Nonthermal fixed points and superfluid turbulence in 2D ultracold Bose gases

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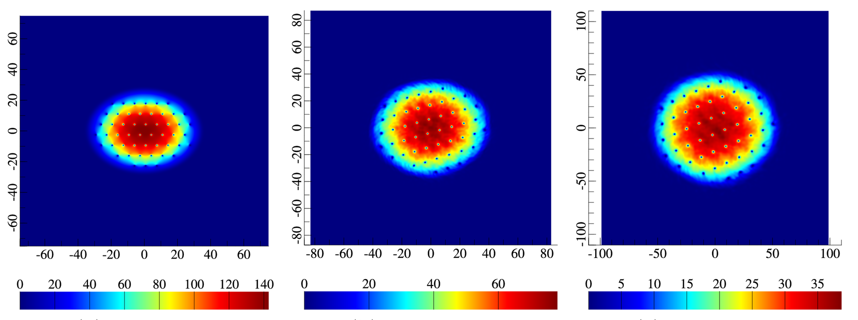
GPE Simulation with expanding coordinates

We study the influence of turbulence (vortices) on free expansion using the Gross-Pitaevskii equation.

$$\bar{x}' = \Lambda(t) \cdot \bar{x}$$

The expanding coordinates ease the computational difficulties of expanding systems on a grid.

$$i\partial_t \Psi = \left(-\tfrac{1}{2}\Lambda^{-2}\Delta + g|\Psi|^2 + i\dot{\Lambda}\Lambda^{-1}\bar{x}\bar{\nabla}\right)\Psi$$

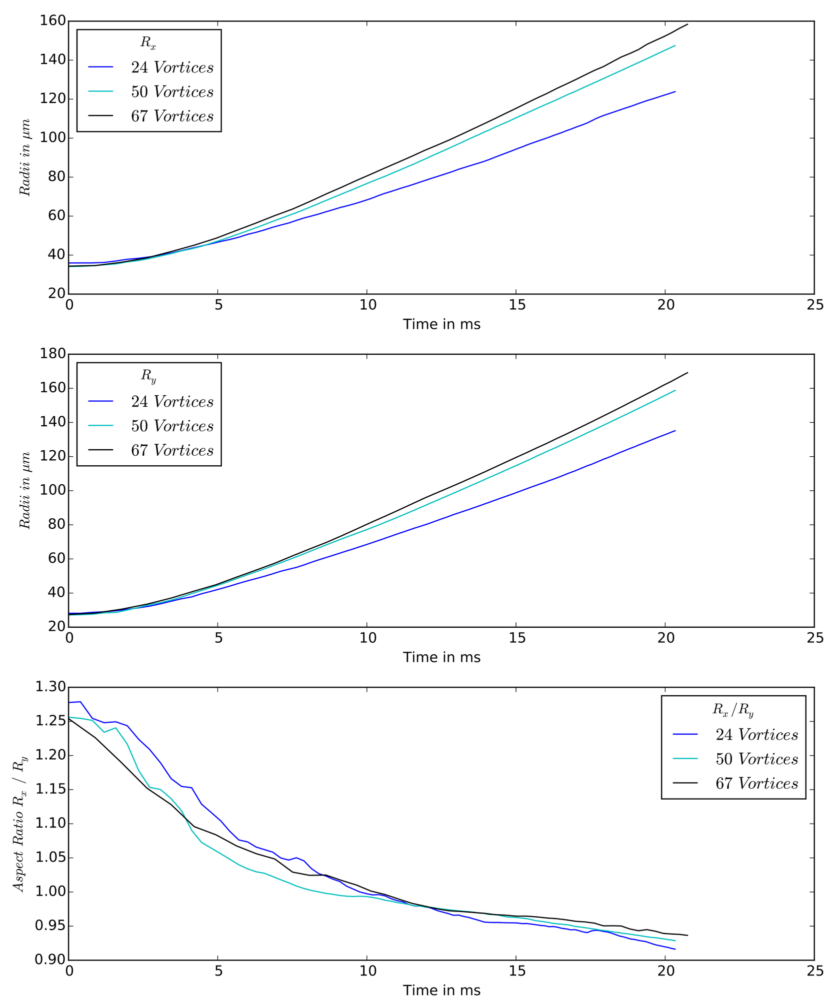


(a) t = 0 ms (b) t = 3.8 ms (c) t = 6.8 ms

(d) t = 11.4 ms (e) t = 16.4 ms (f) t = 20.3 ms

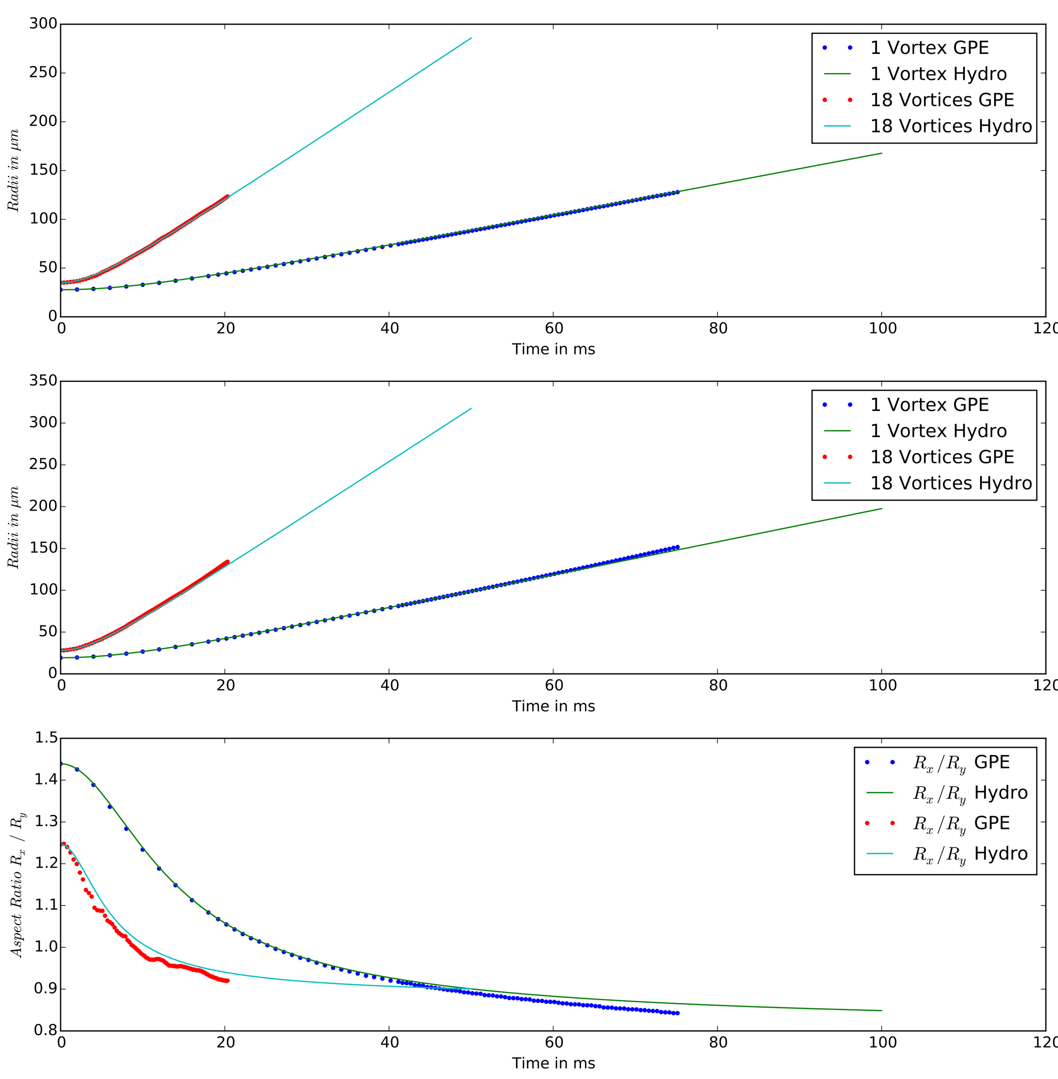
Results of GPE Simulations

The inversion of the aspect ratio during free expansion influenced by different quantities of vortices present:



Both models in comparison

Combination of both simulations show good compliance for given vortex numbers.



One Component Bose Gas

We study the influences and dynamics of vortices on Bose-Einstein condensates with one component in two dimensions.

Hamiltonian for ultra cold Bose-Einstein condensates.

$$H = \int d^dx[-\Phi^\dagger \tfrac{\nabla^2}{2m}\Phi + \tfrac{g}{2}\Phi^\dagger\Phi^\dagger\Phi\Phi]$$

Important quantities:

$$n(k,t) = \int d\Omega \, \langle \Psi^*(k,t)\Psi(k,t)\rangle \ (\textbf{Occupation number})$$

$$|\rho(k,t)|^2 = \int d\Omega \, \langle \rho^*(k,t)\rho(k,t)\rangle \ (\textbf{Density-Density Correlation})$$

Turbulence as a Nonthermal Fixed Point

"Big whirls have little whirls
that feed on their velocity,
and little whirls
have lesser whirls
and so on to viscosity."

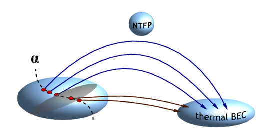
L.F. Richardson's cascade, 1920

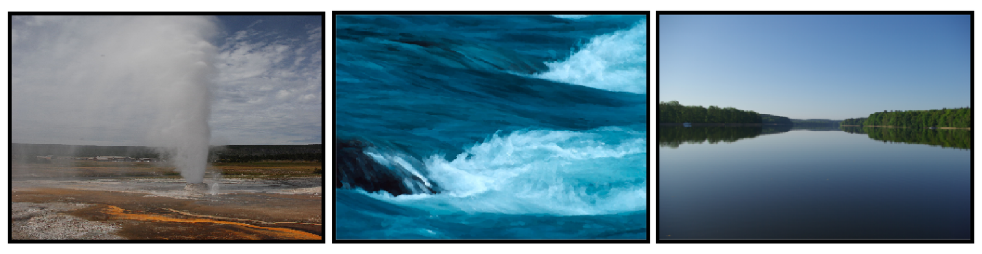
Quantum field theory scaling analysis

Radial **particle density** flux, $k^{D-1}\mathbf{Q}(k,t)$, is k-independent if:

$$\begin{cases} D-1+3D-D-2-3\zeta+2(2-D+\zeta+z/2) & = -1 \\ D-1+3D-D-2-3\zeta & = -1 \end{cases}$$

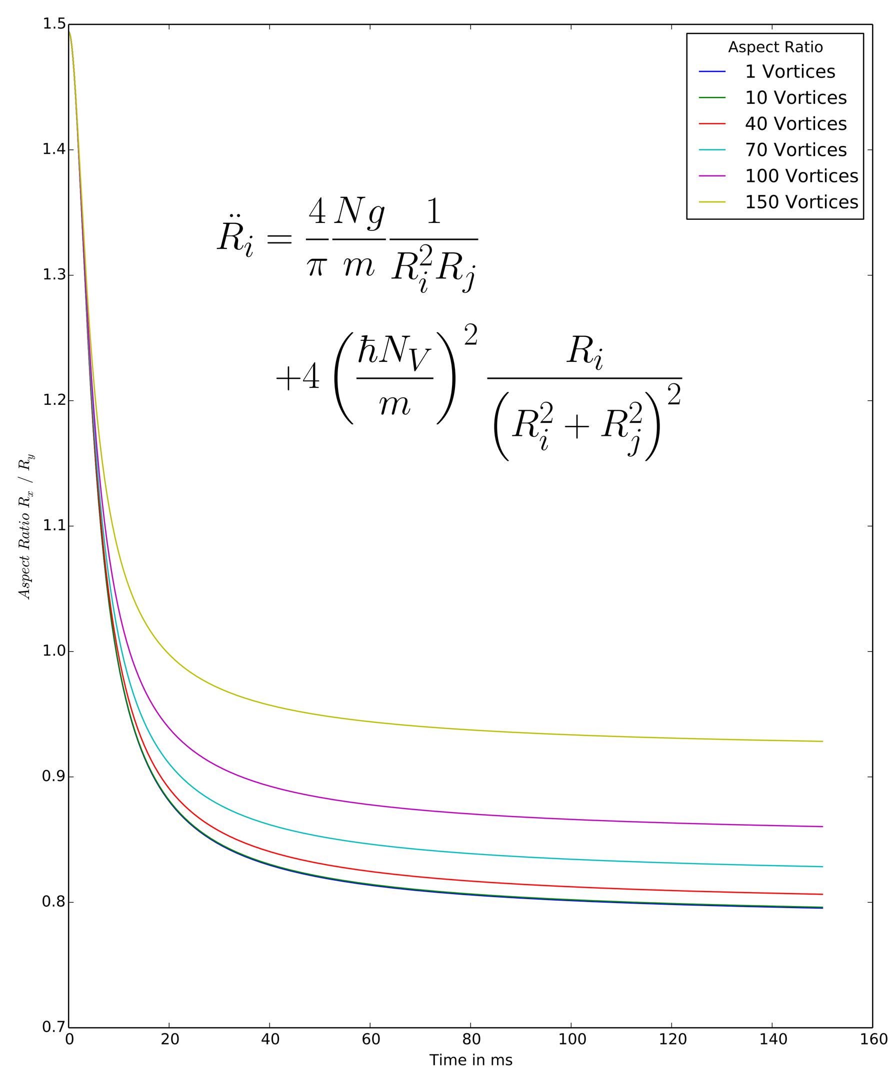
For the radial **energy density** flux, $k^{D-1}k^z\mathbf{Q}(k,t)$: $\zeta = D+2+z$ and $\zeta = D$





Hydrodynamic Model

A set of differential Equations for the Thomas-Fermi Radii during free expansion of elliptic BECs.
Influence of turbulence on the inversion of the aspect ratio boils down to the number of vortices in BEC.



$$\ddot{R}_i = \frac{4}{\pi}\frac{Ng}{m}\frac{1}{R_i^2 R_j} + 4\left(\frac{\hbar N_V}{m}\right)^2 \frac{R_i}{\left(R_i^2+R_j^2\right)^2}$$

References

M. Caracanhas, A. L. Fetter, V. S. Bagnato **JLTP 166**, (40)
W. Bao **JCP 187**
B. Nowak, Jan Schole, and T. Gasenzer **New. J. Phys. 16: 093052, 2014**
M. Karl, B. Nowak, and T. Gasenzer **Sci. Rep. 3: 2394, 2013**
B. Andrews **MSci Thesis**
J. Berges, A. Rothkopf, J. Schmidt, **PRL 101**, 041603 (08)
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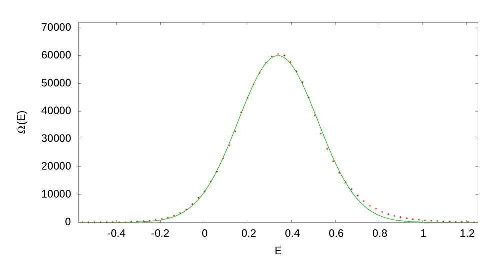
C. Scheppach, J. Berges, T. Gasenzer **PRA 81**, 033611 (10)

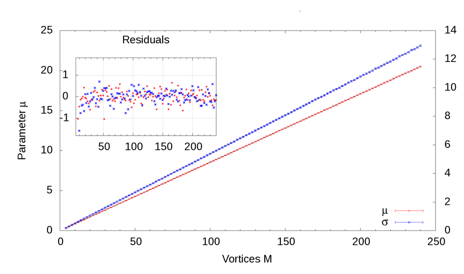
Onsager Model

The Onsager Energy is given by:

$$H = -\frac{1}{2\pi}\sum_{i>j}^{M}\kappa_i\kappa_j \ln(|r_i - r_j|)$$

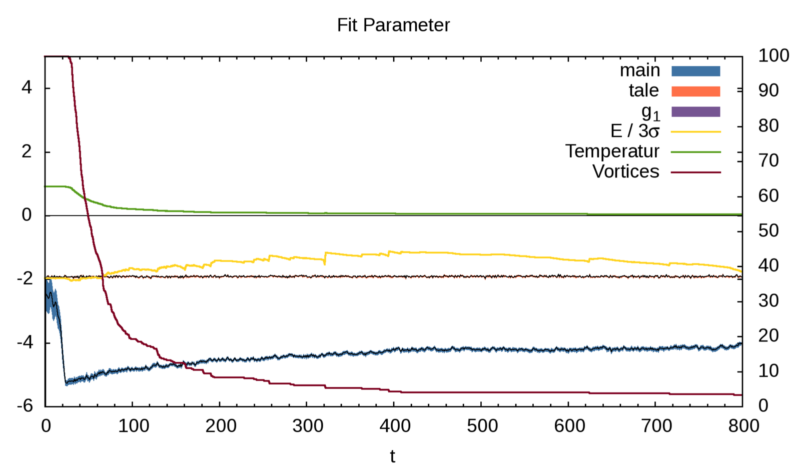
Energy Histogram **Fit Parameters**





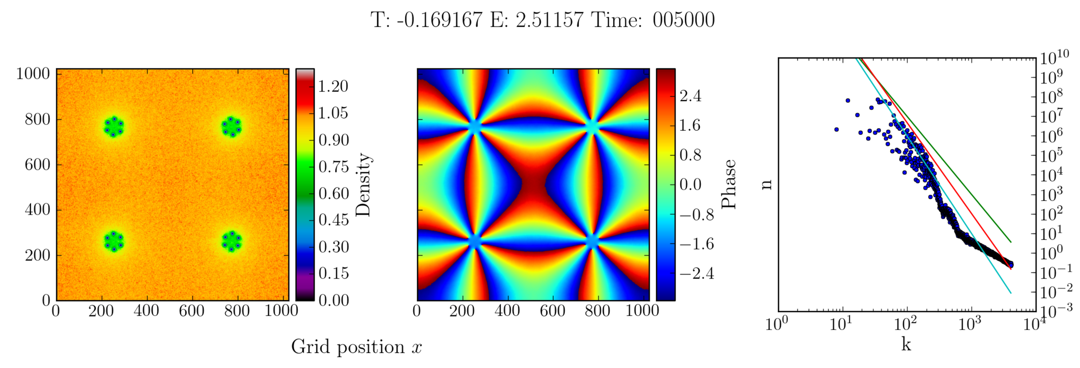
We use the phase space volume $\Omega(E)$ to calc. the Temperature via:

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{\partial k_B \ln(\Omega(E)) = -k_B\frac{E-\mu}{\sigma^2}}{\partial E}$$

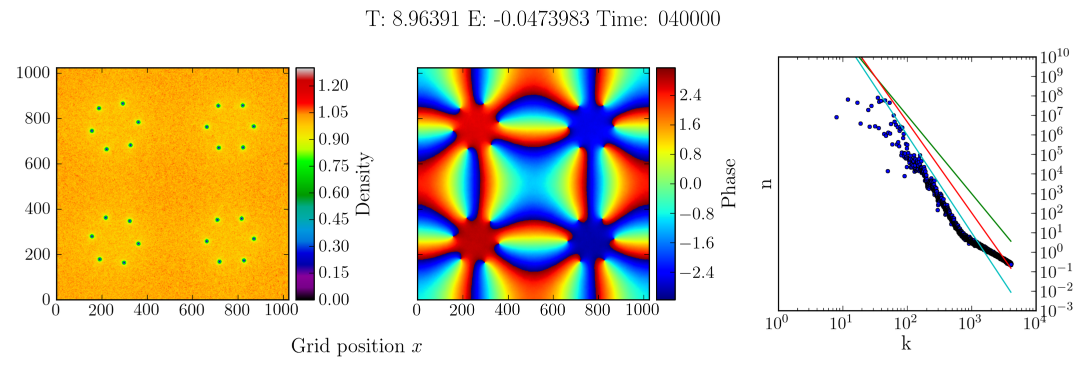


Results for a simulation with a regular vortex grid as initial condition:

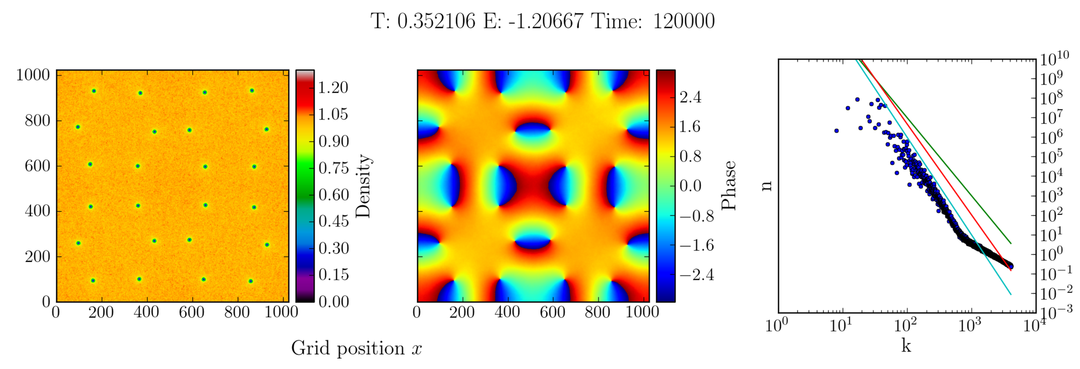
Clustering vs. Pair production



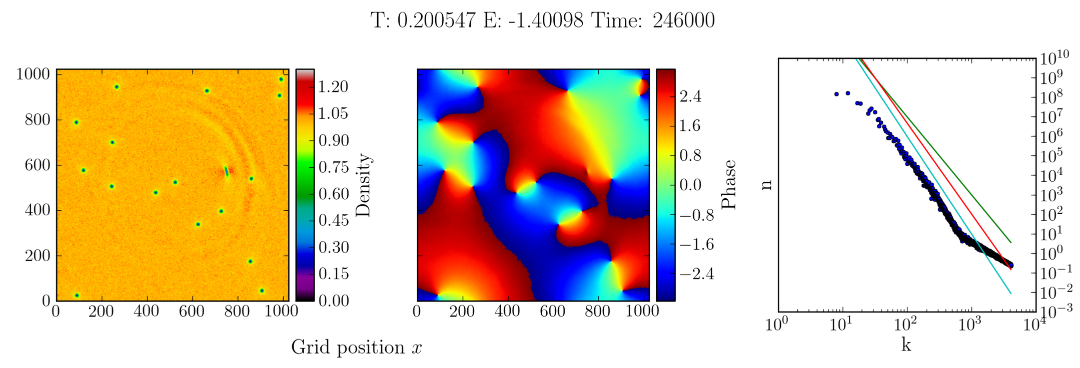
Initial state: highly clustered vortex configuration.



The vortices repel each other and the clusters spread apart.



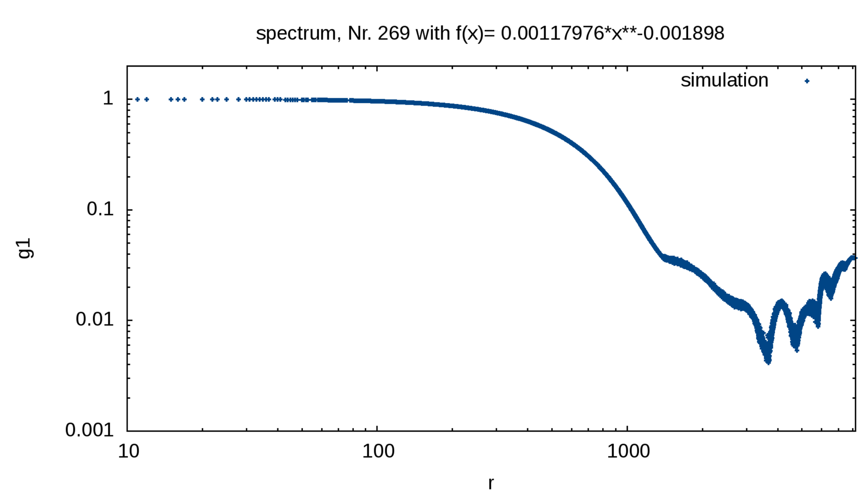
The vortices and antivortices start to "feel" each other.



The clusters are completely destroyed and we get pair annihilation again

Ongoing Projects

Correlation function in position space



exponential decay, or power law?

for a **review** see: B. Nowak, S. Erne, M. Karl, J. Schole, D. Sexty, and T. Gasenzer, **Non-thermal fixed points: universality, topology, & turbulence in Bose gases** arXiv:1302.1448