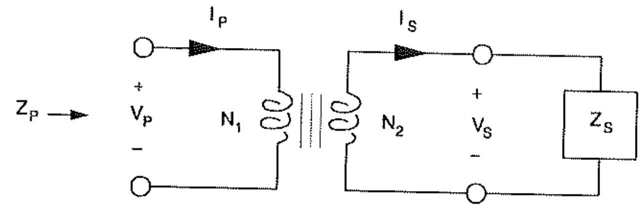
$$\omega_o L = \frac{1}{\omega_o C} \quad \text{and}$$

$Z = R$ at resonance.

$$Q = \omega_o RC = \frac{R}{\omega_o L}$$

$$BW = \omega_o/Q \text{ (rad/s)}$$

TRANSFORMERS



Turns Ratio

$$a = N_1/N_2$$

$$a = \left|\frac{\boldsymbol{V}_P}{\boldsymbol{V}_S}\right| = \left|\frac{\boldsymbol{I}_S}{\boldsymbol{I}_P}\right|$$

The impedance seen at the input is

$$Z_P = a^2 Z_S$$

ALGEBRA OF COMPLEX NUMBERS

Complex numbers may be designated in rectangular form or polar form. In rectangular form, a complex number is written in terms of its real and imaginary components.

$z = a + \mathrm{j}b$, where

a = the real component,

b = the imaginary component, and

j = $\sqrt{-1}$

In polar form

$z = c \angle \theta$, where

c = $\sqrt{a^2 + b^2}$,

θ = $\tan^{-1}(b/a)$,

a = $c\cos\theta$, and

b = $c\sin\theta$.

Complex numbers are added and subtracted in rectangular form. If

$$z_1 = a_1 + \mathrm{j}b_1 = c_1(\cos\theta_1 + \mathrm{j}\sin\theta_1)$$
$$= c_1 \angle \theta_1 \text{ and}$$
$$z_2 = a_2 + \mathrm{j}b_2 = c_2(\cos\theta_2 + \mathrm{j}\sin\theta_2)$$
$$= c_2 \angle \theta_2, \text{ then}$$
$$z_1 + z_2 = (a_1 + a_2) + \mathrm{j}(b_1 + b_2) \text{ and}$$
$$z_1 - z_2 = (a_1 - a_2) + \mathrm{j}(b_1 - b_2)$$

While complex numbers can be multiplied or divided in rectangular form, it is more convenient to perform these operations in polar form.

$$z_1 \times z_2 = (c_1 \times c_2) \angle \theta_1 + \theta_2$$
$$z_1/z_2 = (c_1/c_2) \angle \theta_1 - \theta_2$$

The complex conjugate of a complex number $z_1 = (a_1 + \mathrm{j}b_1)$ is defined as $z_1^* = (a_1 - \mathrm{j}b_1)$. The product of a complex number and its complex conjugate is $z_1 z_1^* = a_1^2 + b_1^2$.

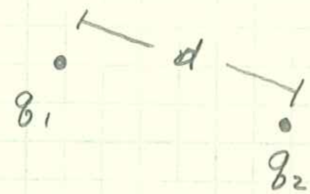
ELECTROSTATICS

CHARGE: UNIT IS COULOMB

- 1 electron has a charge of 1.6×10^{-19} C
- 1 COULOMB IS THE CHARGE OF 6.24×10^{16} e

FORCE A FORCE OF ONE CHARGE ON ANOTHER

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi \varepsilon d^2} \vec{a}_{12}$$



$q_i \equiv i^{th}$ POINT CHARGE COULOMBS

$\varepsilon :=$ PERMITIVITY OF THE MEDIUM (HOW CAPABLE IS THE MATERIAL WHERE THE CHARGE EXISTS TO ALLOW THE FORCES TO EXIST)

FREE SPACE VS TEFLON, METALS

$d :=$ distance between two charges q_1, q_2

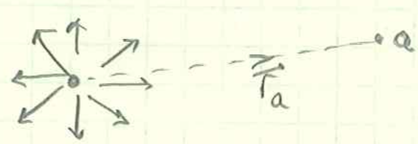
$\vec{a}_{12} :=$ UNIT VECTOR FROM q_1 TO q_2

ELECTRIC FIELD

$$\vec{F}_{12} = q_1 \vec{E} \qquad \vec{E}_2 = \frac{q_2}{4\pi \varepsilon r^2} \vec{r}$$

ELECTRIC FIELD SURROUNDING A CHARGE

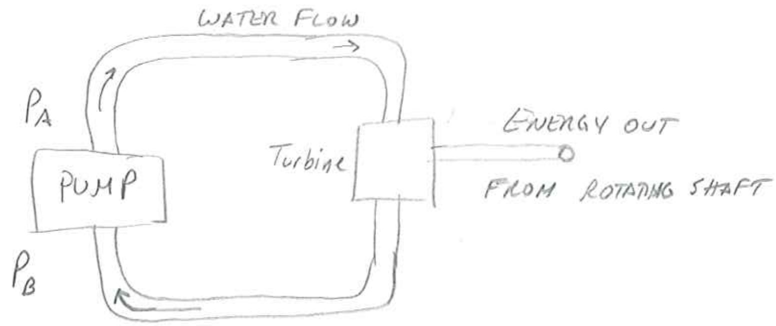
$\vec{r} :=$ UNIT VECTOR POINTING FROM THE CHARGE TO THE $\vec{E}$ FIELD EVALUATION POINT



EVALUATION POINT "a"

$$\vec{E}_a = \frac{q}{4\pi \varepsilon d^2} \vec{r}_a$$

ANALOGY OF FLUID FLOW AND CURRENT FLOW

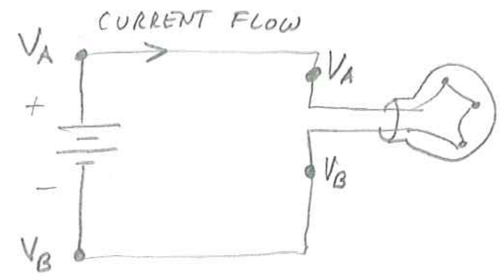


WATER EXITS PUMP AT HIGHER PRESSURE P_A

WATER ENTERS PUMP AT LOWER PRESSURE P_B

FLOW RATE INTO TURBINE =
FLOW RATE OUT

PRESSURE INTO TURBINE > PRESSURE OUT



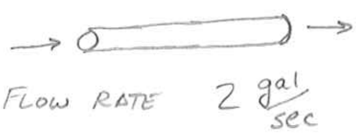
CURRENT EXITS BATTERY FROM
HIGHER VOLTAGE

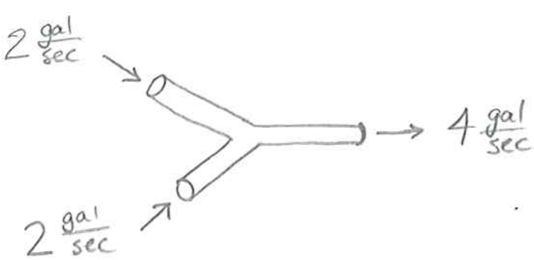
CURRENT ENTERS BATTERY AT
LOWER VOLTAGE

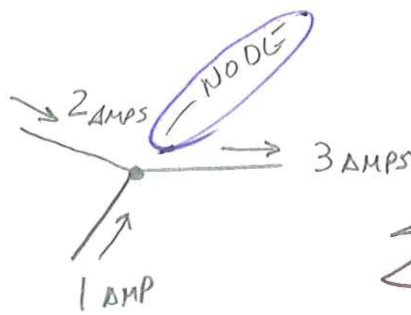
CURRENT INTO BULB = CURRENT OUT

VOLTAGE IN $\geq$ VOLTAGE OUT

FLOW RATE ANALOGY TO CURRENT FLOW:







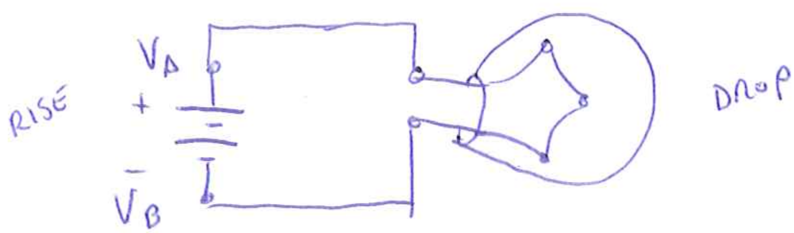
KCL

$$\sum i = 0$$

$$I_{IN} = I_{OUT}$$

KVL FOR A CLOSED PATH

$$\sum V_{rises} = \sum V_{Drops}$$



VOLTAGE POTENTIAL DIFFERENCE BTWN 2 POINTS

WORK DONE PER UNIT CHARGE REQ'D TO MOVE CHARGE BTWN 2 POINTS

$$V_A - V_B = \int_a^b \vec{E} \cdot d\vec{l}$$ LINE INTEGRAL OF $\vec{E}$

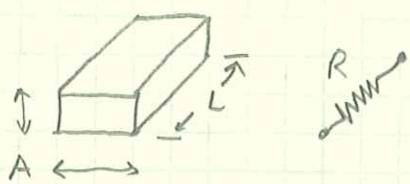
CURRENT RATE OF CHANGE PASSING ACROSS A SURFACE OF POSITIVE CHARGE.

$$I = i(t) = \frac{dq}{dt}$$

RESISTANCE MEASURES ABILITY OF CHARGE TO MOVE FROM ONE POINT TO ANOTER

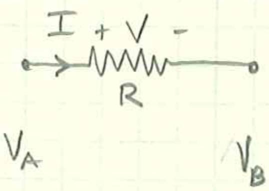
$$R = \frac{\rho L}{A}$$ $\rho :=$ MATERIAL RESISTIVITY

$\sigma = \frac{1}{\rho} :=$ CONDUCTIVITY



OHMS LAW

DC	AC
$V = IR$	$v(t) = i(t)R$



$V_R = V_A - V_B$ "VOLTAGE DROP ACROSS R."

PASSIVE SIGN CONVENTION: POSITIVE CHARGE FLOWS FROM HIGH VOLTAGE TO LOW.

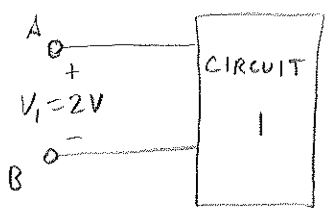
SOLVING ELECTRIC CIRCUITS

SIMILAR TO A MECHANICAL SYSTEM WE SET UP A COORDINATE SYSTEM THAT WILL ESTABLISH THE POSITIVE DIRECTION.

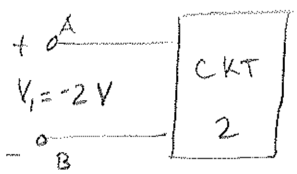
AFTER CALCULATING VELOCITY (FOR EXAMPLE) IF THE ANSWER IS A POSITIVE NUMBER, OUR ARBITRARY SELECTION OF THE POSITIVE DIRECTION WERE CORRECT.

IF THE ANSWER WAS NEGATIVE, THIS MEANS MOVEMENT IS ACTUALLY IN THE OPPOSITE DIRECTION.

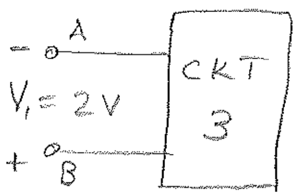
FOR ELECTRIC CKTS WE WILL ALSO ASSIGN AN ARBITRARY DIRECTION OF CURRENT FLOW AND VOLTAGE DIFFERENCE (POLARITY)



TERMINAL A IS 2 V HIGHER THAN TERMINAL B

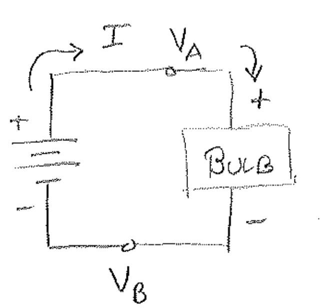


TERMINAL B IS 2 V HIGHER THAN TERMINAL A



TERMINAL B IS 2 V HIGHER THAN TERMINAL A

POWER DELIVERED VS. POWER ABSORBED



BATTERY DELIVERS POWER TO BULB

BULB ABSORBS POWER FROM BATTERY

CONVENTION FOR CURRENT

POSITIVE CURRENT IS THE FLOW OF POSITIVE CHARGE (OPPOSITE OF ELECTRON FLOW)

NEGATIVE CURRENT IS THE OPPOSITE FLOW OF POSITIVE CHARGE.

BOTH BRANCHES HAVE CURRENT FLOW NODE A TO B

CONVENTION FOR VOLTAGE

VOLTAGE BETWEEN 2 POINTS IS THE DIFFERENCE IN THE ENERGY LEVEL OF A UNIT CHARGE LOCATED AT EACH OF THE TWO POINTS.

VOLTAGE BETWEEN NODE A AND B IS $V_{AB} = -V_{BA}$

ENERGY AT NODE A IS GREATER THAN THAT AT NODE B.

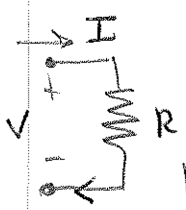
IF $V_{AB} = 5$ VOLTAGE AT NODE A IS 5 VOLTS MORE THAN

IF $V_{AB} = -5$ 5 VOLTS LESS THAN.

VOLTAGE AT GROUND ALWAYS = 0.

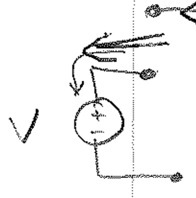
CONVENTION FOR POWER

POWER IS ABSORBED OR SUPPLIED.



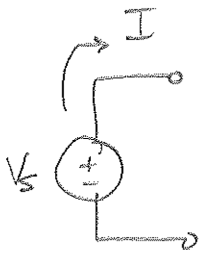
IF POSITIVE CURRENT FLOWS INTO A HIGHER VOLTAGE NODE : POWER IS ABSORBED

$V = IR$



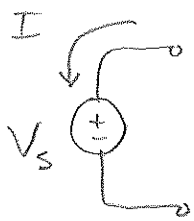
IF POSITIVE CURRENT FLOWS OUT OF A HIGHER VOLTAGE NODE : POWER IS SUPPLIED.

FIND POWER ABSORBED/SUPPLIED BY EACH ELEMENT.



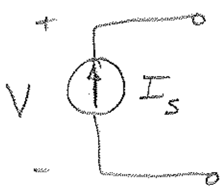
POWER SUPPLIED $= VI$

ABSORBED $= -VI$



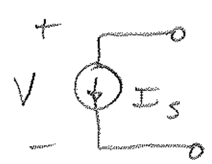
POWER ABSORBED $= +VI$

(CHARGING BATTERY)

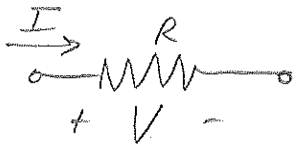


POWER SUPPLIED $= +VI$

ABSORBED $= -VI$



POWER ABSORBED. $= -VI$

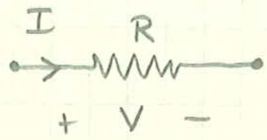


POWER ABSORBED.

$P_A = VI$

$P_s = -VI$

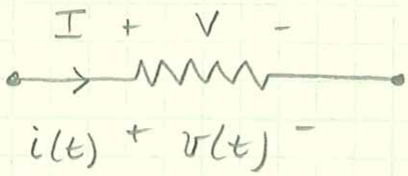
POWER ABSORBED $\quad P = VI = v(t)\,i(t)$



UNITS WATTS

PASSIVE DEVICE ABSORBS "POSITIVE" POWER EQUAL TO CURRENT THROUGH TIMES VOLTAGE ACROSS.

DC	DIRECT	V	I	CONSTANT
AC	ALTERNATING	$v(t)$	$i(t)$	TIME VARYING



EX

$V = 10\ V \qquad I = 5$ AMPS

FIND R, POWER ABSORBED.

SOLN $\quad V = IR$

$$R = \frac{V}{I} = \underline{\underline{2\ \Omega}}$$

$P = VI$
$= \underline{\underline{50\ \text{WATTS}}}$

EX $\quad v(t) = 10\cos(t) \qquad R = 2$

FIND $i(t) \quad P(t)$

SOLN $\quad v(t) = i(t)\,R$

$$i(t) = \frac{v(t)}{R} = 5\cos(t)\ \text{Amps}$$

$$p(t) = 10\cos(t) \times 5\cos(t)$$

$$= 50\cos^2(t) = 50\left[\frac{1}{2} + \frac{1}{2}\cos(2(t))\right]$$

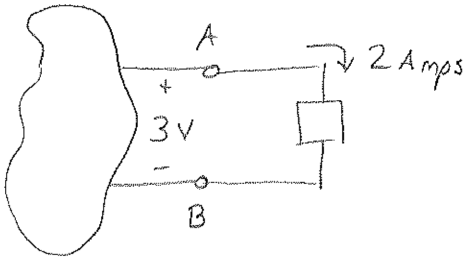
$$p(t) = 25 + 25\cos 2t \quad \text{WATTS}$$

$p(t)$ ↑ INSTANTANEOUS POWER. $\quad$ 25 ↑ AVERAGE POWER

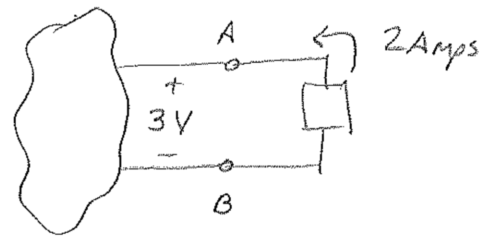
$$P_{AVE} = \frac{VI}{2}$$

AC AVERAGE POWER.

- MORE COMPLICATED CRTS ARE NOT AS EASY TO PREDICT BEFORE PERFORMING CRT ANALYSIS.
- ONCE VOLTAGES AND CURRENTS ARE CALCULATED, WE CAN DETERMINE POWER DELIVERED AND ABSORBED.



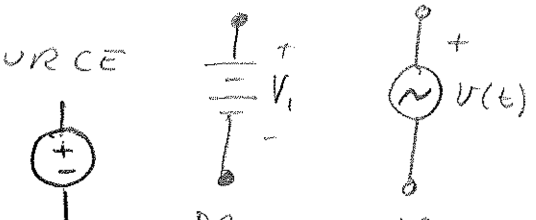
POWER ABSORBED
(BULB)



POWER SUPPLIED
(BATTERY)

CIRCUIT ELEMENTS

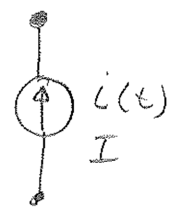
SOURCES: INDEPENDENT VOLTAGE SOURCE



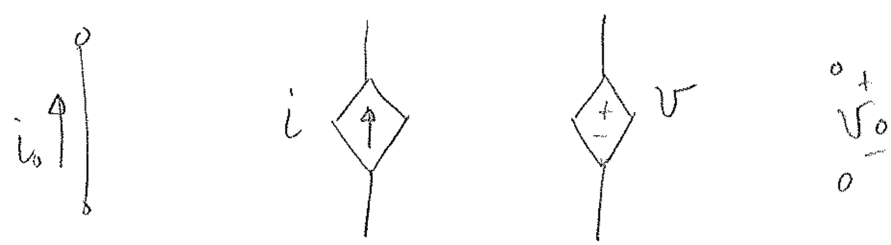
DC CONSTANT

AC SINUSOIDAL

INDEPENDENT CURRENT SOURCE



DEPENDENT SOURCES



CURRENT

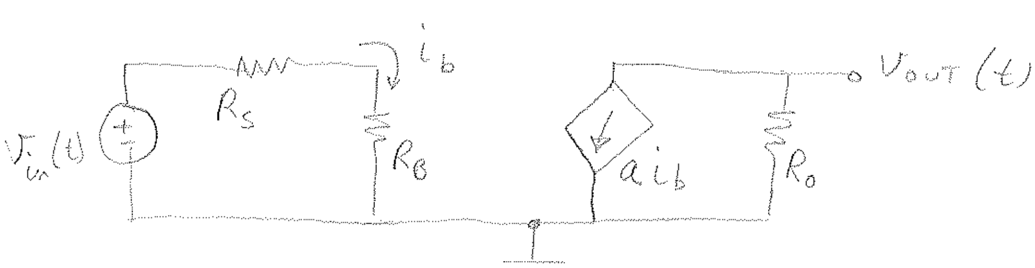
$i = \beta i_0$

$i = g v_0$

VOLTAGE

$v = a v_0$

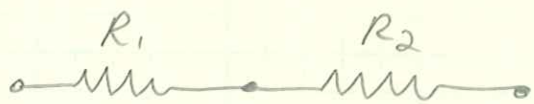
$v = r i_0$

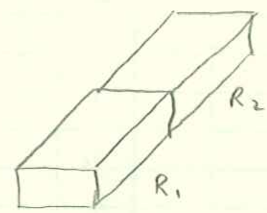


SIMPLIFYING RESISTIVE CIRCUITS.

- RESISTOR SIMPLIFICATION
- THEVENIN EQUIVALENT CIRCUITS
- NORTON EQUIVALENT CIRCUITS.

RESISTORS IN SERIES



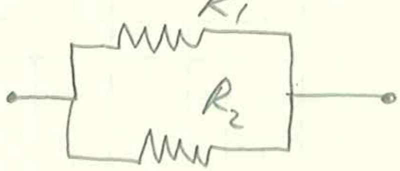


$$R = \frac{\rho L(2)}{A}$$

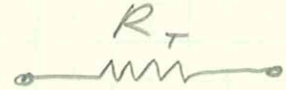
$$R_T = R_1 + R_2$$

$R = R_1 + R_2$

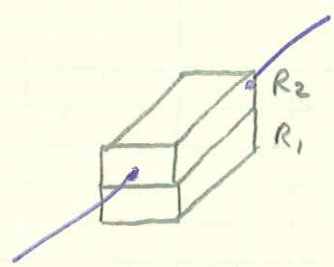
RESISTORS IN PARALLEL



$\Rightarrow$

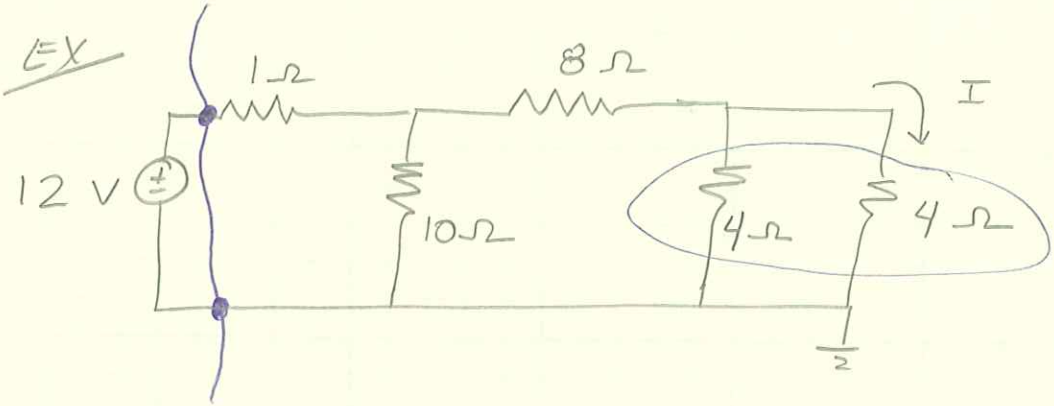


$$R_T = \frac{(R_1)(R_2)}{R_1 + R_2}$$



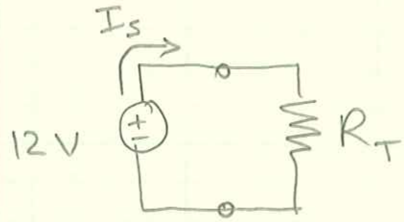
$$R_T = \frac{\rho L}{2A} = \frac{R_1}{2} = \frac{R_2}{2}$$

EX



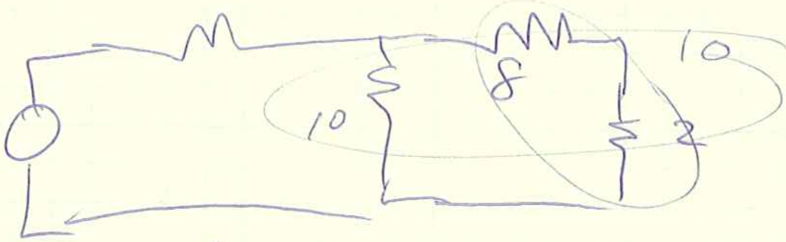
FIND I IN THIS CKT.

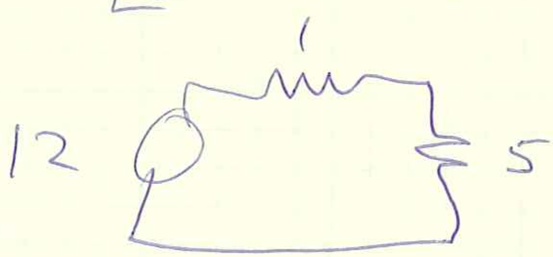
CAN USE LOOP EQUATIONS OR RESISTIVE SIMPLIFICATION

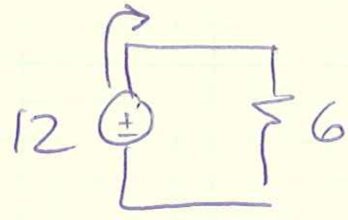


CURRENT FROM SOURCE I_S DELIVERED TO RESISTIVE CIRCUIT.

$$4\Omega \,||\, 4\Omega = \frac{16}{8} = 2\Omega$$





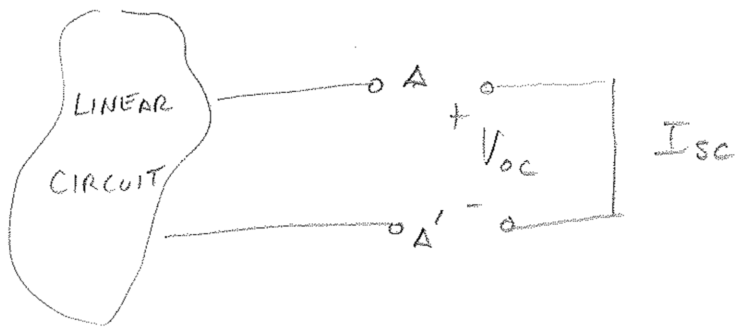


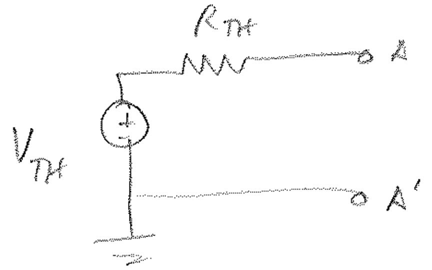
$V = IR$

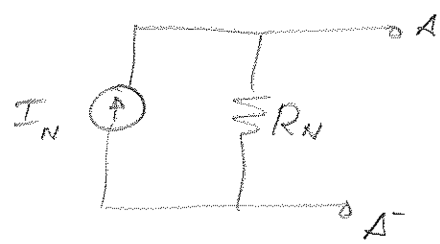
$$\boxed{I = 2\,A}$$

THEVENIN AND NORTON EQUIVALENT CIRCUITS

AT ANY "PORT" OF A LINEAR CIRCUIT WE CAN REPLACE WITH AN IDEAL SOURCE AND A RESISTANCE







$V_{OC} = V_{TH}$

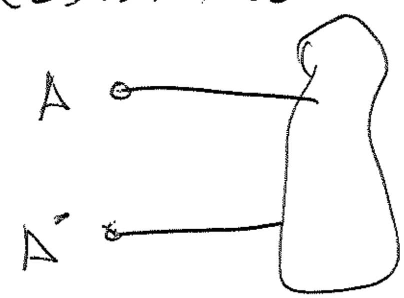
$I_{SC} = I_N$

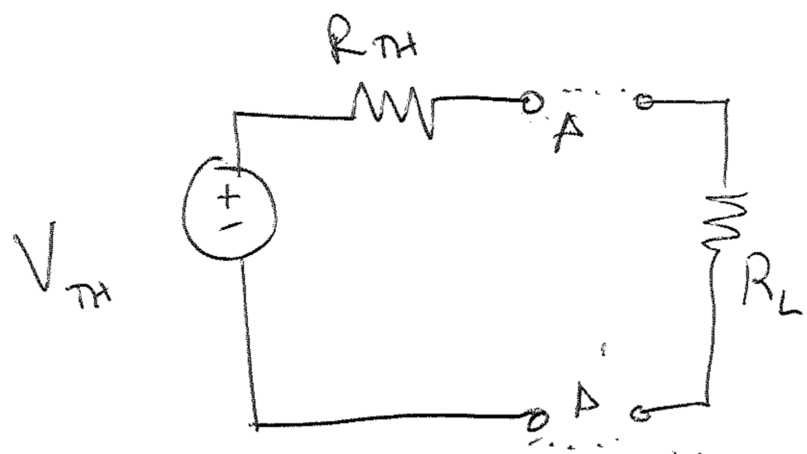
$$R_{TH} = \frac{V_{SC}}{I_N}$$

THEVENIN NORTON

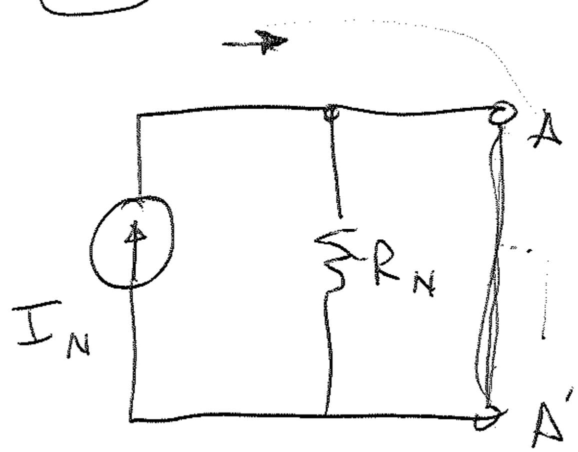
EQUIVALENT CIRCUITS

AT ANY PORT OF A LINEAR CIRCUIT. WE CAN REPLACE ALL ELEMENTS WITH IDEAL SOURCE AND SINGLE RESISTANCE





TEC $I_L = \frac{V_{TH}}{R_{TH}+R_L}$



NEC.

AT NODE C

$$\Sigma I = 0$$

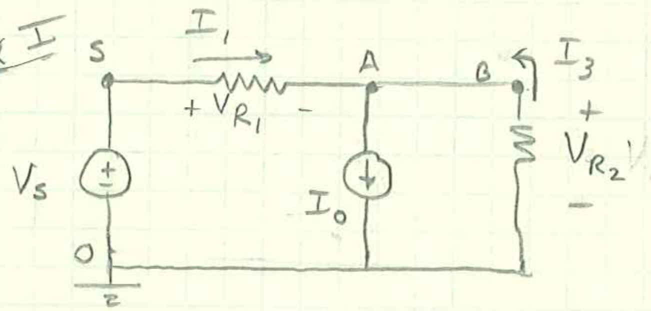
$$I_{BC} + I_{AC} + I_1 + I_{CN} = 0$$

ALL CURRENTS ARE POINTING INTO NODE C,

ONE OR MORE MUST BE NEGATIVE FOR KCL TO HOLD.

* IF A CURRENT OR VOLTAGE ARE FOUND TO BE NEGATIVE, THEN ACTUAL $\{V, I\}$ IS OPPOSITE OF ORIGINAL ARBITRARY ASSIGNMENT.

EX I



FIND I_1, I_2, V_B, V_A, V_{R2}

GIVEN

$V_s = 5\,V$ $\quad I_0 = 1\,A$

$R_1 = 6\,\Omega$ $\quad R_2 = 4\,\Omega$

SOLN $\quad I_1 + I_3 - I_0 = 0$

$I_1 + I_3 = I_0 = 1\,A$ $\qquad$ KCL @ A

$I_1 = 1 - I_3$

$5 - V_{R1} - V_{R2} = 0$

$5 - I_1 R_1 + I_3 R_2 = 0$ $\qquad$ KVL $\qquad V_{R_2} = -I_3 R_2$

NOTE PASSIVE SIGN CONVENTION

$5 - (1 - I_3)R_1 + I_3 R_2 = 0$

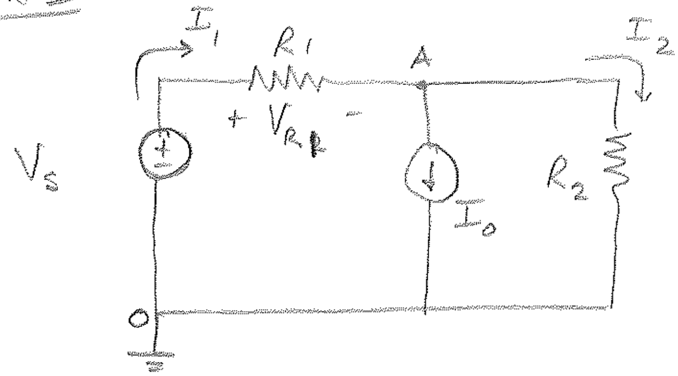
$5 - 1R_1 + R_1 I_3 + R_2 I_3 = 0$

$10\, I_3 = 1$

$I_3 = 0.1\,A$ $\qquad V_B = -0.4\,V = -I_3 R_2$

$I_1 = 0.9\,A$ $\qquad V_A = -0.4$

Ex II



GIVEN

$V_s = 5\ V$ $\quad R_1 = 6\ \Omega$

$I_0 = 1\ A$ $\quad R_2 = 4\ \Omega$

FIND

I_1, I_2, V_B, V_{R_2}

SOLUTION

KCL @ A

$$I_1 - I_0 - I_2 = 0$$

$$I_1 = I_0 + I_2$$

$$I_1 = 1 + I_2$$

$$I_1 = 1 - \frac{1}{10}$$

$$\boxed{I_1 = 0.9\ A}$$

KVL: $V_s - V_{R_1} - V_{R_2} = 0$

$$V_s - I_1 R_1 - I_2 R_2 = 0$$

$$V_s - (1 + I_2) R_1 - I_2 R_2 = 0$$

$$V_s - R_1 - I_2 R_1 - I_2 R_2 = 0$$

$$I_2 = \frac{-(V_s - R_1)}{-(R_1 + R_2)}$$

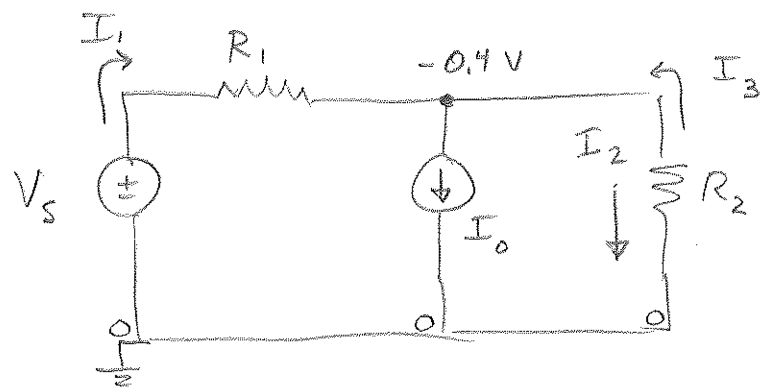
$$I_2 = \frac{-(5-6)}{-(6+4)} = -\frac{1}{10}\ A$$

$$\boxed{I_2 = -\frac{1}{10}\ A}$$

$$V_B = V_A = I_2 R_2 = \underline{-0.4\ V}$$

$$V_{R_2} = I_1 R_1 = \underline{5.4\ V}$$

FINDING POWER



$I_3 = -I_2 = 0.1\ A$

$V_A = -0.4\ V$

$V_S = 5\ V$

$I_1 = 0.9\ A$

$P_{V_S} = 5(0.9) = 4.5\ W$ SUPPLIED

$P_{R1} = VI = \frac{V^2}{R} = I^2 R_1 = (0.9)^2\, 6 = 4.86\ W$ ABSORBED

$P_{R2} = VI = (0.4)(0.1) = .04\ W$ ABSORBED

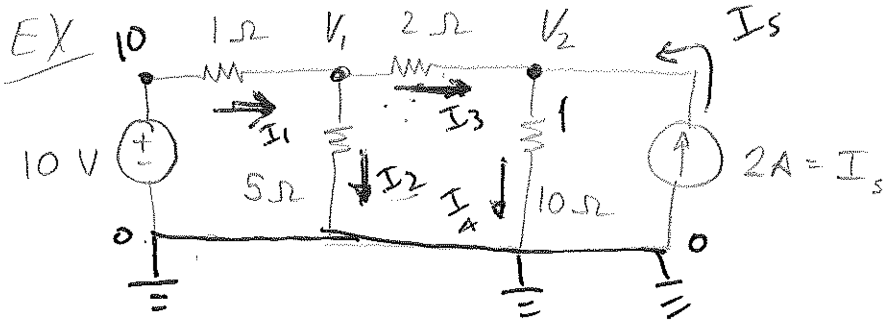
$P_{I_0} = VI = (0.4)(1) = 0.4\ W$ SUPPLIED

SUPPLIED = ABSORBED

$4.5 + .4 = 4.86 + .04$

$4.9 = 4.9$

CONSERVED



① FIND V_1 V_2

② ALL BRANCH CURRENTS

③ POWER

SOLVE USING

KCL NODE VOLTAGE (ANALYSIS) EQUATIONS

KVL LOOP CURRENT (ANALYSIS) EQUATIONS

SUPERPOSITION

THEVENIN EQUIVALENT EQNS

KCL @ V_1

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$$

@ V_2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0$$

SOLVE SIMULTANEOUSLY

$$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0$$

$$17V_1 - 5V_2 = 100$$

$$V_1 = \frac{100 + 5V_2}{17}$$

$$5V_2 - 5V_1 + V_2 = 20$$

$$6V_2 - 5V_1 = 20$$

$$6V_2 - \frac{5}{17}(100 + 5V_2) = 20$$

$$102V_2 - 500 - 25V_2 = 340$$

$$77V_2 = 834$$

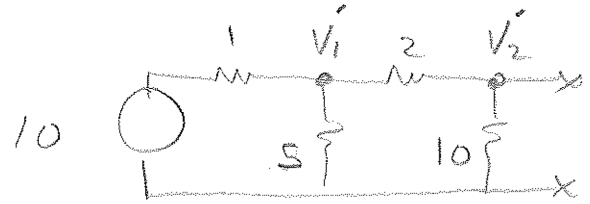
$$\boxed{V_2 = \frac{834}{77} = 10.83}$$

$$6(10.83) - 5V_1 = 20$$

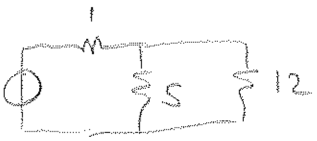
$$\boxed{V_1 = 9.0}$$

SUPER POSITION (1 IND. SOURCE AT A TIME)

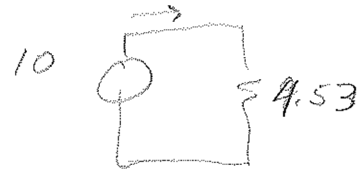
SET $I_s = 0$ (OPEN)



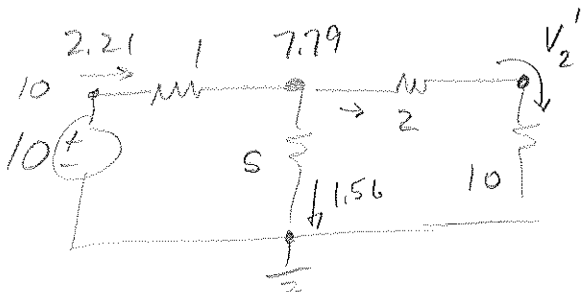
RESISTOR SIMPLIFICATION



$\frac{60}{17} = 3.53$



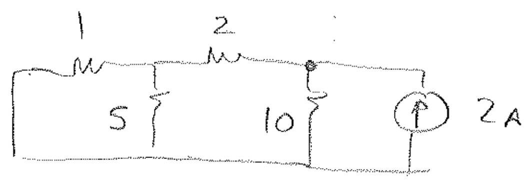
$I = 2.21$

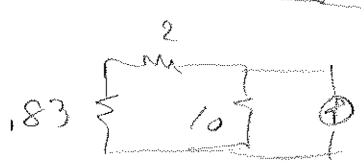


$V_1' = 10 - 1(2.21) = \boxed{7.79}$

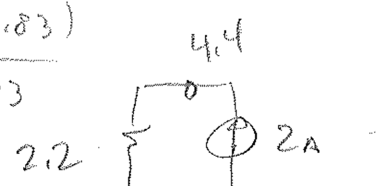
$I_5' = \frac{7.79}{5} = 1.56$

$I_2' = 2.21 - 1.56$
$= 0.652$ $\boxed{V_2' = 6.52\,V}$

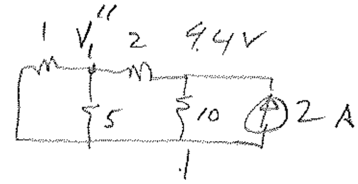




$\frac{10(2.83)}{12.83}$



$\boxed{V_2'' = 4.4}$



$I_{10}'' = .44$

$-I_2'' = 2 - .44$
$= 1.56$

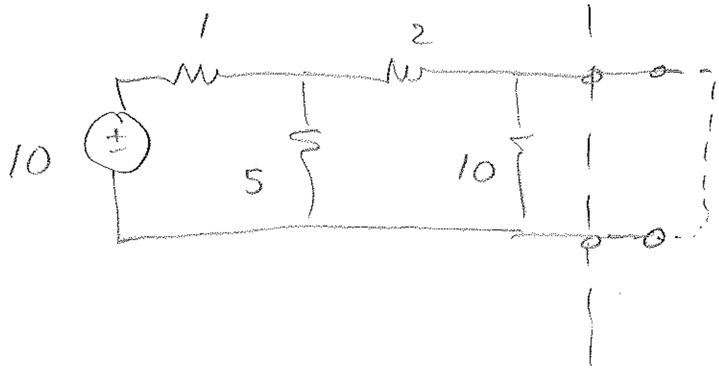
$V_1'' = 4.4V - (2)(1.56)$

$\boxed{V_1'' = 0.88}$

$V_2 = 6.52 + 4.4 = 10.9$

$V_1 = 7.79 + .88 = 8.7$

THEVENIN EQUIVALENT

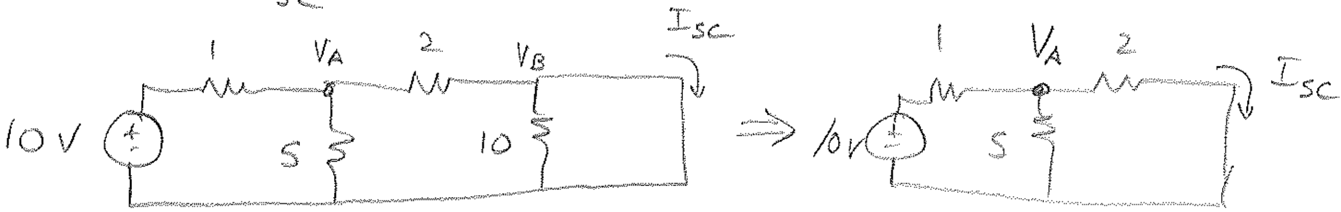


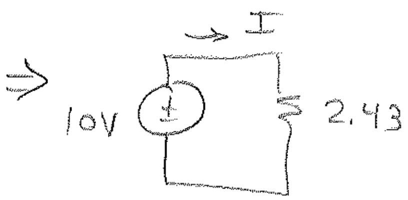
$V_{OC} =$

I_{SC}

V_{OC} FOUND IN SUPERPOSITION $= \underline{6.52\ V}$

FINDING I_{SC}

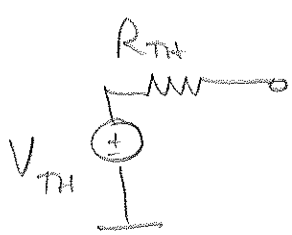




$I = 4.11\ A$

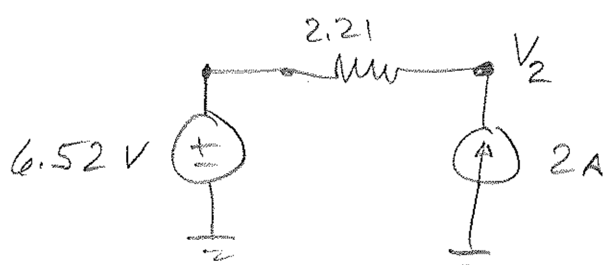
$$V_A = 10 - 1(4.11) = 5.89\ V$$

$$I_{SC} = \frac{5.89V}{2\Omega} = \underline{2.95\ A}$$



$$V_{TH} = V_{OC} = 6.52\ V$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = 2.21\ \Omega$$

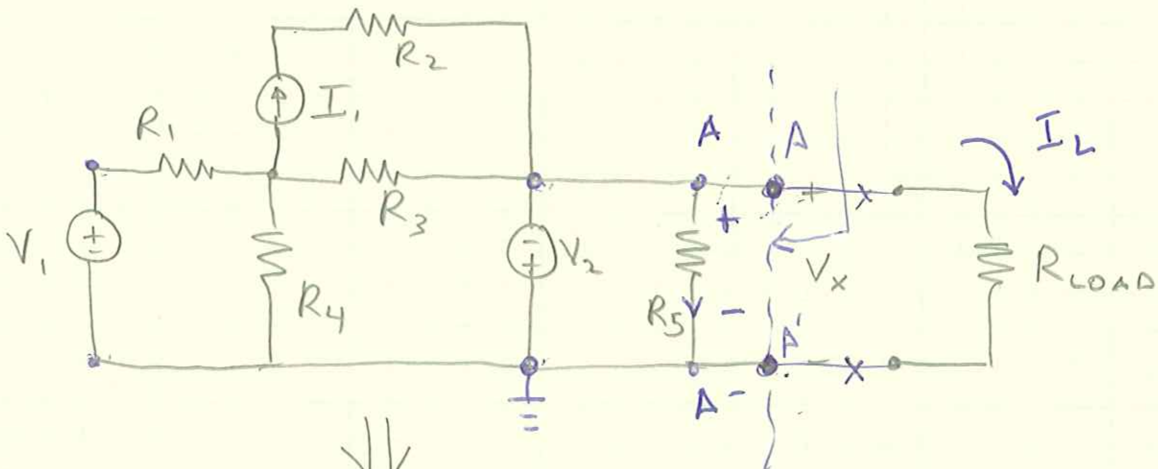


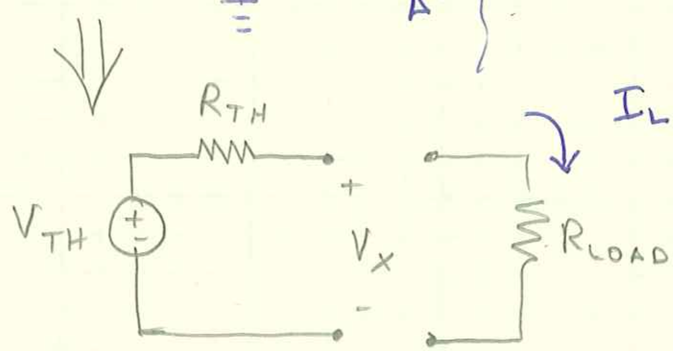
$$V_2 = 6.52 + 2(2.21)$$

$$\boxed{V_2 = 10.9}$$

THEVENIN EQUIVALENT CIRCUIT

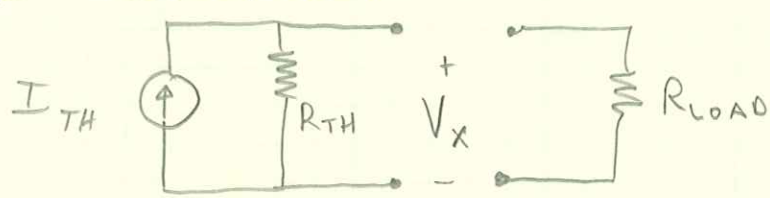
ANY RESISTIVE CIRCUIT, NO MATTER HOW COMPLEX, CAN BE REDUCED TO A SERIES VOLTAGE AND RESISTANCE, AT ANY TERMINAL IN THAT CIRCUIT.





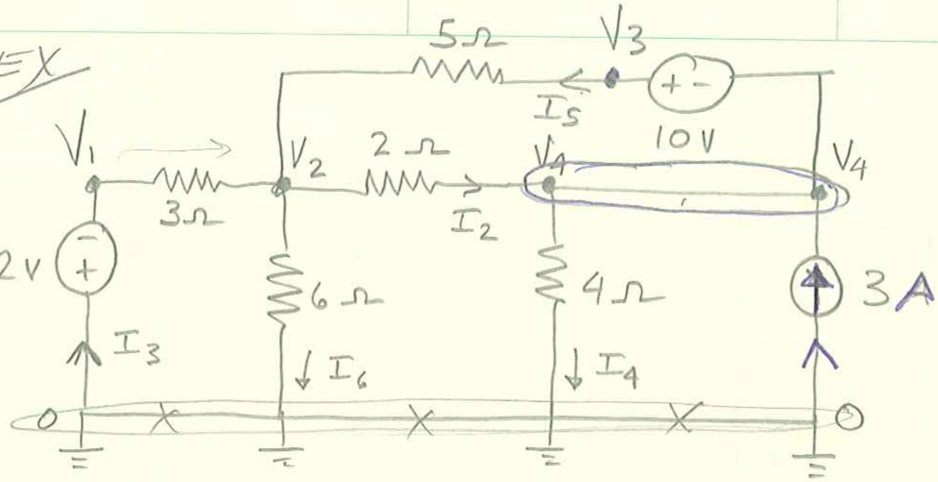
OR

NORTON EQUIVALENT.



$$V_x = I_{TH}\left(\frac{R_{TH}R_L}{R_{TH}+R_L}\right)$$

EX



FIND ALL BRANCH CURRENTS AND NODE VOLTAGES.

STEP 1) LABEL ALL BRANCHES AND NODES

BRANCH CURRENTS $\quad I_3 , I_6 \quad I_4 \quad I_5 , I_2$

FIND VOLTAGES AT ALL NODES
THEN USE Ohm's LAW

$$I_2 = \frac{V_2 - V_4}{2\Omega} \qquad I_6 = \frac{V_2 - 0}{6}$$

$$\boxed{V_1 = -12\text{ V}} \qquad \boxed{V_3 = 10 + V_4}$$

KCL @ NODE 2 $\qquad I_3 + I_5 = I_6 + I_2$

$\sum I = 0$

$$I_3 = \frac{V_1 - V_2}{3} \qquad I_5 = \frac{V_3 - V_2}{5}$$

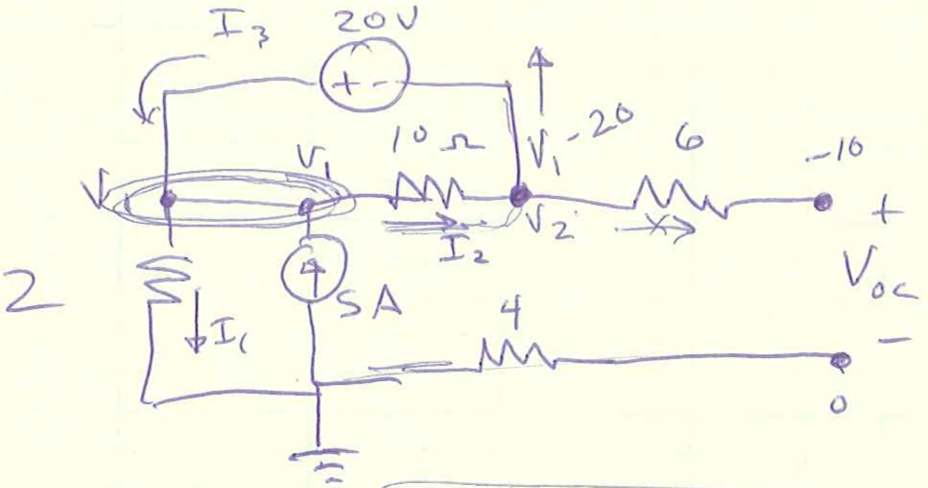
$$\frac{V_1 - V_2}{3} + \frac{V_3 - V_2}{5} = \frac{V_2}{6} + \frac{V_2 - V_4}{2}$$

KCL @ NODE 4

$$I_2 + 3 = I_4 + I_5 \qquad I_4 = \frac{V_4 - 0}{4}$$

4 EQN. 4 UNKNOWNS CAN BE SOLVED SIMULTANEOUSLY

FIND V_{oc}



KCL @ V_1 $\boxed{\cancel{I_2} + I_1 = 5 + \cancel{I_3}}$

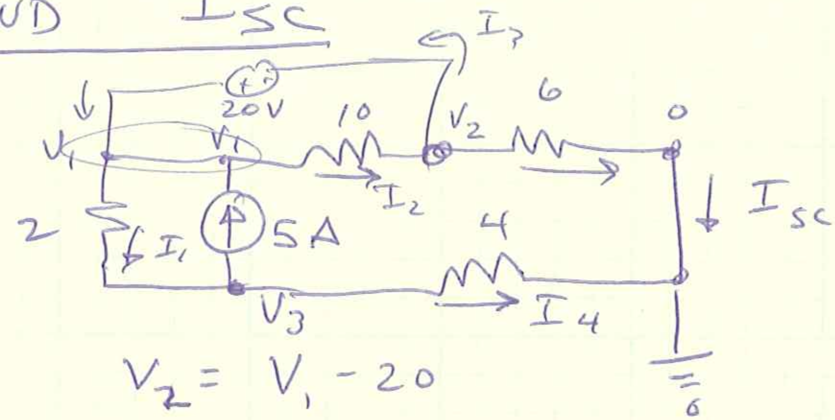
$$\frac{V_1 - (V_1 - 20)}{10} + \frac{V_1 - 0}{2} = 5 + I_2$$

$$\boxed{\frac{V_1}{2} = 5}$$

$V_1 = 10$ $\boxed{V_2 = -10}$

$\boxed{V_{oc} = -10}$

FIND I_{SC}



$V_2 = V_1 - 20$

$$I_{sc} = \frac{V_2}{6} = \frac{V_1 - 20}{6}$$

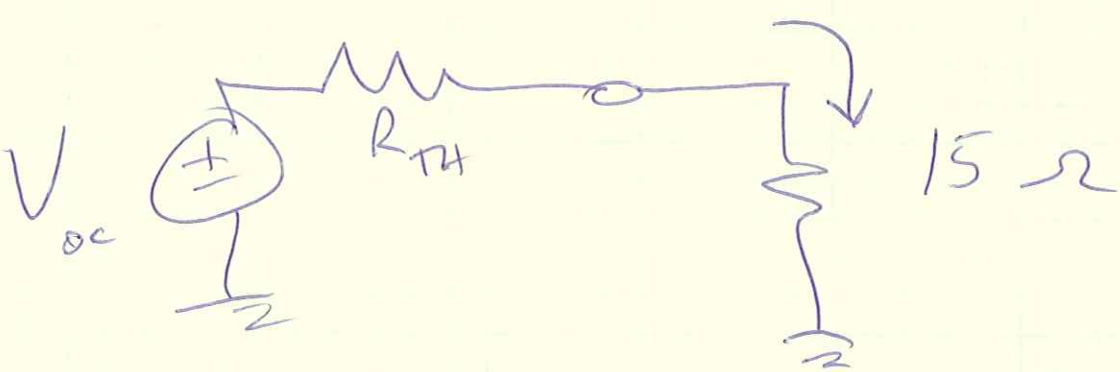
KCL $I_1 + I_2 = I_3 + 5A$

V_1 $\quad \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{10} =$

KCL @ V_2 $\quad I_2 = I_3 + I_{sc}$

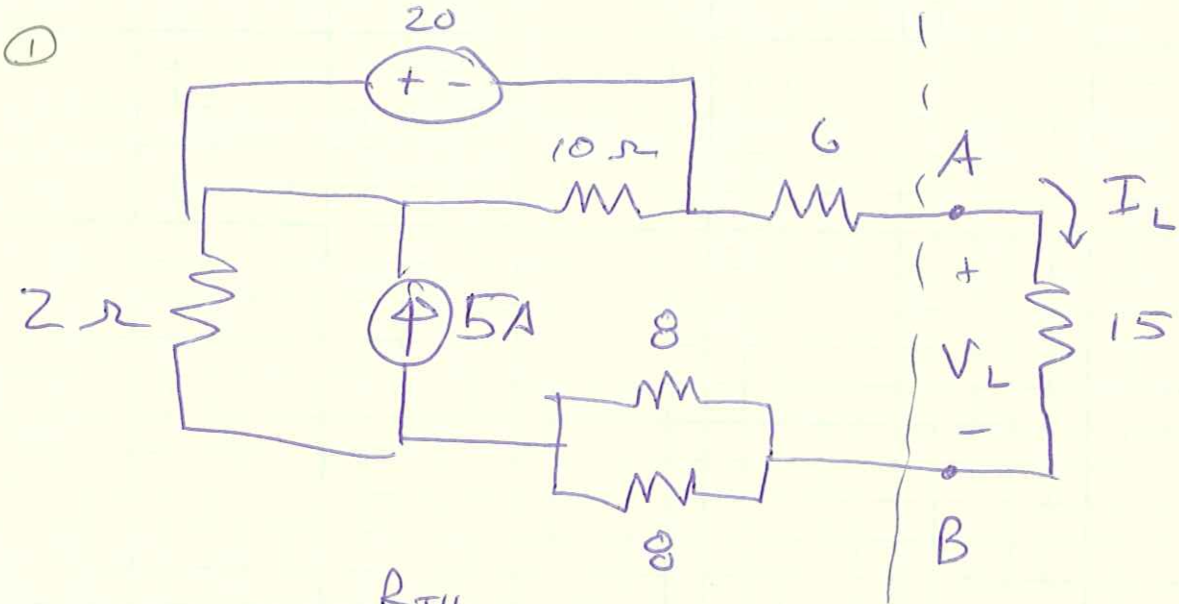
I_{sc}

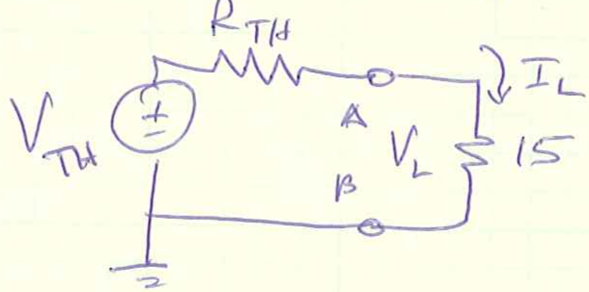
$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$



FINDING T.E.C

(1)

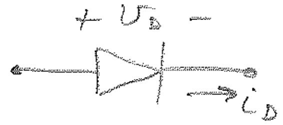




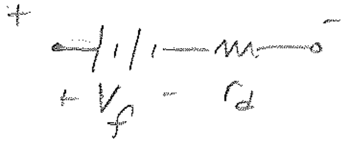
$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$V_L = 15\, I_L$$

DIODES



FORWARD



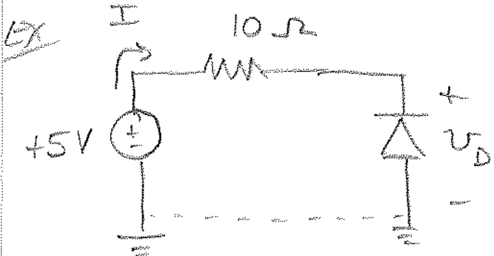
$v_D = V_f + i_D r_d$

$r_d = \frac{25 mV}{I_D'}$

REVERSE

I_D' = APPROXIMATE CURRENT SHORT CIRCUIT

EX

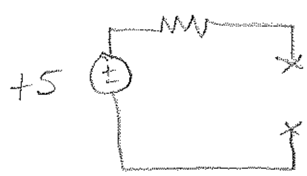


FIND : I, v_D, i_D

GIVEN : $V_f = 0.7 V$

WHAT DO WE KNOW? $i_D = I$

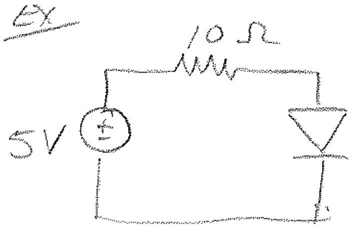
DIODE REVERSE



NO PATH $I = i_D = 0$

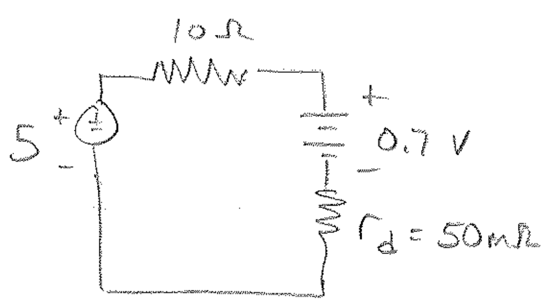
$V_D = 5 V$

EX

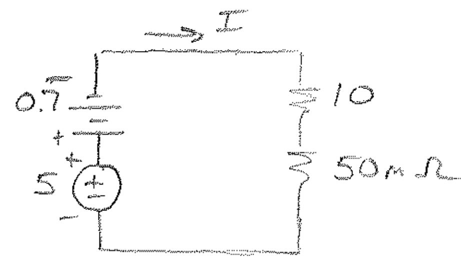


DIODE FWD. $I_D' = \frac{5 - 0}{10} = 0.5 A$

$r_d = \frac{25 mV}{0.5 A} = 50 m\Omega$



=



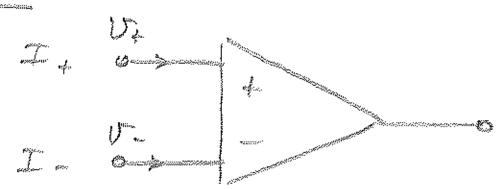
$4.3 = I(10.05)$

$\boxed{I = .428 A}$

$v_D = .7 + .428(.05)$

$\boxed{= .721 V}$

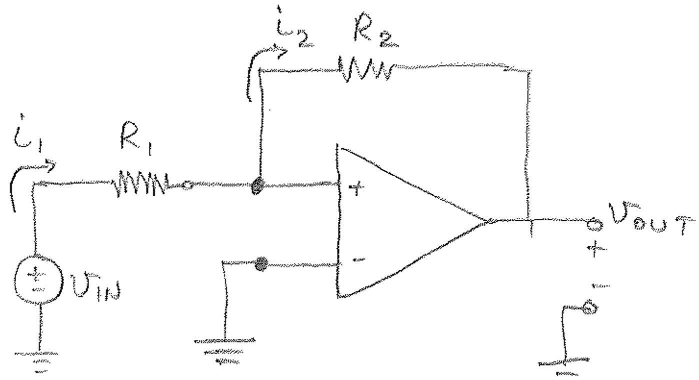
IDEAL OP-AMPS



$A_o = \infty$

$I_+ = I_- = 0$

$v_+ = v_-$



FIND: $\dfrac{v_{OUT}}{v_{IN}}$

i_1, i_2 IF $v_{IN} = 5V$

$R_1 = 1K\Omega$

$R_2 = 10K\Omega$

ASSUME IDEAL OP-AMP

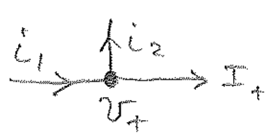
WHAT DO WE KNOW?

I) $v_- = v_+$ $\quad I_+ = I_- = 0$

II) $v_1 = 0$ (CONNECTED TO GROUND) $\quad v_+ = 0$

III) $i_1 = \dfrac{v_{IN} - v_+}{R_1}$ OHMS LAW $= \underline{5mA}$

IV)



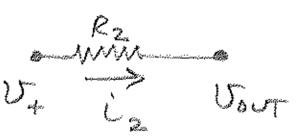
KCL @ NODE v_+

$\sum_i i_{IN} = \sum_i i_{OUT}$

$i_1 = i_2 + I_+$

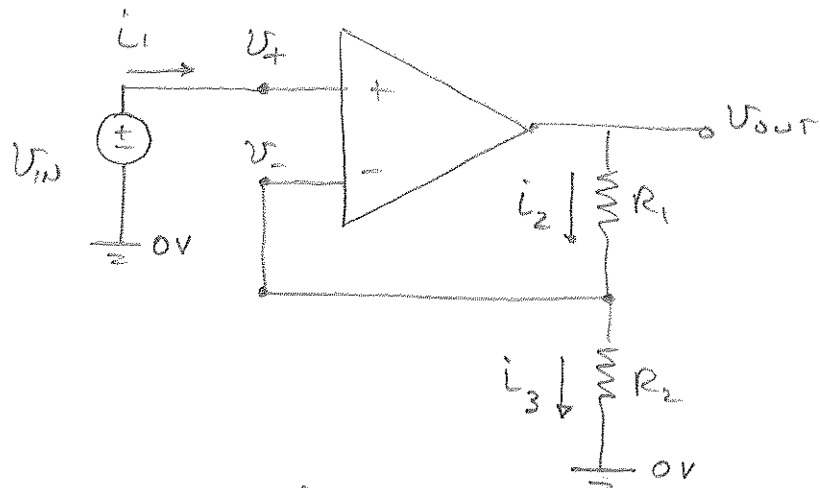
$i_1 = i_2 = 5mA$

V)



$i_2 = \dfrac{v_+ - v_{OUT}}{R_2} = \dfrac{-v_{OUT}}{R_2} = i_2$

$i_1 = i_2$ $\qquad \dfrac{-v_{OUT}}{R_2} = \dfrac{v_{IN}}{R_1}$ $\qquad \boxed{\dfrac{v_{OUT}}{v_{IN}} = \dfrac{-R_2}{R_1}}$



ASSUME IDEAL OP-AMP

FIND $\frac{v_{OUT}}{v_{IN}}$ i_1, i_2, i_3 IF $v_{IN} = 5\,V$ $R_1 = 5\,k\Omega$

$R_2 = 1\,k\Omega$

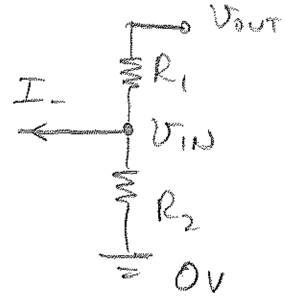
WHAT DO WE KNOW?

I) $v_+ = v_-$ $\quad I_+ = I_- = 0$

II) $v_+ = v_{IN}$ $\quad i_1 = I_+$ $\quad i_2 = i_3$ (KCL)

$v_- = v_+ = v_{IN}$

III)



R_1 AND R_2 SHARE CURRENT

$i_2 = i_3$

$$i_3 = \frac{v_{IN} - 0}{R_2}$$

$$i_2 = \frac{v_{OUT} - v_{IN}}{R_1}$$

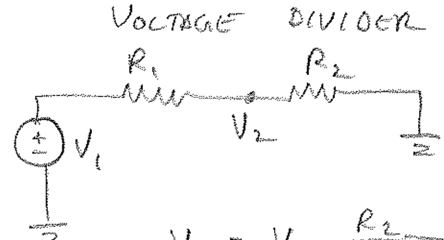
$$i_3 = i_2$$

$$\frac{v_{IN}}{R_2} = \frac{v_{OUT} - v_{IN}}{R_1}$$

$$\frac{R_1}{R_2} v_{IN} + v_{IN} = v_{OUT}$$

$$\frac{R_1}{R_2} + 1 = \boxed{\frac{v_{OUT}}{v_{IN}} = \frac{R_1 + R_2}{R_1}}$$

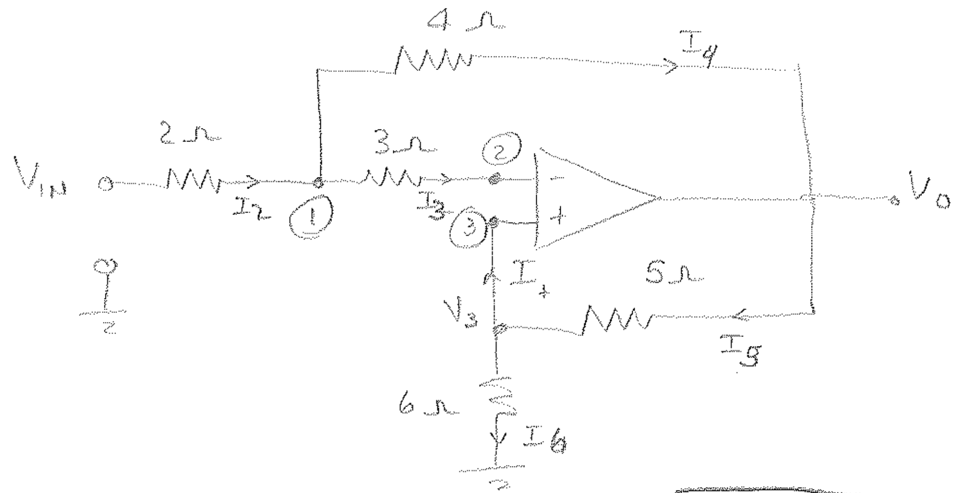
VOLTAGE DIVIDER



$$V_2 = V_1 \frac{R_2}{R_1 + R_2}$$

$V_2 = V_{IN}$

$V_1 = V_{OUT}$



FIND $\dfrac{V_o}{V_{IN}}$:

USING IDEAL PROPERTIES:

$I_3 = 0 \qquad I_2 = I_4$

$I_+ = 0 \qquad I_5 = I_6$

$V_3 = V_1$

NODE 3: $I_5 = I_6$

$$\frac{V_o - V_3}{5} = \frac{V_3 - 0}{6}$$

$$6V_o - 6V_3 = 5V_3$$

$$6V_o = 11V_3$$

$$\boxed{V_3 = \frac{6}{11} V_o}$$

ALSO BY VOLTAGE DIVISION

$$V_3 = V_o\left(\frac{6}{5+6}\right) = V_1$$

NODE 1

$$\frac{V_{IN} - V_1}{2} = \frac{V_1 - V_o}{4} \rightarrow \frac{V_{IN} - \frac{6}{11}V_o}{2} = \frac{\frac{6}{11}V_o - V_o}{4}$$

$$2V_{IN} - \frac{12}{11}V_o = \frac{-5}{11}V_o$$

$$2V_{IN} = \frac{7}{11}V_o$$

$$\boxed{\frac{V_o}{V_{IN}} = \frac{22}{7}}$$

TIME DOMAIN	PHASOR
$R \longrightarrow$	R
$v(t) = L\dfrac{di(t)}{dt}$ $\quad L \longrightarrow$	$j\omega L = Z_L$
$C \longrightarrow$	$Z_C = \dfrac{1}{j\omega C}$

$$\vec{V} = L\, j\omega\, \vec{I}$$

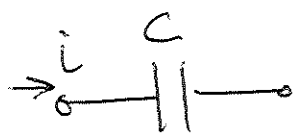
$$V \cancel{e^{j(\omega t+\theta)}} = L j\omega I_m \left(\cancel{e^{j(\omega t+\theta)}}\right)$$

$$V = j\omega I_m L$$

$$V = \underbrace{(j\omega L)}\, I_m$$

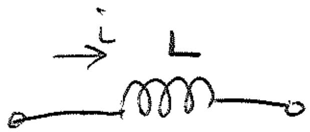
AC CIRCUITS

CAPACITORS

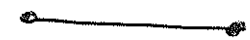


$$i(t) = C \frac{dv(t)}{dt}$$

INDUCTORS



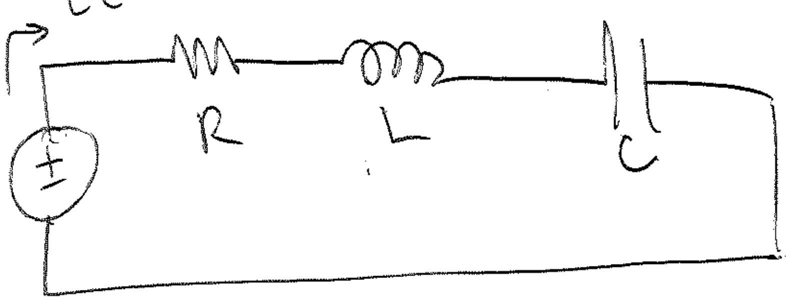
$$v(t) = L \frac{d\,i(t)}{dt}$$



RESISTOR:

$$V = IR$$

$$v(t) = i(t) R$$



$$v(t) = 10 \cos(\omega t + \theta)$$

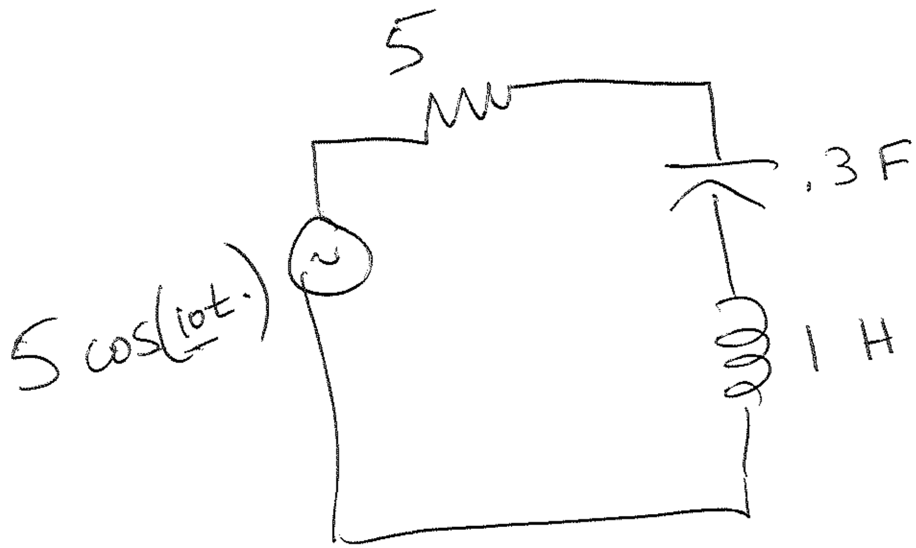
$$\vec{V} = 10 e^{j(\omega t + \theta)}$$ PHASOR

$$v(t) = Re[\vec{V}]$$

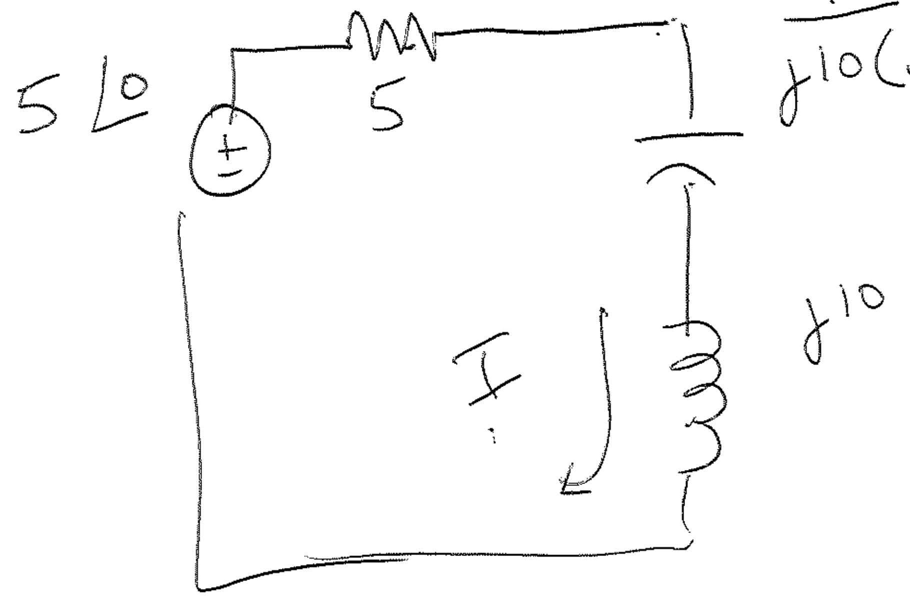
$$\vec{I} = I_M e^{j(\omega t + \theta)}$$

$$j\omega I_M e^{j(\omega t + \theta)}$$

FIND $i(t)$



$V = I Z_T$



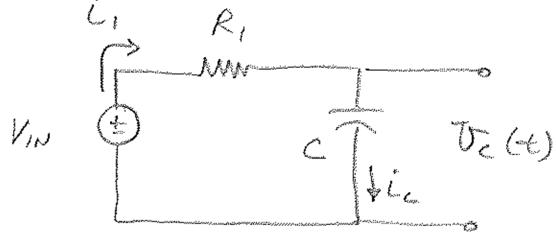
$$\frac{1}{j10(.3)} = \frac{-j}{3}$$

$$I = \frac{5\angle 0^\circ}{5 + \frac{-j}{3} + j10} = \frac{5\angle 0^\circ}{5 + j(10 - \frac{1}{3})} = \frac{5\angle 0^\circ}{5 + j9.67} = \frac{5\angle 0^\circ}{10.89\angle 62.6^\circ}$$

$$I = .459\angle -62.6^\circ$$

$$\boxed{i(t) = .459\cos(10t - 62.6^\circ)\ \text{A}}$$

TRANSIENT CIRCUITS



FIND $v_c(t)$

$i_1(t)$

WHAT DO WE KNOW?

$$i_c = C \frac{d\,v_c(t)}{dt}$$ v-i LAW OF CAPACITORS

$$i_c = i_1$$ KCL

$$i_1 = \frac{V_{IN} - v_c(t)}{R_1}$$ OHMS LAW

I) $$\frac{V_{IN} - v_c(t)}{R_1} = C \frac{d\,v_c(t)}{dt}$$

$$\frac{d\,v_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} V_{IN}(t)$$ FIRST ORDER CONSTANT COEFF. DIFFEQ

IF $v_{IN}(t) = A e^{3t}$ $\qquad v_c(t) = K e^{st}$

HOMOGENEOUS SOLN

$$\frac{d v_c(t)}{dt} + \frac{1}{RC} v_c(t) = 0$$

$$K s e^{st} + \frac{K}{RC} e^{st} = 0$$

$$K e^{st}\left(s + \frac{1}{RC}\right) = 0 \qquad \boxed{s = \frac{-1}{RC}} \qquad v_c(t) = K e^{-t/RC}$$

PARTICULAR SOLUTION

$$v_c(t) = V_c e^{3t}$$

$$3 V_c e^{3t} + \frac{V_c}{RC} e^{3t} = A e^{3t}$$

$$3 V_c + \frac{V_c}{RC} = A$$

$$V_c = \frac{A}{\left(3 + \frac{1}{RC}\right)} \qquad v_c(t) = \frac{A}{\left(3 + \frac{1}{RC}\right)} e^{3t}$$

TOTAL

$$v_c(t) = K e^{-t/RC} + \frac{A}{\left(3 + \frac{1}{RC}\right)} e^{3t}, \qquad t > 0$$

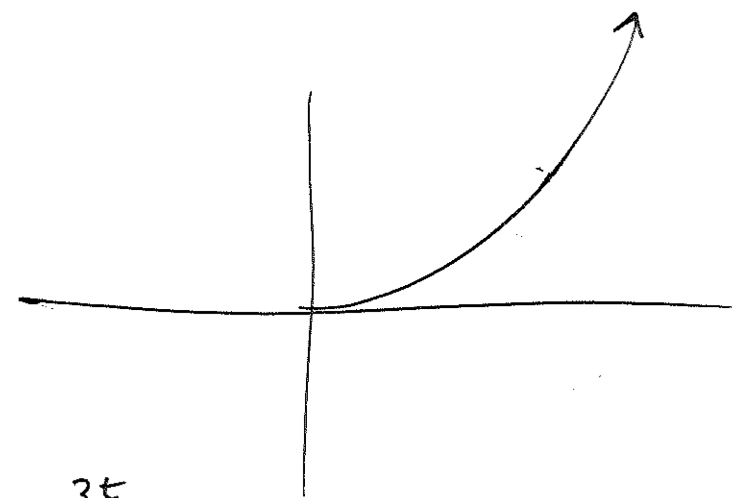
ASSUME $v_c(t) = 0$ FOR $t < 0$ (INITIAL CONDITIONS)

$$0 = K + \frac{A}{\left(3 + \frac{1}{RC}\right)} \qquad K = \frac{-A}{\left(3 + \frac{1}{RC}\right)}$$

$$v_c(t) = \frac{A}{\left(3 + \frac{1}{RC}\right)}\left[e^{3t} - e^{-t/RC}\right], \qquad t > 0$$

AS $t \to \infty$ STEADY STATE $\quad v_c(t) \to \frac{A}{\left(3 + \frac{1}{RC}\right)} e^{3t}$

$$v_c(t) = \frac{A}{3+\frac{1}{RC}}\left[e^{3t} - e^{-t/RC}\right], \quad t > 0$$



Ex

$R = 1\,k\Omega$

$C = 1\,\mu F$

$v_{IN}(t) = 5e^{3t}$

Steady State Expression

as $t \to \infty$ $\quad v_c(t) \to \left[\frac{A}{3+\frac{1}{RC}}\right]e^{3t}$

$$\frac{d\,x(t)}{dt} + a\,x(t) = f(t)$$

IF $x_p(t)$ IS a soln, AND $X_c(t)$ IS A SOLN

TO THE HOMOGENEOUS EQN:

$$\frac{d\,x(t)}{dt} + a\,x(t) = 0$$

THEN

$$x(t) = x_p(t) + x_c(t)$$

LET $f(t) = A$ constant

$$\frac{d\,x_p(t)}{dt} + a\,x_p(t) = A \qquad x_p(t) = K_1$$

$$\frac{d\,x_c(t)}{dt} + a\,x_c(t) = 0$$

$$x_p(t) = K_1 = \frac{A}{a} \qquad x_c(t) = K_2 e^{-at}$$

$$\boxed{x(t) = K_1 + K_2 e^{-at}}$$

USING LAPLACE TRANSFORMS

$$v_c(t) + CR\frac{dv_c}{dt} = 0 \qquad v(0^-) = V_s$$

$$V(s) + CR\left(sV(s) - v(0^-)\right) = 0$$

$$V(s)(1 + CRs) - CRv(0^-) = 0$$

$$V(s) = \frac{CR\,V_s}{1 + CRs} = \frac{V_s}{\frac{1}{RC} + s}$$

$$v(t) = V_s e^{-\frac{1}{RC}t}$$

$$\frac{1}{s+a} \Leftrightarrow e^{-at}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{V_s}{RC}$$

$$sV(s) + \frac{V(s)}{RC} = \frac{V_s}{sRC}$$

$$V(s)\left(s + \frac{1}{RC}\right) = \frac{\frac{V_s}{RC}}{s(s+\frac{1}{RC})} = \frac{k_1}{s} + \frac{k_2}{s+\frac{1}{RC}}$$

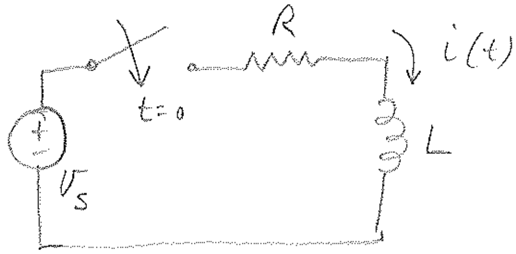
$$\frac{\frac{V_s}{RC}}{\frac{1}{RC}} = \boxed{k_1 = V_s}$$

$$\left.\frac{\frac{V_s}{RC}}{s}\right|_{s=-\frac{1}{RC}} = \boxed{k_2 = -V_s}$$

$$v(t) = V_s - V_s e^{-t/RC}$$

AC SS CKT ANALYSIS INIT. COND $= 0$

SINUSOIDAL RESPONSE



$$V_s(t) = V_m \cos(\omega t + \phi_v)$$

$$L\frac{di(t)}{dt} + Ri(t) = V_m \cos(\omega t + \phi_v)$$

$$i(t) = \frac{-V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\theta_v - \theta)e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t + \phi_v - \theta)$$

Homogeneous (transient)

as $t \to \infty$
term $\to 0$

Particular (Sinusoidal)

Steady state

$$i_{ss}(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t + \phi_v - \theta)$$

$$I_{max} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \qquad \text{Let } \phi_v - \theta = \theta_i$$

$$i_{ss}(t) = I_M \cos(\omega t + \theta_i)$$

AC CIRCUIT ANALYSIS - STEADY STATE ANALYSIS

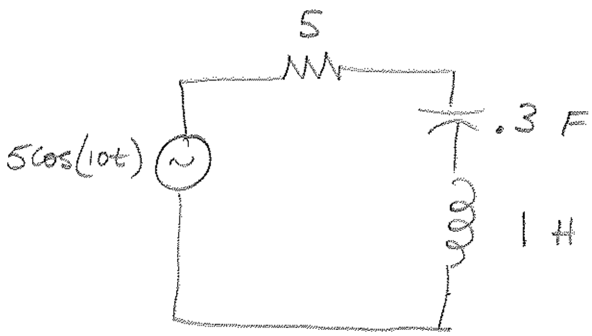
CONVERT TO PHASOR DOMAIN

$$v(t) = A\cos(\omega t + \theta) = Re\left[A e^{j(\omega t + \theta)}\right]$$

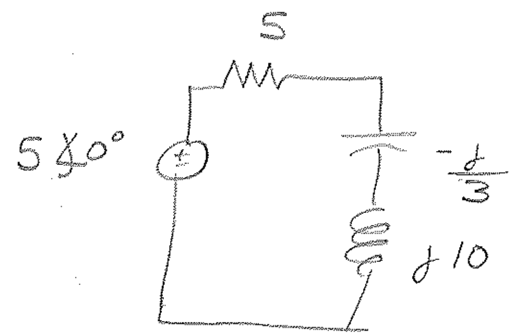
ALSO WRITTEN AS A PHASOR $\mathbf{V} = A \angle \theta$

CONVERT COMPONENTS

TIME DOMAIN		PHASOR
R	$\longrightarrow$	R
L	$\longrightarrow$	$j\omega L$
C	$\longrightarrow$	$\frac{1}{j\omega C}$



$\longrightarrow$



$$I = \frac{5\angle 0°}{5 - \frac{j}{3} + j10}$$

$$I = \frac{5\angle 0°}{5 + j\left(10 - \frac{1}{3}\right)}$$

PHASOR NOTATION - A COMPLEX NUMBER THAT CARRIES AMPLITUDE AND PHASE ANGLE OF A SINUSOIDAL FUNCTION

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta \qquad Re\{e^{j\phi}\} = \cos\theta$$

$$Im\{e^{j\phi}\} = \sin\theta$$

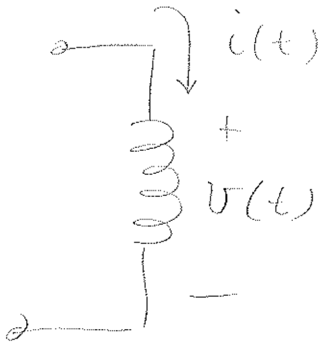
$$v(t) = V_m \cos(\omega t + \phi)$$

$$= V_m Re\{e^{j(\omega t + \phi)}\} = V_m Re\{e^{j\omega t} e^{j\phi}\}$$

$$= Re\{\underbrace{V_m e^{j\phi}}_{\text{PHASOR OF } v(t) \ \vec{V}} e^{j\omega t}\}$$

$$\vec{V} = V_m e^{j\phi} = V_m \cos\phi + j V_m \sin\phi = V_m \angle\phi$$

EX $\vec{V} = 100 \angle -26^\circ \rightarrow v(t) = 100\cos(\omega t - 26^\circ)$



$$v(t) = L\frac{d\,i(t)}{dt}$$

$$\boxed{\begin{aligned} i(t) &= I_M\cos(\omega t+\theta_i) \\ v(t) &= V_M\cos(\omega t+\theta_v) \end{aligned}}$$

$$i(t) = \mathrm{Re}\{\vec{I}\}$$

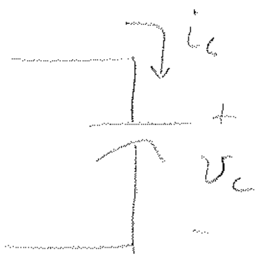
$$V = V_M e^{j\theta_v} e^{j\omega t} \qquad I = \underbrace{I_M e^{j\theta_i}}_{\vec{I}} e^{j\omega t}$$

$$V_M e^{j\theta_v} e^{j\omega t} = L\, I_M e^{j\theta_i} j\omega e^{j\omega t}$$

$$V_M e^{j\theta_v} = I_M e^{j\theta_i} j\omega L$$

$$\vec{V} = \vec{I}\,\underbrace{j\omega L}_{Z_L}$$

INDUCTOR IMPEDANCE IS NOW COMPLEX AND FREQUENCY DEPENDENT.



$$i_c = C\frac{dv_c(t)}{dt}$$

$$I_M e^{j\theta_i} e^{j\omega t} = C\, V_M e^{j\theta_v} j\omega e^{j\omega t}$$

$$I_M e^{j\theta_i} = j\omega C\, V_M e^{j\theta_v}$$

$$V_M e^{j\theta_v} = \frac{I_M e^{j\theta_i}}{j\omega C}$$

$$\vec{V} = \vec{I} Z_c \qquad \boxed{Z_c = \frac{1}{j\omega C}}$$

$$I = \frac{5\angle 0^\circ}{5 + j9.67} = \frac{5\angle 0^\circ}{10.89\angle 62.6^\circ} = 0.459\angle -62.6^\circ$$

$$\boxed{i(t) = 0.459\cos(10t - 62.6^\circ)}$$

SUMMARY OF COMPLEX NUMBERS

$$x + jy = re^{j\theta}$$

$$Ae^{jx} = A(\cos x + j\sin x)$$

$$r = \sqrt{x^2 + y^2}$$

$$\mathrm{Re}\{e^{jx}\} = \cos x$$

$$\theta = \tan^{-1}\frac{y}{x}$$

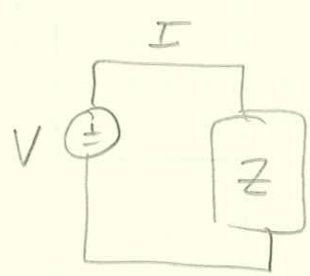
$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\frac{1}{e^{j\theta}} = e^{-j\theta} \quad \text{or} \quad \frac{1}{5\angle 30} = 0.2\angle -30^\circ$$

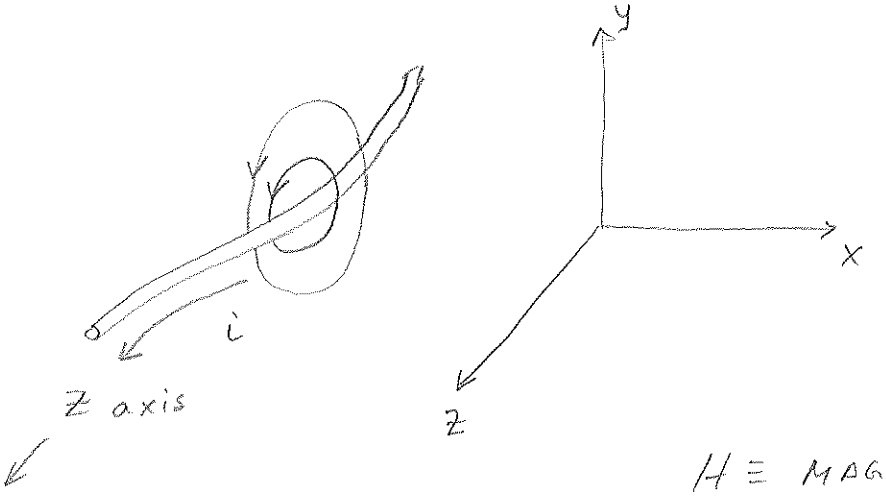
COMPLEX IMPEDANCE

$$Z = \underset{\substack{\uparrow \\ \text{Real}}}{R} + j\underset{\substack{\uparrow \\ \text{reactive}}}{X}$$



$$V = IZ$$

MAGNETIC FIELDS



ϕ pos

$$H = \frac{B}{\mu} = \frac{I \, \vec{a}_\phi}{2\pi r}$$

H = MAG FIELD STRENGTH (A/m)

B = MAG FLUX DENSITY (Tesla)

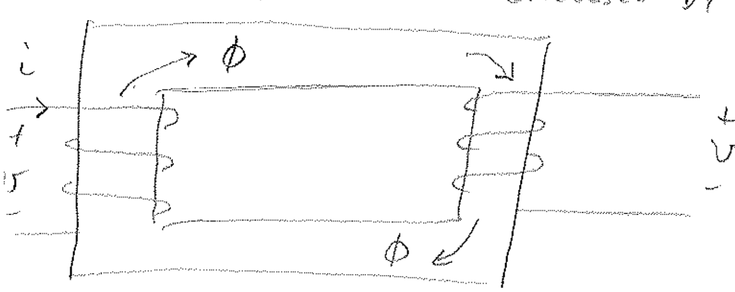
$\vec{a}_\phi$ = unit vector in pos ϕ direction in cylindrical coordinates

I = current

μ = permeability of medium (air, iron core)

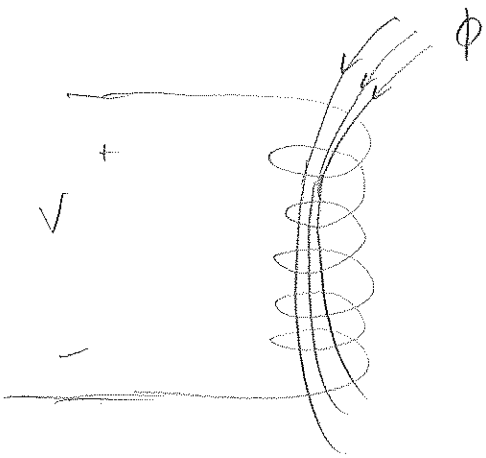
air $4\pi \times 10^{-7}$ H/m

MAGNETIC ϕ FLUX ENCLOSED BY N TURNS OF WIRE



$$v = -N \frac{d\phi}{dt}$$ INDUCED VOLTAGE

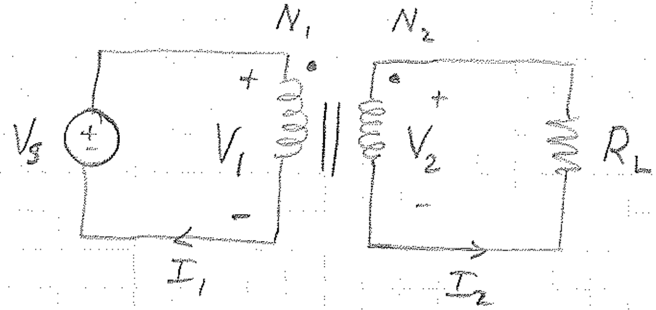
$$\phi = \oint_S B \cdot dS$$ = TOTAL FLUX IN A VOLUME



TOTAL VOLUME OF MATERIAL DENSITY IS WEIGHT

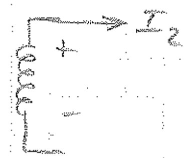
TOTAL VOLUME OF B IS ϕ

THE IDEAL TRANSFORMER

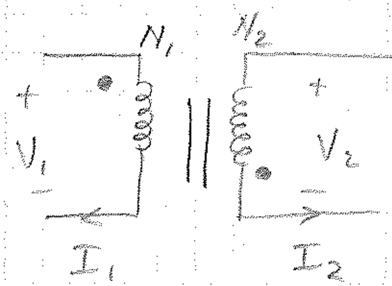


coil 1 $P_1 = V_1 I_1$ ABSORBED

coil 2 $P_2 = V_2 I_2$ supplied

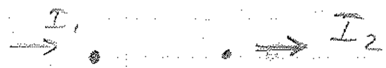


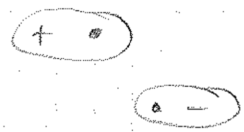
$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \qquad N_1 I_1 = -N_2 I_2$$

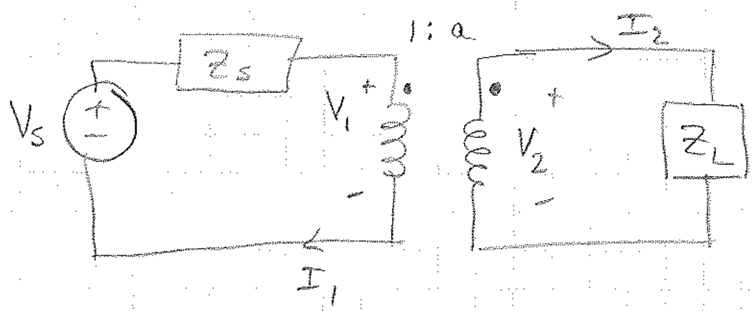


$$a = \frac{N_2}{N_1}$$

$$\frac{V_1}{N_1} = -\frac{V_2}{N_2} \qquad N_1 I_1 = N_2 I_2$$





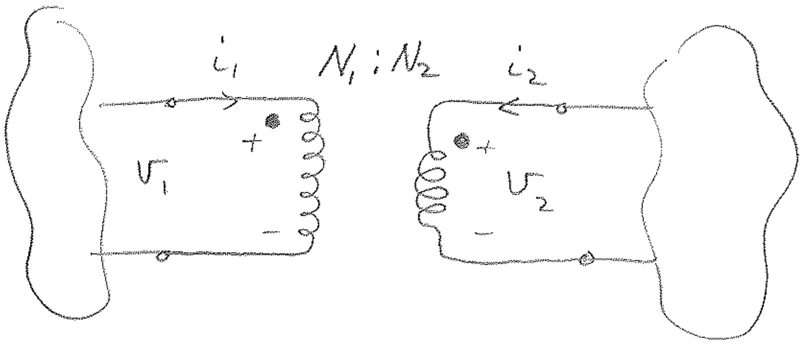


$$V_2 = a V_1$$

$$I_2 = \frac{I_1}{a}$$

$$Z_{IN} = \frac{Z_L}{a^2}$$

IDEAL TRANSFORMERS



$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \qquad \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

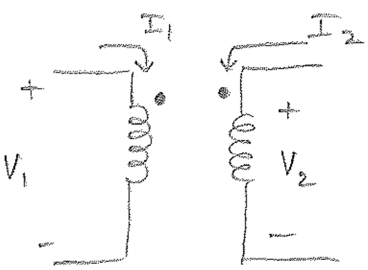
$$v_1 i_1 + v_2 i_2 = 0$$

NO POWER LOST IN AN IDEAL XFORMER

$$n = \frac{N_2}{N_1} := \text{TURNS RATIO}$$

$$V_1 = \frac{V_2}{n} \qquad I_1 = n I_2 \qquad Z_1 = \frac{Z_L}{n^2}$$

IDEAL TRANSFORMERS

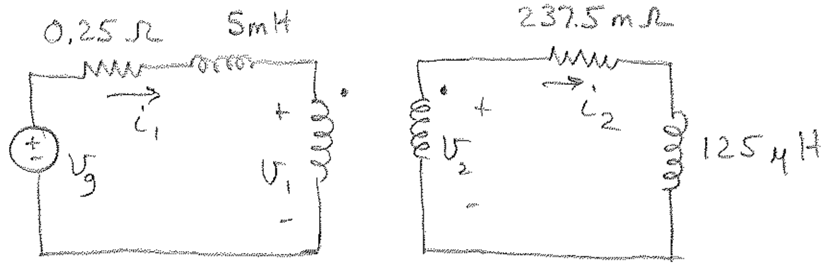


$N_1 : N_2$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{-N_2}{N_1}$$

EX



$v_g = 2500 \cos 400t$

10 : 1

FIND STEADY STATE i_1, v_1, i_2, v_2

$v_g \Rightarrow \vec{V}_G = 2500 \angle 0°$

$L_1 = 5mH \quad Z_{L1} = j(5mH)(400) = j2$

$L_2 = 125\mu H \quad Z_{L2} = j(125\mu H)(400) = j0.05$

$$\boxed{\frac{V_G - V_1}{.25 + j2} = I_1}$$

$$\frac{I_1}{-I_2} = \frac{-N_2}{N_1}$$

$$\frac{I_1}{I_2} = \frac{1}{10}$$

$$10 I_1 = I_2$$

$V_2 = I_2(.2375 + j.05)$

$V_2 = \frac{V_1}{10}$

$V_1 = 10 I_2 (.2375 + j0.05)$

$V_1 = (10)(10) I_1 (.2375 + j.05)$

$V_1 = I_1 (23.75 + j5)$

*

STEADY STATE $\Rightarrow$ PHASORS

DROP FREQUENCY

$v(t) = V \cos(\omega t + \theta)$

$\vec{V} = V \angle \theta$

$Z_L = j\omega L \qquad Z_R = R$

$Z_C = \frac{1}{j\omega C}$

$$V_G = I_1(.25 + j2) + V_1$$

$$= I_1(.25 + j2) + I_1(23.75 + j5)$$

$$V_G = I_1(24 + j7)$$

$$\frac{2500 \angle 0^\circ}{25 \angle 16.26^\circ} = I_1$$

$$\boxed{I_1 = 100 \angle -16.26^\circ} \text{ A}$$

$$\boxed{i_1(t) = 100 \cos(400t - 16.26^\circ) \text{ A}}$$

$$I_2 = 10 I_1 = \boxed{1000 \cos(400t - 16.26^\circ) \text{ A} = i_2(t)}$$

$$V_1 = I_1(23.75 + j5) = (100 \angle -16.26)(24.27 \angle 11.89)$$

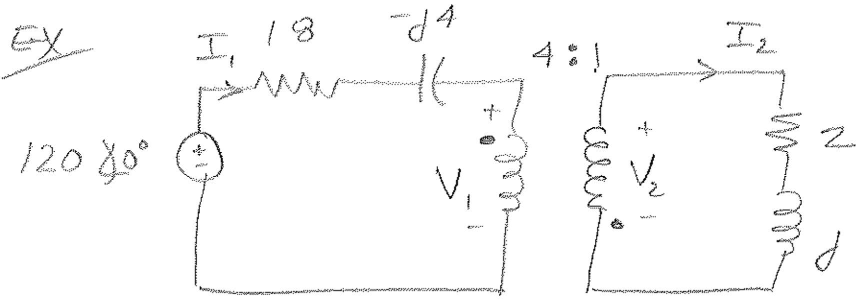
$$= 2427 \angle -4.37^\circ$$

$$\boxed{v_1(t) = 2427 \cos(400t - 4.37^\circ) \text{ V}}$$

$$V_2 = \frac{V_1}{10} = 242.7 \angle -4.37^\circ$$

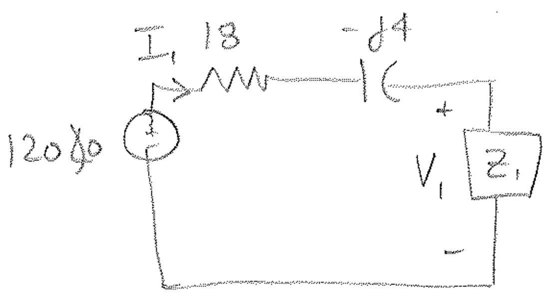
$$\boxed{v_2(t) = 242.7 \cos(400t - 4.37^\circ) \text{ V}}$$

Ex



$$V_1 = \frac{-V_2}{n} \qquad I_1 = -n I_2 \qquad n = \frac{1}{4}$$

$$Z_1 = \frac{Z_L}{n^2} = 4^2 Z_L$$



$$4^2(2+j1) = 32 + j16$$

$$I_1 = \frac{120\angle 0}{18 - j4 + 32 + 16j} = 2.33\angle -13.5°$$

$$V_1 = I_1 Z_1 = (2.33\angle -13.5°)(32 + j16)$$

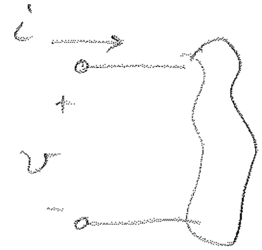
$$= 83.49\angle 13.07°$$

$$V_2 = -V_1 n = \frac{1}{4}\, 83.49\angle 13.07° = 20.87\angle 193.07°$$

$$I_2 = -\frac{I_1}{n} = 4\,(2.33\angle 13.5°) = 9.33\angle 166.50°$$

POWER

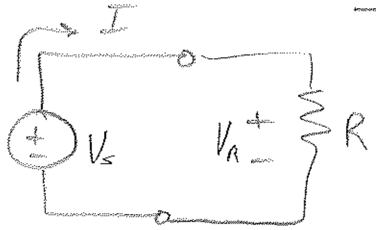
$p = v\,i$



PASSIVE SIGN CONVENTION

POSITIVE POWER (absorbed)

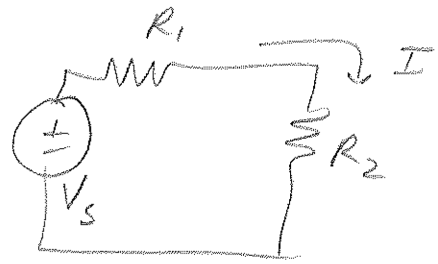
DC



$$P_S = -VI$$

$$P_R = V_R I = I^2 R = \frac{V_R^2}{R}$$

$$V_R = IR$$



$$P_{R2} = V_{R2}\,I = \frac{V_{R2}^2}{R_2}$$

AC

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

USING AN ARBITRARY REFERENCE TIME WE CAN WRITE

$$i(t) = I_m \cos \omega t$$

$$v(t) = V_m \cos(\omega t + \theta_v - \theta_i)$$

$$p = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t$$

USING: $\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2wt + \theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2wt$$

$$- \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2wt$$

AVERAGE (REAL) POWER $\quad P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$

REACTIVE POWER $\quad Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$

$$p(t) = P + P\cos 2wt - Q\sin 2wt$$

P IS NOT TIME DEPENDENT. (CONSTANT, LIKE DC)

$$P = (V_{rms})(I_{rms})(pf)$$

$\left.\begin{matrix} V_{rms} \\ I_{rms} \end{matrix}\right\}$ DC EQUIVALENT root MEAN SQUARED

FOR SINUSOIDAL $v(t)$, $i(t)$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

pf: power factor $\cos(\theta_v - \theta_i)$

$-1 < pf < 1$

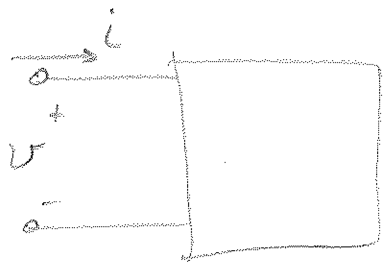
$pf > 0$ lagging pf

CURRENT LAGS VOLTAGE

$pf < 0$ leading pf

CURRENT LEADS VOLTAGE

EX



$v = 100\cos(\omega t + 15°)$ V

$i = 4\sin(\omega t - 15°)$ A

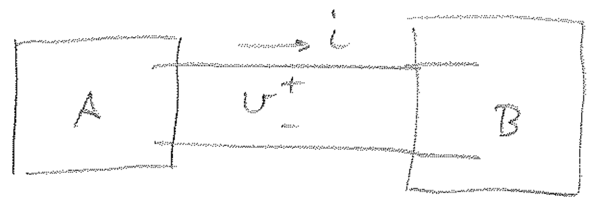
FIND AVG POWER AND REACTIVE POWER AT TERMINALS

$V = 100\angle 15 \qquad I = 4\angle -15-90 = 4\angle -105$

$P = \frac{100(4)}{2}\cos(15+105) = 200(-.5) = -100$ W

SUPPLIED !

$Q = \frac{100(4)}{2}\sin(120) = +173.2$ VAR



a)

$v = 100 \cos(\omega t - 45^\circ)$

$i = 20 \cos(\omega t + 15^\circ)$

$P = \frac{100(20)}{2} \cos(-45 - 15)$

$= 100(.5) = +50\ W$

POWER FROM A to B

$Q = \frac{100(20)}{2} \sin(-60)$

$= -866.03\ VAR$

DELIVERING (MAGNETIZING VARS) FROM B to A

$p(t) = 50 + 50 \cos 2\omega t + 866.03 \sin 2\omega t$

ENERGY BEING STORED IN MAGNETIC FIELDS ASSOCIATED WITH INDUCTIVE ELEMENTS.

COMPLEX POWER

AC

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

INSTANTANEOUS POWER $\quad p(t) = v(t)\, i(t) = I_m V_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$

$$p(t) = \frac{V_m I_m}{2}\left[\underbrace{\cos(\theta_v - \theta_i)}_{\text{TIME IND.}} + \underbrace{\cos(2\omega t + \theta_v + \theta_i)}_{\text{TIME DEP.}}\right]$$

AVERAGE POWER

$$P_{AV} = \frac{V_M I_M}{2} \cos(\theta_v - \theta_i) = \frac{V_M}{\sqrt{2}}\angle\theta_v \, \frac{I_M}{\sqrt{2}}\angle{-\theta_i}$$

$pf \equiv$ power factor $= \cos(\theta_v - \theta_i)$ $\quad = V_{rms} I_{rms}(pf)$

DIFFERENCE IN PHASE ANGLE OF V and I

SINUSOIDAL R.M.S.

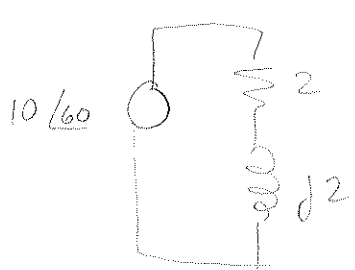
$V_{rms} I_{rms} \equiv$ APPARENT POWER

$1 \le \cos(\theta_v - \theta_i) \le 1$

$0 < pf < 1$ lagging pf

$-1 < pf < 0$ leading pf

EX



FIND P_{ave}, pf

$V = 10\angle 60$

$$I = \frac{10\angle 60}{2 + j2} = 3.53\angle 15°$$

$$P_{AVE} = \frac{10(3.53)}{2}\cos(60 - 15) = 12.48 \text{ Watts}$$

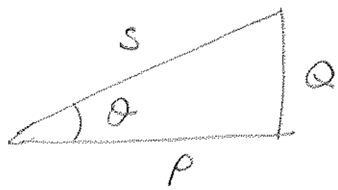
$pf = \cos(45°) = .707$ lagging (INDUCTIVE LOAD)

COMPLEX POWER

$S = P + jQ$ $\qquad P = Re\{S\}$ WATTS

$Q = Im\{S\}$ VARS

S VOLT-AMPS



$\theta = \theta_v - \theta_i$

$|S| = \sqrt{P^2 + Q^2}$ $\qquad \tan\theta = \frac{Q}{P}$

$|S| \equiv$ APPARENT POWER VOLT-AMPS

EX LOAD @ 240 V_{rms} ABSORBS 8 KW w/ pf .8 lag

a) FIND S

b) FIND IMPEDANCE OF THE LOAD

SOLN

(a) $P = |S| \cos\theta$ $\qquad \cos\theta = 0.8$ $\qquad \theta = 36.87$

$\sin\theta = 0.6$

$8\ KW = |S|(0.8)$

$|S| = 10\ KVA$

$Q = |S| \sin\theta = 10\ KVA(.6) = 6\ KVAR$

$S = (8 + j6)\ KVA$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$8000 = 240V \; I_{rms} \; (0.8)$$

$$I_{rms} = \frac{8000}{240(.8)} = 41.67 \text{ Amps}$$

$$|Z| = \left|\frac{V_{rms}}{I_{rms}}\right| = \frac{240}{41.67} = 5.76$$

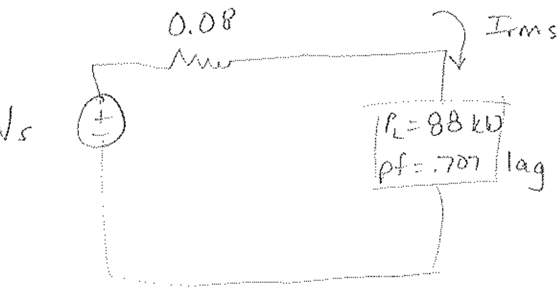
$$\measuredangle\theta_z = \measuredangle\theta_v - \theta_i = 36.87$$

$$Z = 5.76 \angle 36.87 = \boxed{4.608 + j3.456 \,\Omega = Z}$$

EX. INDUSTRIAL LOAD USES 88 kW @ pf = .707 lagg.
FROM 480 Vrms LINE

a) FIND POWER NEEDED FROM Power Co.

b) if pf = 0.90



$P_L = 88\ kW = I_{rms} V_{rms} (pf)$

$88\ kW = I_{rms} (480)(.707)$

$\boxed{I_{rms} = 259.3\ A}$

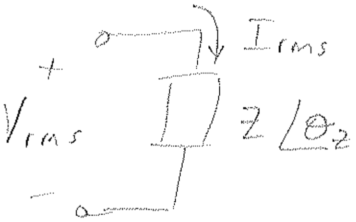
$P_s = I_{rms}^2 (R) + 88\ kW$

$\boxed{P_s = 93.38\ kW}$

b) $pf = 0.90$ $\quad I_{rms} = 203.7\ A$

$\boxed{P_s = 91.32\ kW}$

COMPLEX POWER



$S = V_{rms} I_{rms}^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$

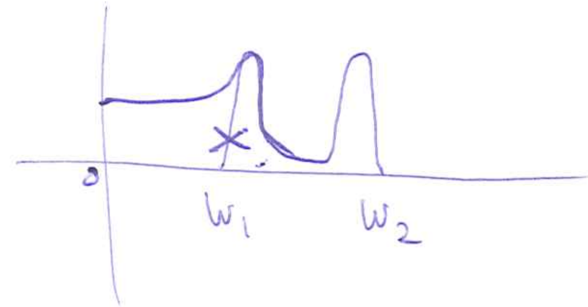
$= \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{\text{real power}} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{\text{reactive quadrature.}}$

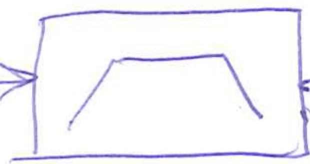
$S = P + jQ$

$P = Re(S) = V_{rms} I_{rms}$

FREQUENCY CONTENT OF A SIGNAL

- HOW FAST SAMPLE?
- BANDWIDTH

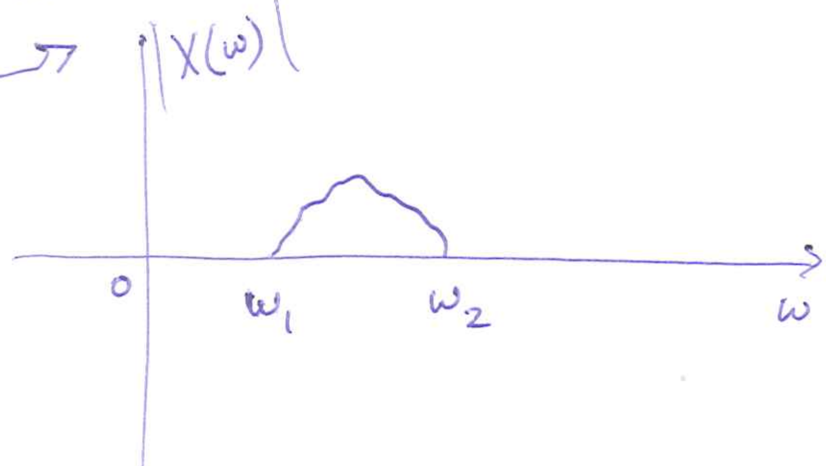


$x(t)$ →  → $y(t)$

↑ SIGNAL

BANDWIDTH
RANGE OF
FREQUENCIES
VOICE → 60 → 12KHz

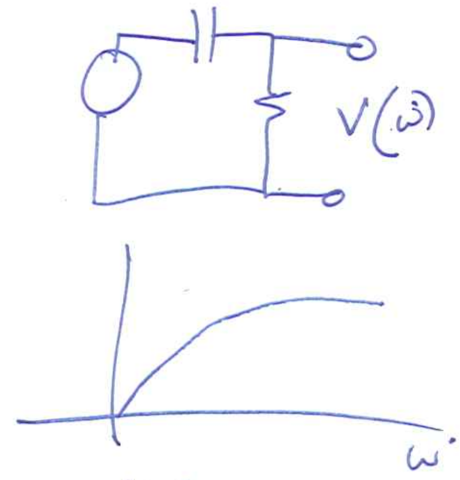
↑ SYSTEM
FILTER
BANDWIDTH
$f_{LO} \rightarrow f_{HI}$
$BW = f_{HI} - f_{LO}$



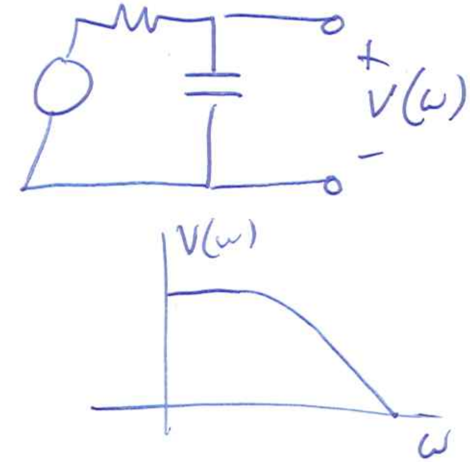
$$Z_L = j\omega L = sL$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{sC}$$

RC

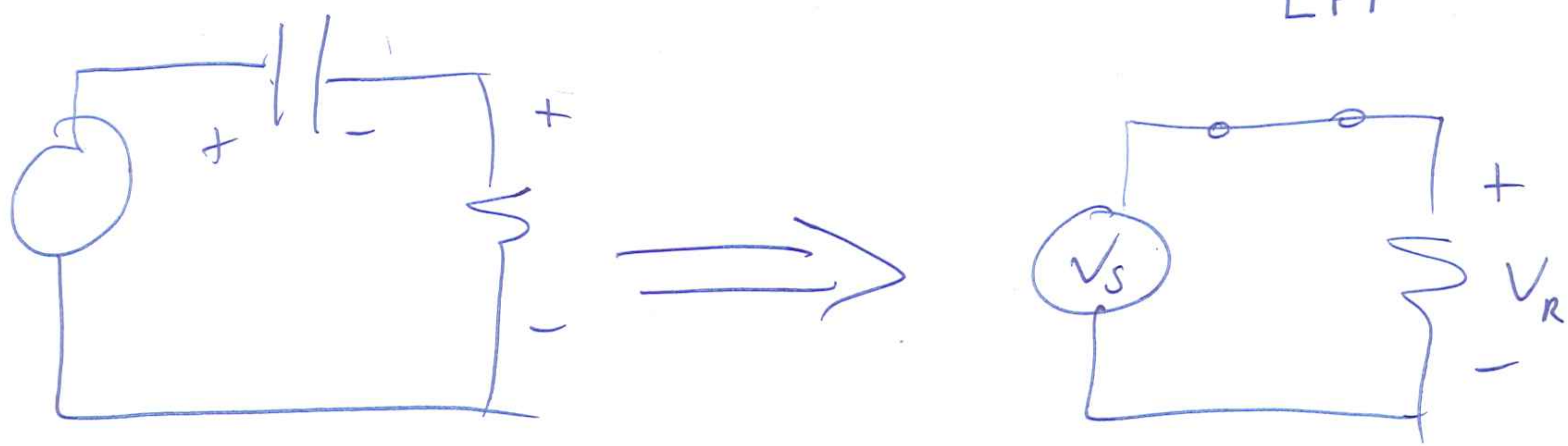


HPF

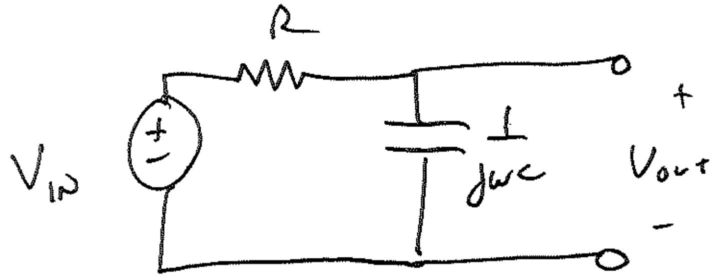


LPF

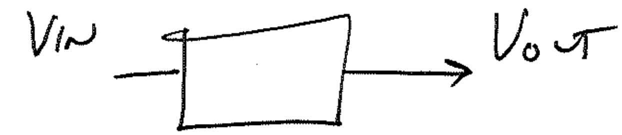
RESONANT CIRCUITS



$\omega \uparrow \quad Z_C \to 0 \qquad V_R \to V_S$



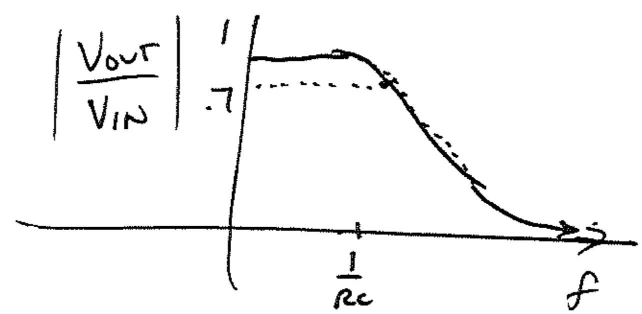
$$\frac{V_{OUT}}{V_{IN}}(\omega) =$$



VOLTAGE DIVIDER

$$V_{OUT} = V_{IN}\left(\frac{Z_C}{Z_C + R}\right)$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot \frac{j\omega C}{j\omega C} = \frac{1}{1 + j\omega RC}$$



$\omega \to 0$ $\quad \frac{V_{OUT}}{V_{IN}} = 1$

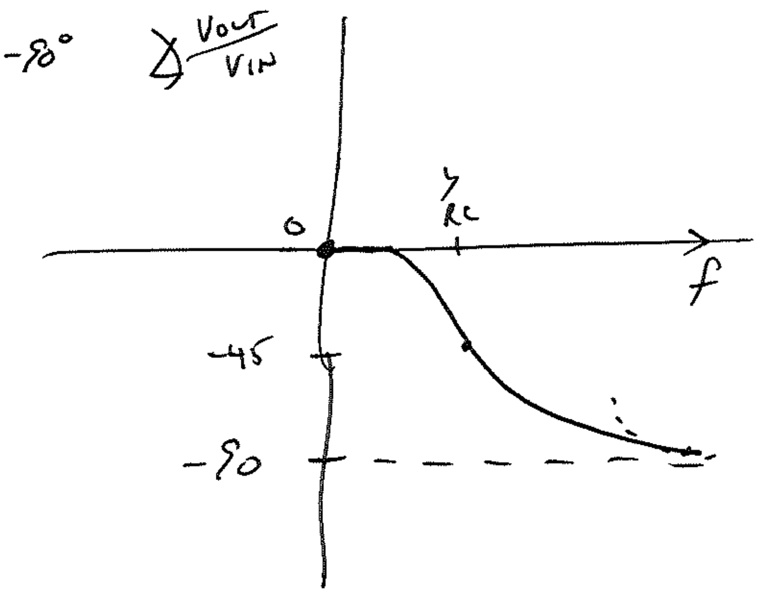
$\omega \to \infty$ $\quad \frac{V_{OUT}}{V_{IN}} \to 0$

$$\measuredangle \frac{1}{j\infty} \quad \frac{\measuredangle 1 = 0^\circ}{\angle 90^\circ} = 0^\circ - 90^\circ = -90^\circ$$

$\omega = \frac{1}{RC}$ $\quad \left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{1+j} = \frac{|1|}{|1+j|} = \frac{1}{\sqrt{2}} = .707$

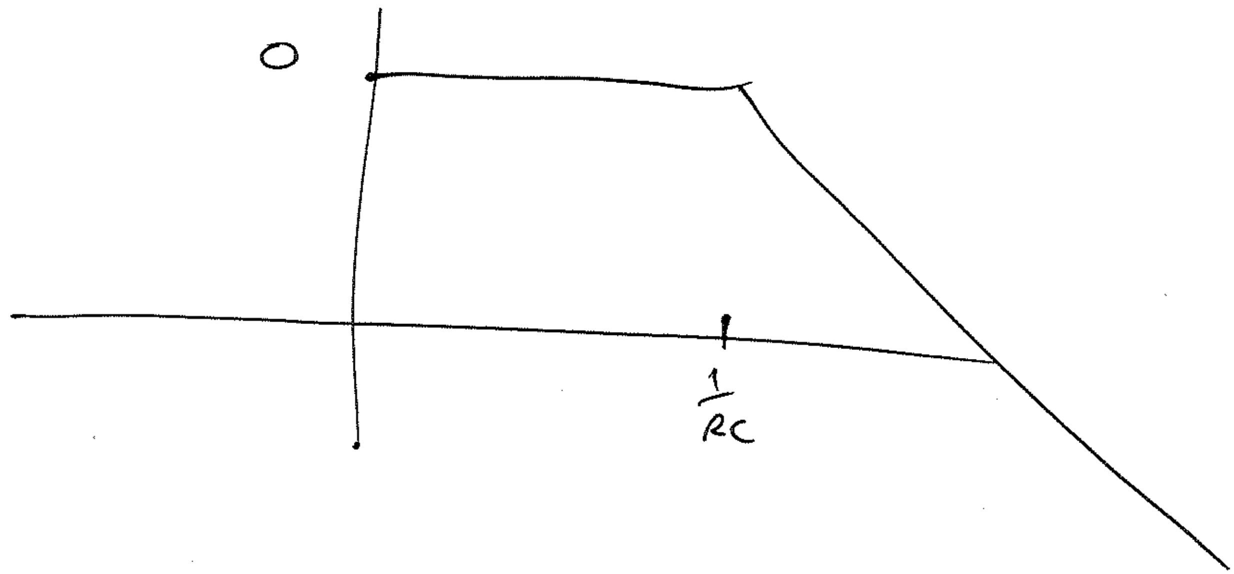
$\omega = \frac{1}{RC} \equiv$ Breakpoint

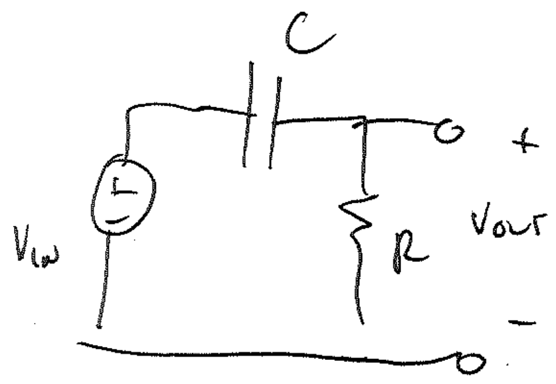
$$\measuredangle \frac{1}{1+j} = \frac{0^\circ}{45^\circ} = 0^\circ - 45^\circ = -45^\circ$$



Low Pass Filter.

$$\frac{1}{1+j\omega RC}$$





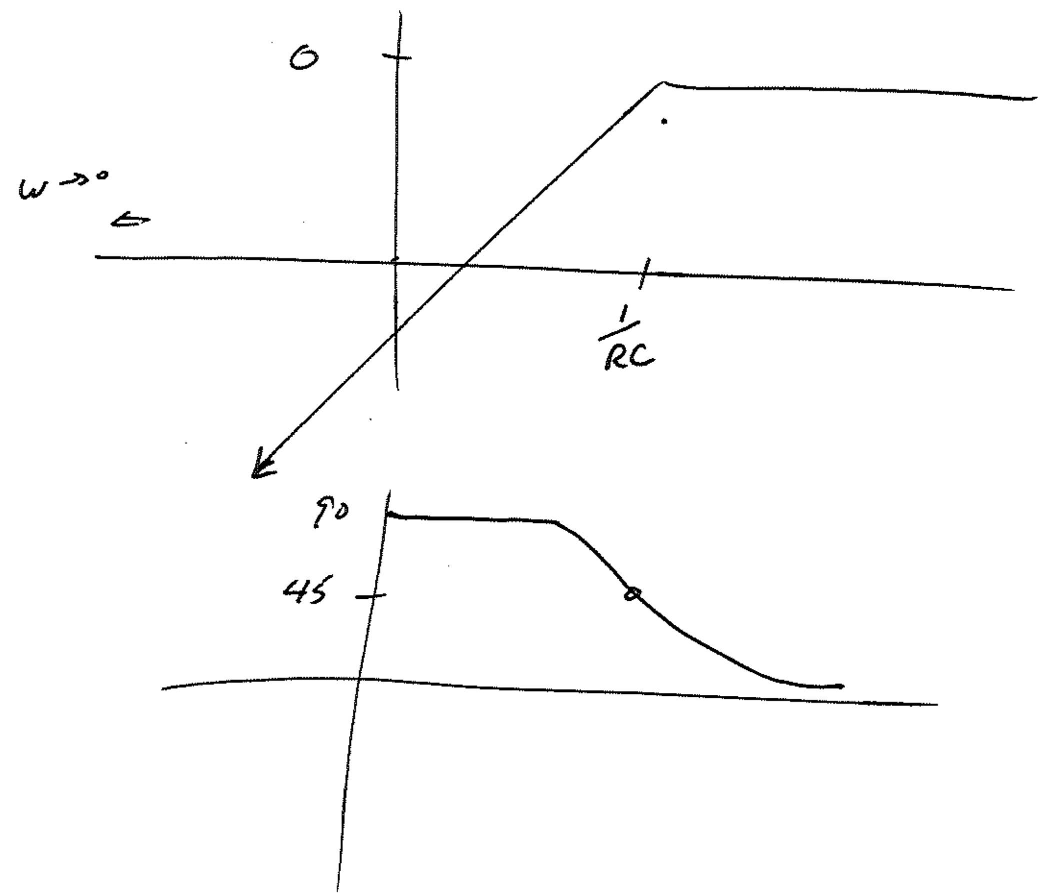
$$V_{OUT} = V_{IN} \frac{R}{R + \frac{1}{j\omega C}}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{j\omega RC}{j\omega RC + 1}$$

$\omega \to 0$

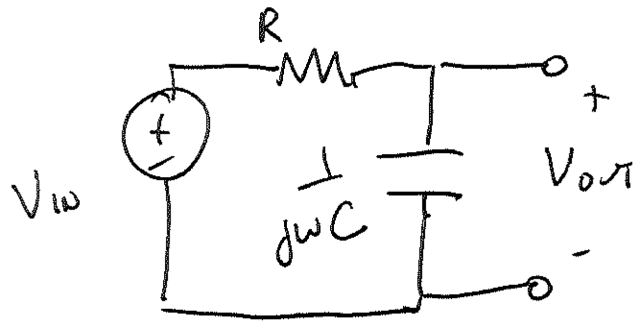
$\omega \to \infty$

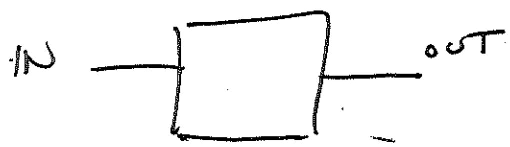
$\omega \to \frac{1}{RC}$



MIDTERM THURS July 2
HW 5 Due Wed July 1

5 - Power 6 7





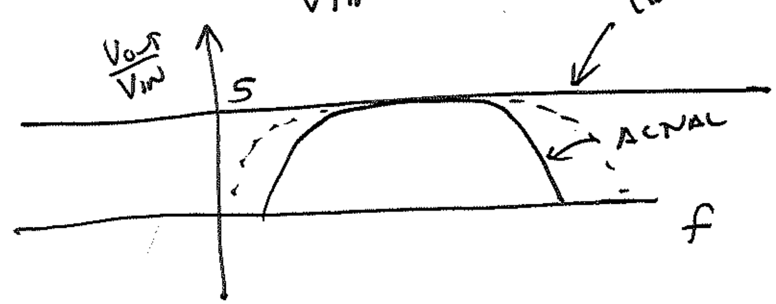
$\frac{V_{OUT}}{V_{IN}}$ = TRANSFER FCN

= 1 UNITY GAIN CONSTANT

= 5

$V_{OUT} = 5 V_{in}$ AMPLIFIER

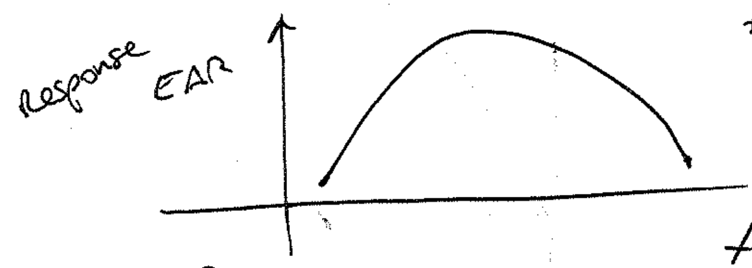
PLOT $\frac{V_{OUT}}{V_{IN}}$ vs f

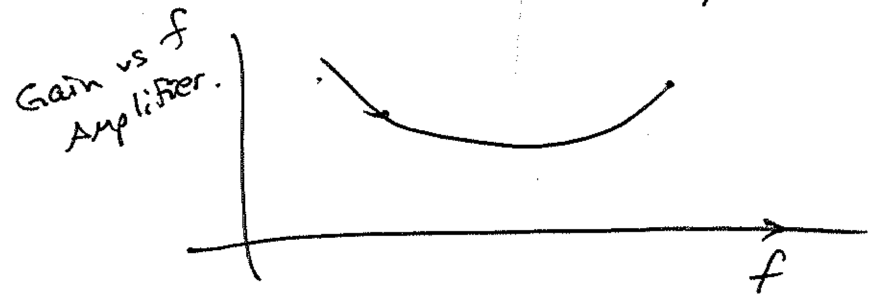


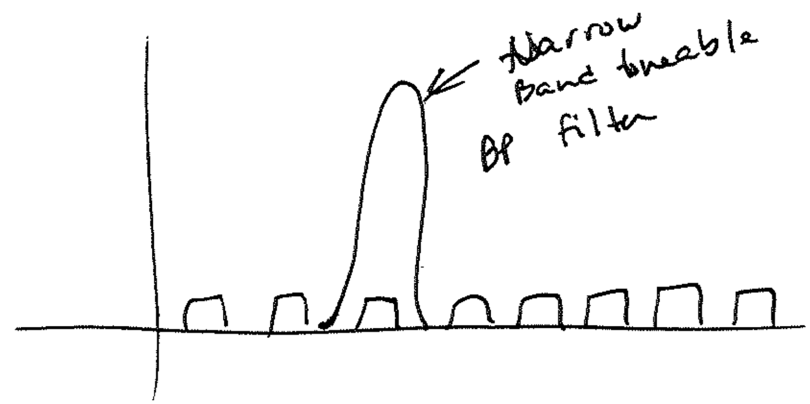
HIGH Fidelity - clean, quality

attributes of sound

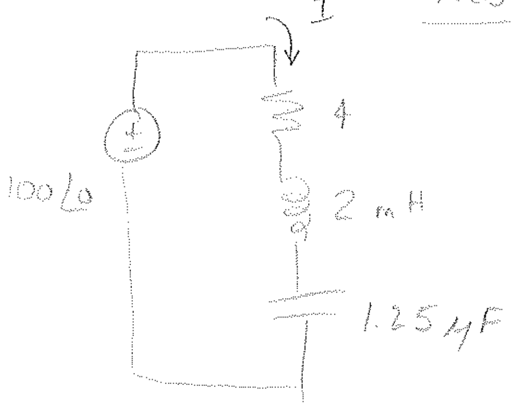
transfer functions

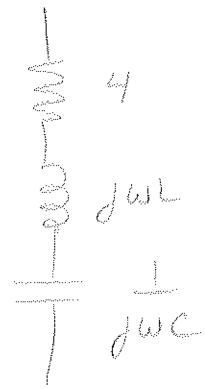






SERIES Resonance Ckts





$$Z_T = R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$|V_L| = |V_C|$ BUT ARE 180° OUT OF PHASE

@ $\omega L = \frac{1}{\omega C}$

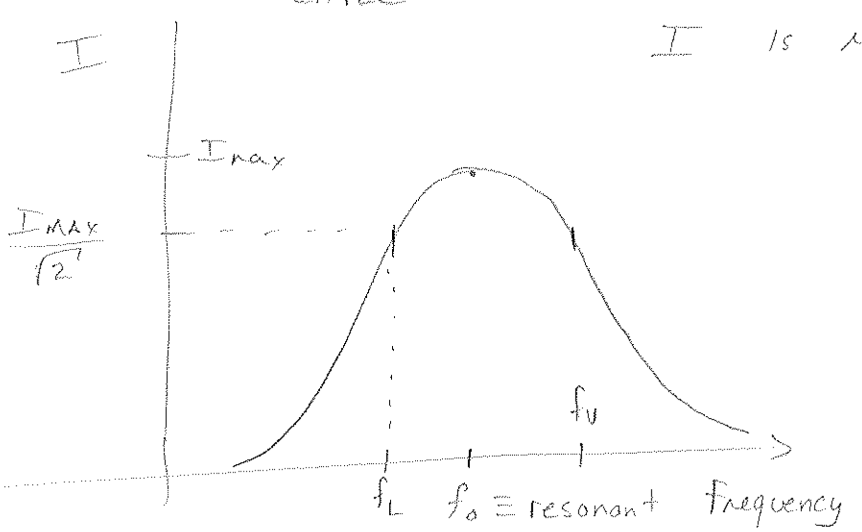
$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \implies Z_T = R$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$|Z_T|$ IS MINIMUM

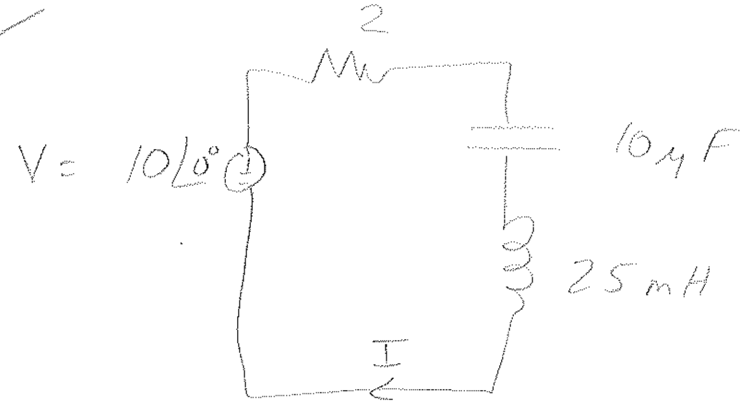
I IS MAXIMUM



$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$ Quality factor

$$BW = \frac{\omega_0}{Q}$$ BANDWIDTH $= f_U - f_L$

EX



FIND ω_o, Q, V_C, V_L, I @ ω_o

$$Z_T = R + j\omega L - \frac{1}{j\omega C}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 2000$$

@ ω_o $\quad I = \dfrac{V}{R}$ $\quad$ B/c $\quad j\omega L - \dfrac{1}{j\omega C} = 0$

$$= 5\angle 0°$$

$$V_R = I(2) = 10\angle 0°$$

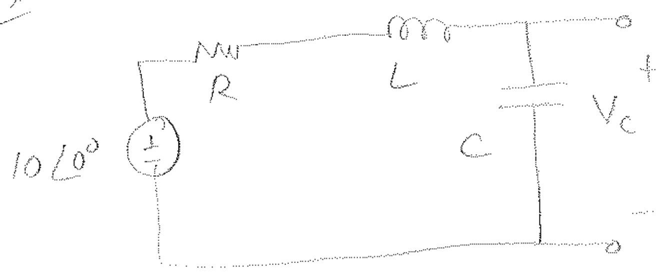
$$V_L = j\omega L\, I = 250\angle 90°$$

$$V_C = \frac{1}{j\omega C}(I) = 250\angle -90°$$

$$Q = \frac{\omega_o L}{R} = 25$$

NOTICE:
$$|V_L| = \omega_o L|I| = \frac{\omega_o L}{R}|V_S| = Q|V_S|$$
$$|V_C| = \frac{1}{\omega_o C}|I| = \frac{1}{\omega_o C R}|V_S| = Q|V_S|$$

EX



CKT MUST HAVE:

$f_0 = 1000$ Hz

$L = 0.02$ H

$R = $ ~~[illegible]~~ 0.63

CHOOSE C   FIND V_C @ Resonance

Q

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad C = 1.27\,\mu F \qquad \omega_0 = 6275 \text{ rad/s}$$

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi(1000)(0.02\ H)}{0.63} = 200$$

AT Resonance:

$$j\omega L - \frac{1}{j\omega C} = 0 \qquad Z_C = \frac{1}{j\omega C} = \frac{1}{2\pi f C}$$

$$Z_T = R$$

$$I = \frac{10\angle 0^\circ}{0.63} = 15.87\angle 0^\circ$$

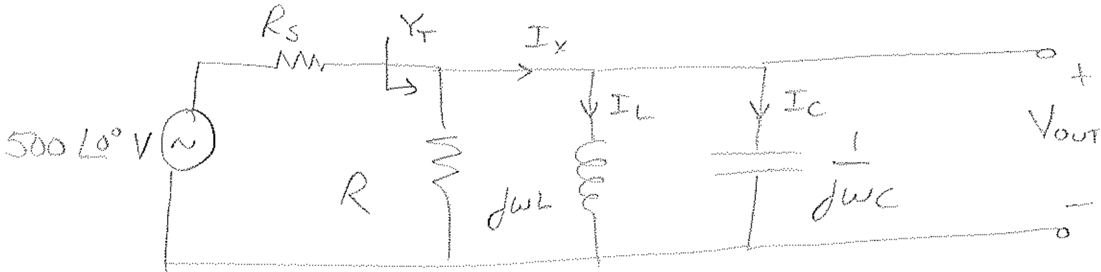
$$V_C = I\,Z_C = 6.28\angle 0^\circ \left(\frac{1}{2\pi(1000)(1.27\,\mu F)}\right) \measuredangle -90$$

$$= 15.87\angle 0^\circ\,(125.3\angle -90)$$

$$V_C = \frac{V_S}{R}\left(\frac{1}{2\pi f C}\right) = 2000$$

$$\left(\frac{1}{2\pi f C}\right)$$

Parallel Resonance



$$Y_T = \frac{1}{R} + \frac{1}{jwL} + jwC$$

$$Y_T = \frac{1}{R} + j\left(wC - \frac{1}{wL}\right)$$

$$@\ \omega C = \frac{1}{\omega L}$$

$$\omega_0^2 = \frac{1}{LC}$$

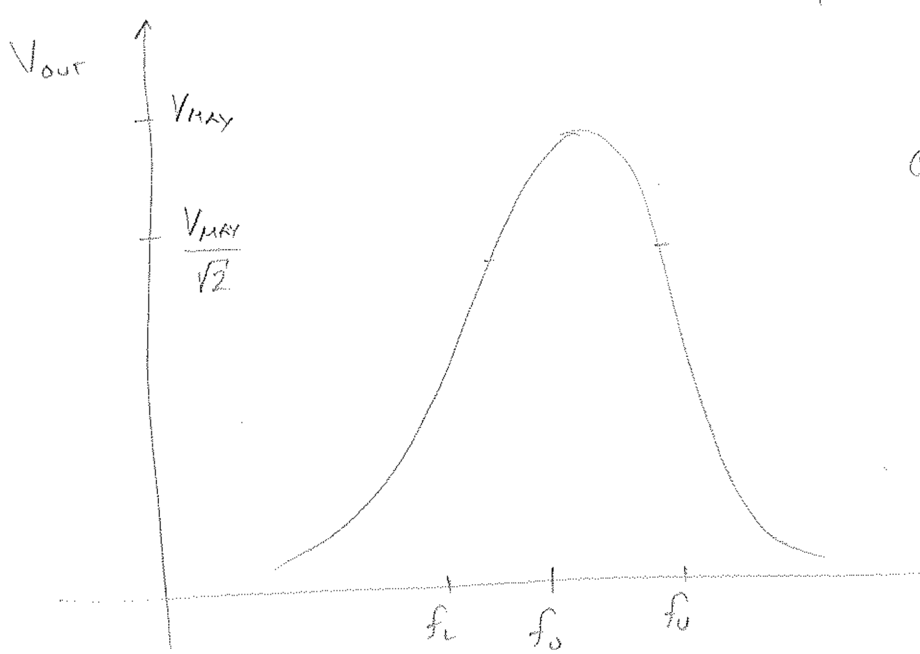
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi LC} \implies$$

Y_T IS MINIMUM
Z_T IS MAXIMUM

$|I_L| = |I_C|$ BUT ARE 180° OUT OF PHASE
$\therefore I_X = 0$

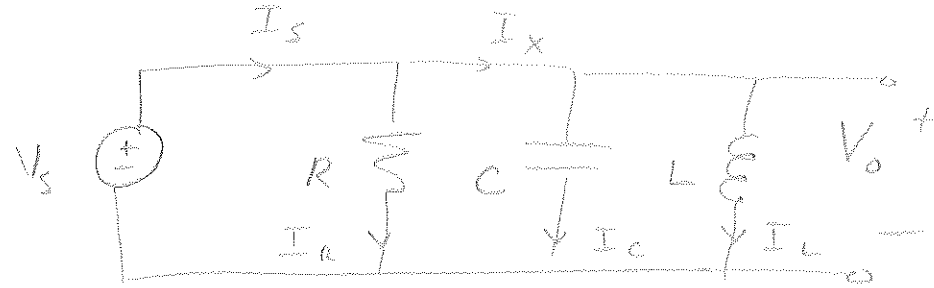
V_{OUT} IS MAXIMUM



$$Q = \frac{R}{\omega_0 L} = \omega_0 RC$$

$$BW = \frac{\omega_0}{Q}$$

$$BW = f_U - f_L$$



$V_S = 120\angle 0°$ $R = 100$ $C = 600\mu F$ $L = 120\,mH$

FIND ALL I, V_o @ Resonance

$$\omega_o = \frac{1}{\sqrt{LC}} = 117.85 \text{ rad/s}$$

$$Y_C = j\omega_o C = j70.7\times 10^{-3}$$

$$Y_L = \frac{1}{j\omega_o L} = -j70.7\times 10^{-3}$$

$$I_R = \frac{V_S}{R} = \frac{120\angle 0°}{100} = 1.2\angle 0°$$

$$I_C = V_S Y_C = 8.49\angle 90°$$

$$I_L = V_S Y_L = 8.49\angle -90°$$

$$I_S = I_R + I_C + I_L = 1.2\angle 0° = I_S.$$

$V_o = V_S$ (OF COURSE)