Coding in R: Generate Half Gaussian Table, Optimal Markowitz portfolio

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14.06.2021

Problem 2

```
# 1. phi function ----
phi <- function(x = NULL){</pre>
  if(is.null(x)) stop("Stop! Null vector not allowed!") # test for non-null vector
  x = scale(x) # obtain standardization of vector x
  out <- 1/sqrt(2 * pi) * exp(-x^2/2) # probability density of vector x
  return(as.vector(out))
}
# output vector of densities
# e.g. phi(x = seq(100)) outputs 100 densities
# 2. quantile vector, integrate to obtain phi vector values ------
quant = seq(0, 5.5, by = 1/100) # for half-table values
f = function(x) 1/sqrt(2 * pi) * exp(-x^2/2) # integrand for Phi(x) value
probs <- sapply(quant,</pre>
                FUN = function(x) integrate(f, lower = 0, upper = x)$value)
# check if probs and pnorm() agree to 4 decimal places
identical(round(probs, 4), round(pnorm(q = quant)-0.5, 4))
## [1] TRUE
# matrix form (subset first 400 to draw half-table)
half_table <- as.data.frame(matrix(probs[1:400], ncol = 10, byrow = TRUE))
rownames(half_table) = sprintf('\%0.1f', seq(0, 3.9, by = 1/10))
colnames(half_table) = sprintf('\%0.2f', seq(0, 0.09, by = 1/100))
# output Gaussian table (half-table)
knitr::kable(half_table,
             digits = 4,
             caption = "Standard normal table: $P(0 < Z < z )$")</pre>
```

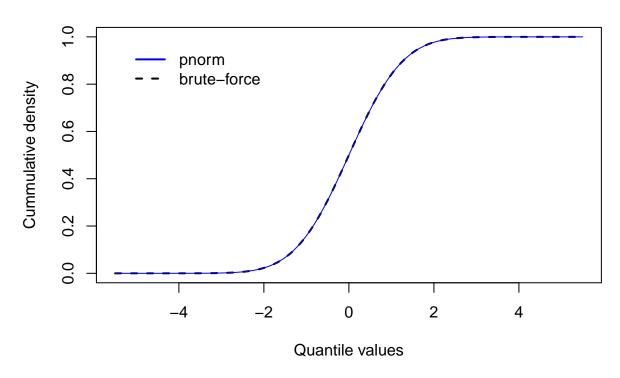
Table 1: Standard normal table: P(0 < Z < z)

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

```
# 3. Distribution curve (full table plot) -----
x = c(-rev(quant), quant)
y = sapply(x,
           FUN = function(x) integrate(f, lower = -5.5, upper = x)$value)
plot(x, y, type = 'l',
     xlab = "Quantile values",
     ylab = "Cummulative density",
     1wd = 2, 1ty = 2,
     main = expression(
       paste("Standard Gaussian distribution function:")~Phi(x))
     # x lim = c(-5, 5), y lim = c(0, 1), axes = TRUE,
curve(expr = pnorm(x), add = TRUE, col = 'blue', lty = 1.5)
legend('topleft',
       inset = 0.05,
       legend = c("pnorm", "brute-force"),
       bty = "n", # Removes the legend box
       lty = c(1, 2),
       col = c('blue', 'black'),
       lwd = 2)
```

Standard Gaussian distribution function: $\Phi(x)$



Problem 3

```
options (digits=4, width=70)
library("PerformanceAnalytics")
library("tseries")
library("zoo")
### TASK 1: Retrieving 10 stock prices (Yahoo S&P500)
tickers <- c("AAPL", "MSFT", "AMZN", "GOOGL", "BRK-B",
           "JPM", "JNJ", "NVDA", "BAC", "PFE")
lst <- list() # To hold adjusted monthly returns</pre>
retrieve_stock <- function(){</pre>
   for(i in tickers){
     lst[[i]] <- get.hist.quote(instrument = i,</pre>
                                     start = "2010-01-01",
                                     end = "2020-01-01",
                                     quote = "AdjClose",
                                     provider = "yahoo",
                                     origin = "2000-09-01",
                                     compression = "m",
                                     retclass = "zoo")
     }
   return(lst)
stock_prices = retrieve_stock() # Stock prices from S&P500 (Yahoo)
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
                     2019-12-01
## time series ends
## time series ends
                     2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
## time series ends 2019-12-01
sapply(stock_prices, length) # Check lengths of downloaded RETURNs (i.e 120)
## AAPL MSFT AMZN GOOGL BRK-B
                                  JPM
                                        JNJ NVDA
                                                   BAC
                                                         PFE
##
    120
          120
                120
                      120
                            120
                                  120
                                        120
                                              120
                                                   120
                                                         120
# Transformation and Summary Statistics -----
### TASK 2: mean, variance-covariance and weights
N = length(tickers) # number of securities (10)
ones <- matrix(data = 1, nrow = N) # matrix of ones
descriptive_stat <- function(){</pre>
 lr <- lapply(X = stock_prices, FUN = function(x) diff(log(x)))</pre>
 mat <- sapply(lr, unclass) # obtain matrix of 10 securities</pre>
 ln = nrow(mat) # off by one after differencing
  # random weights
 wts <- runif(n = N) # random weights (not scaled)
 wts <- wts/sum(wts) # scaled weights (NB: this sums to One!)
 names(wts) <- tickers</pre>
 mu <- 1/ln * t(mat) %*% matrix(1, nrow = ln) # mean vector</pre>
  mu <- as.vector(mu)</pre>
 names(mu) <- tickers</pre>
 mu_mat <- matrix(data = mu, ncol = N, nrow = ln, byrow = TRUE) # mean matrix</pre>
  vcv \leftarrow 1/(ln - 1) * t((mat - mu_mat)) %*% (mat - mu_mat) # var-cov
  vcv <- as.matrix(vcv)</pre>
  dimnames(vcv) <- list(tickers, tickers)</pre>
  class(vcv) <- "matrix"</pre>
  list(mu_cap = mu, cov_mat = vcv, weights = wts) # mean, variance, weights
}
descript <- descriptive_stat() # Get and print values from descriptive_stat()</pre>
descript$mu_cap
##
       AAPL
               MSFT
                        AMZN
                                GOOGL
                                         BRK-B
                                                    JPM
                                                             JNJ
```

0.021089 0.016483 0.022607 0.013608 0.009129 0.012748 0.009606

sd.port <- sqrt(weights %*% cov_mat %*% weights)</pre>

results

```
list(g.er = as.vector(er.port),
      g.sd = as.vector(sd.port),
      weights = weights
}
# Get values from get_portfolio()
get_port <- get_portfolio(er = descript$mu,</pre>
                        cov_mat = descript$cov_mat,
                        weights = descript$weights)
# portfolio with random weights
cat("Portfolio expected return: ", get_port$g.er)
## Portfolio expected return: 0.01428
cat("\nPortfolio standard deviation: ", get_port$g.sd)
##
## Portfolio standard deviation: 0.04258
get_port$weights
##
                     AMZN
                           GOOGL
                                   BRK-B
                                                           NVDA
     AAPL
             MSFT
                                             JPM
                                                     JNJ
## 0.14918 0.09048 0.08194 0.01321 0.16660 0.07758 0.08977 0.09000
##
      BAC
              PFE
## 0.10077 0.14046
# Efficient portfolio (Resolution 1) -----
### TASK 4: Compute minimum variance portfolio
# NB: in this section, target return, c* = mu_cap for the ith return
efficient_portfolio <- function(er, cov_mat, target_return){</pre>
 ### Note the following
  # er: expected returns vector (N x 1)
  # cov_mat: variance-covariance matrix of returns (N x N)
  # weights: random weights vector, summing to 1 (N x 1)
  # target.return: targeted return (scalar)
  # output is portfolio object with the following elements
  # call
                          original function call
                          portfolio expected return
  # er
  # sd
                          portfolio standard deviation
  # weights
                     N x 1 vector of portfolio weights
  ## inputs
 er <- as.vector(er)</pre>
                                      # assign names if none exist
 cov_mat <- as.matrix(cov_mat)</pre>
 if(length(er) != nrow(cov_mat))
   stop("invalid inputs")
  if(any(diag(chol(cov_mat)) <= 0))</pre>
   stop("Covariance matrix not positive definite")
  # compute efficient portfolio
  # forming system of equation
 ones <- rep(1, length(er))
```

```
top <- cbind(2*cov_mat, er, ones)</pre>
 bot <- cbind(rbind(t(er), t(ones)), matrix(0,2,2)) # edited by me (no error)
 A <- rbind(top, bot)
 b.target <- as.matrix(c(rep(0, length(er)), target_return, 1))</pre>
 x <- solve(A, b.target)</pre>
 w \leftarrow x[1:length(er)]
 names(w) <- tickers</pre>
  # compute portfolio expected returns and variance
 er.port <- crossprod(er,w)</pre>
  sd.port <- sqrt(w %*% cov_mat %*% w)
  # output
 list(exp_ret_port = as.vector(er.port),
      sd_port = as.vector(sd.port),
      weights_ef = w)
}
efficient_portfolio(er = descript$mu_cap,
                   cov_mat = descript$cov_mat,
                   target_return = 0.0211) ### mu_cap for AAPL is 0.0211
## $exp_ret_port
## [1] 0.0211
##
## $sd_port
## [1] 0.05075
##
##
  $weights_ef
##
      AAPL
               MSFT
                        AMZN
                                GOOGL
                                         BRK-B
                                                    JPM
                    0.28213 -0.11812 0.17346 0.37663 -0.07312
##
   0.33510 0.28451
##
      NVDA
                BAC
                         PFE
   0.04852 -0.44763 0.13852
##
# To compute for efficient portfolios for all tickers, run the code below
\# sapply(1:N, FUN = function(x))
  efficient_portfolio(er = descrip$mu_cap,
#
                      cov_mat = desc_vals$cov_mat,
#
                      target_return = descript$mu_cap[x])})
# Lagrangian method -----
### TASK 5: Lagrangian method (lambda1 and lambda2)
markowitz = function(mu,cov_mat,er) {
 A = t(ones) %*% solve(cov_mat) %*% mu
 B = t(mu) \%*\% solve(cov_mat) \%*\% mu
 C = t(ones) %*% solve(cov_mat) %*% ones
 D = B*C - A^2
 lam1 = (C*er-A)/D
 lam2 = (B-A*er)/D
 wts = lam1[1]*(solve(cov_mat) %*% mu) + lam2[1]*(solve(cov_mat) %*% ones)
 g = (B[1]*(solve(cov_mat) %*% ones) - A[1]*(solve(cov_mat) %*% mu))/D[1]
 h = (C[1]*(solve(cov_mat) %*% mu) - A[1]*(solve(cov_mat) %*% ones))/D[1]
 wts = g + h*er
}
# output (computed for only one of the 10 portfolios)
```

```
# Optimal portfolio weights: Assuming expected returns = c*, eq. AAPL
(markowitz(mu = descript$mu_cap, cov_mat = descript$cov_mat, er = 0.0211))
##
            [,1]
## AAPL
         0.33510
## MSFT
        0.28451
## AMZN
        0.28213
## GOOGL -0.11812
## BRK-B 0.17346
## JPM
        0.37663
## JNJ
        -0.07312
## NVDA
       0.04852
## BAC
        -0.44763
## PFE
        0.13852
# Global Minimum variance (Resolution 2) -----
### TASK 6: Global minimum variance portfolio
global_min_portfolio <- function(er, cov_mat){</pre>
 #
 cov_mat_inv <- solve(cov_mat)</pre>
 one_vec <- rep(1,length(er))</pre>
 w_gmin <- rowSums(cov_mat_inv) / sum(cov_mat_inv)</pre>
 w_gmin <- as.vector(w_gmin)</pre>
 names(w_gmin) <- tickers</pre>
 er_gmin <- crossprod(w_gmin, er)</pre>
 sd_gmin <- sqrt(t(w_gmin) %*% cov_mat %*% w_gmin)</pre>
 # output
 list(er_gmin = as.vector(er_gmin),
      sd_gmin = as.vector(sd_gmin),
      w_gmin = w_gmin)
glmin <- global_min_portfolio(er = descript$mu_cap, cov_mat = descript$cov_mat)</pre>
# Global minimum variance
cat("Portfolio expected return (global): ", glmin$er_gmin)
## Portfolio expected return (global): 0.01059
cat("Portfolio standard deviation (global): ", glmin$sd_gmin)
## Portfolio standard deviation (global): 0.03062
glmin$w_gmin
##
              MSFT
                              GOOGL
                                      BRK-B
                                                JPM
                                                        TN T
      AAPT.
                      AMZN
   0.11713 0.12599 -0.03203
                           0.09569 0.44496 -0.07798 0.17196
##
      NVDA
                       PFE
               BAC
## -0.05228 -0.03597 0.24254
# Compute efficient frontier -----
### TASK 7: Compute Markowitz bullet
efficient_frontier <- function(er, cov_mat,</pre>
                            nport = N, alpha_min = -0.5,
                            alpha_max = 1.5){
 # create portfolio names
```

```
port_names <- rep("port", nport)</pre>
 ns <- seq(1, nport)</pre>
 port_names <- paste(port_names, ns)</pre>
  # compute global minimum variance portfolio
  cov_mat_inv <- solve(cov_mat)</pre>
  one_vec <- rep(1, length(er))
  port_gmin <- global_min_portfolio(er, cov_mat)</pre>
  w_gmin <- port_gmin$w_gmin</pre>
  # compute efficient frontier as convex combinations of two efficient ports
  # 1st efficient port: global min var portfolio
  # 2nd efficient port: min var port with ER = max of ER for all assets
  er.max <- max(er)
 port.max <- efficient_portfolio(er, cov_mat, er.max)</pre>
 w.max <- port.max$weights_ef</pre>
  a <- seq(from = alpha_min, to = alpha_max,length = nport)# convex combinations
  we.mat <- a %o% w_gmin + (1-a) %o% w.max # rows are efficient portfolios
  er.e <- we.mat %*% er
                                                  # expected returns of efficient portfolios
  er.e <- as.vector(er.e)
 names(er.e) <- port_names</pre>
  cov.e <- we.mat %*% cov_mat %*% t(we.mat) # cov mat of efficient portfolios
  sd.e <- sqrt(diag(cov.e))</pre>
                                             # std devs of efficient portfolios
  sd.e <- as.vector(sd.e)</pre>
  names(sd.e) <- port_names</pre>
  dimnames(we.mat) <- list(port_names, tickers)</pre>
  # summarize results
  list(markowitz_er = er.e,
       markowitz_sd = sd.e,
       markowitz_weights = we.mat)
}
efficient_frontier(er = descript$mu_cap,
                   cov_mat = descript$cov_mat,
                   nport = N,
                   alpha_min = -0.5,
                   alpha_max = 1.5)
## $markowitz_er
##
    port 1 port 2 port 3 port 4 port 5 port 6
## 0.030120 0.027226 0.024333 0.021439 0.018545 0.015652 0.012758
    port 8 port 9 port 10
## 0.009864 0.006971 0.004077
##
## $markowitz_sd
## port 1 port 2 port 3 port 4 port 5 port 6 port 7 port 8
## 0.08118 0.07099 0.06113 0.05179 0.04331 0.03630 0.03174 0.03075
## port 9 port 10
## 0.03364 0.03957
##
## $markowitz_weights
                       MSFT
                                AMZN
                                          GOOGL
                                                               JPM
##
             AAPL
                                                   BRK-B
          0.52213 0.42054 0.55170 -0.301584 -0.05950 0.76671
## port 1
## port 2 0.46213 0.37691 0.46522 -0.242729 0.01524 0.64157
## port 3
           0.40213 0.33327 0.37875 -0.183874 0.08997
```

0.34213 0.28963 0.29227 -0.125019 0.16471 0.39129

port 4

```
## port 5 0.28213 0.24599 0.20579 -0.066164 0.23944 0.26615
## port 6 0.22213 0.20235 0.11931 -0.007308 0.31417 0.14101
## port 7  0.16213  0.15871  0.03283  0.051547  0.38891  0.01587
## port 8  0.10213  0.11508 -0.05365  0.110402  0.46364 -0.10927
## port 9 0.04213 0.07144 -0.14013 0.169257 0.53838 -0.23441
## port 10 -0.01787 0.02780 -0.22661 0.228112 0.61311 -0.35955
##
              JNJ
                      NVDA
                                BAC
## port 1 -0.28341 0.135002 -0.800866 0.04926
## port 2 -0.21595 0.107257 -0.687548 0.07789
## port 3 -0.14848 0.079513 -0.574230 0.10653
## port 4 -0.08102 0.051768 -0.460912 0.13516
## port 5 -0.01356 0.024023 -0.347594 0.16379
## port 6 0.05390 -0.003722 -0.234275 0.19243
## port 7 0.12136 -0.031467 -0.120957 0.22106
## port 8 0.18882 -0.059212 -0.007639 0.24970
## port 9 0.25628 -0.086957 0.105679 0.27833
## port 10 0.32375 -0.114702 0.218997 0.30696
```

Extraneous: Obtain S&P500 tickers (symbols)

```
### Get S&P500 stickers
# Approach I
library(BatchGetSymbols)
sp500 <- GetSP500Stocks()</pre>
head(sp500$Tickers, 8)
## [1] "MMM" "ABT" "ABBV" "ABMD" "ACN" "ATVI" "ADBE" "AMD"
# Approach II
library(rvest)
url <- "https://en.wikipedia.org/wiki/List_of_S%26P_500_companies"
SP500 <- url %>%
  read html() %>%
 html_nodes(xpath='//*[@id="mw-content-text"]/div/table[1]') %>%
 html_table()
SP500 <- SP500[[1]]
head(SP500$Symbol, 8)
## [1] "MMM" "ABT" "ABBV" "ABMD" "ACN" "ATVI" "ADBE" "AMD"
```