Instantaneous Spectral Analysis (ISA) MATLAB® Toolbox User Guide Version 1.0

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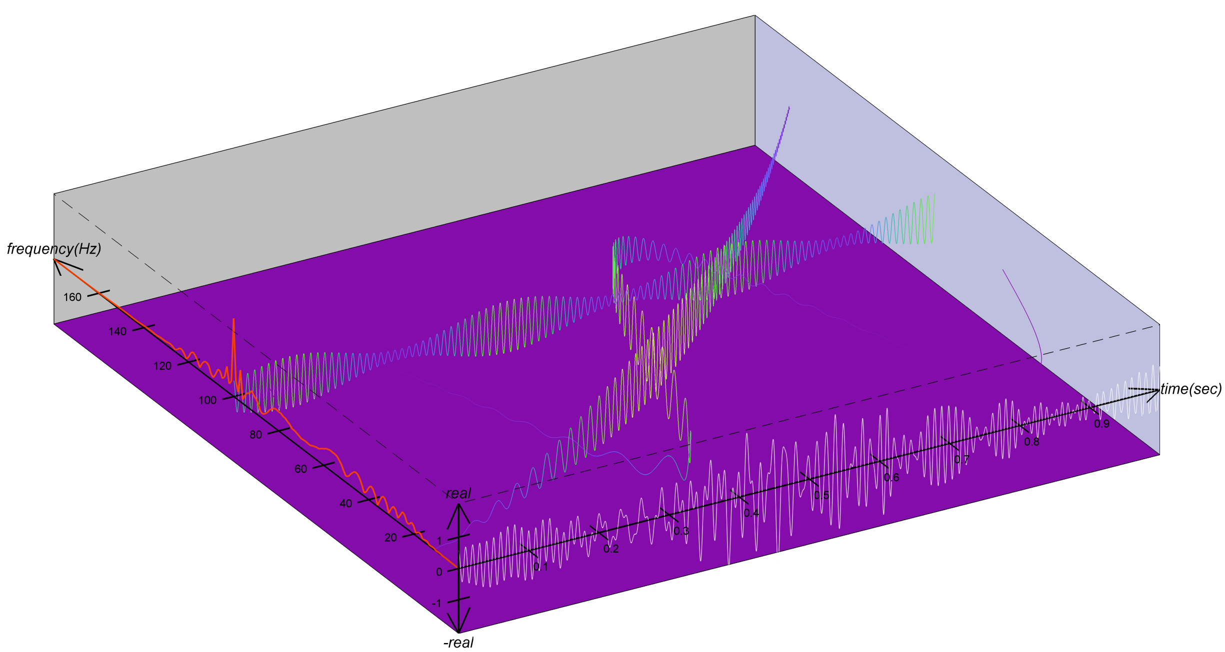


Contents

Preface

ISA Toolbox is a software library for MATLAB implementing algorithms related to AM–FM modeling and the Instantaneous Spectrum. The most important research paper related to the theoretical foundation for the Instantaneous Spectral Analysis (ISA) is [1]. The ISA Toolbox has been has been developed for research purposes, and while the reuse of this code can be seen as good practice, copying other peoples computer code without citing it correctly may be a plagiarism violation. If you use this code in your research please cite the following work:

```
@article{ISA2018_Sandoval,
           title = {The Instantaneous Spectrum: A General Framework for
                    Time-Frequency Analysis},
           author = {S.~Sandoval and P.~L.~De~Leon},
           journal = {{IEEE Trans.~Signal Process.}},
           volume = {66},
           year = {2018},
           month = {Nov},
           pages = {5679-5693}
}
```

Chapter 1

Introduction

Table of Principle Symbols

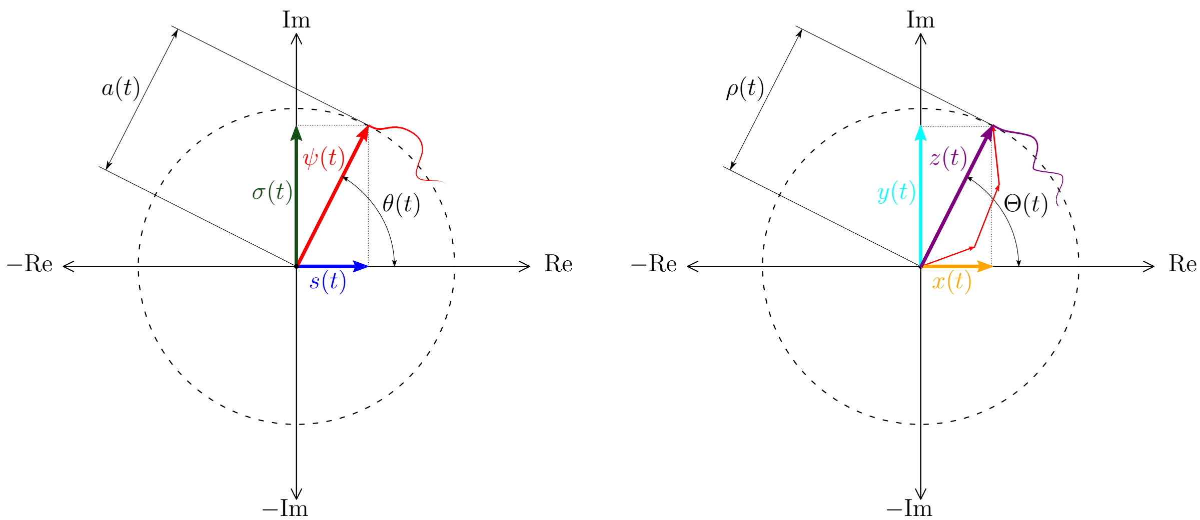
Symbol	**Description**
t	time instants
$\mathscr{S}$	the component set, $\mathscr{S} \triangleq \{\mathscr{C}_0, \mathscr{C}_1, \cdots, \mathscr{C}_{K-1}\}$
$\mathscr{C}_k$	canonical triplet for the kth AM–FM component, $\mathscr{C}_k \triangleq (a_k(t), \omega_k(t), \phi_k)$
$\psi_k(t)$	the kth AM–FM component, $\psi_k(t) \triangleq a_k(t)\mathrm{e}^{\mathrm{j}\left[\int_{-\infty}^{t} \omega_k(\uptau)\,\mathrm{d}\uptau + \phi_k\right]}$
$s_k(t)$	real part of the kth component, $s_k(t) = \mathrm{Re}\{\psi_k(t)\}$
$\sigma_k(t)$	imaginary part of the kth component, $\sigma_k(t) = \mathrm{Im}\{\psi_k(t)\}$
$a_k(t)$	instantaneous amplitude (IA) of the kth component, $a_k(t) = \pm\mathrm{abs}\{\psi_k(t)\}$
$\theta_k(t)$	phase, or instantaneous angle of the kth component, $\theta_k(t) = \arg\{\psi_k(t)\}$
$\omega_k(t)$	instantaneous frequency (IF) of the kth component, $\omega_k(t) = \frac{\mathrm{d}}{\mathrm{d}t}\theta_k(t)$
$m_k(t)$	frequency modulation (FM) message of the kth component, $m_k(t) = \omega_k(t) - \varpi_k$
$M_k(t)$	phase modulation (PM) message of the kth component, $M_k(t) = \int_{-\infty}^{t} m_k(\uptau)\,\mathrm{d}\uptau$
ϕ_k	phase reference of the kth component, $\int_{-\infty}^{0} m_k(t)\,\mathrm{d}t = M_k(0) = 0 \implies \phi_k = \theta_k(0)$
ϖ_k	frequency reference of the kth component, $\varpi_k = \omega_k(0) - m_k(0)$
$\mathcal{S}(t, \omega)$	the instantaneous spectrum (IS)
$z(t)$	the complex signal, $z(t) = \sum_{k=0}^{K-1} \psi_k(t)$
$x(t)$	real part of the complex signal, $x(t) = \mathrm{Re}\{z(t)\}$
$y(t)$	imaginary part of the complex signal, $y(t) = \mathrm{Im}\{z(t)\}$
$\rho(t)$	instantaneous amplitude (IA) of the signal, $\rho(t) = \pm\lvert z(t)\rvert$
$\Theta(t)$	phase, or instantaneous angle of the signal, $\Theta(t) = \arg\{z(t)\}$
$\Omega(t)$	instantaneous frequency (IF) of the signal, $\Omega(t) = \frac{\mathrm{d}}{\mathrm{d}t}\Theta(t)$
$\uptau$	a dummy time variable for integration
τ	a time-shift variable
$Z(\mathrm{j}\omega)$	the Fourier spectrum, $Z(\mathrm{j}\omega) = \int_{-\infty}^{\infty} z(t)\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}t$

1.1 The Complex AM–FM Component

In the most general sense, a complex AM–FM component is any complex-valued signal that can be expressed as

$$\psi(t;\mathscr{C}) \triangleq a(t)\exp\left(\mathrm{j}\left[\int_{-\infty}^{t}\omega(\tau)\,\mathrm{d}\tau+\phi\right]\right) \tag{1.1}$$

where $\mathscr{C} \triangleq (a(t), \omega(t), \phi)$ is a canonical triplet. This definition is useful because it guarantees differentiability of the phase function, ensuring both a well-defined IA and IF.



(a) (b)

Figure 1.1.1: (a) Argand diagram of an AM–FM component in (1.1) at some time instant. Each component, $\psi(t)$ (→) is interpreted as a vector: the IA $a(t)$ (↔) is interpreted as the component vector's length, the phase $\theta(t)$ (↶) is interpreted as a component vectors's angular position. Although not shown, the IF $\omega(t)$ is interpreted as a component vector's angular velocity and phase reference ϕ is interpreted the angular position at $t = 0$. The real part of the component $s(t)$ (→) and the imaginary part of the component $\sigma(t)$ (→) are interpreted as orthogonal projections of $\psi(t)$. We have included an example path (—) taken by $\psi(t)$. (b) Argand diagram of the signal $z(t)$ (→) in (1.2) at some time instant, composed of a superposition of components (→). The signal is interpreted as a vector: the IA $\rho(t)$ (↔) is interpreted as the signal vector's length, the phase $\Theta(t)$ (↶) is interpreted as a signal vector's angular position, and although not shown the IF $\Omega(t)$ is interpreted as a signal vector's angular velocity. The real part of the signal $x(t)$ (→) and the imaginary part of the signal $y(t)$ (→) are interpreted as orthogonal projections of $z(t)$. We have included an example path (—) taken by $z(t)$.

1.2 The Complex AM–FM Model

We meticulously parameterize the complex AM–FM model for a complex signal $z(t)$ as a superposition of K (possibly infinite) complex AM–FM components

$$
\begin{aligned}
z\big(t;\mathscr{S}\big) &\triangleq \sum_{k=0}^{K-1}\psi_k\big(t;\mathscr{C}_k\big) && (1.2\text{a})\\
&= \rho(t)\mathrm{e}^{\mathrm{j}\left[\int_{-\infty}^{t}\Omega(\tau)\,\mathrm{d}\tau+\Phi\right]} && (1.2\text{b})\\
&= \rho(t)\mathrm{e}^{\mathrm{j}\Theta(t)} && (1.2\text{c})\\
&= x(t)+\mathrm{j}y(t) && (1.2\text{d})
\end{aligned}
$$

where $\mathscr{S} \triangleq \{\mathscr{C}_0, \mathscr{C}_1, \cdots, \mathscr{C}_{K-1}\}$ is the component set; $\mathscr{C}_k \triangleq (a_k(t), \omega_k(t), \phi_k)$ is the canonical triplet for the kth component; and for the signal $z(t)$, $\rho(t)$ is the IA, $\Theta(t)$ is the phase function, $\Omega(t) = \frac{\mathrm{d}}{\mathrm{d}t}\Theta(t)$ is the IF, Φ is the phase reference, $x(t)$ is the real part, and $y(t)$ is the imaginary part. The kth complex AM–FM component in (1.2a) is defined as

$$
\begin{aligned}
\psi_k\big(t;\mathscr{C}_k\big) &\triangleq a_k(t)\mathrm{e}^{\mathrm{j}\left[\int_{-\infty}^{t}\omega_k(\tau)\,\mathrm{d}\tau+\phi_k\right]} && (1.3\text{a})\\
&= a_k(t)\mathrm{e}^{\mathrm{j}\theta_k(t)} && (1.3\text{b})\\
&= s_k(t)+\mathrm{j}\sigma_k(t) && (1.3\text{c})
\end{aligned}
$$

where for the kth AM–FM component $\psi_k(t)$, $a_k(t)$ is the IA, $\theta_k(t)$ is the phase function,

$$\omega_k(t) = \frac{\mathrm{d}}{\mathrm{d}t}\theta_k(t) \tag{1.4}$$

is the IF, ϕ_k is the phase reference, $s_k(t)$ is the real part, and $\sigma_k(t)$ is the imaginary part. Rewriting $\omega_k(t) = \varpi_k + m_k(t)$, the phase $\theta_k(t)$ may be expressed in terms of frequency reference ϖ_k and FM message $m_k(t)$ as $\theta_k(t) = \varpi_k t + \int_{-\infty}^{t} m_k(\tau)\,\mathrm{d}\tau + \phi_k$ or in terms of the phase modulation message $M_k(t)$ as $\theta_k(t) = \varpi_k t + M_k(t) + \phi_k$. The geometric interpretations of the AM–FM component in (1.1) and the AM–FM model in (1.2) are illustrated with the Argand diagrams in Fig. 1.1.1. The AM–FM component can be visually interpreted as a single rotating vector in the complex plane with time-varying length and time-varying angular velocity. The time evolution of the AM–FM model may be interpreted directly in terms of mechanics—where the sometimes misunderstood concept of IF may be conveniently interpreted as angular velocity.

We assume that the phase and frequency references are selected at $t = 0$, i.e. $\int_{-\infty}^{0}\omega_k(t)\,\mathrm{d}t = 0$ which implies $\int_{-\infty}^{0} m_k(t)\,\mathrm{d}t = M_k(0) = 0$, $\phi_k = \theta_k(0)$, and $\varpi_k = \omega_k(0) - m_k(0)$. We define a *monocomponent signal* as any signal expressed with $K = 1$ and thus with a single canonical triplet. A *multicomponent signal* can then be defined as any signal expressed with $K > 1$ and thus with a set of canonical triplets.

1.3 Instantaneous Spectral Analysis

We use the term instantaneous spectral analysis (ISA) to refer to a very general framework for TFA consisting of three parts: 1) a parameter set, 2) an instantaneous spectrum, and 3) a signal model. Specifically, in the ISA framework: 1) a signal is represented by a set of canonical triplets $\mathscr{S} = \{\mathscr{C}_0, \mathscr{C}_1, \cdots, \mathscr{C}_{K-1}\}$, 2) each component set has a single-valued mapping to an IS $\mathscr{S} \mapsto \mathcal{S}(t,\omega)$, and 3) each IS has a single-valued mapping to a signal $\mathcal{S}(t,\omega) \mapsto z(t)$.

For the signal model we use the complex AM–FM model as parameterized in Section 1.2. Thus using terminology by Flandrin, the complex AM–FM signal model is a formal *model* because the structure of the analyzed signal is incorporated in the parameterization and is also a formal *decomposition* because the construction process is a linear superposition of time-frequency atoms [2]. In fact, we consider the AM–FM component parameterized by $\mathscr{C}$ in (1.1) as the most general form of a time-frequency atom.

The IS is both *moving and joint* because it consists of a local description in both time and frequency and *evolutionary* because the coefficients are explicitly time-dependent [2]. The IS is also "*causal*" in the sense of Page and Gupta [3, 4], i.e. the value at time t_0 does not require the signal for $t > t_0$. Although the IS may be considered a "*distribution*" in the sense that it describes the energy allocation in time and frequency, it is not a formal TFD [5] because it is not obtained via an integral transform.

At its heart, the TFA problem is that of signal representation. ISA provides a mathematical framework which is similar to a coordinate system. When using IS framework the TFA problem may be stated as follows. For a given signal $z(t)$ or $x(t)$ find $\mathscr{S}$ subject to

$$\mathscr{S} \overset{\text{Eqn. (1.5)}}{\longmapsto} \mathcal{S}(t,\omega) \overset{\text{Eqn. (1.6)}}{\longmapsto} z(t) \overset{\text{Eqn. (1.7)}}{\longmapsto} x(t).$$

The problem is not, in general, uniquely solvable because the problem is under-constrained—providing few general mathematical constraints and allowing for infinite degrees of freedom. This under-determinedness manifests as an infinite number of ways to decompose a signal into a sum of parts, each leading to a different IS.

1.3.1 Definition of the Instantaneous Spectrum

We define the IS in the *time-frequency coordinates* for a signal expressed with set of canonical triplets $\mathscr{S} = \{\mathscr{C}_0, \mathscr{C}_1, \cdots, \mathscr{C}_{K-1}\}$ as

$$\begin{aligned}\mathcal{S}(t,\omega;\mathscr{S}) &\triangleq 2\pi \sum_{k=0}^{K-1} \int_{-\infty}^{\infty} \psi_k\big(\tau;\mathscr{C}_k\big)\, {}^2\delta\big(t-\tau, \omega-\omega_k(\tau)\big)\,\mathrm{d}\tau \\ &= 2\pi \sum_{k=0}^{K-1} \psi_k\big(t;\mathscr{C}_k\big)\,\delta\big(\omega-\omega_k(t)\big)\end{aligned} \tag{1.5}$$

where $\delta(\cdot)$ and ${}^2\delta(\cdot,\cdot)$ are 1-D and 2-D Dirac deltas and we have used the well-known sifting property $\int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,\mathrm{d}\tau = f(t)$ and ${}^2\delta(t,\omega) = \delta(t)\delta(\omega)$ [6].

The IS, $\mathcal{S}(t,\omega;\mathscr{S})$ maps to signal $z(t;\mathscr{S})$ with

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{S}(t,\omega;\mathscr{S})\,\mathrm{d}\omega = z(t;\mathscr{S}). \tag{1.6}$$

One consideration with the IS is that in general, it is not unique for a particular signal under analysis. That is, in general, there are an infinite number of different sets of canonical triplets $\mathscr{S} = \{\mathscr{C}_0, \mathscr{C}_1, \cdots, \mathscr{C}_{K-1}\}$ and subsequently an infinite number of instantaneous spectra that can be associated with a given complex signal. Despite this non-uniqueness, this framework is advantageous because the inherent ambiguities allow for flexibility in the model. Furthermore, with the proper constraints placed on the model, a unique IS will arise.

1.3.2 Visualization of the Instantaneous Spectrum

For purposes of interpretation and visualization, we extend (1.5) by defining a 3-D IS in the *time-frequency-real coordinates* as

$$\mathcal{S}(t,\omega,s;\mathscr{S}) = 2\pi \sum_{k=0}^{K-1} \psi_k\big(t;\mathscr{C}_k\big)\,{}^2\delta\big(\omega-\omega_k(t), s-s_k(t)\big)$$

where $s_k(t)$ is the real part of the kth component, as shown in (1.3c), and it is understood that the sifting property has been used to rewrite the 3-D Dirac delta as a 2-D Dirac delta similar to (1.5). We consider the time-frequency-real space as the most intuitive space for interpretation. Integrating out the real dimension, it can be shown that

$$\int_{-\infty}^{\infty} \mathcal{S}(t,\omega,s;\mathscr{S})\,\mathrm{d}s = \mathcal{S}(t,\omega;\mathscr{S}).$$

We can visualize $\mathcal{S}(t,\omega,s)$ by plotting $\omega_k(t)$ vs. $s_k(t)$ vs. t as a line in a 3-D space and coloring the line with respect to $|a_k(t)|$ for each component. Thus, the simultaneous visualization of multiple parameters for each component in the *time-frequency-real* space is possible. Further, orthographic projections of $\mathcal{S}(t,\omega,s)$ yield common plots: the *time-real plane* (the real component waveforms), the *time-frequency plane* (the IS), and the *frequency-real plane* (analogous to the Fourier magnitude spectrum). We consider the 3-D visualization as a type of phase space plot as illustrated in Fig. 1.3.1.

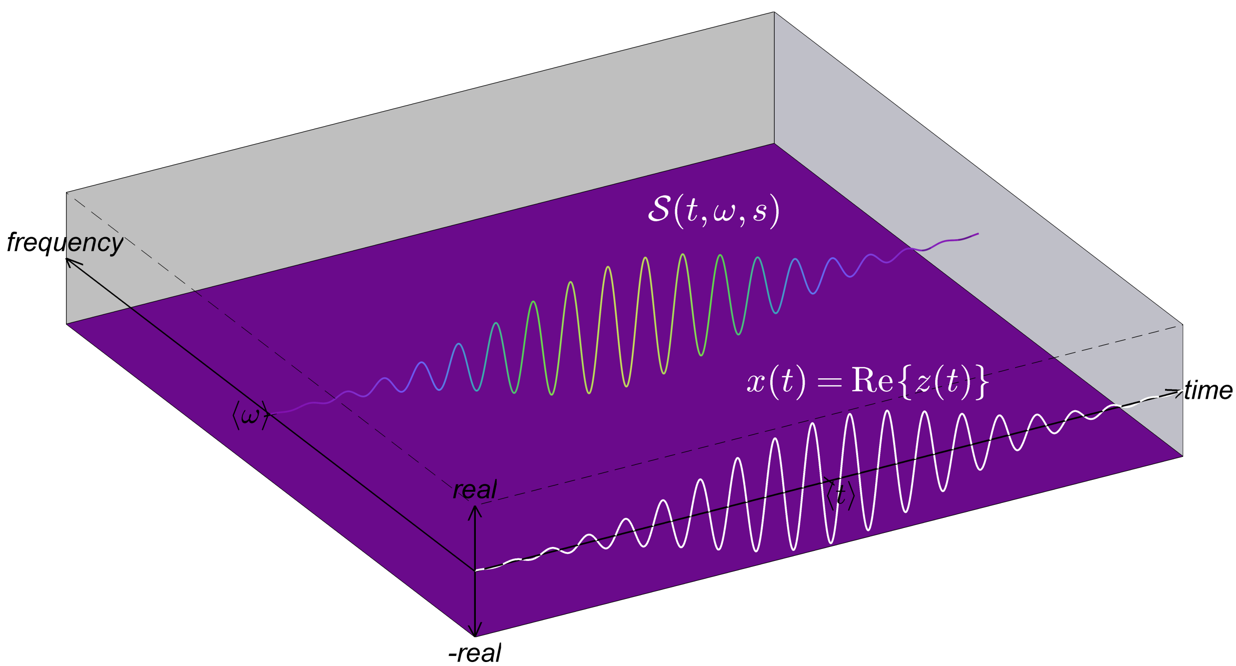


Figure 1.3.1: The 3-D instantaneous spectrum $\mathcal{S}(t,\omega,s)$ (—) for a signal consisting of a single Gabor atom $z(t) = a_0\mathrm{e}^{-\pi(t-\langle t\rangle)^2/(2\sigma_t^2)}\mathrm{e}^{\mathrm{j}\langle\omega\rangle(t-\langle t\rangle)}$ with real part shown along the time axis (—).

1.4 Latent Signal Analysis

The complex-valued nature of the proposed model leads to a new, possibly radical view of signals. This view is that all signals are in fact complex-valued and that in reality, only the real part $x(t)$, is observed (or measured) and the imaginary part $y(t)$, is *latent*, i.e. the act of observation corresponds to

$$z(t) \mapsto x(t) \quad \text{according to} \quad x(t) = \mathrm{Re}\{z(t)\}. \tag{1.7}$$

We term this view Latent Signal Analysis (LSA). The problem considered in LSA is to determine a complex signal extension, i.e. to determine the (total) *latent signal* $z(t) = x(t) + \mathrm{j}y(t)$ when the imaginary part $y(t)$ is hidden, given an observation $x(t)$.

Traditionally, there are two choices for $y(t)$ that are made in the literature. If one chooses $y(t) = 0$, then conjugate symmetry is imposed in frequency, i.e. $Z(\mathrm{j}\omega) = Z^*(-\mathrm{j}\omega)$. If one chooses $y(t) = \mathcal{H}\{x(t)\}$, then single-sidedness is imposed in frequency, $Z(\mathrm{j}\omega) = 0$ for $\omega < 0$, which may also be considered a symmetry in frequency, as we will show. We note that both of these choices may artificially impose symmetry.

In the context of this work, determining the IA/IF of a signal becomes that of determining $y(t)$ and hence $z(t)$ from the observation (or measurement) of $x(t)$, in the most general way possible *without imposing unnecessary constraints*. In the frequency domain, this problem can alternatively be stated as that of determining the *latent spectrum*, $Z(\mathrm{j}\omega) = X(\mathrm{j}\omega) + \mathrm{j}Y(\mathrm{j}\omega)$ from $X(\mathrm{j}\omega)$ where both $X(\mathrm{j}\omega)$ and $Y(\mathrm{j}\omega)$ are complex-valued.

Chapter 2

Modulation/Demodulation Algorithms

The functions in this chapter, are used to create complex AM–FM signal components (modulate) and extract component parameters from complex AM–FM signal components (demodulate).

Contents

2.1 amfmmod.m

Call Syntax: [psi, s, sigma, fi, t, theta] = amfmmod(a, m, fc, fs, phi, method)

Description: This function synthesizes an AM-FM component based on a specified instantaneous amplitude, FM message, frequency reference, sampling frequency, and phase reference. Additionally, there is an optional parameter to specify the method for integration approximation.

Algorithm 1 Synthesis of a complex AM–FM component from parameters.

procedure $[\psi_0(t), s_0(t), \sigma_0(t), \frac{\omega_0(t)}{2\pi}, t, \theta_0(t)] = \text{amfmmod}(\ a_0(t), \frac{m_0(t)}{2\pi}, \frac{\varpi_0}{2\pi}, f_s, \phi_0\ )$
    $t \in (0, T)$
    $M_0(t) = \int_0^t m_0(\tau)\,\mathrm{d}\tau$
    $\omega_0(t) = 2\pi[\varpi_0 + M_0(t)]$
    $\theta_0(t) = \varpi_0 + M_0(t) + \phi_0$
    $\psi_0(t) = a_0(t)\exp[\mathrm{j}\theta_0(t)]$
    $s_0(t) = \mathrm{Re}\{\psi_0(t)\}$
    $\sigma_0(t) = \mathrm{Im}\{\psi_0(t)\}$
end procedure

Input Arguments:

Name: $\text{a} = \begin{bmatrix} \top \\ a_0(t) \\ \bot \end{bmatrix}$
Type: vector (real)
Description: instantaneous amplitude

Name: $\text{m} = \frac{1}{2\pi}\begin{bmatrix} \top \\ m_0(t) \\ \bot \end{bmatrix}$
Type: vector (real)
Description: FM message in Hertz

Name: $\text{fc} = \frac{\varpi_0}{2\pi}$
Type: scalar
Description: initial frequency in Hertz

Name: $\text{fs} = f_s$
Type: scalar
Description: sampling frequency in Hertz

Name: $\text{phi} = \phi_0$
Type: scalar
Description: phase reference in radians

Name: method
Type: string
Description: numerical integration method: 'left' or 'right' or 'center' or 'trapz'[default] or 'simps'

Output Arguments:

Name: psi $= \begin{bmatrix} \top \\ \psi_0(t) \\ \bot \end{bmatrix}$
Type: vector (complex)
Description: AM-FM component

Name: s $= \begin{bmatrix} \top \\ s_0(t) \\ \bot \end{bmatrix}$
Type: vector
Description: real part of the AM-FM signal

Name: sigma $= \begin{bmatrix} \top \\ \sigma_0(t) \\ \bot \end{bmatrix}$
Type: vector
Description: imaginary part of the AM-FM signal

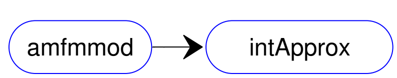
Name: fi $= \frac{1}{2\pi} \begin{bmatrix} \top \\ \omega_0(t) \\ \bot \end{bmatrix}$
Type: vector
Description: instantaneous frequency of the AM-FM signal

Name: t $= \begin{bmatrix} 0 \\ \frac{1}{f_s} \\ \vdots \\ \bot \end{bmatrix}$
Type: vector
Description: time instants

References:

Notes:

Function Dependencies:



2.2 amfmdemod.m

Call Syntax: [A,IF,S,SIGMA,THETA] = amfmdemod(PSI,varargin)

Description: This function demodulates a complex AM-FM component.

Algorithm 2 Demodulation of a complex AM–FM component.

procedure $[a_k(t), \omega_k(t), s_k(t), \sigma_k(t), \theta_k(t)] = \text{amfmdemod}(\ \psi_k(t)\ )$
  for $k \in \{0, 1, \cdots, K-1\}$ **do**
    $\theta_k(t) = \text{unwrap}\Big(\angle\psi_k(t)\Big)$
    $\omega_k(t) = \dfrac{\mathrm{d}}{\mathrm{d}t}\theta_k(t)$
    $a_k(t) = |\psi_k(t)|$
    $s_k(t) = \text{Re}\{\psi_k(t)\}$
    $\sigma_k(t) = \text{Im}\{\psi_k(t)\}$
  end for
end procedure

Input Arguments:

Name: $\text{PSI} = \begin{bmatrix} \underset{\bot}{\overset{\top}{\psi_0(t)}} & \underset{\bot}{\overset{\top}{\psi_1(t)}} & \cdots & \underset{\bot}{\overset{\top}{\psi_{K-1}(t)}} \end{bmatrix}$
Type: complex matrix (or vector)
Description: the kth column is the kth component $\psi_k(t)$

Optional Input Arguments:

Name: demodPhaseDeriv
Type: string
Default: 'center9'
Description: numerical differentiation methodmethod: 'forward' or 'backward' or 'center3' or 'center5' or 'center7' or 'center9' [default] or 'center11' or 'center13' or 'center15'

Name: fs
Type: scalar
Default: 1
Description: sampling freq

Output Arguments:

Name: $\mathrm{A} = \begin{bmatrix} \top & \top & & \top \\ a_0(t) & a_1(t) & \cdots & a_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IA of $\psi_k(t)$

Name: $\mathrm{IF} = \dfrac{1}{2\pi} \begin{bmatrix} \top & \top & & \top \\ \omega_0(t) & \omega_1(t) & \cdots & \omega_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IF of $\psi_k(t)$ in Hertz

Name: $\mathrm{S} = \begin{bmatrix} \top & \top & & \top \\ s_0(t) & s_1(t) & \cdots & s_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the real part of $\psi_k(t)$

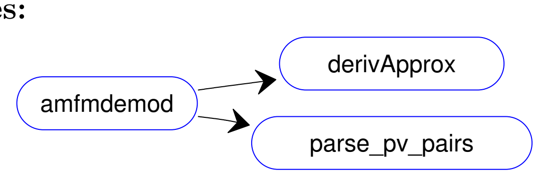
Name: $\mathrm{SIGMA} = \begin{bmatrix} \top & \top & & \top \\ \sigma_0(t) & \sigma_1(t) & \cdots & \sigma_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the imaginary part of $\psi_k(t)$

Name: $\mathrm{THETA} = \begin{bmatrix} \top & \top & & \top \\ \theta_0(t) & \theta_1(t) & \cdots & \theta_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the instantaneous angle of $\psi_k(t)$

References:

Notes:

Function Dependencies:



Chapter 3

Decomposition Algorithms

This Chapter is in Development

Chapter 4

Visualization

The functions in this chapter are used to visualize and interpret the instantaneous spectrum. Additionally, some functions are provided to help facilitate the comparison to other time-frequency methods.

Contents

4.1 Argand.m

Call Syntax: h = Argand(t,psi)

Description: This function plots an AM-FM component as a rotating vector.

Input Arguments:

Name: t
Type: vector (real)
Description: time instants

Name: psi = $\begin{bmatrix} \top \\ \psi_0(t) \\ \bot \end{bmatrix}$
Type: vector (complex)
Description: AM-FM component

Output Arguments:

Name: h
Type: handle
Description: figure handle

References:

Notes:

Function Dependencies:

4.2 ISA2dPlot.m

Call Syntax: h = ISA2dPlot(t, S, IF, A, fs, fMax)

Description: This function plots a 2D instantenous spectrum

Name: t
Type: vector (real)
Description: time instants

Name: S = $\begin{bmatrix} \top & \top & & \top \\ s_0(t) & s_1(t) & \cdots & s_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the real part of $\psi_k(t)$

Name: IF $= \dfrac{1}{2\pi} \begin{bmatrix} \top & \top & & \top \\ \omega_0(t) & \omega_1(t) & \cdots & \omega_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IF of $\psi_k(t)$ in Hertz

Name: A = $\begin{bmatrix} \top & \top & & \top \\ a_0(t) & a_1(t) & \cdots & a_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IA of $\psi_k(t)$

Name: fs
Type: scalar
Description: sampling frequency

Name: fMax or [fMax, fMin] (optional)
Type: scalar or [scalar,scalar]
Description: maximum plotting frequency and optionally the minimum plotting frequency

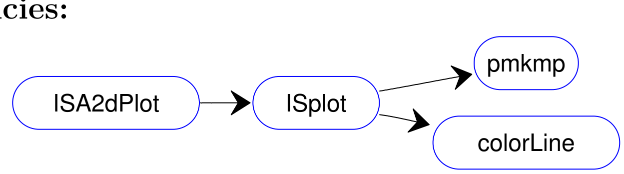
Output Arguments:

Name: h
Type: handle
Description: figure handle

References:

Notes:

Function Dependencies:



4.3 ISA3dPlot.m

Call Syntax: h = ISA3dPlot(t, PSI, IF, A, fs, fMax, STFTparams)

Description: This function plots a 3D instantaneous spectrum.

Name: t
Type: vector (real)
Description: time instants

Name: PSI $= \begin{bmatrix} \top & \top & & \top \\ \psi_0(t) & \psi_1(t) & \cdots & \psi_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: complex matrix (or vector)
Description: the kth column is the kth component $\psi_k(t)$

Name: IF $= \frac{1}{2\pi} \begin{bmatrix} \top & \top & & \top \\ \omega_0(t) & \omega_1(t) & \cdots & \omega_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IF of $\psi_k(t)$ in Hertz

Name: A $= \begin{bmatrix} \top & \top & & \top \\ a_0(t) & a_1(t) & \cdots & a_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IA of $\psi_k(t)$

Name: fs
Type: scalar
Description: sampling frequency

Name: fMax or [fMax, fMin] (optional)
Type: scalar or [scalar,scalar]
Description: maximum plotting frequency and optionally the minimum plotting frequency

Name: STFTparams (optional)
Type: vector (1x2)
Description: STFT parameters: [N_FFT, frame_advance]

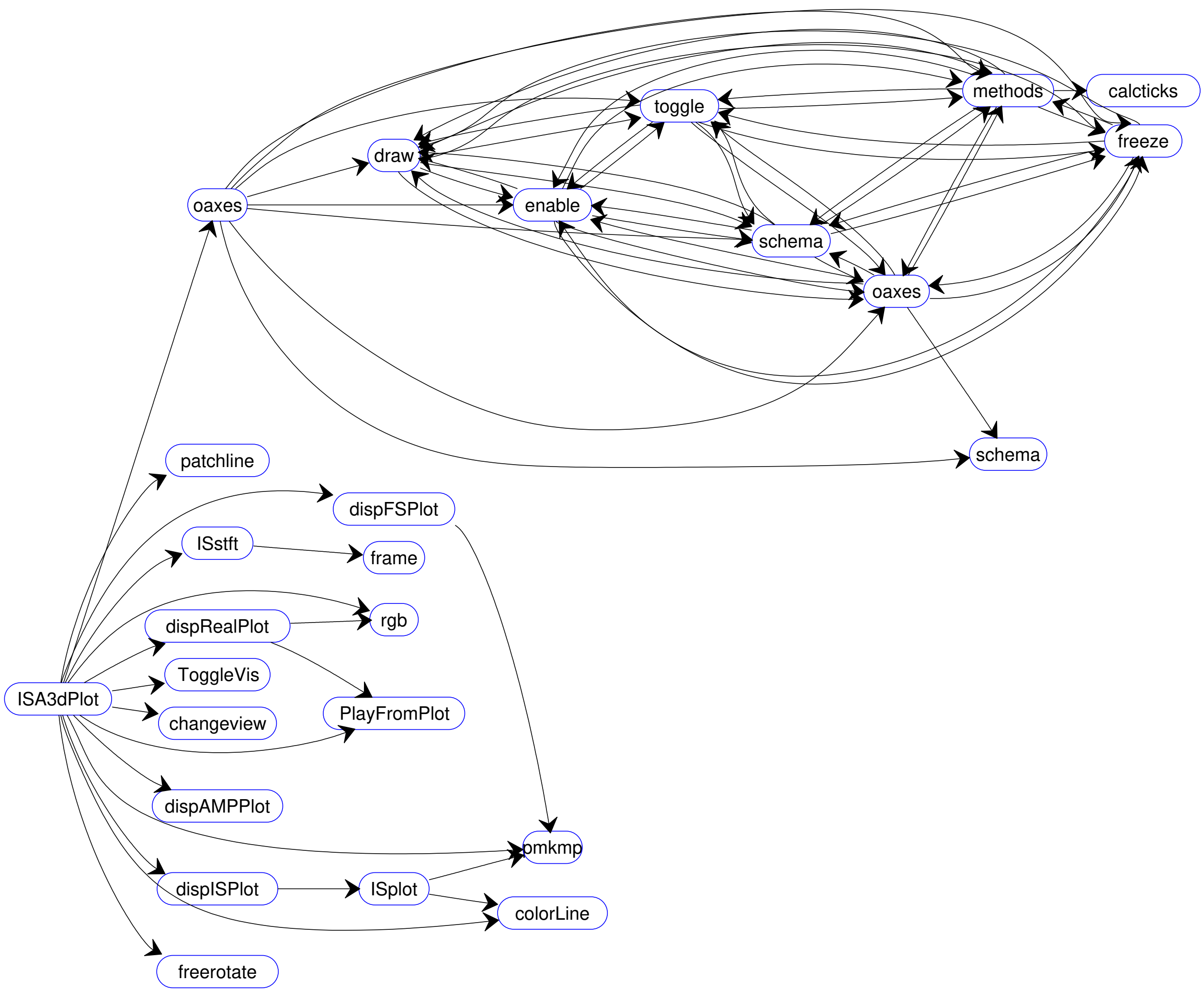
Output Arguments:

Name: h
Type: handle
Description: figure handle

References:

Notes:

Function Dependencies:



4.4 ISA3dPlotPrint.m

Call Syntax: h = ISA3dPlotPrint(t, PSI, IF, A, fs, fMax, STFTparams)

Description: This function plots a 3D instantaneous spectrum without the GUI buttons.

Name: t
Type: vector (real)
Description: time instants

Name: PSI = $\begin{bmatrix} \top & \top & & \top \\ \psi_0(t) & \psi_1(t) & \cdots & \psi_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: complex matrix (or vector)
Description: the kth column is the kth component $\psi_k(t)$

Name: IF = $\frac{1}{2\pi}\begin{bmatrix} \top & \top & & \top \\ \omega_0(t) & \omega_1(t) & \cdots & \omega_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IF of $\psi_k(t)$ in Hertz

Name: A = $\begin{bmatrix} \top & \top & & \top \\ a_0(t) & a_1(t) & \cdots & a_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: real matrix (or vector)
Description: the kth column is the IA of $\psi_k(t)$

Name: fs
Type: scalar
Description: sampling frequency

Name: fMax or [fMax, fMin] (optional)
Type: scalar or [scalar,scalar]
Description: maximum plotting frequency and optionally the minimum plotting frequency

Name: STFTparams (optional)
Type: vector (1x2)
Description: STFT parameters: [N_FFT, frame_advance]

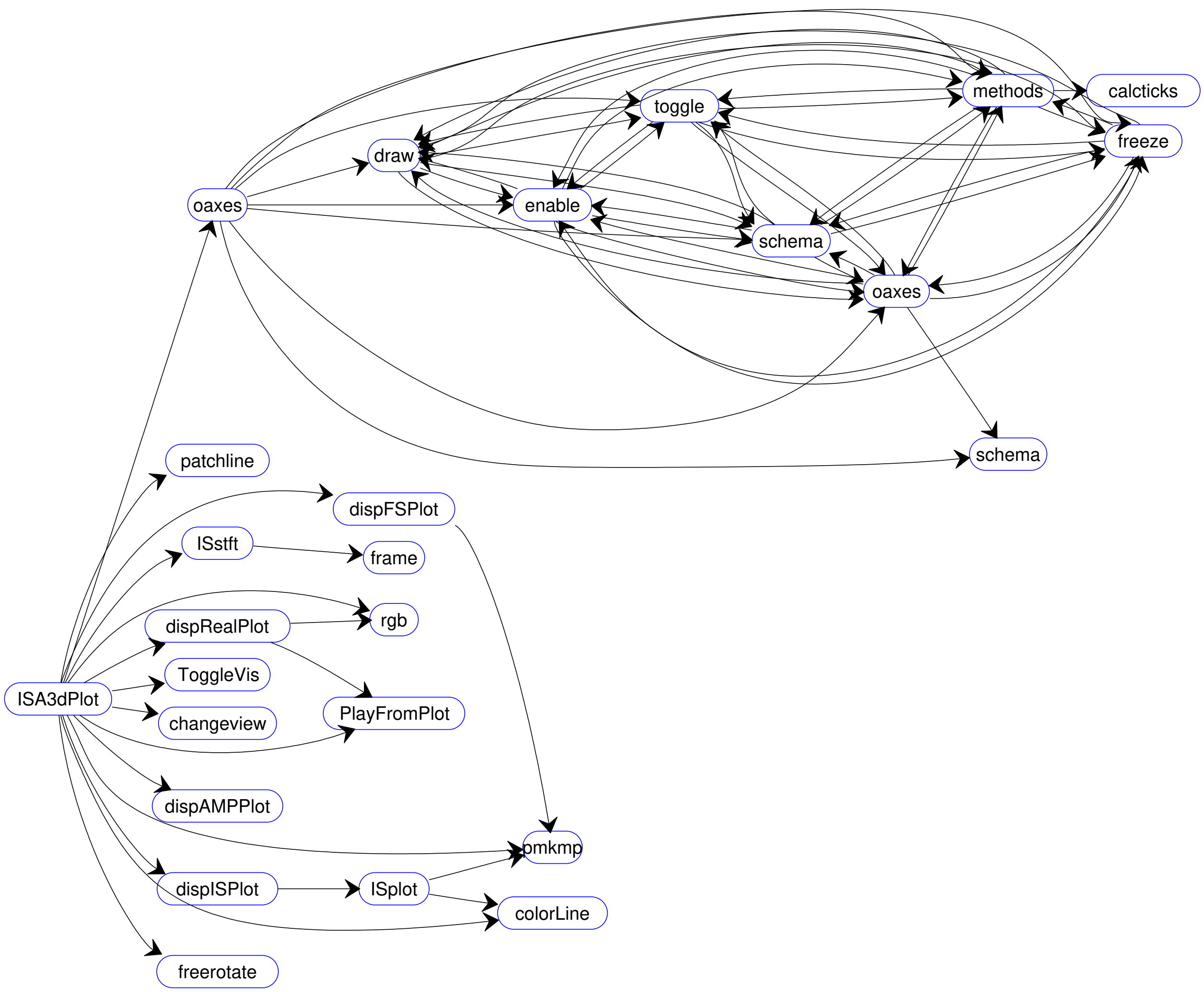
Output Arguments:

Name: h
Type: handle
Description: figure handle

References:

Notes:

Function Dependencies:



4.5 STFT2dPlot.m

Call Syntax: h = STFT2dPlot(z, fs, fMax, STFTparams)

Description: This function plots the short-time Fourier transform magnitude using a Hamming window.

Input Arguments:

Name: z
Type: vector
Description: complex signal

Name: fs
Type: scalar
Description: sampling frequency

Name: fMax or [fMax, fMin] (optional)
Type: scalar or [scalar,scalar]
Description: maximum plotting frequency and optionally the minimum plotting frequency

Name: STFTparams
Type: vector (1x2)
Description: STFT parameters: [N_FFT,frame_advance]

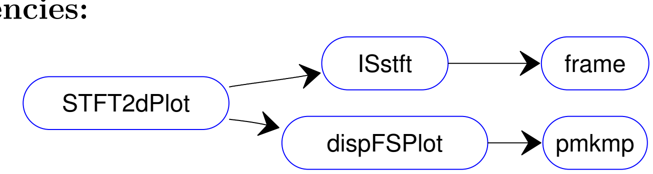
Output Arguments:

Name: h
Type: handle
Description: figure handle

References:

Notes:

Function Dependencies:



Chapter 5

Other Algorithms

The functions in this chapter, perform supporting roles and are required by other functions in the toolbox.

Contents

5.1 derivApprox.m

Call Syntax: Y = derivApprox(X,fs,method)

Description: This function estimates the approximate computation of a derivative using various numerical techniques.

Input Arguments:

Name: X = $\begin{bmatrix} \top & \top & & \top \\ x_0(t) & x_1(t) & \cdots & x_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: matrix (or vector)
Description: the kth column is the kth signal

Name: fs (optional)
Type: scalar
Description: sampling frequency [default = 1]

Name: method (optional)
Type: string
Description: numerical differentiation method

'forward'	-	forward difference
'backward'	-	backward difference
'center3'	-	3-pt stencil central difference
'center5'	-	5-pt stencil central difference
'center7'	-	7-pt stencil central difference
'center9'	-	9-pt stencil central difference
'center11'	-	11-pt stencil central difference
'center13'	-	13-pt stencil central difference
'center15'	-	15-pt stencil central difference [default]

Output Arguments:

Name: Y = $\begin{bmatrix} \top & \top & & \top \\ \frac{\mathrm{d}}{\mathrm{d}t}x_0(t) & \frac{\mathrm{d}}{\mathrm{d}t}x_1(t) & \cdots & \frac{\mathrm{d}}{\mathrm{d}t}x_{K-1}(t) \\ \bot & \bot & & \bot \end{bmatrix}$
Type: matrix (or vector)
Description: the kth column is the integral estimate of the kth signal

References:

[1] http://www.holoborodko.com/pavel/numerical-methods/numerical-derivative/central-differences/
[2] http://web.media.mit.edu/~crtaylor/calculator.html

Notes: See DEMO_derivApprox.m

Function Dependencies: None

5.2 intApprox.m

Call Syntax: Y = intApprox(X, fs, method)

Description: This function evaluates an approximate computation of an integral using various numerical techniques.

Input Arguments:

Name: X = $\begin{bmatrix} x_0(t) & x_1(t) & \cdots & x_{K-1}(t) \end{bmatrix}$
Type: matrix (or vector)
Description: the kth column is the kth signal

Name: fs (optional)
Type: scalar
Description: sampling frequency [default = 1]

Name: method (optional)
Type: string
Description: numerical integration method

‘left’	-	use the left rectangle rule
‘right’	-	use the right rectangle rule
’center’	-	use the midpoint rule
‘trapz’	-	use the trapezoidal rule [default]
‘simps’	-	use Simpson’s rule

Output Arguments:

Name: Y = $\begin{bmatrix} \int_0^t x_0(\tau)\,d\tau & \int_0^t x_1(\tau)\,d\tau & \cdots & \int_0^t x_{K-1}(\tau)\,d\tau \end{bmatrix}$
Type: matrix (or vector)
Description: the kth column is the cumulative integral estimate of the kth signal up to time t

References:

Notes: See DEMO_intApprox.m

Function Dependencies: None

Chapter 6

Demo Scripts

Contents

6.1 DEMO_1_SimpleHarmicComponent.m

Description: This script demonstrates the functionality of ‘amfmmod.m’, ‘Argand.m’, ‘ISA2dPlot.m’, and ‘ISA3dPlot.m’ by synthesizing a simple harmonic component and generating several visualizations.

A simple harmonic component has both constant IA and constant IF.

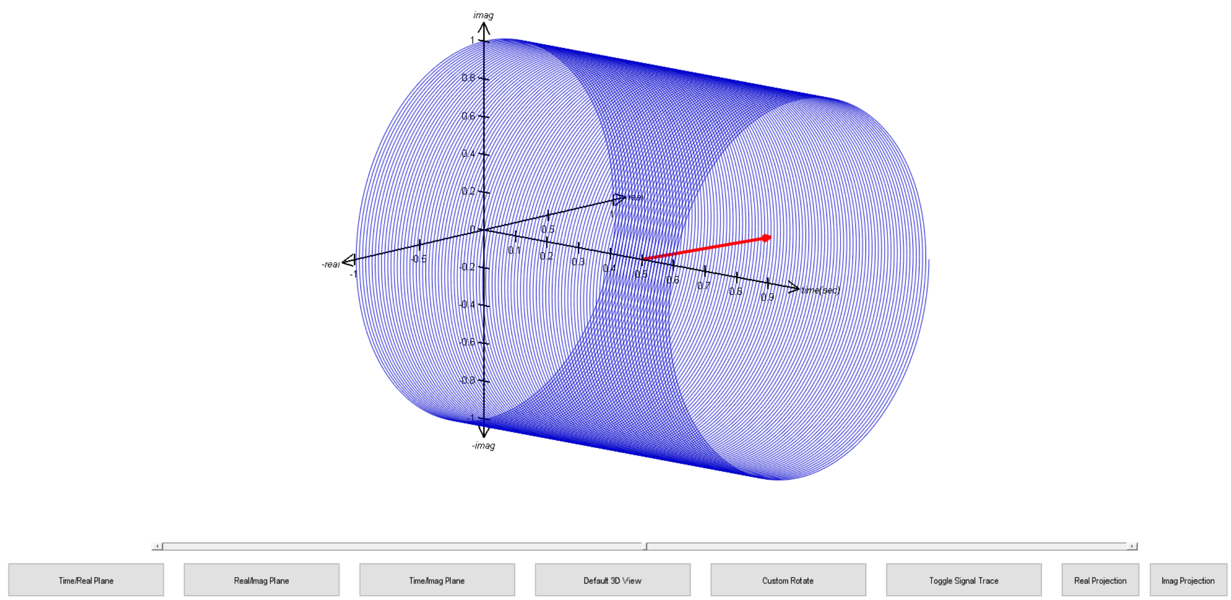


Figure 6.1.1: The interactive figure generated by running ‘Argand.m’ for a simple harmonic component.

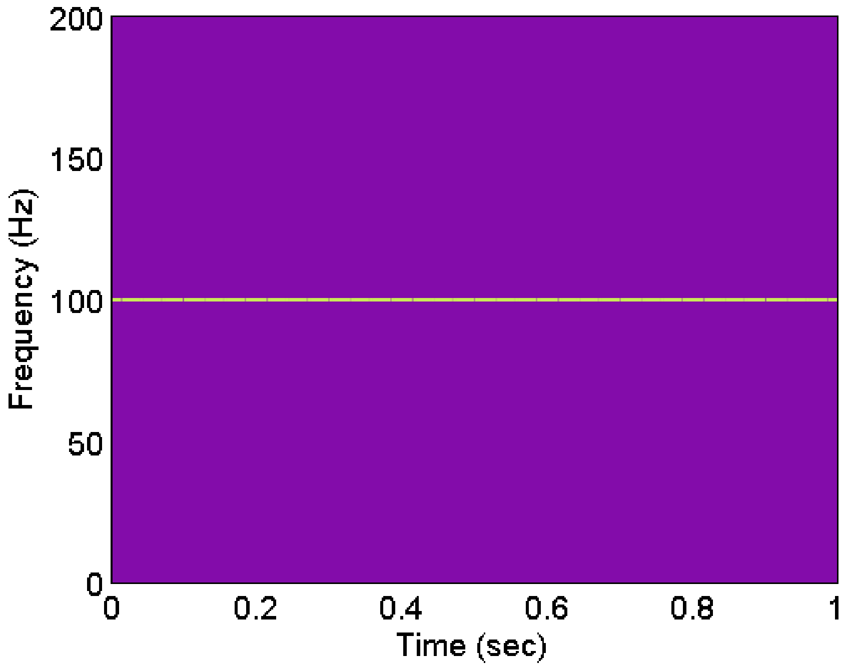


Figure 6.1.2: The IS plot generated by running 'ISA2dPlot.m' for a simple harmonic component.

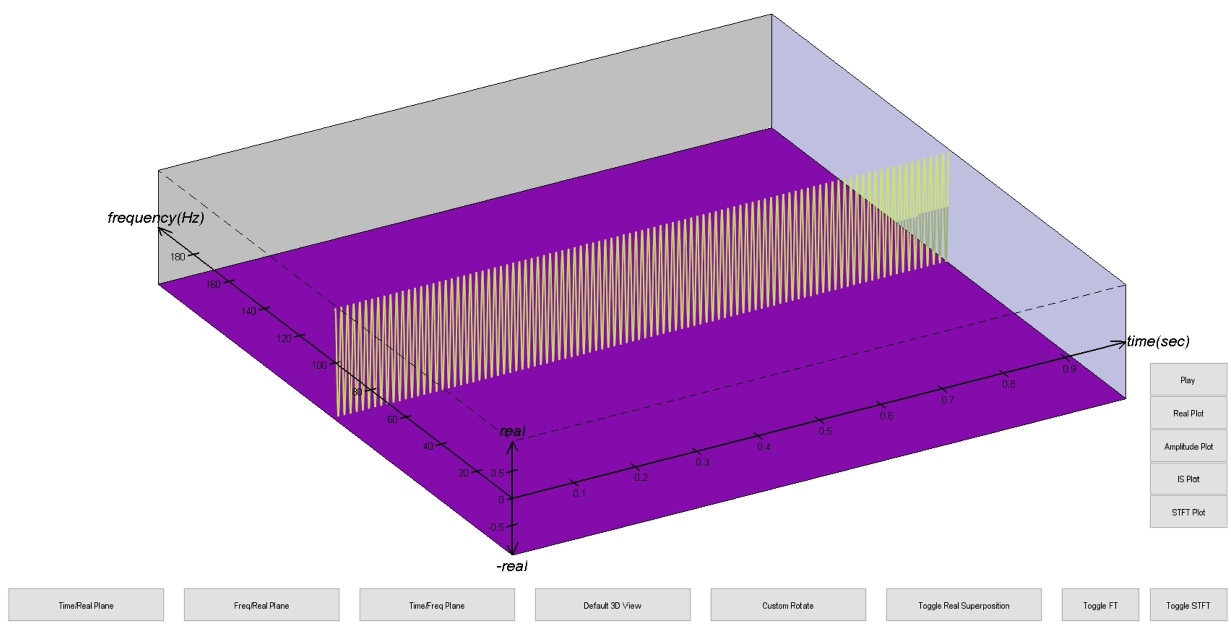


Figure 6.1.3: The interactive 3D figure generated by running 'ISA3dPlot.m' for a simple harmonic component.

6.2 DEMO_2_SimpleAMComponent.m

Description: This script demonstrates the functionality of 'amfmmod.m', 'Argand.m', 'ISA2dPlot.m', and 'ISA3dPlot.m' by synthesizing a simple AM component and generating several visualizations.

A simple AM component has constant IF but time-varying IA. Lines 40-43 in the demo file allow the user to switch between three predefined IA functions. The default is sinusoidal AM.

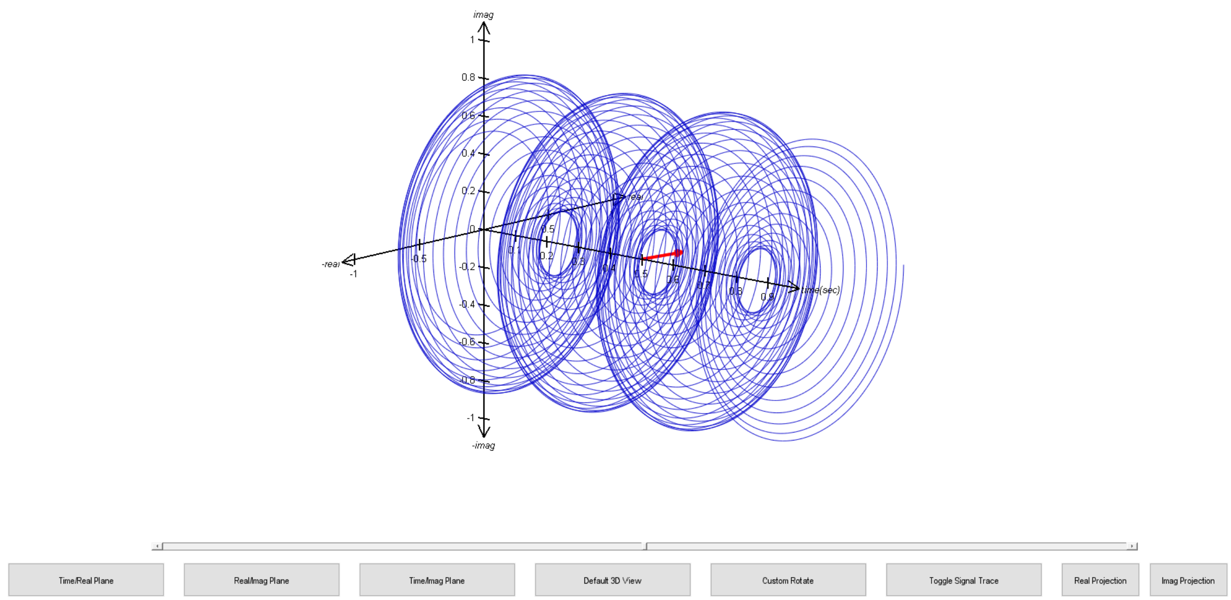


Figure 6.2.1: The interactive figure generated by running 'Argand.m' for a simple AM component with sinusoidal AM.

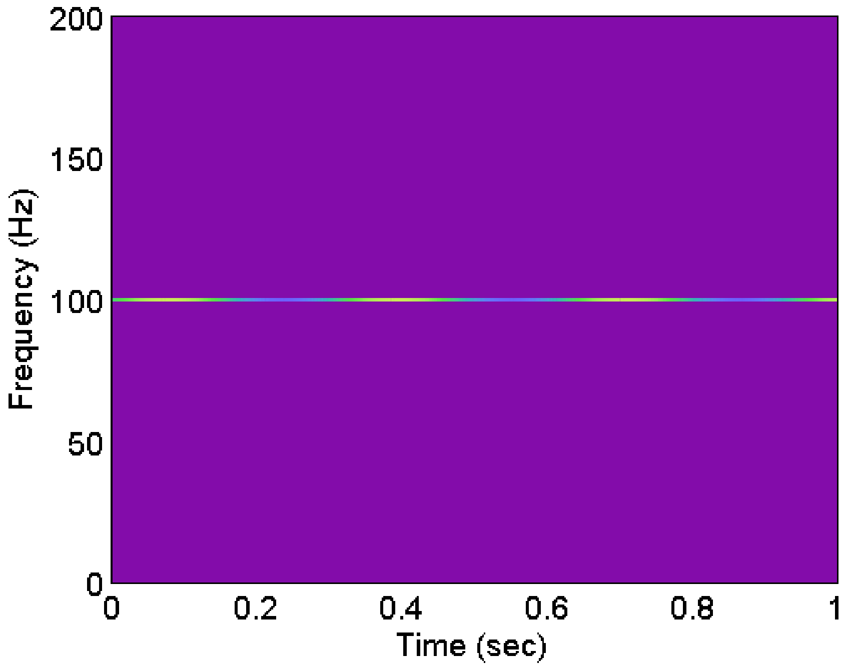


Figure 6.2.2: The IS plot generated by running ‘ISA2dPlot.m’ for a simple AM component with sinusoidal AM.

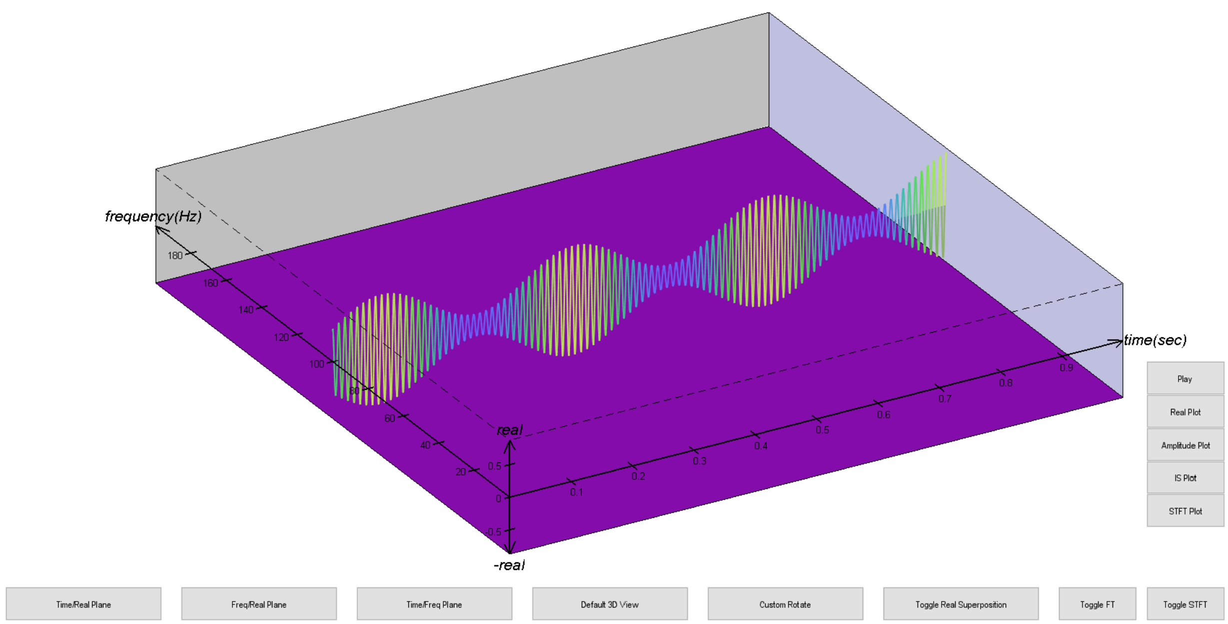


Figure 6.2.3: The interactive 3D figure generated by running ‘ISA3dPlot.m’ for a AM harmonic component with sinusoidal AM.

6.3 DEMO_3_SimpleFMComponent.m

Description: This script demonstrates the functionality of ‘amfmmod.m’, ‘Argand.m’, ‘ISA2dPlot.m’, and ‘ISA3dPlot.m’ by synthesizing a simple FM component and generating several visualizations.

A simple FM component has constant IA but time-varying IF. Lines 39-41 in the demo file allow the user to switch between three predefined IF functions. The default is an FM chirp.

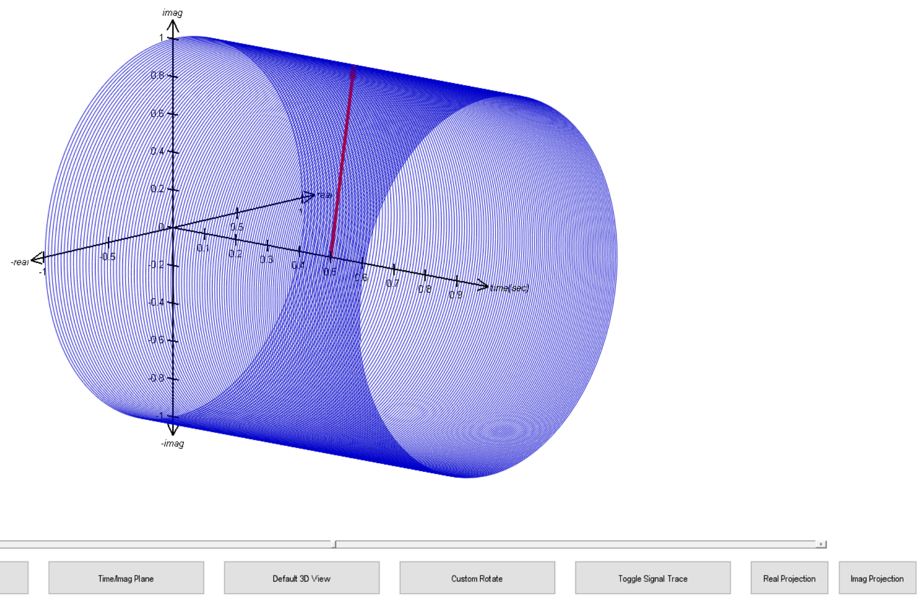


Figure 6.3.1: The interactive figure generated by running ‘Argand.m’ for a simple FM component chirp.

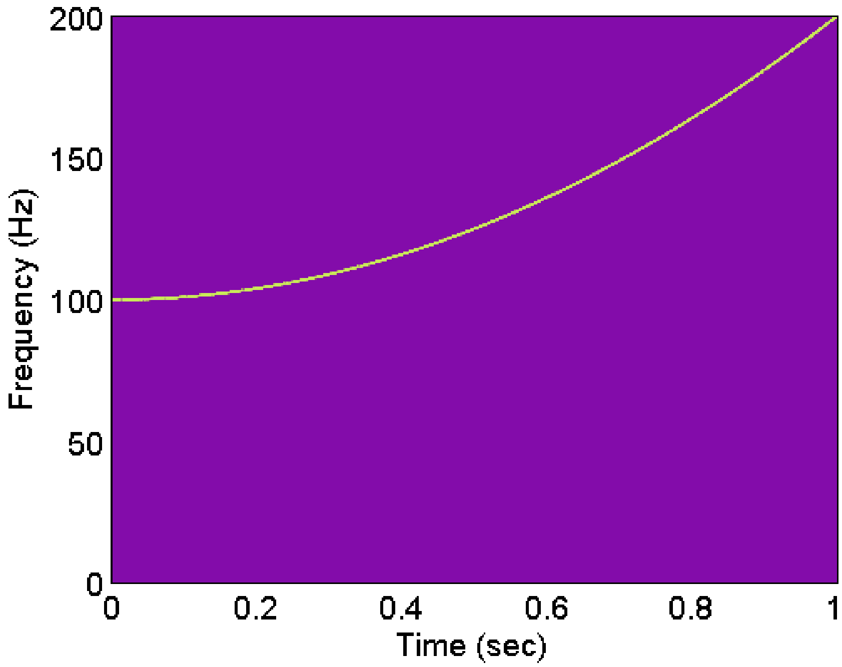


Figure 6.3.2: The IS plot generated by running ‘ISA2dPlot.m’ for a simple FM component chirp.

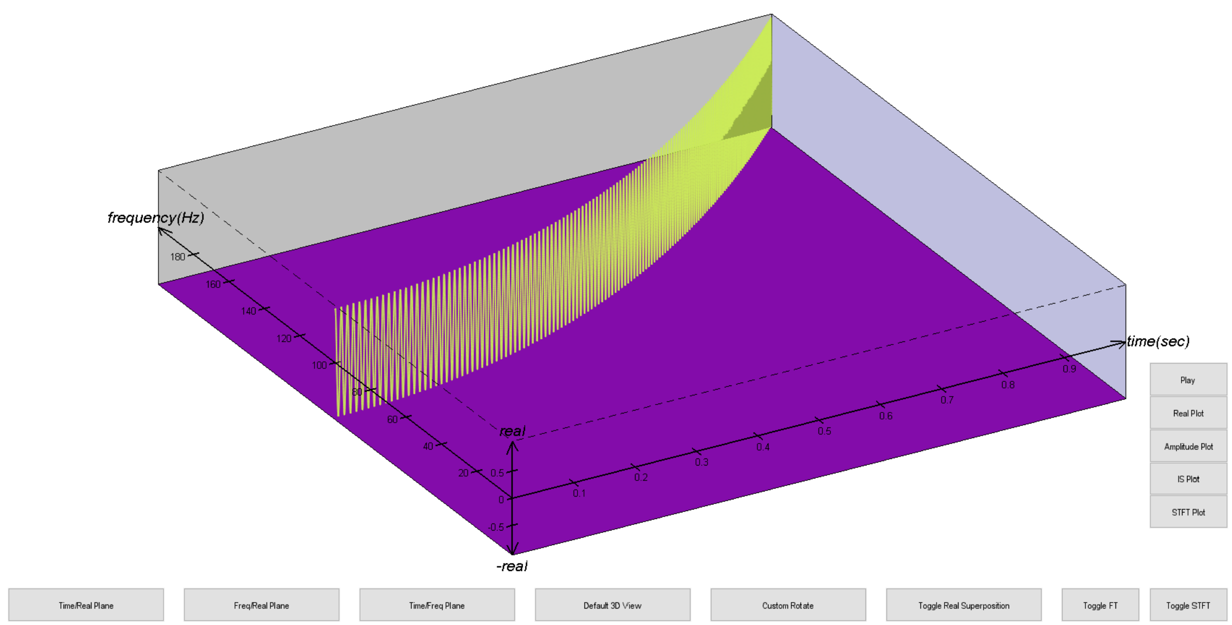


Figure 6.3.3: The interactive 3D figure generated by running ‘ISA3dPlot.m’ for a simple FM component chirp.

6.4 DEMO_4_AMFMComponent.m

Description: This script demonstrates the functionality of 'amfmmod.m', 'Argand.m', 'ISA2dPlot.m', and 'ISA3dPlot.m' by synthesizing an AM–FM component and generating several visualizations.

An AM–FM component has both a time-varying IA and IF.

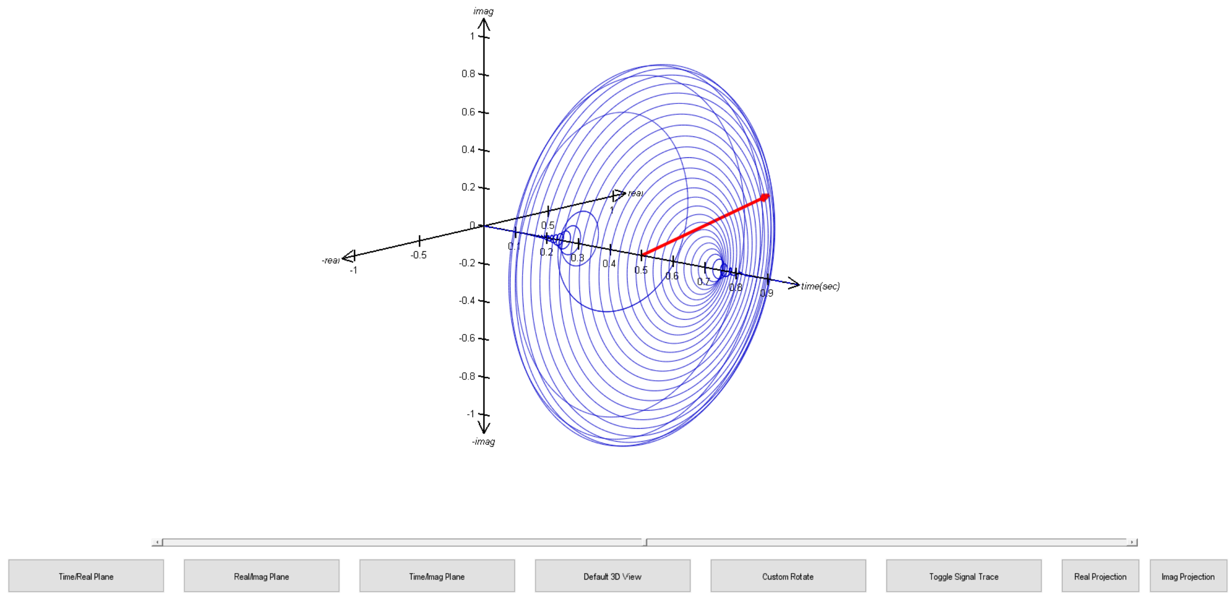


Figure 6.4.1: The interactive figure generated by running 'Argand.m' for an AM–FM component.

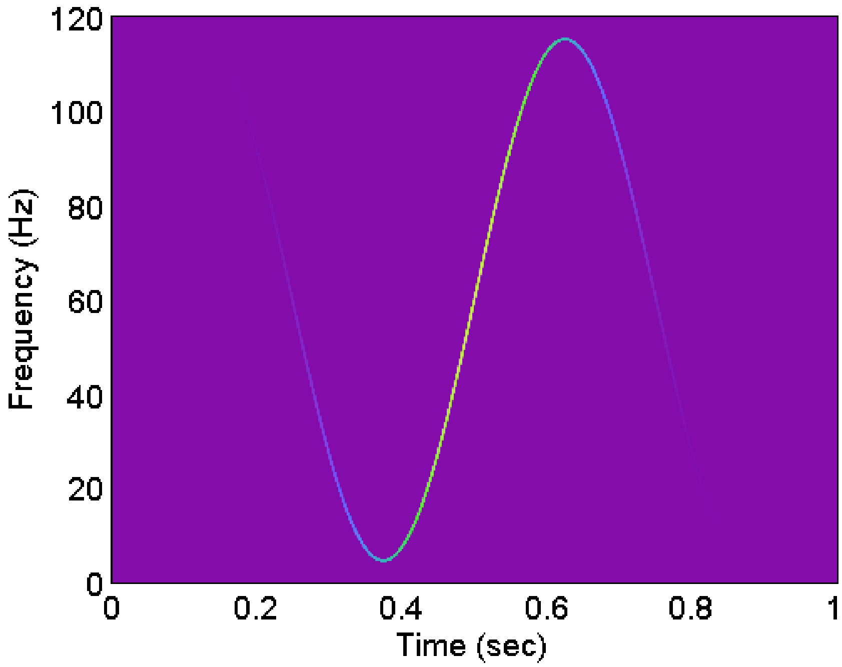


Figure 6.4.2: The IS plot generated by running ‘ISA2dPlot.m’ for an AM–FM component.

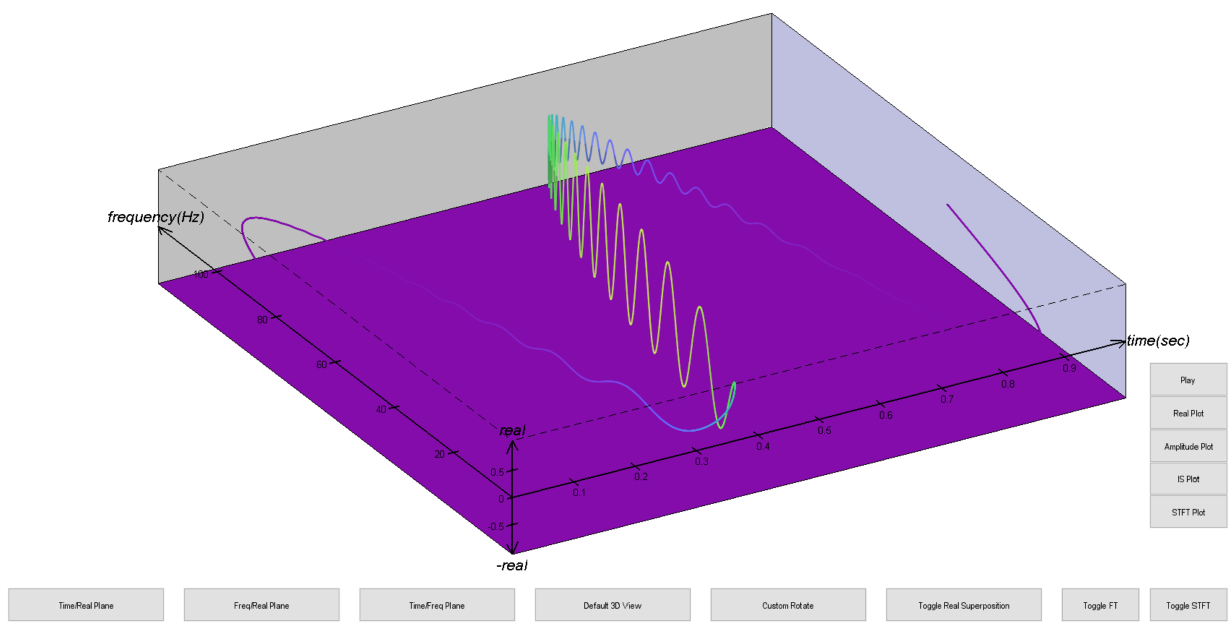


Figure 6.4.3: The interactive 3D figure generated by running ‘ISA3dPlot.m’ for an AM–FM component.

6.5 DEMO_5_MultiComponentSignal.m

Description: This script demonstrates the functionality of 'amfmmod.m', 'Argand.m', 'ISA2dPlot.m', and 'ISA3dPlot.m' by synthesizing a multi-component AM–FM signal and generating several visualizations.

A multi-component signal consists of multiple AM–FM components.

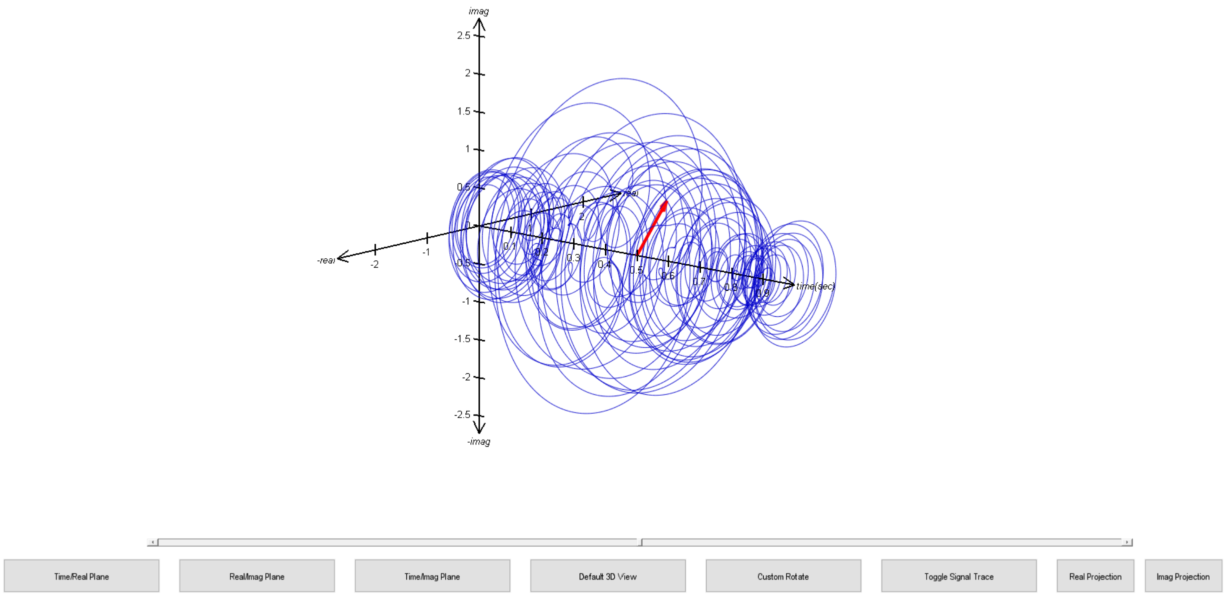


Figure 6.5.1: The interactive figure generated by running 'Argand.m' for a multi-component AM–FM signal.

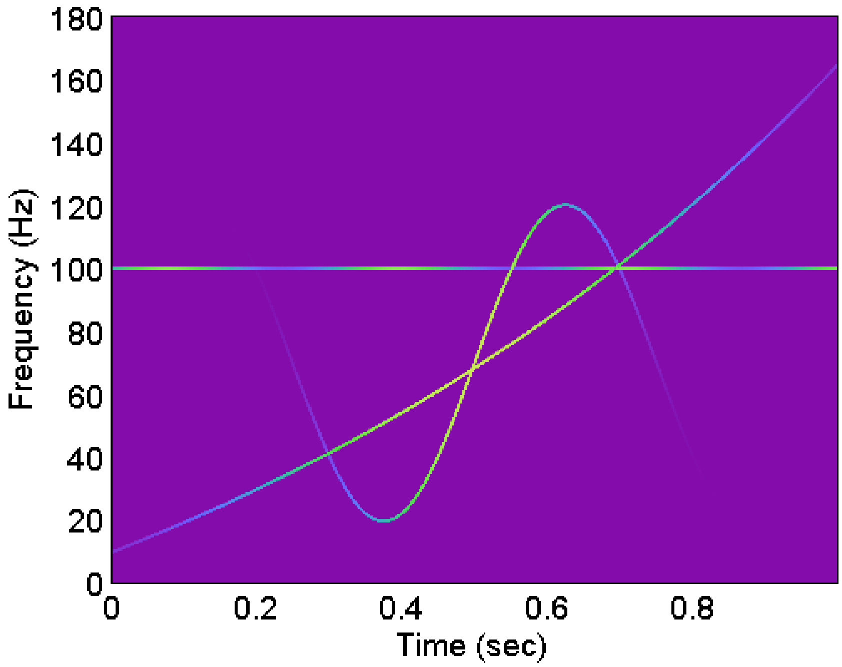


Figure 6.5.2: The IS plot generated by running ‘ISA2dPlot.m’ for a multi-component AM–FM signal.

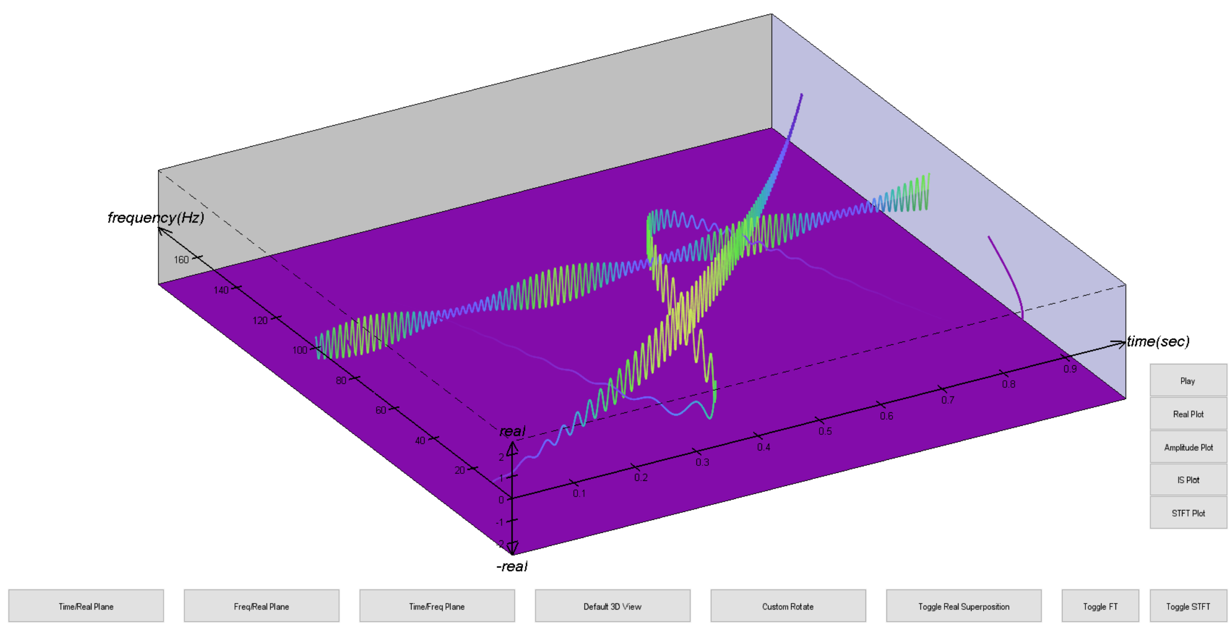


Figure 6.5.3: The interactive 3D figure generated by running ‘ISA3dPlot.m’ for a multi-component AM–FM signal.

6.6 FIGURE_ISA2018_SinusoidalAM.m

Description: This script generates Figure 5 (a) and (b) in [1].

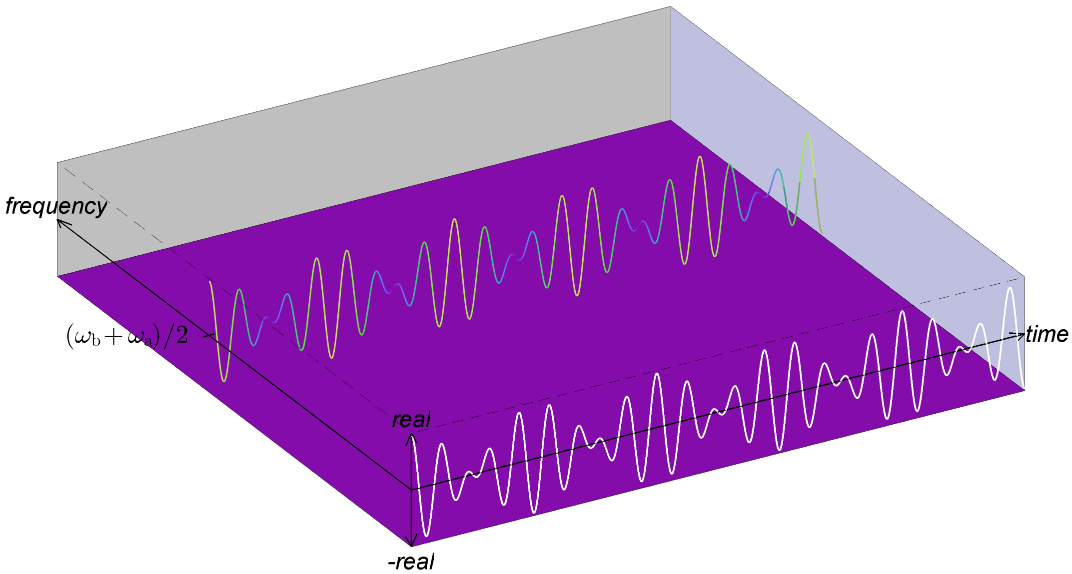


Figure 6.6.1: Figure 5 (a) in [1].

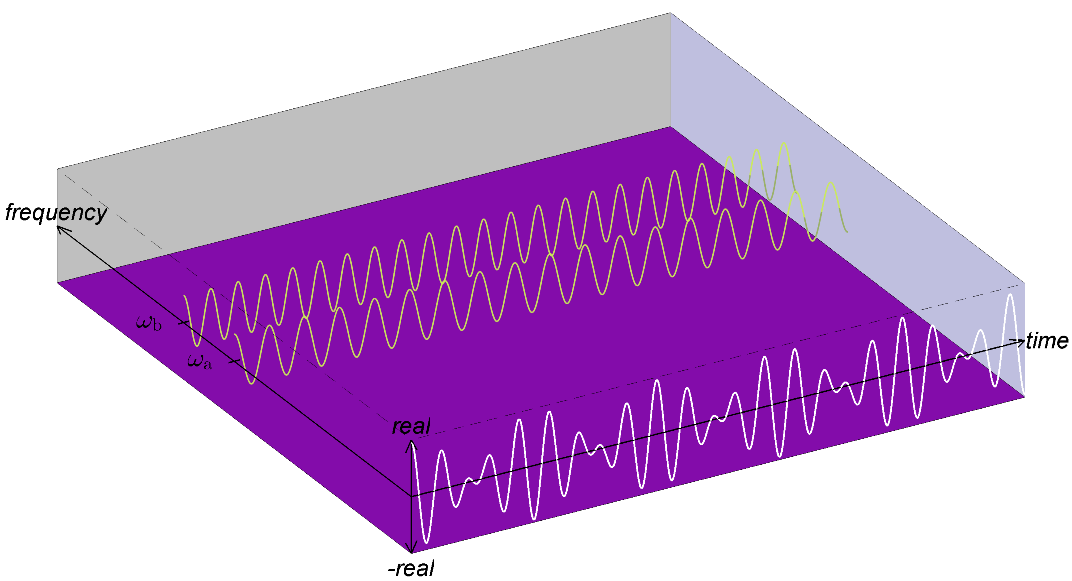


Figure 6.6.2: Figure 5 (b) in [1].

6.7 FIGURE_ISA2018_SinusoidalFM.m

Description: This script generates Figure 6 (a) and (b) in [1].

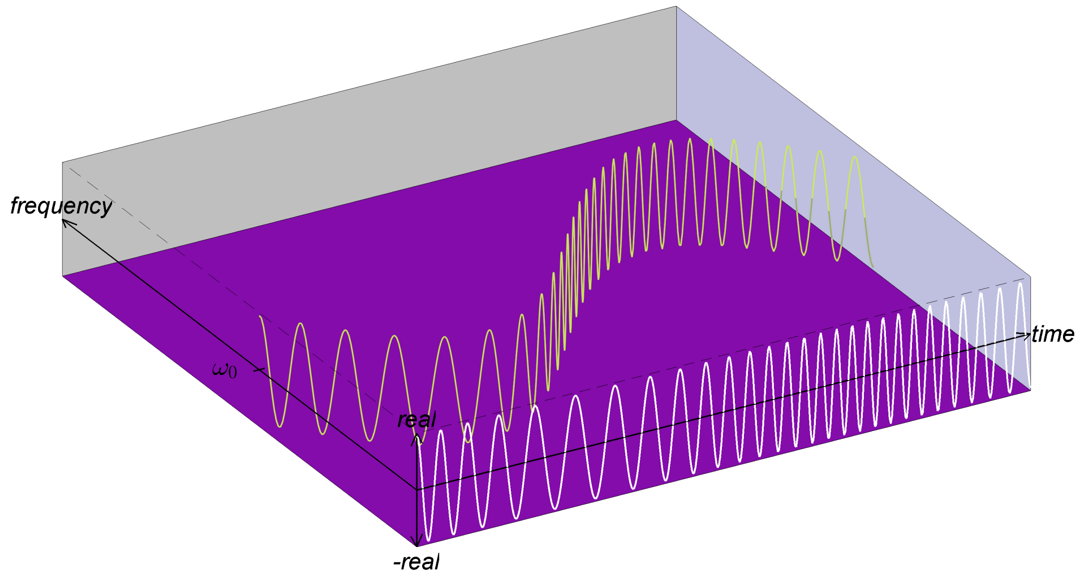


Figure 6.7.1: Figure 6 (a) in [1].

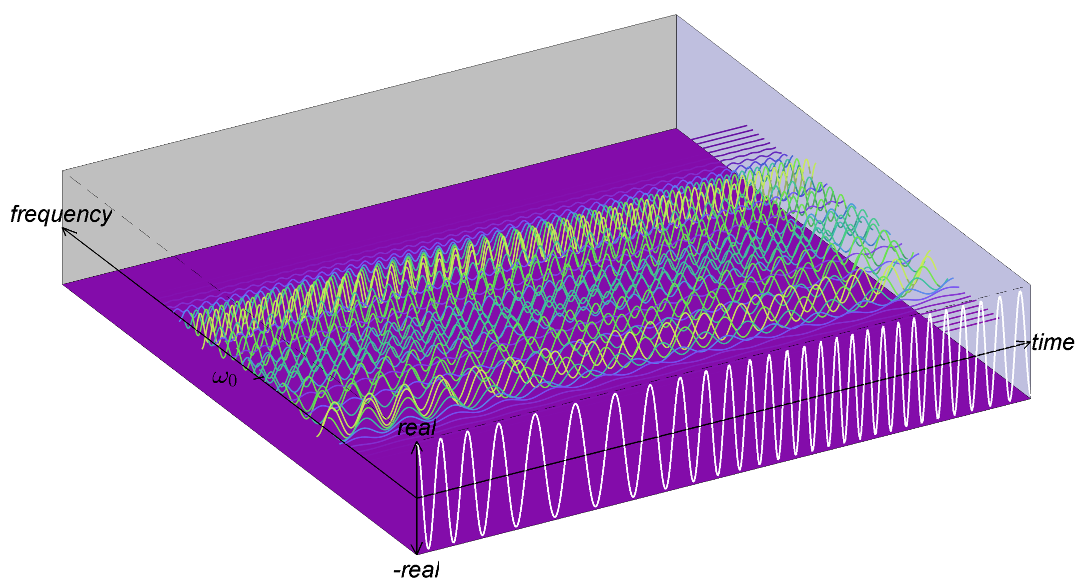


Figure 6.7.2: Figure 6 (b) in [1].

6.8 FIGURE_ISA2018_GaussianAMchirpFMchirp.m

Description: This script generates Figure 7 (a) in [1].

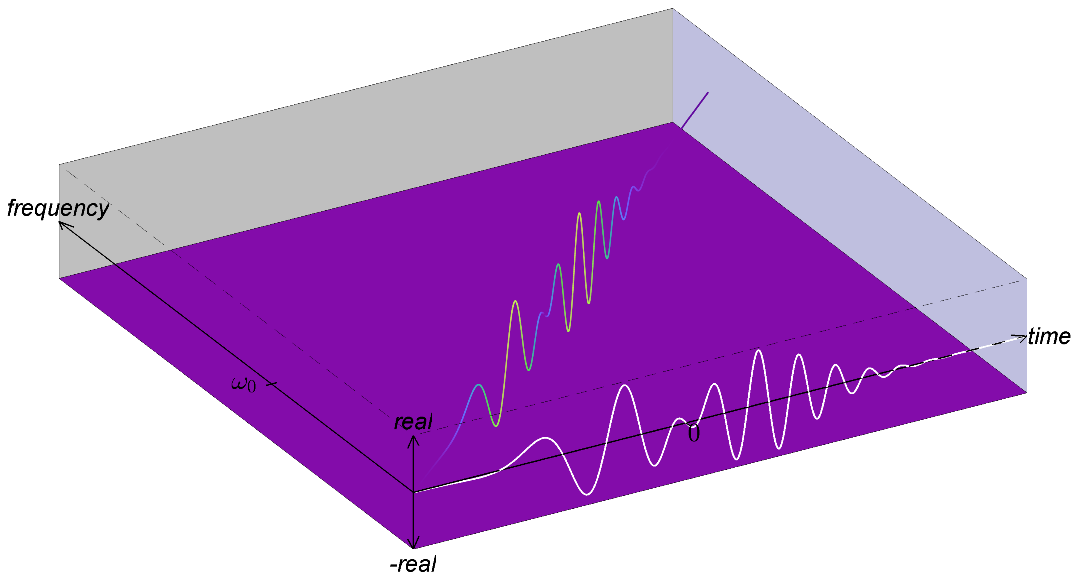


Figure 6.8.1: Figure 7 (a) in [1].

6.9 DEMO_intApprox.m

Description: This script demonstrates the 'intApprox.m', which estimates the integral of a signal using one of several methods.

A time-domain comparison of the integration approximation methods is given in Figure 6.9.1. Figure 6.9.1(a) shows the various integration approximations for one period of a sinusoid. The corresponding error at each instant, given in dB, in shown in Figure 6.9.1(b).

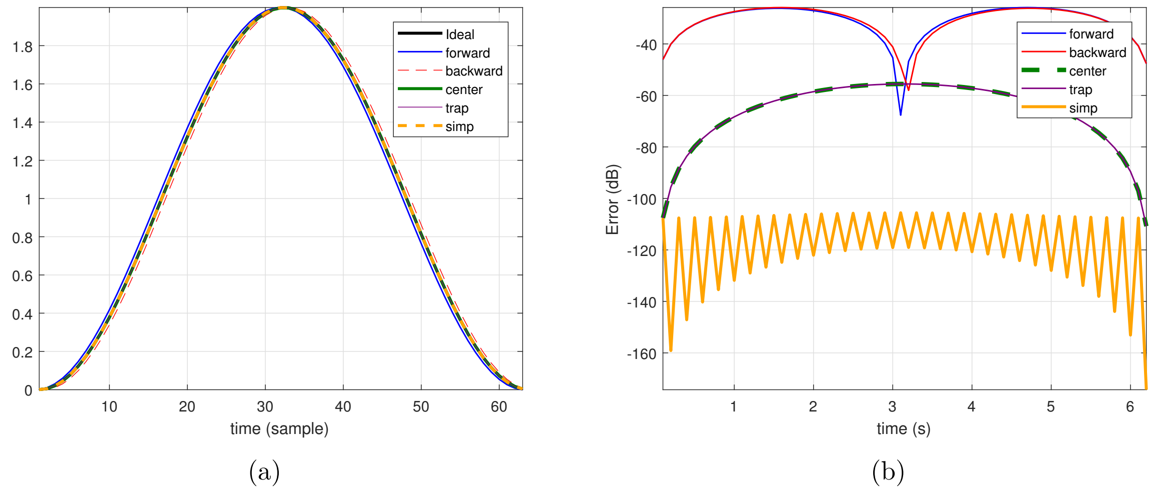
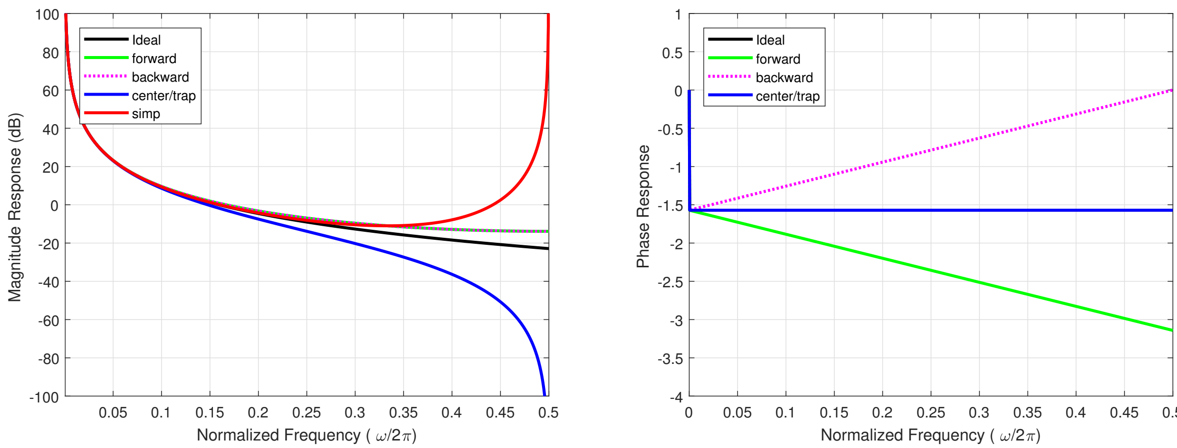


Figure 6.9.1: Comparison of the various integral approximations methods for one period of a of a sinusoid. As the order of the approximation increases, the approximation error decreases.

A frequency-domain comparison of the magnitude responses of the approximations is given in Figure 6.9.2. Figure 6.9.2(a) shows the magnitude response of the ideal integrator alongside the various approximation methods. Based on the magnitude response, the forward and backward method is the one closest to the ideal signal. However, the phase response shown in Figure 6.9.2(b) needs to be considered as well. Here it is apparent that the forward and backward differences introduce phase distortions while the center and trap differences do not



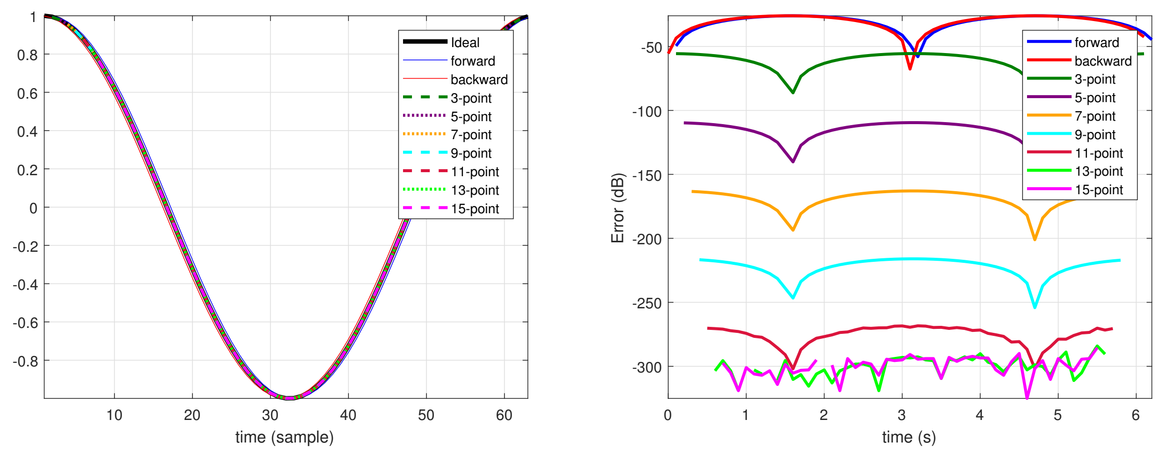
(a) (b)

Figure 6.9.2: (a) Magnitude response and (b) phase response of the ideal integrator along side the various integral approximation methods. As the methods of integration changes, the frequency response approaches the ideal integrator

6.10 DEMO_derivApprox.m

Description: This script demonstrates the functionality of 'derivApprox.m', which estimates the derivative of a signal using one of several methods.

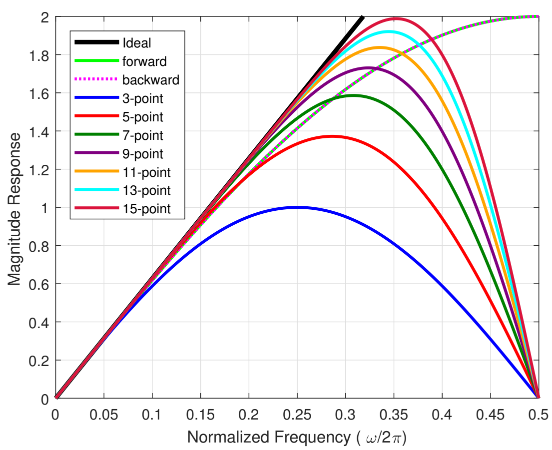
A time-domain comparison of the derivative approximation methods is given in Figure 6.10.1. Figure 6.10.1(a) shows the various derivative approximations for one period of a sinusoid. The corresponding error at each instant, given in dB, in shown in Figure 6.10.1(b). It appears that the 13-point and 15-point central differences are limited by the numerical precision of the computer.



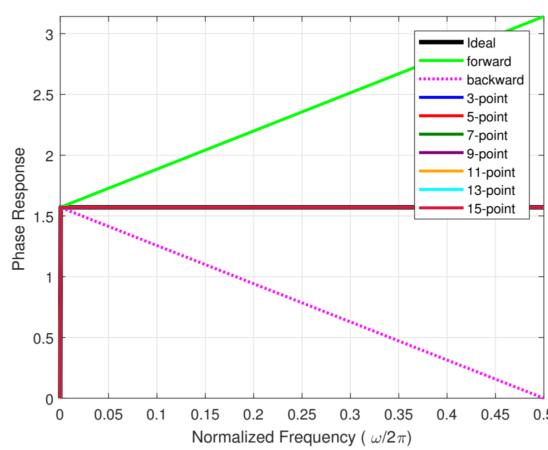
(a) (b)

Figure 6.10.1: Comparison of the various derivative approximations methods for one period of a of a sinusoid. As the order of the approximation increases, the error decreases and performance increases.

A frequency-domain comparison of the magnitude responses of the approximations is given in Figure 6.10.2. Figure 6.10.2(a) shows the magnitude response of the ideal differentiator alongside the various approximation methods. Based on the magnitude response, the performance of the central differences improves as the order of the approximation is increased. From the magnitude response plot, it may appear that the forward and backward differences outperform the central differences at certain frequency ranges, however, consider the phase responses show in Figure 6.10.2(b). Here it is apparent that the forward and backward differences introduce phase distortions while the central differences do not.



(a)



(b)

Figure 6.10.2: (a) Magnitude response and (b) phase response of the ideal differentiator for various derivative approximation methods. As the order of the approximation increases, the frequency response approaches the ideal differentiator.

Chapter 7

Bibliography

[1] S. Sandoval and P. L. D. Leon, "The instantaneous spectrum: A general framework for time-frequency analysis," *IEEE Trans. Signal Process.*, vol. 66, pp. 5679–5693, Nov 2018.

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