# Probability

## Glossary

1. **Prevalence:** Proportion of the population having the condition being considered.
2. **False Positive:** Wrong test result that incorrectly indicates that the subject has a condition when the subject does not have the condition.
3. **False Negative**: Wrong test result that incorrectly indicates that the subject does not have a condition when the subject has the condition.
4. **True Positive**: Correct result that indicates that the subject has a condition when subject has the condition.
5. **True Negative:** Correct test result that indicates that a subject does not have a condition when the subject does not have a condition.
6. **Test Sensitivity**: The probability of a true positive test result given that the subject actually has the condition being tested.
7. **Test Specificity:** The probability of a true negative test result given that the subject does not have any condition being tested.
8. **Positive Predictive value**: Probability that the subject does not actually have the condition given that the test yield a negative result

9. An **event** is any collection of results or outcomes of a procedure.
10. A **simple event** is an outcome or an event that cannot be further broken down into simpler components.
11. The **sample space** for a procedure consists of all possible *simple* events. That is, the sample space consists of all outcomes that cannot be broken down any further.
12. Simple Event: only one combination is possible
    Non simple event: combination of events is possible
13. Complement: The complement of event A, denoted by A, consists of all outcomes in which event A does not occur
14. Significantly low number of successes: x successes among n trials is a significantly low number of successes if the probability of x or fewer successes is unlikely with a probability of 0.05 or less. That is, x is a significantly low number of successes if P(x or fewer)≤ 0.05.*

    Significantly high number of successes: x successes among n trials is a significantly high number of successes if the probability of x or more successes is unlikely with a probability of 0.05 or less. That is, x is a significantly high number of successes if P(x or more2)≤0.05.*
15. Among 100 births, 75 girls is significantly high because the probability of 75 or more girls is 0.0000003, which is less than or equal to 0.05 (so the gender selection method appears to be effective).

Among 100 births, 55 girls is not significantly high because the probability of 55 or more girls is 0.184, which is greater than 0.05 (so

the gender selection does not appear to be effective).

16. The probability of an event is a fraction or decimal number between 0 and 1 inclusive.
    - The probability of an impossible event is 0.
    - The probability of an event that is certain to occur is 1.
    - Notation: P1A2 = the probability of event A.
    - Notation: P1A2 = the probability that event A does not occur.
17. The actual odds against event A occurring are the ratio P1A2 >P1A2, usually expressed in the form of a:b (or “a to b”), where a and b are integers. (Reduce using

the largest common factor; if a = 16 and b = 4, express the odds as 4:1 instead of 16:4.)

The actual odds in favour of event A occuring are the ratio P(A)/P(A'), which is the reciprocal of the actual odds againstthat event. If the odds against an event are a:b, then odds are in favour are b:a.

18. The payoff odds against event A occuring are the ratio of the net profit
    Payoff odds against event A = (net profit):(amount bet)
19. P(A or B): p(A) + p(B) - p(A and B)
20. P(A and B): A occurs in first trial and B occurs in second trial. P(B|A); B after A has occured.
21. p(A and B): P(A) * P(B|A)
22. Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If A and B are not independent, they are said to be dependent.
23. Sampling with replacement: Selections are independent events.
    Sampling without replacement: Selections are dependent events.
24. TREATING DEPENDENT EVENTS AS INDEPENDENT:
    5% GUIDELINE FOR CUMBERSOME CALCULATIONS
    When sampling without replacement and the sample size is no more than 5% of the size of the population, treat the selections as being independent (even though they are actually dependent).
25. “At least one” has the same meaning as “one or more.”
    The complement of getting “at least one” particular event is that you get no occurrences of that event.
26.

**Finding the probability of getting *at least one* of some event:**

1. **Let $A$ = getting *at least one* of some event.**
2. **Then $\overline{A}$ = getting *none* of the event being considered.**
3. **Find $P(\overline{A})$ = probability that event $A$ does not occur.** (This is relatively easy using the multiplication rule.)
4. **Subtract the result from 1. That is, evaluate this expression:**

$$P(\textit{at least one} \text{ occurrence of event } A) = 1 - P(\textit{no} \text{ occurrences of event } A)$$

27. Conditional Probability

**FORMAL APPROACH FOR FINDING $P(B|A)$**

The probability $P(B|A)$ can be found by dividing the probability of events $A$ and $B$ both occurring by the probability of event $A$:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

28. P(B|A) are generally not equal to P(A|B)
29. Bayes Theorem

**TABLE 4-2** Test Results

| | **Positive Test Result** (Test shows cancer.) | **Negative Test Result** (Test shows no cancer.) | **Total** |
|---|---|---|---|
| **Cancer** | 8 (True Positive) | 2 (False Negative) | 10 |
| **No Cancer** | 99 (False Positive) | 891 (True Negative) | 990 |

The solution in Example 4 is not very difficult. Another approach is to compute the probability using this formula commonly given with Bayes' theorem:

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{[P(A) \cdot P(B|A)] + [P(\overline{A}) \cdot P(B|\overline{A})]}$$

If we replace $A$ with $C$ and replace $B$ with "positive," we get this solution for Example 4:

$$P(\text{C}|\text{positive}) = \frac{P(\text{C}) \cdot P(\text{positive}|\text{ C})}{P(\text{C}) \cdot P(\text{positive}|\text{ C}) + P(\overline{\text{C}}) \cdot P(\text{positive}|\text{ }\overline{\text{C}})}$$

$$= \frac{0.01 \cdot 0.80}{(0.01 \cdot 0.80) + (0.99 \cdot 0.10)} = 0.0748$$

30. Factorial Rule: The number of different arrangements( order matters ) of n different items when all n of them are selected is n!

31. Permutations and Combinations

**DEFINITIONS**

**Permutations** of items are arrangements in which different sequences of the same items are counted *separately*. (The letter arrangements of abc, acb, bac, bca, cab, and cba are all counted *separately* as six different permutations.)

**Combinations** of items are arrangements in which different sequences of the same items are counted as being the *same*. (The letter arrangements of abc, acb, bac, bca, cab, and cba are all considered to be *same* combination.)

32. How to remember P or C

**Mnemonics for Permutations and Combinations**

- Remember "**P**ermutations **P**osition," where the alliteration reminds us that with permutations, the positions of the items makes a difference.
- Remember "**C**ombinations **C**ommittee," which reminds us that with members of a committee, rearrangements of the same members result in the same committee, so order does not count.

33. Permutation Rule

**PERMUTATIONS RULE**

When $n$ different items are available and $r$ of them are selected without replacement, the number of different permutations (order counts) is given by

$$_nP_r = \frac{n!}{(n-r)!}$$

34. Combination Rule

**COMBINATIONS RULE:**

When $n$ different items are available, but only $r$ of them are selected *without replacement,* the number of different combinations (order does not matter) is found as follows:

$$_nC_r = \frac{n!}{(n-r)!r!}$$