Model Formulation:

$Y_i = \alpha + \beta \ln(\delta + X_i) + \varepsilon_i;\ \varepsilon_i \sim N(0, \sigma_e^2)$;

Assumption:

$f(\alpha_t, \beta_t) \sim BVN\left(\alpha_{t-1}, \sigma_\alpha^2, \beta_{t-1}, \sigma_\beta^2, \rho\right);\ \sigma_e^2 \sim IG(p, q);\ \delta_t \sim U(X_{min} - \delta_{t-1}, X_{min} + \delta_{t-1});$

$\boldsymbol{\theta}_t = \left(\alpha_t, \sigma_\alpha^2, \beta_t, \sigma_\beta^2, \rho, \delta_t\right)$

Likelihood:

$$\boldsymbol{L}(\boldsymbol{Y}|\boldsymbol{\theta}_t, \boldsymbol{X}, \sigma_e^2) = \frac{1}{(2\pi\sigma_e^2)^{n/2}} \exp\left[-\frac{1}{2\sigma_e^2}\sum_{i=1}^{n}\{Y_i - \alpha_t - \beta_t \ln(\delta_t + X_i)\}^2\right]$$

Prior Distribution:

$$\boldsymbol{h}(\boldsymbol{\theta}_t|\boldsymbol{\theta}_{t-1}) \propto \frac{1}{\sigma_\alpha^2\sigma_\beta^2\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{\alpha_t-\alpha_{t-1}}{\sigma_\alpha}\right)^2 + \left(\frac{\beta_t-\beta_{t-1}}{\sigma_\beta}\right)^2 - 2\rho\left(\frac{\alpha_t-\alpha_{t-1}}{\sigma_\alpha}\right)\left(\frac{\beta_t-\beta_{t-1}}{\sigma_\beta}\right)\right\}\right] + \delta_t$$

Posterior Distribution:

$\boldsymbol{L}(\boldsymbol{Y}|\boldsymbol{\theta}_t, \boldsymbol{X}, \sigma_e^2) \times \boldsymbol{h}(\boldsymbol{\theta}_t|\boldsymbol{\theta}_{t-1}) \propto$

$$\sigma_e^{-(n+p+2)} \times \exp\left[-\frac{1}{2\sigma_e^2}\{\textstyle\sum_{i=1}^{n}\{Y_i - \alpha_t - \beta_t \ln(\delta_t + X_i)\}^2 + q\}\right] \times \left(\exp\left[-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{\alpha_t-\alpha_{t-1}}{\sigma_\alpha}\right)^2 + \left(\frac{\beta_t-\beta_{t-1}}{\sigma_\beta}\right)^2 - 2\rho\left(\frac{\alpha_t-\alpha_{t-1}}{\sigma_\alpha}\right)\left(\frac{\beta_t-\beta_{t-1}}{\sigma_\beta}\right)\right\}\right] + \delta_t\right)$$

Step by Step Estimation Algorithm:

1. Draw $\alpha_0, \beta_0 \sim BVN\left(\mu_\alpha, \sigma_\alpha^2, \mu_\beta, \sigma_\beta^2, \rho\right);\ \delta_0 \sim U\left(X_{(0)}, X_{(1)}\right);\ \sigma_e^2 \sim IG(p = 0.01, q = 0.01)$
2. Estimate equivalent posterior distribution function using (1); let say $\boldsymbol{L_0}$
3. For $\boldsymbol{t} = 1$ to $\boldsymbol{N}$(=500,000); Draw the $\boldsymbol{\theta}_t = \left(\alpha_t, \sigma_\alpha^2, \beta_t, \sigma_\beta^2, \rho, \delta_t\right)$ as following
    a. $\alpha_t,\ \beta_t \sim BVN\left(\alpha_{t-1}, \sigma_\alpha^2, \beta_{t-1}, \sigma_\beta^2, \rho\right)$
    b. $\delta_t \sim U(X_{min} - \delta_{t-1}, X_{min} + \delta_{t-1})$
    c. $\sigma_e^2 \sim IG(p = 0.01, q = 0.01)$
4. Estimate posterior distribution for $\boldsymbol{t}^{th}$ draw; let say $\boldsymbol{L_t}$
5. Compute $\boldsymbol{\tau} = min\left(1, \frac{\boldsymbol{L_t}}{\boldsymbol{L_{t-1}}}\right)$ and draw $\boldsymbol{\varphi} \sim U(0, 1)$
6. If $\boldsymbol{\tau} > \boldsymbol{\varphi}$; then $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t$ *else* $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_{t-1}$
7. Following criteria used for estimation:
    a. Eliminate first 50,000 sample as burn-in sample.
    b. Consider the 5th sample to eliminate auto-correlation.
    c. Estimation is done based on 10,000 samples.