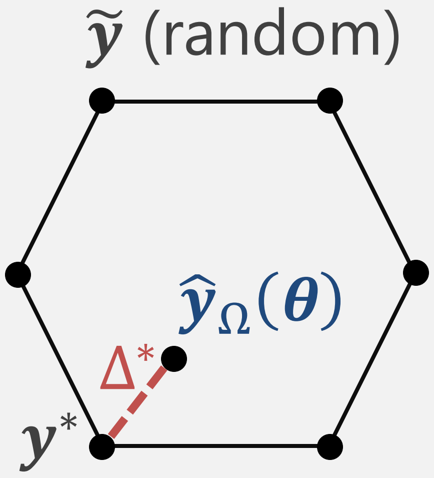
Main result: Finite surrogate regret for online structured prediction

$$\sum_{t=1}^{T} \mathbb{E}[L_t(\widehat{\boldsymbol{y}}_t)] \leq \sum_{t=1}^{T} S_t(\boldsymbol{U}) + \left(1 - \frac{4\gamma}{\lambda\nu}\right)^{-1} \frac{b}{4} \|\boldsymbol{U}\|_F^2.$$

Main idea: randomized decoding w/ regularized prediction

Regularized pred: $\widehat{\boldsymbol{y}}_\Omega(\boldsymbol{\theta}) \coloneqq \operatorname{argmax}_{\boldsymbol{y}' \in \operatorname{conv}(\mathcal{Y})} \langle \boldsymbol{\theta}, \boldsymbol{y}' \rangle - \Omega(\boldsymbol{y}')$.

Return $\psi(\boldsymbol{\theta}) = \begin{cases} \widetilde{\boldsymbol{y}} & \text{w.p. } p \coloneqq \min\{1, 2\Delta^*/\nu\} \ (\mathbb{E}[\widetilde{\boldsymbol{y}}] = \widehat{\boldsymbol{y}}_\Omega(\boldsymbol{\theta})), \\ \boldsymbol{y}^* & \text{w.p. } 1 - p. \end{cases}$



Key lemma For any $(\boldsymbol{\theta}, \boldsymbol{y}) \in \mathbb{R}^d \times \mathcal{Y}$, it holds that $\mathbb{E}[L(\psi(\boldsymbol{\theta}); \boldsymbol{y})] \leq \frac{4\gamma}{\lambda\nu} S(\boldsymbol{\theta}; \boldsymbol{y})$.

Allows for exploiting the gap! (Van der Hoeven 2020)