

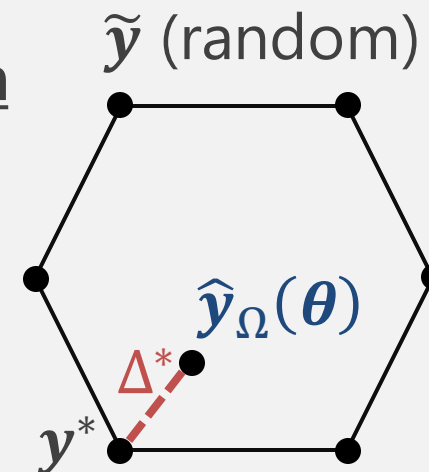
## Main result: Finite surrogate regret bound for online structured prediction

$$\sum_{t=1}^T \mathbb{E}[L_t(\hat{\mathbf{y}}_t)] \leq \sum_{t=1}^T S_t(\mathbf{U}) + \left(1 - \frac{4\gamma}{\lambda\nu}\right)^{-1} \frac{b}{4} \|\mathbf{U}\|_F^2.$$

### Main idea: randomized decoding w/ regularized prediction

Regularized pred:  $\hat{\mathbf{y}}_\Omega(\boldsymbol{\theta}) := \operatorname{argmax}_{\mathbf{y}' \in \operatorname{conv}(\mathcal{Y})} \langle \boldsymbol{\theta}, \mathbf{y}' \rangle - \Omega(\mathbf{y}')$ .

Return  $\psi(\boldsymbol{\theta}) = \begin{cases} \tilde{\mathbf{y}} & \text{w.p. } p := \min\{1, 2\Delta^*/\nu\} \text{ } (\mathbb{E}[\tilde{\mathbf{y}}] = \hat{\mathbf{y}}_\Omega(\boldsymbol{\theta})), \\ \mathbf{y}^* & \text{w.p. } 1 - p. \end{cases}$



### Key lemma

For any  $(\boldsymbol{\theta}, \mathbf{y}) \in \mathbb{R}^d \times \mathcal{Y}$ , it holds that  $\mathbb{E}[L(\psi(\boldsymbol{\theta}); \mathbf{y})] \leq \frac{4\gamma}{\lambda\nu} S(\boldsymbol{\theta}; \mathbf{y})$ .

Allows for exploiting the surrogate gap! (Van der Hoeven 2020)