## Problem setting

For $t = 1, \dots, T$:

🤖 **Learner**

Make prediction $\hat{c}_t \in \mathbb{B}^n$ of $c^*$.

Take action $x_t \in \operatorname{argmax}_{x \in X_t} \langle c^*, x \rangle$.

👩 **Agent**

Observe $(X_t, x_t)$ and update from $\hat{c}_t$ to $\hat{c}_{t+1}$.

## Regret

**Opt. val.** **Val. of learner's action**

$$R_T^{c^*} \coloneqq \sum_{t=1}^T \langle c^*, x_t - \hat{x}_t \rangle = \sum_{t=1}^T \langle c^*, x_t \rangle - \sum_{t=1}^T \langle c^*, \hat{x}_t \rangle \quad \text{for} \quad \hat{x}_t \in \arg\max \{ \langle \hat{c}_t, x \rangle \mid x \in X_t \}.$$

## Our results

- ONS-based method with $R_T^{c^*} = O(n \log T)$; the per-bound complexity is independent of $T$.
  - The first method to have these two desirable properties!
- MetaGrad-based method to handle suboptimality of $x_t$.
- Lower bound of $R_T^{c^*} = \Omega(n)$.