Discrete Probability Distribution

Glossary

1. **Probability Distribution**

> **DEFINITIONS**
>
> A **random variable** is a variable (typically represented by *x*) that has a single numerical value, determined by chance, for each outcome of a procedure.
>
> A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a table, formula, or graph.

2. **Discrete vs Continuous Random Variable**

   A discrete random variable has a collection of values that is finite or countable.
   (If there are infinitely many values, the number of values is countable if it is possible to count them individually, such as the number of tosses of a coin before getting heads.)
   A continuous random variable has infinitely many values, and the collection of values is not countable. (That is, it is impossible to count the individual items because
   at least some of them are on a continuous scale, such as body temperatures.)

3. **Conditions for Probability Distribution**

   1.There is a numerical (not categorical) random variable x, and its number values are associated with corresponding probabilities.
   2. ΣP(x) = 1 where x assumes all possible values. (The sum of all probabilities must be 1, but sums such as 0.999 or 1.001 are acceptable because they result from rounding errors.)
   3. 0<=P(x)<=1 for every individual value of the random variable

4. **Probability Distribution Formula**

$P(x) = \frac{1}{2(2-x)!x!}$ (where x can be 0, 1, or 2). Using that formula, we find that $P(0) = 0.25$, $P(1) = 0.50$, and $P(2) = 0.25$. The probabilities found using this

5. **Parameters of Probability Distribution**

FORMULA 5-1 Mean μ for a probability distribution

$$\mu = \Sigma[x \cdot P(x)]$$

FORMULA 5-2 Variance σ^2 for a probability distribution

$$\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)] \text{ (This format is easier to understand.)}$$

FORMULA 5-3 Variance σ^2 for a probability distribution

$$\sigma^2 = \Sigma[x^2 \cdot P(x)] - \mu^2 \text{ (This format is easier for manual calculations.)}$$

FORMULA 5-4 Standard deviation σ for a probability distribution

$$\sigma = \sqrt{\Sigma[x^2 \cdot P(x)] - \mu^2}$$

When applying Formulas 5-1 through 5-4, use the following rule for rounding results.

Round-Off Rule for μ, σ, and σ^2 from a Probability Distribution

Round results by carrying *one more decimal place* than the number of decimal places used for the random variable x. If the values of x are integers, round μ, σ, and σ^2 to one decimal place.

6. **Expected value = Mean**

7. **Significance**

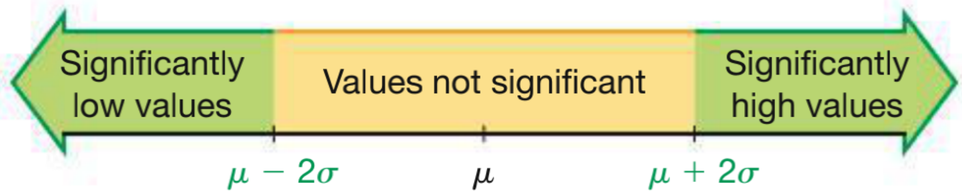
Range Rule of Thumb for Identifying Significant Values

Significantly low values are $(\mu - 2\sigma)$ or lower.

Significantly high values are $\mu + 2\sigma$ or higher.

Values not significant: Between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$

Figure 3-3 from Section 3-2 illustrates the above criteria:



8. Significantly high number of successes: x successes among n trials is a significantly high number of successes if the probability of x or more successes is 0.05 or less. That is, x is a significantly high number of successes if P(x or more) … 0.05.*

   Significantly low number of successes: x successes among n trials is a significantly low number of successes if the probability of x or fewer successes is 0.05 or less. That is, x is a significantly low number of successes if P(x or fewer) … 0.05.*

9. **Binomial Probability Distribution**

   A binomial probability distribution results from a procedure that meets these four requirements:

   1. The procedure has a fixed number of trials. (A trial is a single observation.)
   2. The trials must be independent, meaning that the outcome of any individual trial doesn’t affect the probabilities in the other trials.
   3. Each trial must have all outcomes classified into exactly two categories, commonly referred to as success and failure.
   4. The probability of a success remains the same in all trials.

FORMULA 5-5 Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x} \qquad \text{for } x = 0, 1, 2, \ldots, n$$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial $(q = 1 - p)$

10. **Parameters for Binomial Distribution**

For Binomial Distributions

Formula 5-6	Mean:	$\mu = np$
Formula 5-7	Variance:	$\sigma^2 = npq$
Formula 5-8	Standard Deviation:	$\sigma = \sqrt{npq}$

11. **Poisson Dsitribution**

A Poisson probability distribution is a discrete probability distribution that applies to occurrences of some event over a specified interval. The random variable x is the number of occurrences of the event in an interval. The interval can be time, distance, area, volume, or some similar unit.

FORMULA 5-9 Poisson Probability Distribution

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$$

where $e \approx 2.71828$

μ = mean number of occurrences of the event in the intervals

Requirements for the Poisson Probability Distribution

1. The random variable x is the number of occurrences of an event *in some interval.*
2. The occurrences must be *random.*
3. The occurrences must be *independent* of each other.
4. The occurrences must be *uniformly distributed* over the interval being used.

Parameters of the Poisson Probability Distribution

- The mean is μ.
- The standard deviation is $\sigma = \sqrt{\mu}$.

mean = np