Lecture Map

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Lec. #	Contents
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Lec 1: Charge, Current, Voltage

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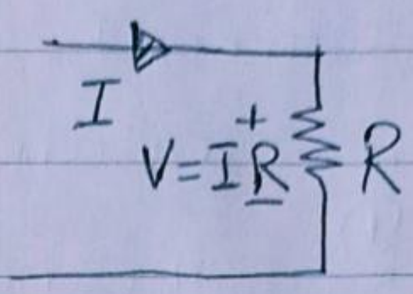
Today's Lecture:

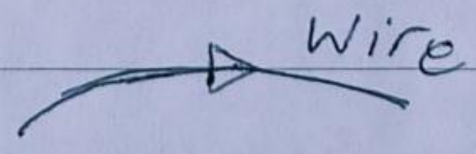
1- Why circuits?
2- Course outline.
3- electric Quantities

ohm's law



- why circuits? "Listen To Razavi" "00:00 To 7:00"
- Course outline
  - electric quantities.
  - electric Devices.
  - circuit Laws.
  - Analysis Techniques.
  - Circuit Theorems.
  - RC, RL, RLC circuits.
  - Op-Amp circuits

- electric Quantities
  - charge, Q, ~~meas~~ in coulomb
  - current occurs when electric charge Moves.



$$I \overset{?}{=} \frac{Q}{t} \xrightarrow[\text{good}]{\text{very}} I = \frac{dQ}{dt} \ (A)$$

- official difinition of current: The rate of which The charge passes Through This wire $\left(\frac{dQ}{dt}\right)$

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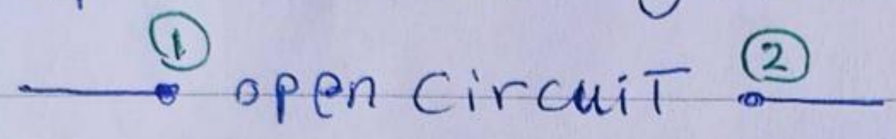
important points

① each current has a value and a direction.

② An open circuit guarantees $I = 0$.



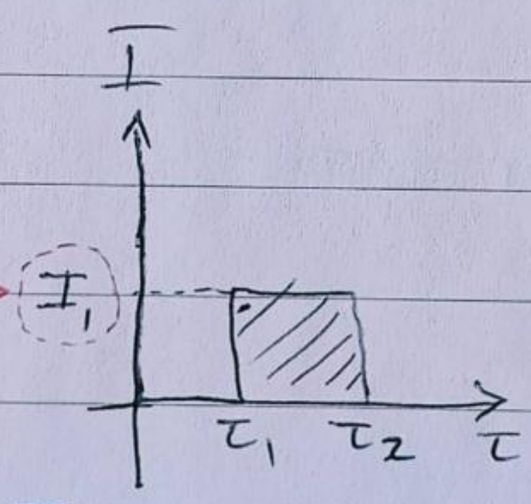
open circuit : There No path for charge To flow from ① To ②, There's No path, There's No current

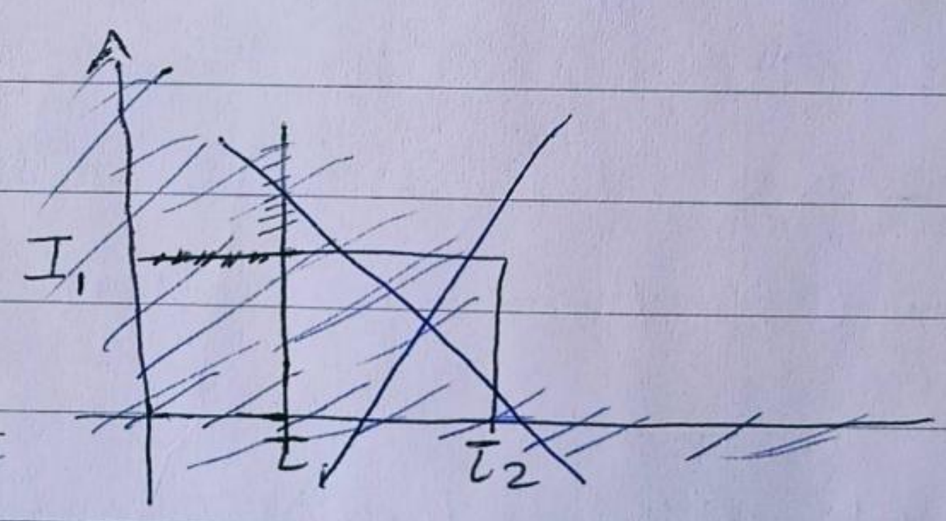
③ The amount of charge passing Through a wire or a branch in certain time T_1 :

$$Q = \int_0^{T_1} I \, dt$$

example

current Through resistor →



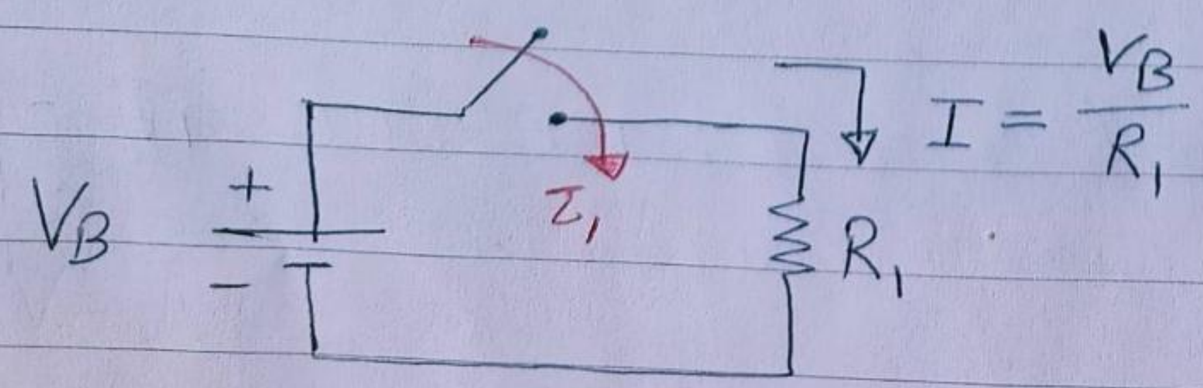


$$Q = \int_{t_1}^{t_2} I_1 \, dt = (t_2 - t_1) I_1$$

Example

- construct a circuit That creates This current.

ans



$$I = \frac{V_B}{R_1}$$

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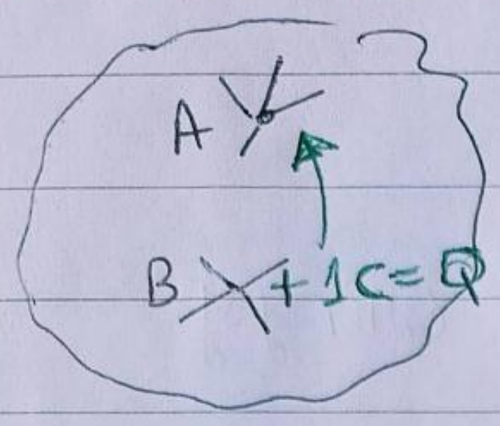
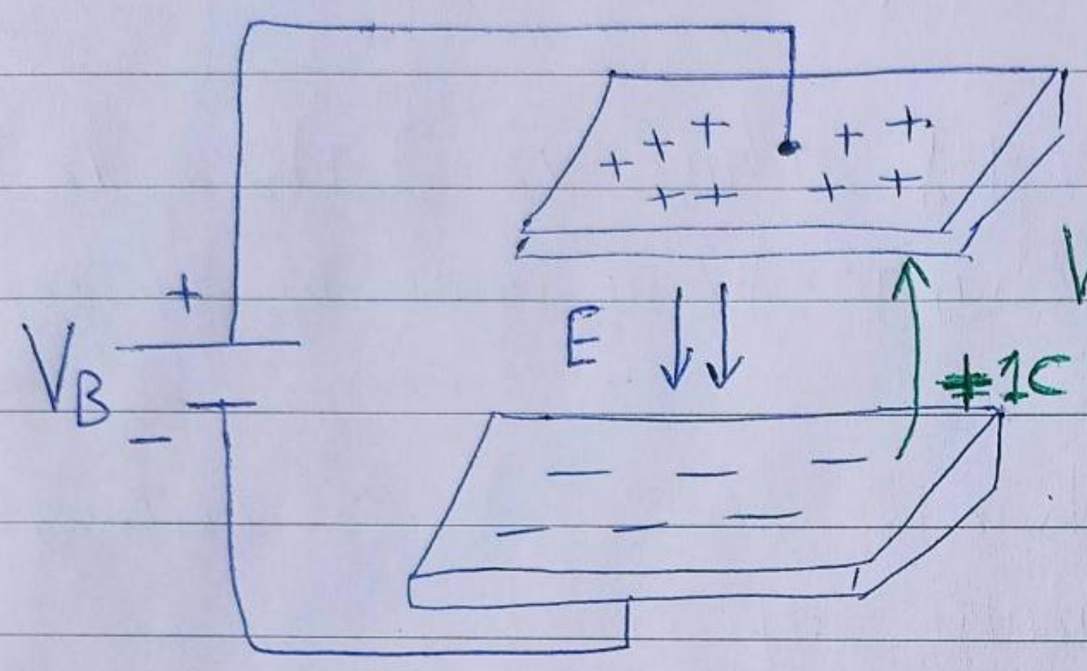
- Voltage (V)



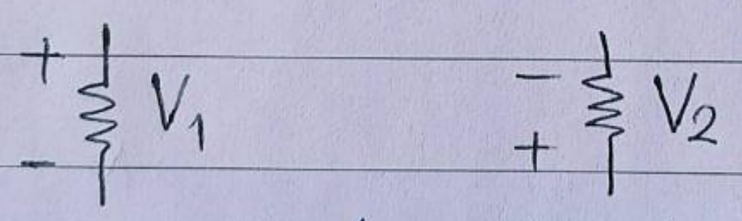
$V_A - V_B$: The amount of energy necessary to take a positive unit charge from B to A.



V_B = energy needed to bring charge from lower plate to upper plate.

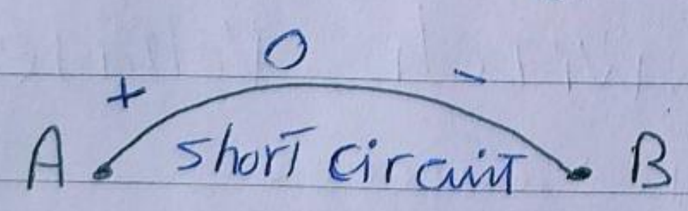
- Important Points

① A Voltage has a value and a sign;

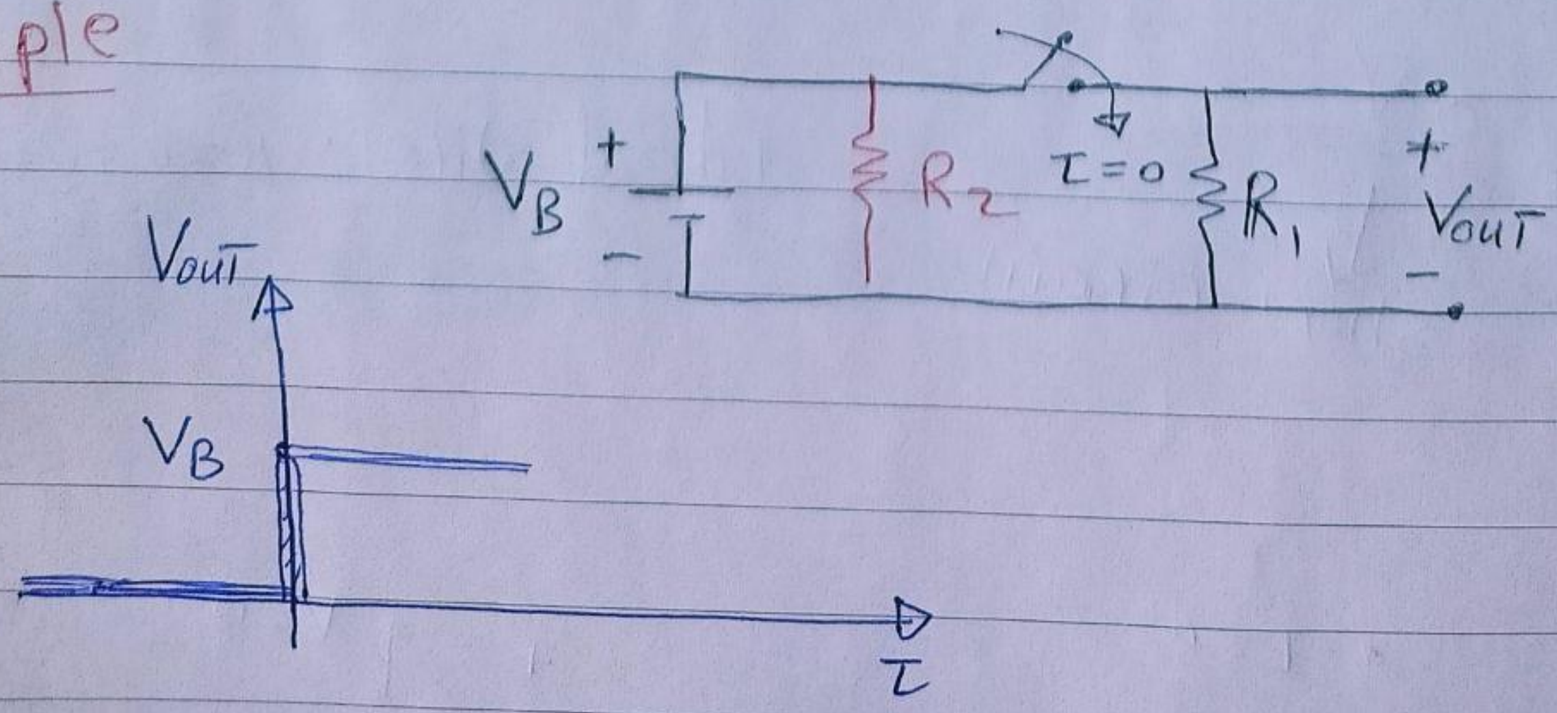


② Voltage is meaningful only between two points.

③ A short circuit guarantees $V=0$;



Example



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Lec 2: Voltage Source, Current Source

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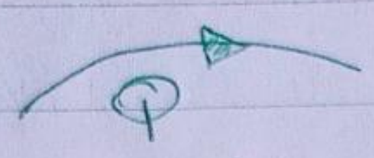
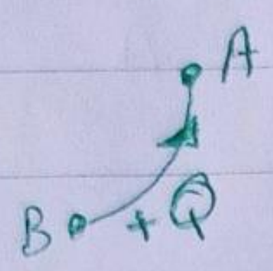
Review of Lecture 1

- Current : 

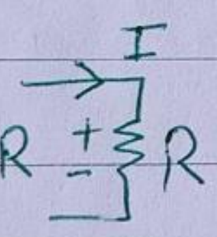
$$I = \frac{dQ}{d\tau}$$

- Voltage : 

$$V_A - V_B = \frac{E}{Q} \rightarrow \text{Total energy}$$

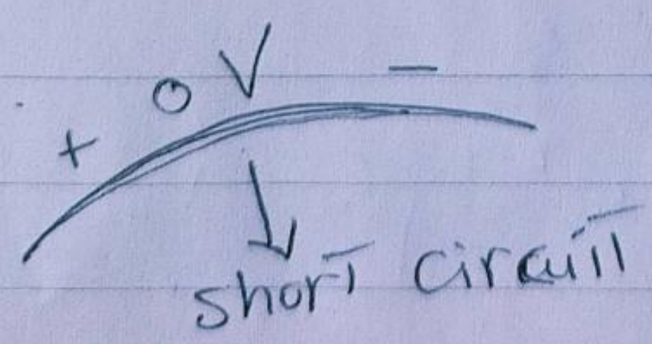
Voltage : The amount of energy that it takes to take a unit positive charge from point to another.

current : - The amount of charge that passes through in wire in a very very amount of time
- The derivative of charge with respect to the time sometimes we call this "The instantaneous current".

- ohm's law $V = IR$ 

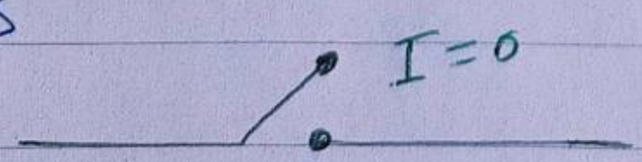
{ Today's lecture }

- electric devices : wire, switch, voltage source, current sources.

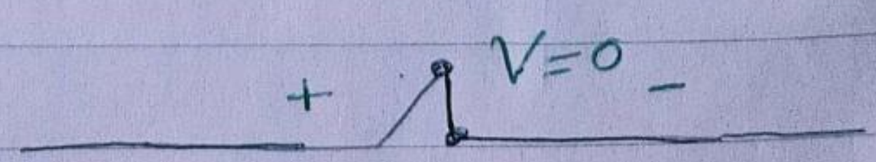
- wire 

Ideal wire : $R=0, C=0, L=0$

$V = IR$

- switches 

open ≡ off ≡ open circuit



closed ≡ on ≡ short circuit

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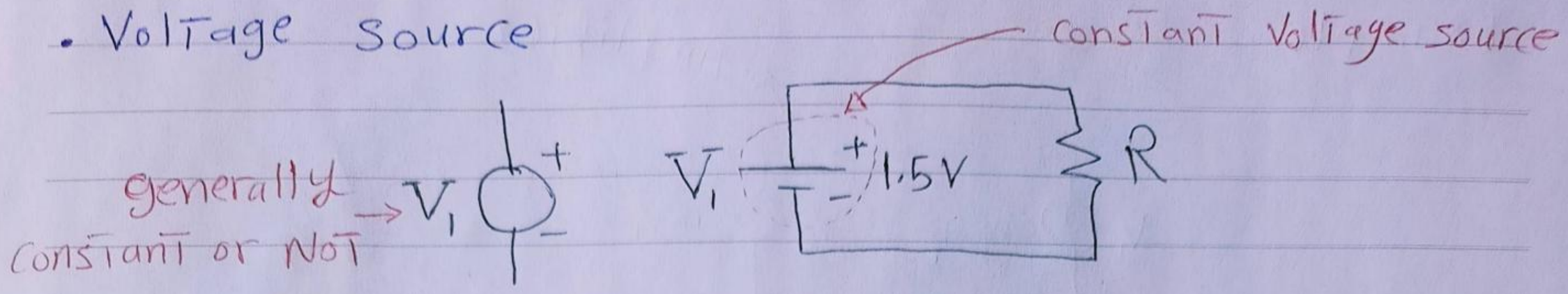
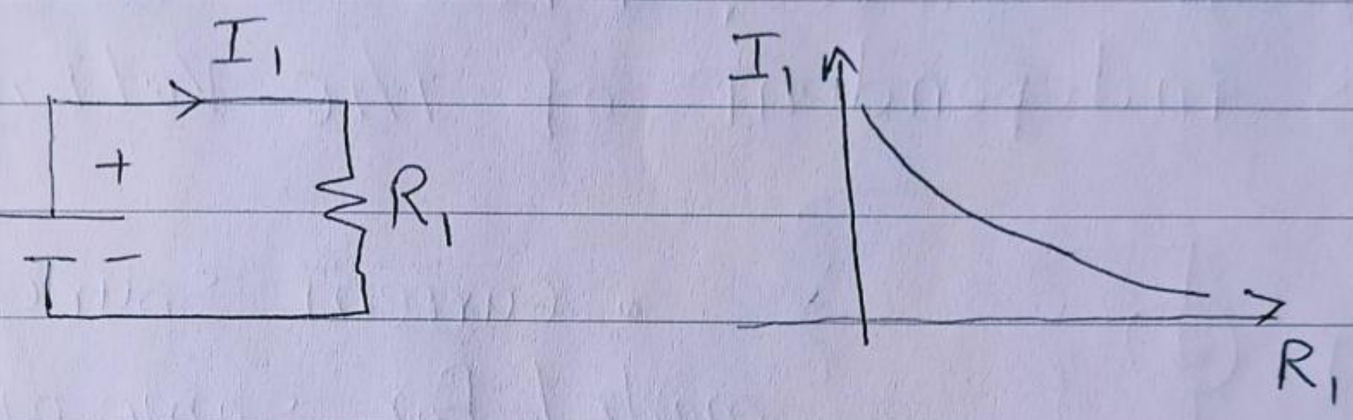
- Voltage source



A Two-Terminal device whose voltage does not depend on the current through it.

Example

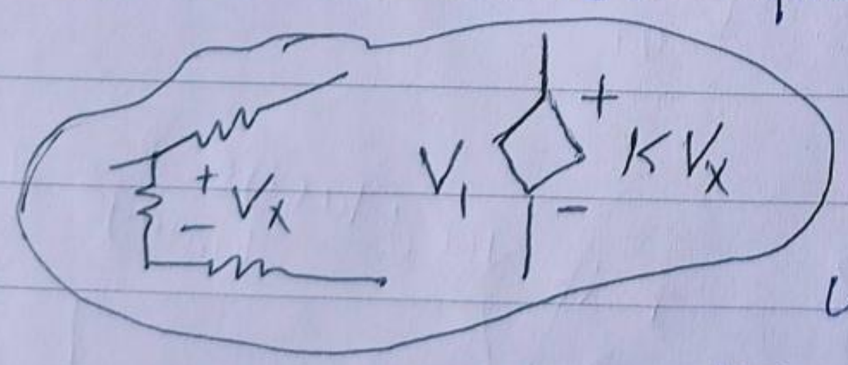
$I_1 = \frac{V_1}{R_1}$



- independent and Dependent Voltage Sources.

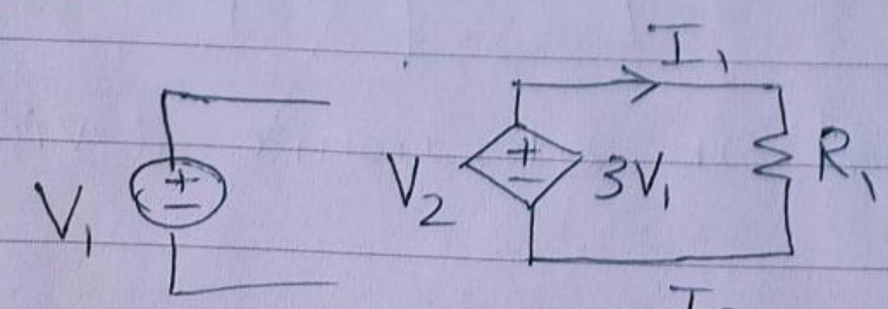
- independent Voltage source is one whose voltage is independent of other quantities in the circuit.

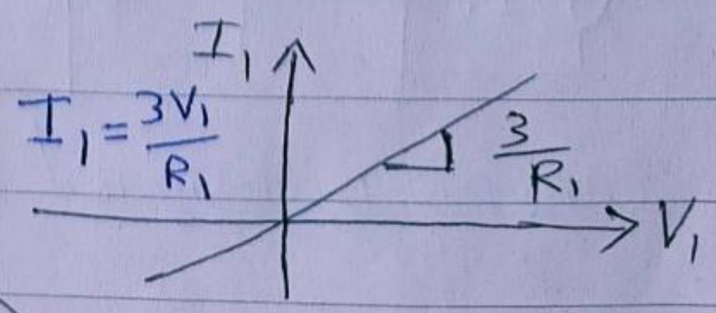
- dependent Voltage source is one whose value depends on some other quantity in the circuit.



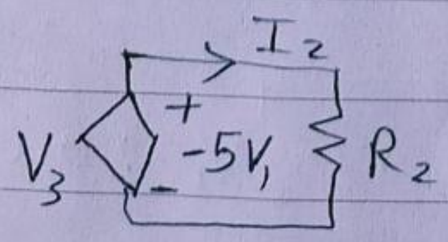
- Dependent voltage sources are useful for modeling more complex circuits, e.g, Amplifiers.

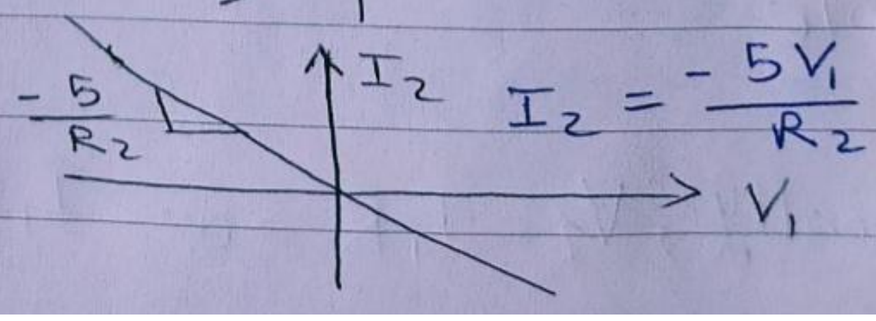
Example





$I_1 = \frac{3V_1}{R_1}$





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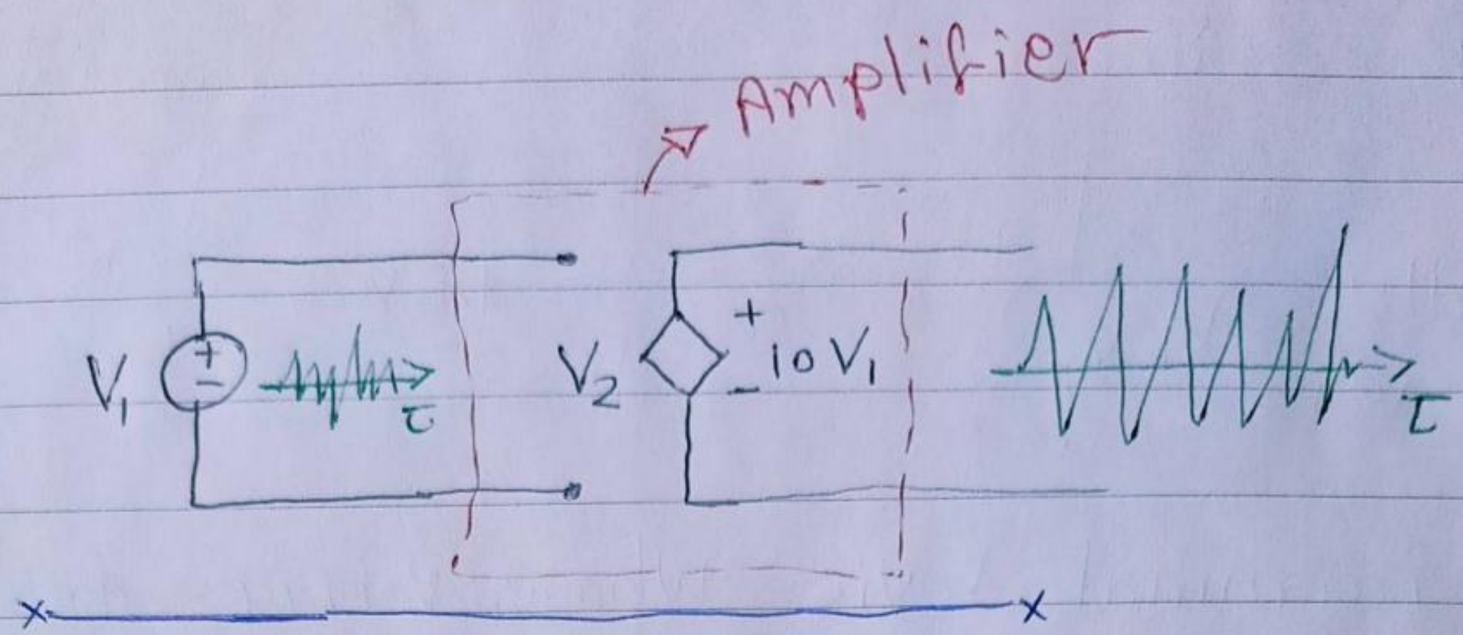
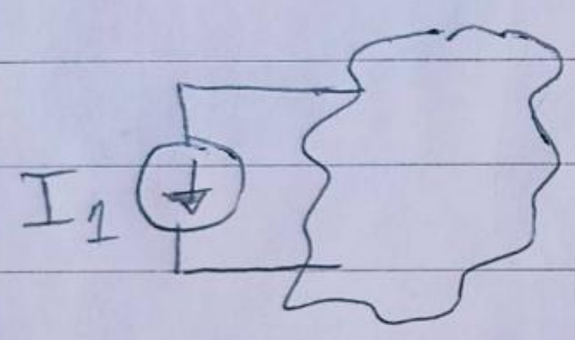
Example



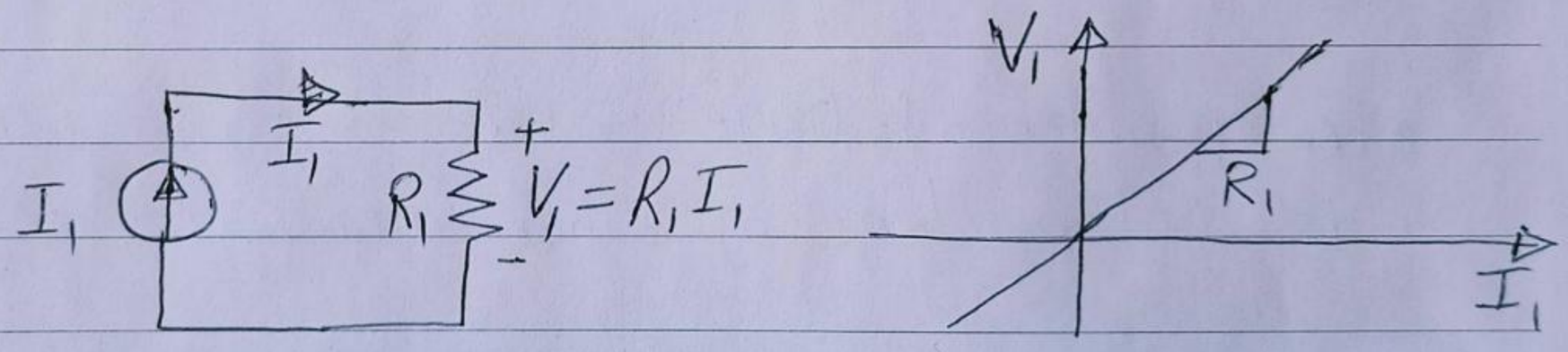
"beauty of circuits → 32:00 To 36:30 in Video"

- Current Source: a Two-Terminal device whose is independent of The Voltage across it.



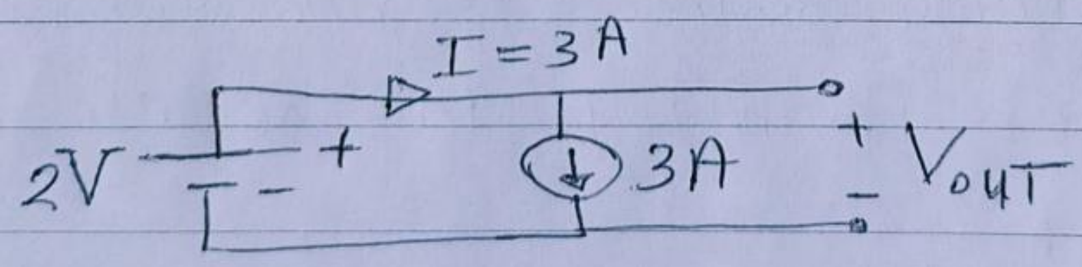
- Current Sources provide a good model for some other devices such as Transistors.

Example



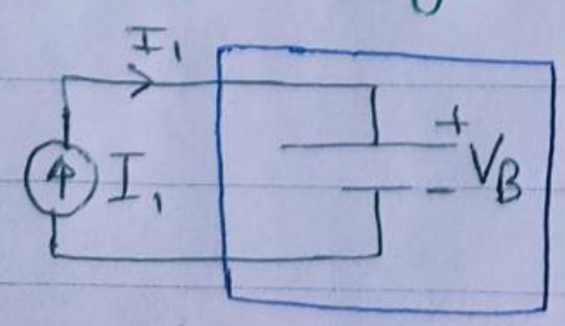
Example

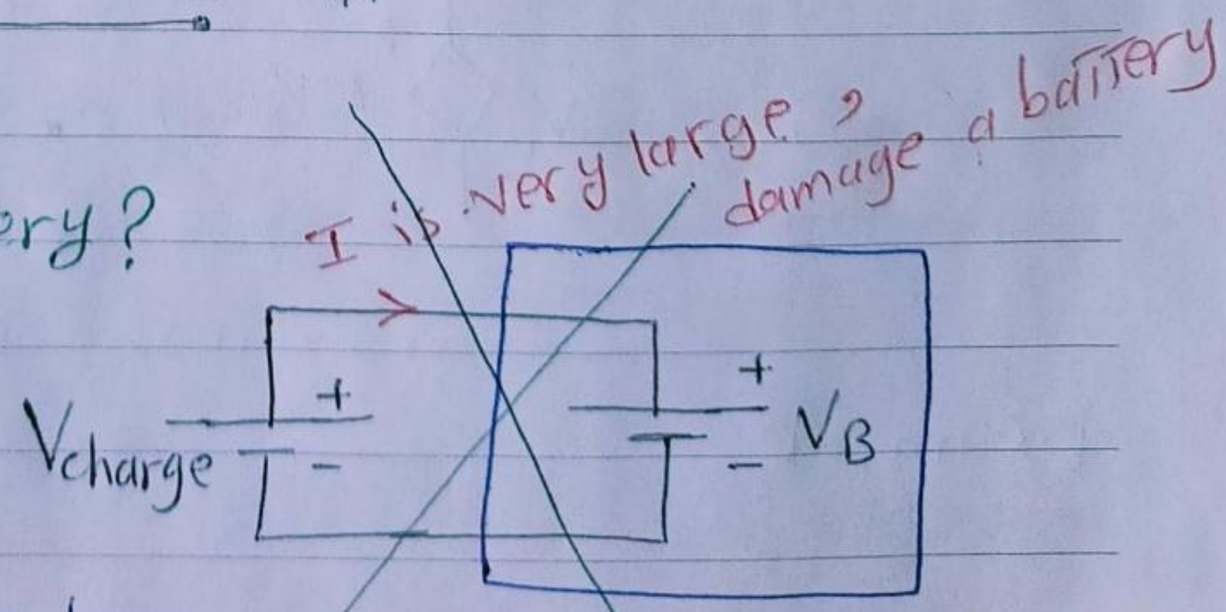
$V_{OUT} = 2V$



Example

How do we charge a battery?

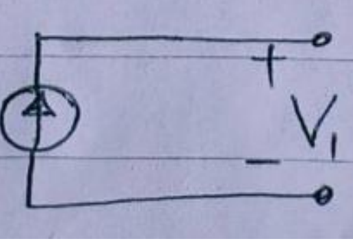




- I is well defined This current is designed based on electronic devices

Example

Mathematically; $V_1 = I_1 R \to \infty$



: o.c ⇒ $R = \infty$

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Lec 3: Dependent Sources, Ohm's Law, I/V Characteristics

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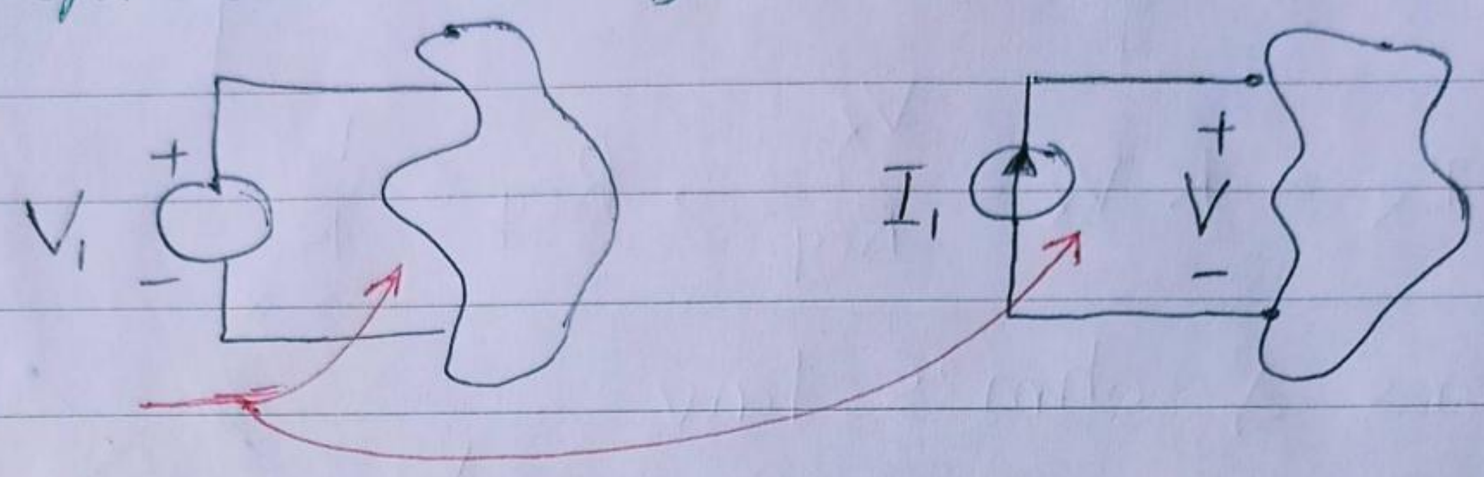
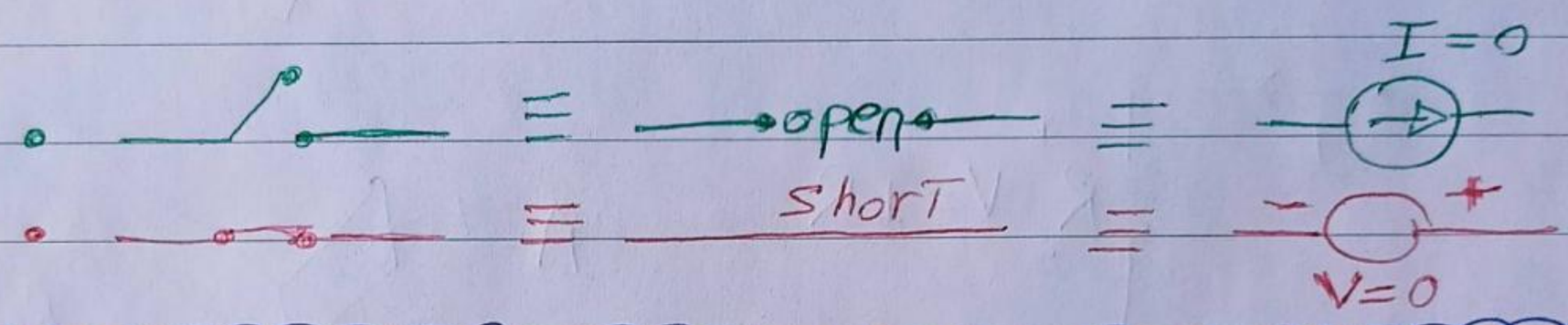
Review of Lecture 2

- electric devices : wire , switch , independent & dependent voltage and current sources.



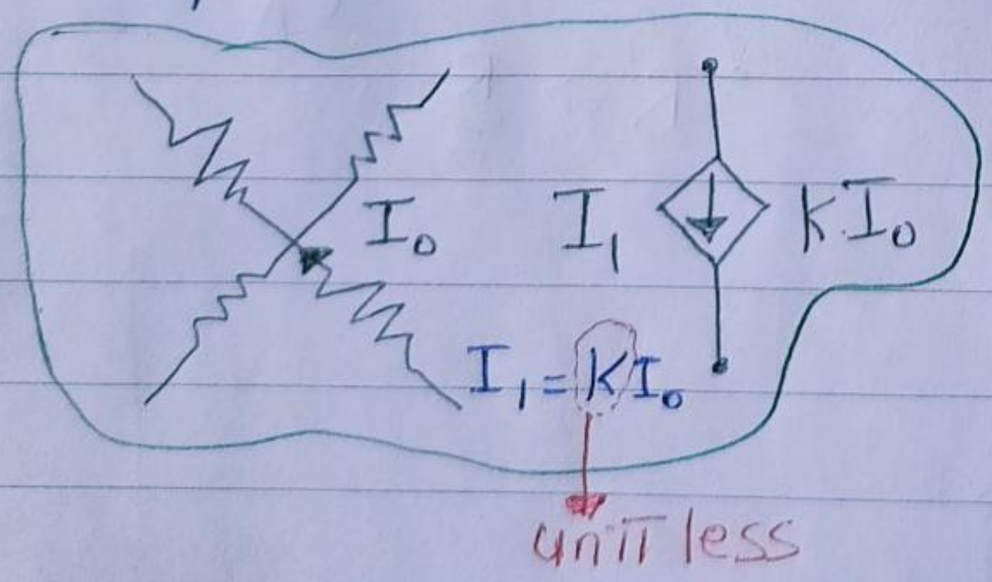
Do not satisfy ohm's law.

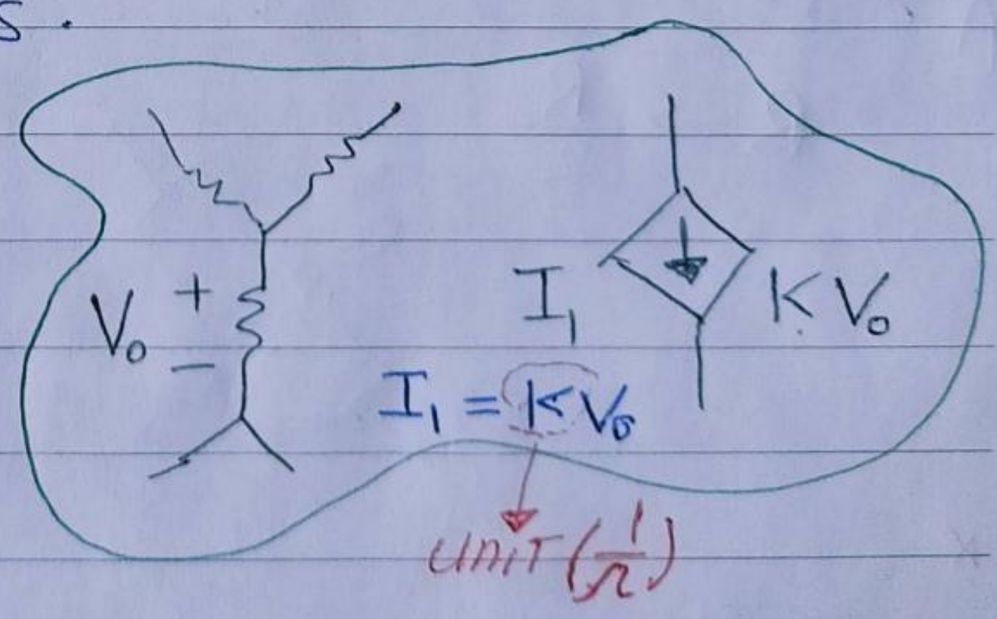


- Today's lecture

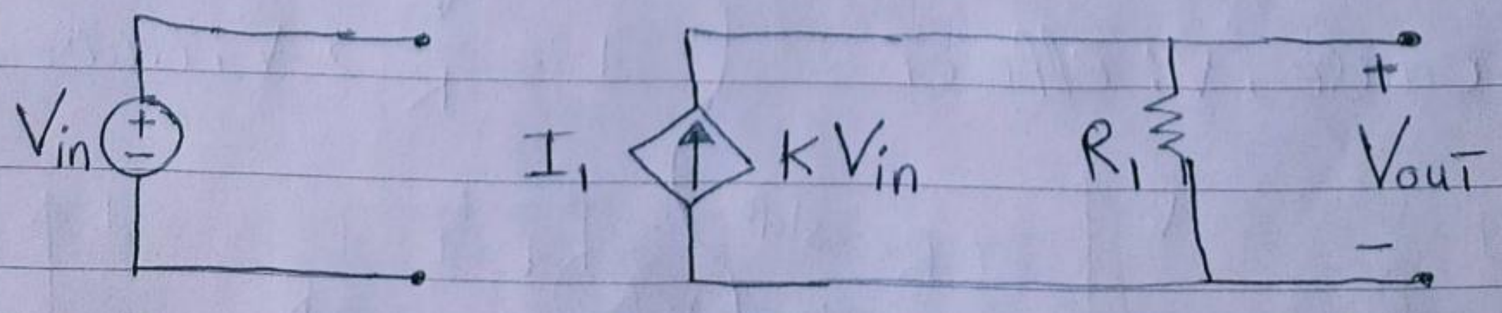
- Dependent current sources.
- Resistors and ohm's law.
- I / V characteristic.

- Dependent current sources.





Example



$V_{out} = R_1 K V_{in}$ ⇒ circuit acts as an Amplifier.

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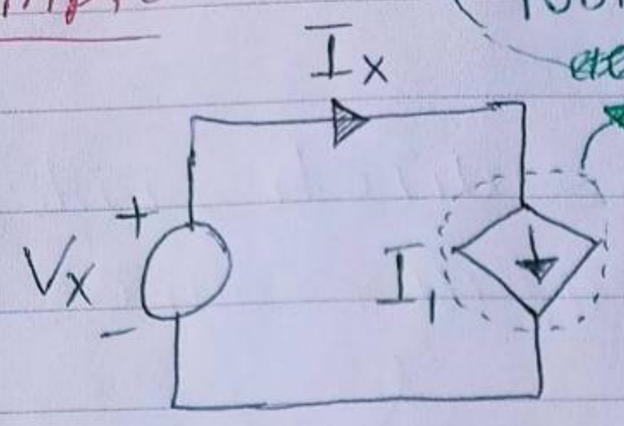
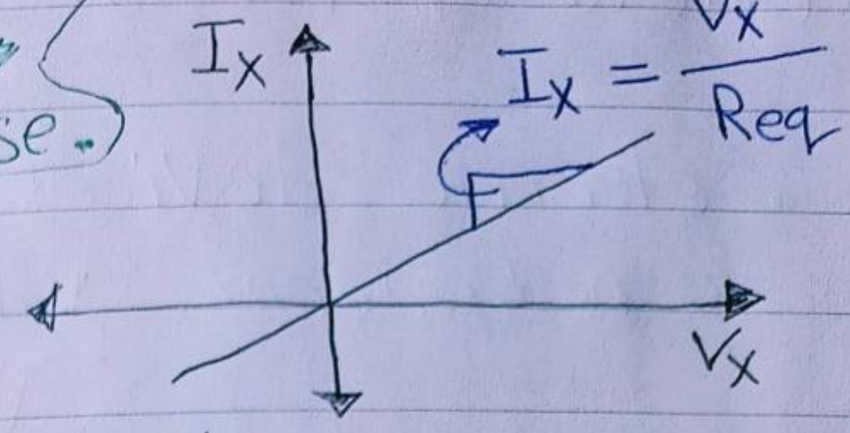
Example

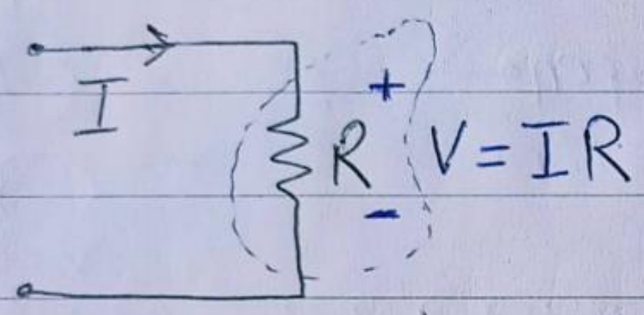


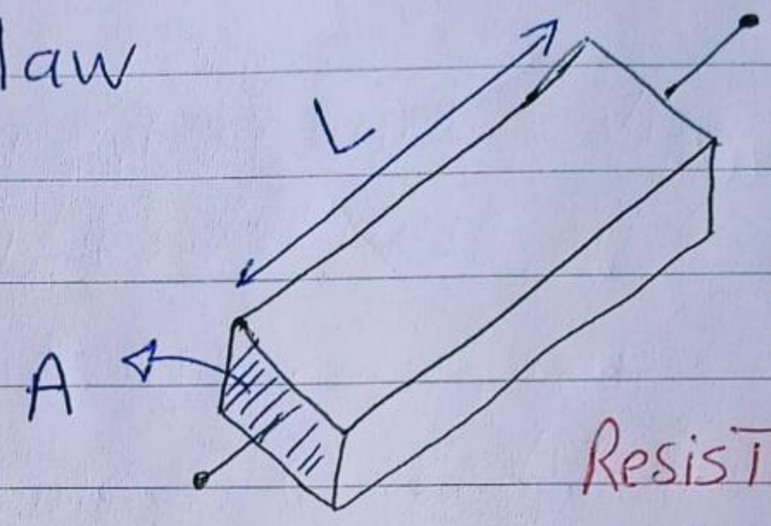
look Like a resistor ~~act as resistor~~ in This case.



$$I_x = KV_x = \frac{V_x}{R_{eq}}, \quad R_{eq} = \frac{1}{K}$$

- Resistors & ohm's law



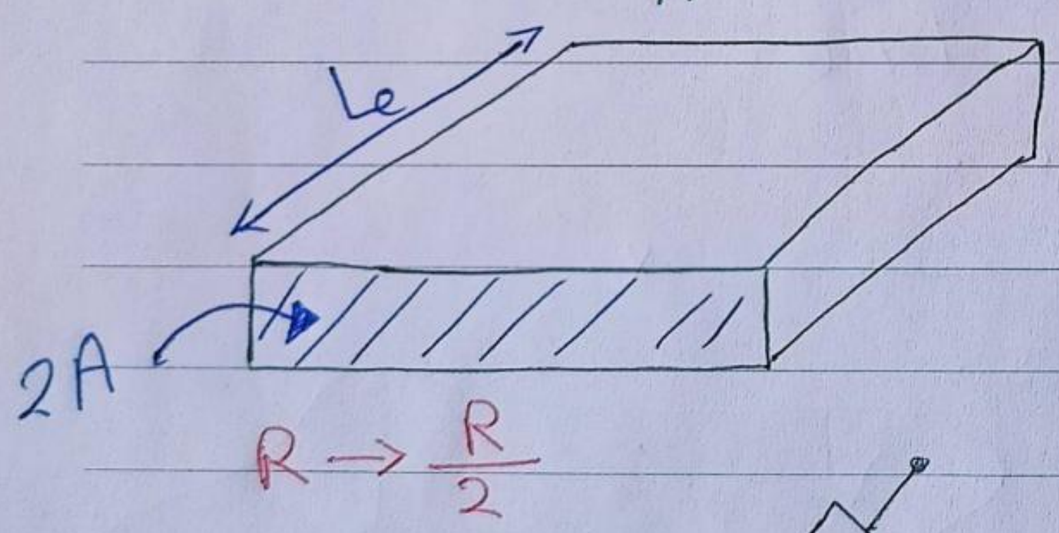


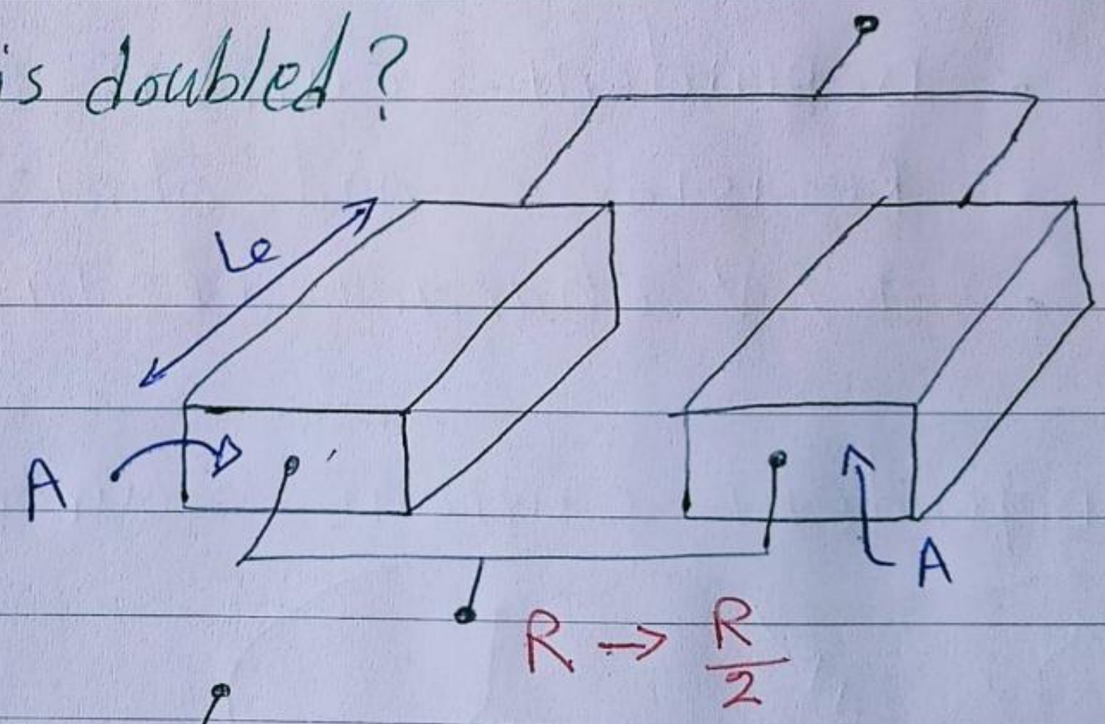
$$R = \rho \frac{L}{A}$$

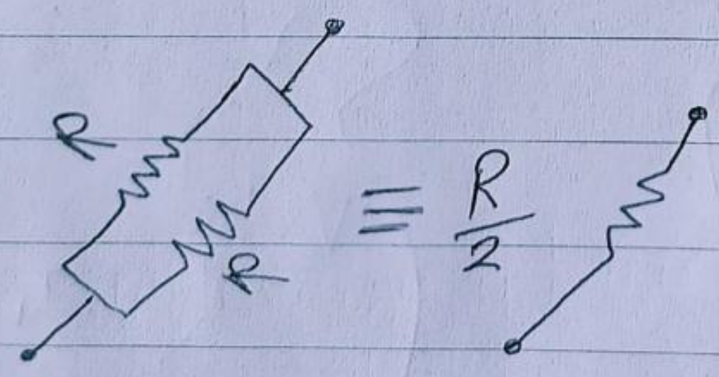
Resistivity of Material

Example

what happens if A is doubled?

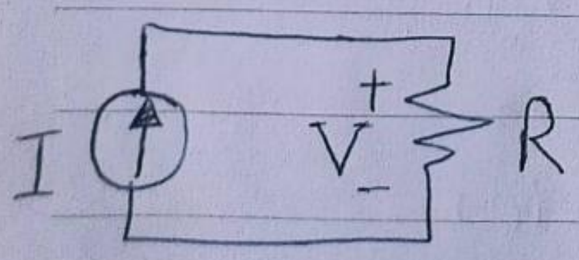


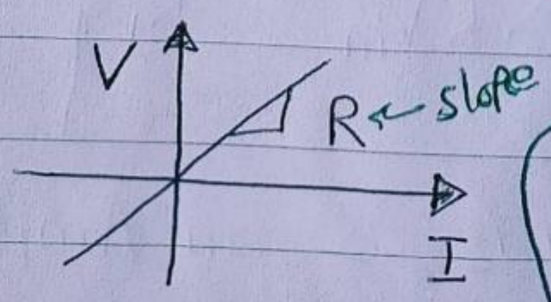




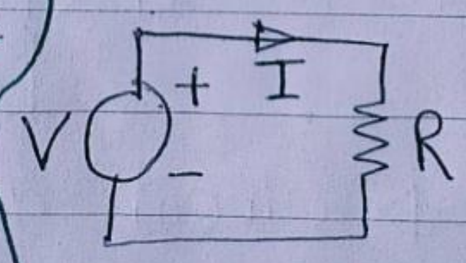
- I/V characteristics.

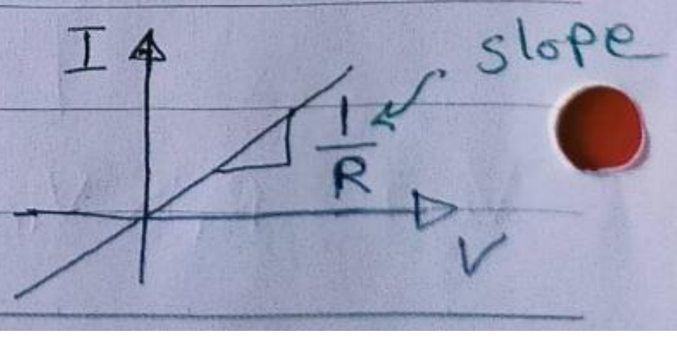
$V = IR$, I: cause, V: effect





$I = \frac{V}{R}$, V: cause, I: effect





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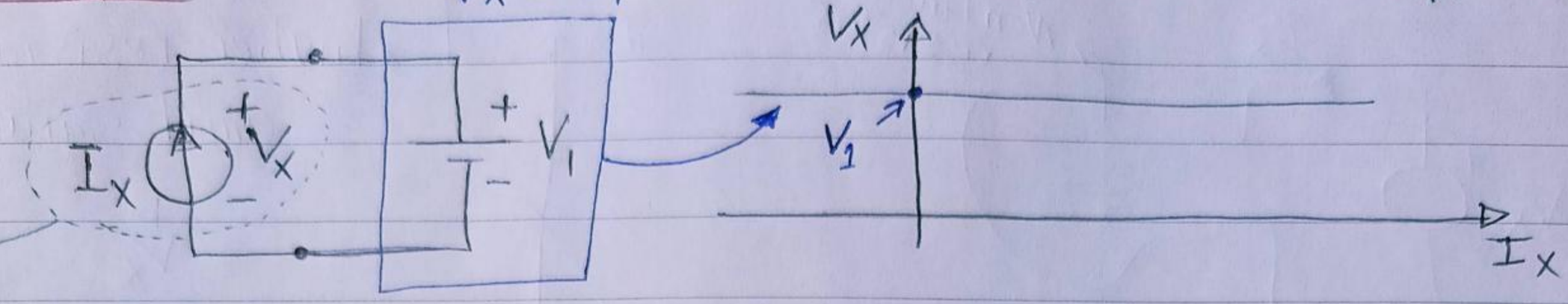
Example

$V_x = V_1$

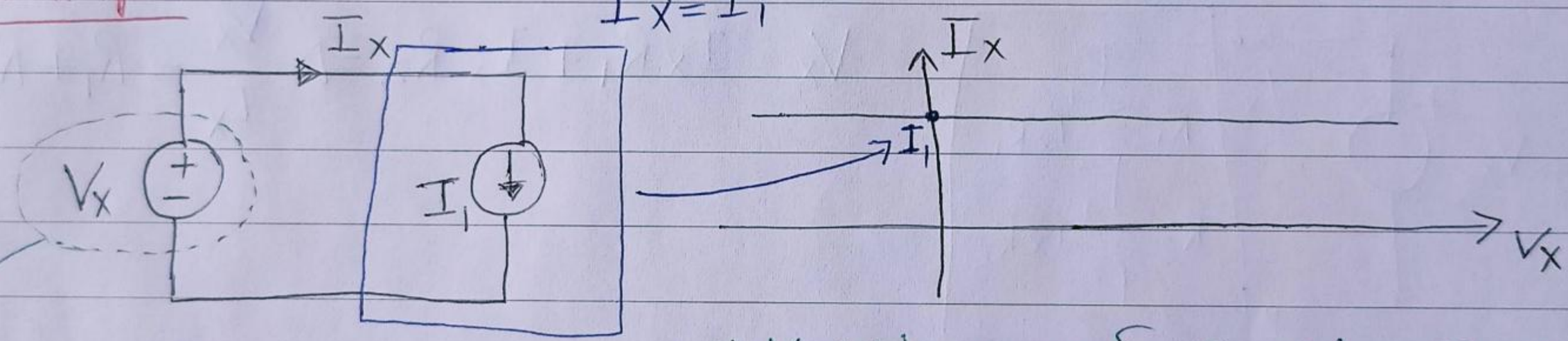


" I_x : The cause & V_x : The effect "

if Voltage source ~~is~~ applied , Voltage source makes no sense ~~we can't~~ due To finding Voltage source , we can't do That , we get an infinite current . so ~~The~~ The current Source is appled .

Example

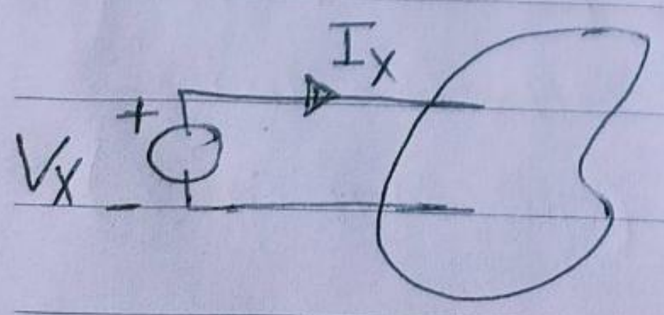
$I_x = I_1$



" V_x : The cause & I_x : The effect "

If current source is applied , ~~Thi~~ it has To fight This current Source " I_1 " , That doesn't make sense , so we have To apply a Voltage source .

- Application of ohm's Law



$$R_{eq} = \frac{V_x}{I_x}$$

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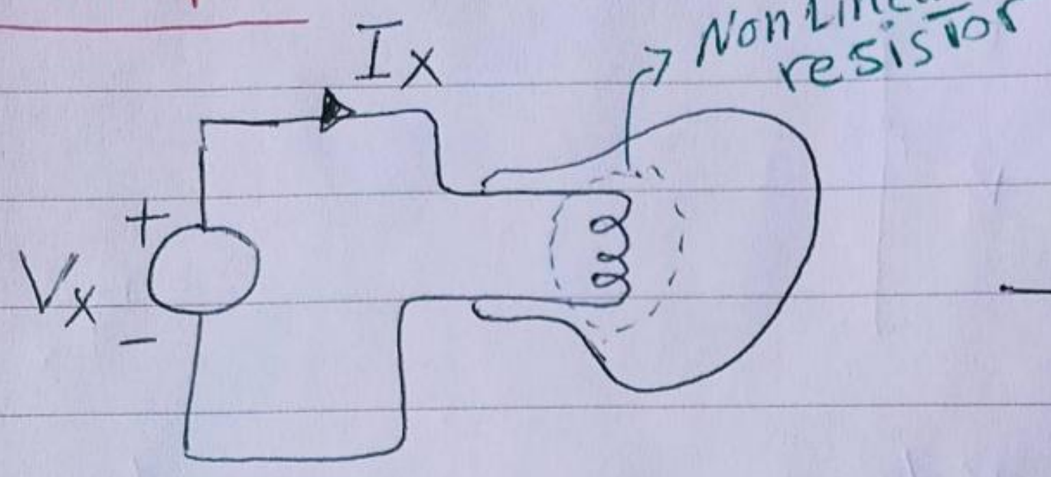
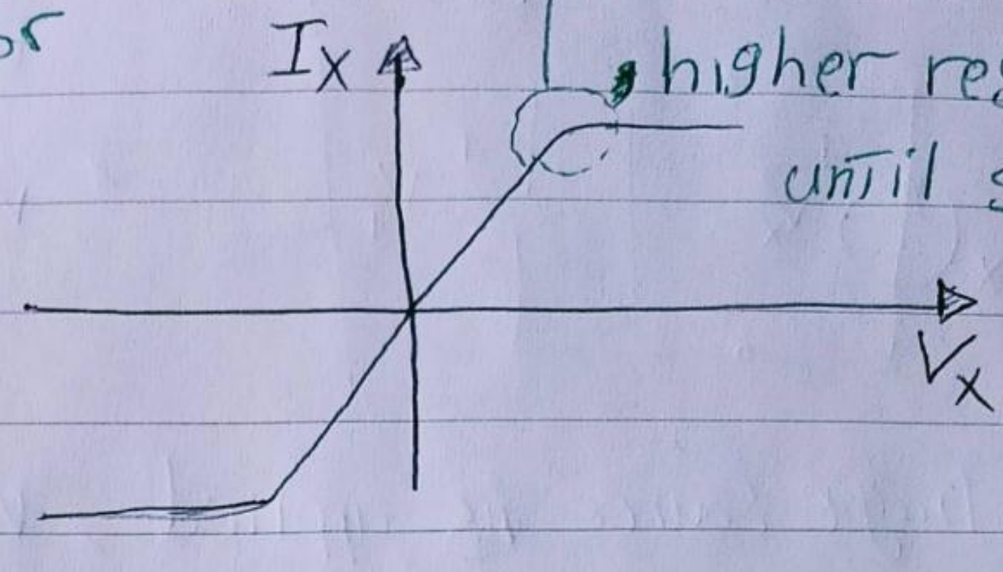
Example

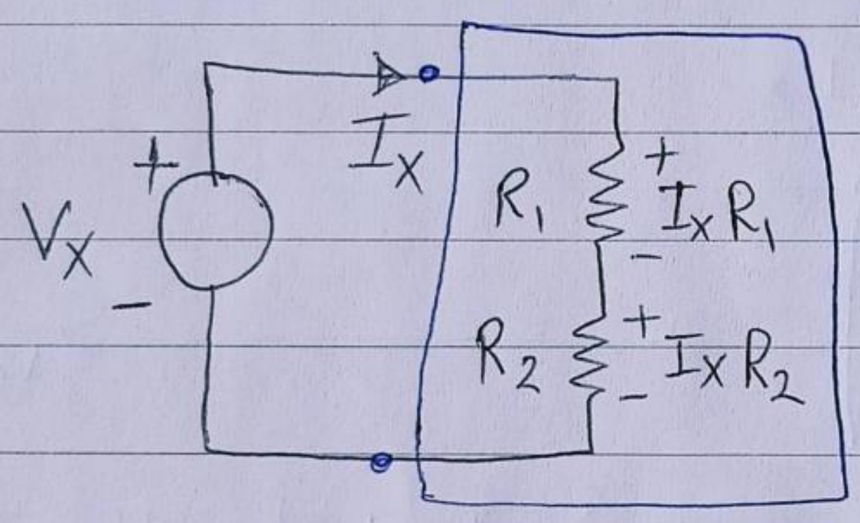




Filament is hotter, higher resistance until saturate.

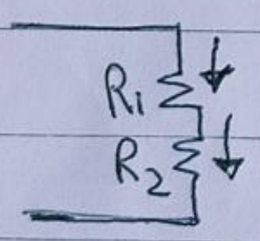
Example

Find the equivalent resistance of this topology ?!

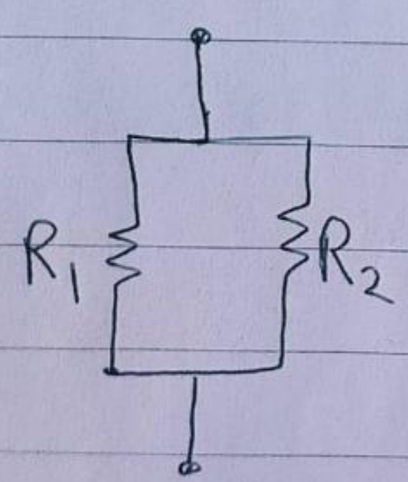


$$V_x = I_x R_1 + I_x R_2 \Rightarrow \frac{V_x}{I_x} = R_1 + R_2$$

Definition: Two Resistors are in series if they share only one Terminal <u>and</u> have the same current.



Definition: Two resistors are in parallel if they share both Terminals and have the same voltage (automatically).



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Lec 4: Kircchoff's Laws (KVL, KCL)

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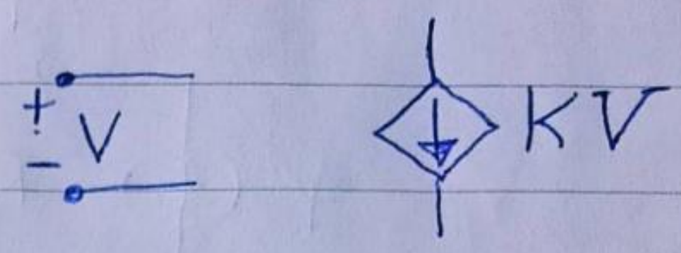
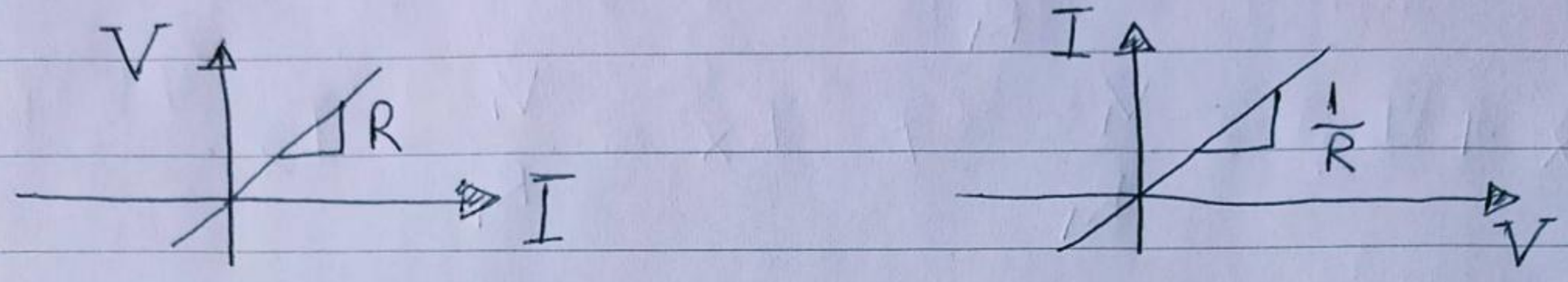
- Review of Lecture 3

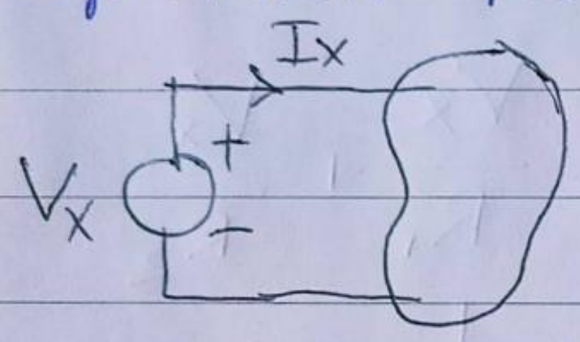
- Dependent current source



- Resistor : $V = IR$
- I / V characteristics

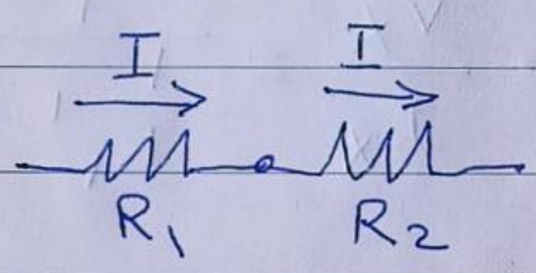


- Equivalent Resistance

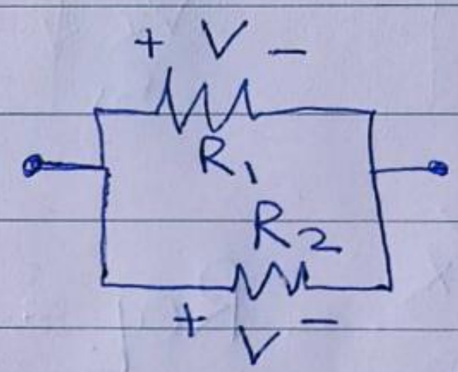


$$R_{eq} = \frac{V_x}{I_x}$$

- series



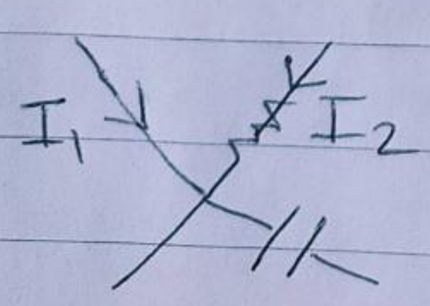
- parallel



- Today's lecture

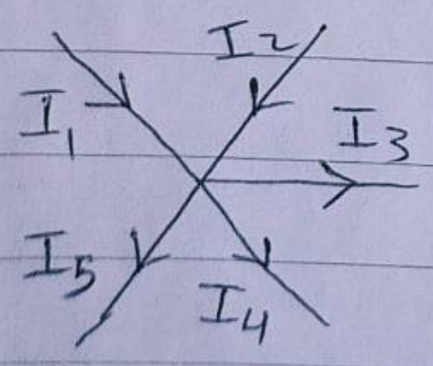
- Circuit Theorems : Kirchoff's current & Voltage laws.

- KCL



$$\sum_j I_j = 0 \text{ (charge conversation)}$$

$$\underbrace{\sum_j I_j}_{\text{entering}} = \underbrace{\sum_j I_j}_{\text{exiting}}$$



$$I_1 + I_2 = I_3 + I_4 + I_5$$

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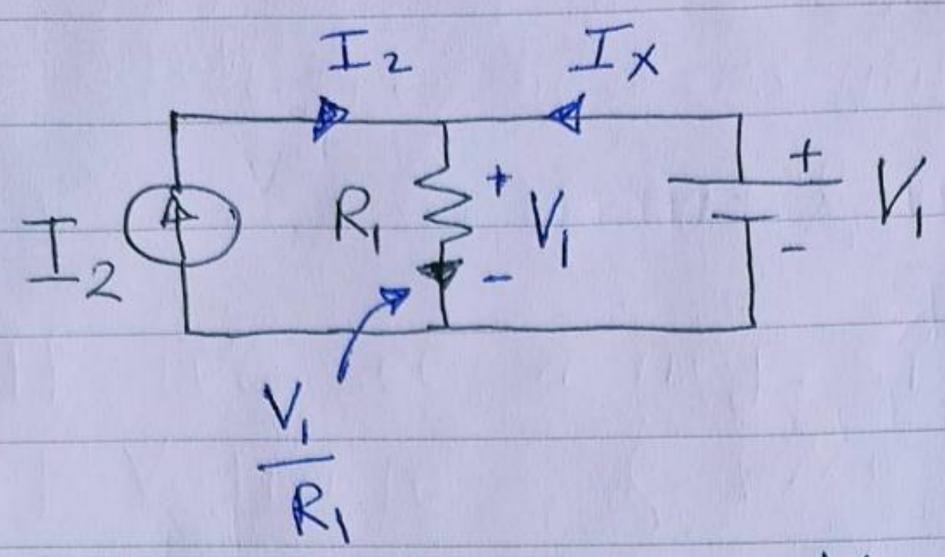
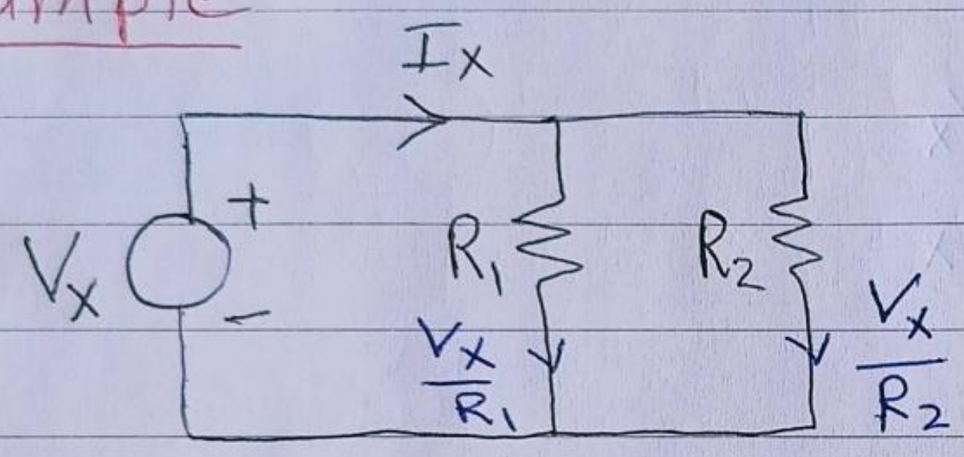
Example



$$I_x + I_2 = \frac{V_1}{R_1} \Rightarrow I_x = \frac{V_1}{R_1} - I_2 .$$

Example

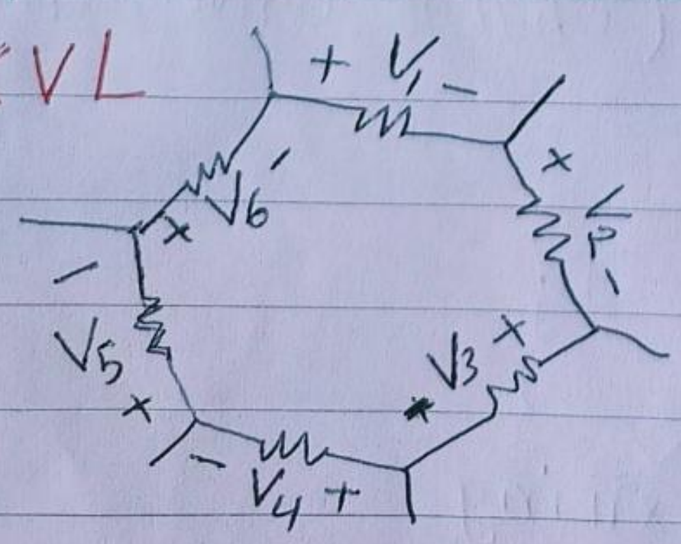


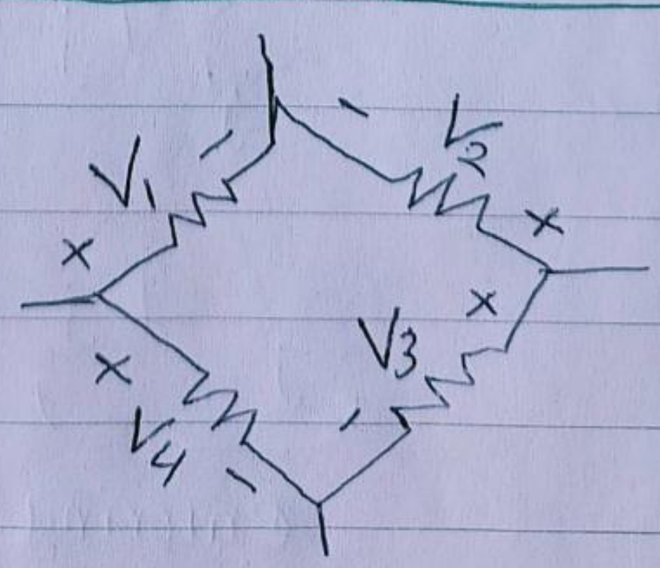
- $I_x = \frac{V_x}{R_1} + \frac{V_x}{R_2}$
- $\frac{V_x}{I_x} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$
- $\frac{V_x}{I_x} = R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

observations:

- $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} < R_1$ or R_2
- $R_1 \to 0 \Rightarrow R_{eq} \to 0$
- $R_1 \to \infty \Rightarrow R_{eq} \to R_2 \quad (R_{eq} = R_2)$

- KVL





$$\sum_j V_j = 0 \text{ around a loop.}$$

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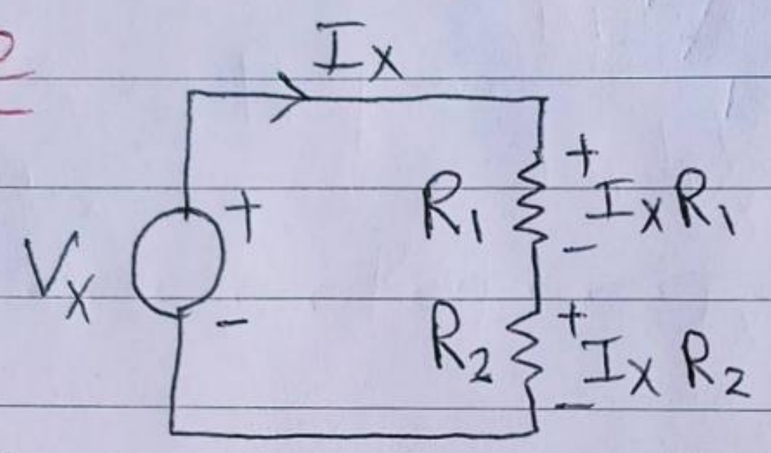
Approach 1 : If we go from high potential to low potential, then assign a neg. voltage & vice versa

$$-V_1 - V_2 - \cdots - V_6 = 0 \quad \& \quad -V_1 + V_2 - V_3 + V_4 = 0$$

Approch 2 : If we see a neg. sign, then assign a neg voltage - & vice versa.

$$V_1 + V_2 + \cdots + V_6 = 0 \quad \& \quad -V_2 + V_3 - V_4 + V_1 = 0$$

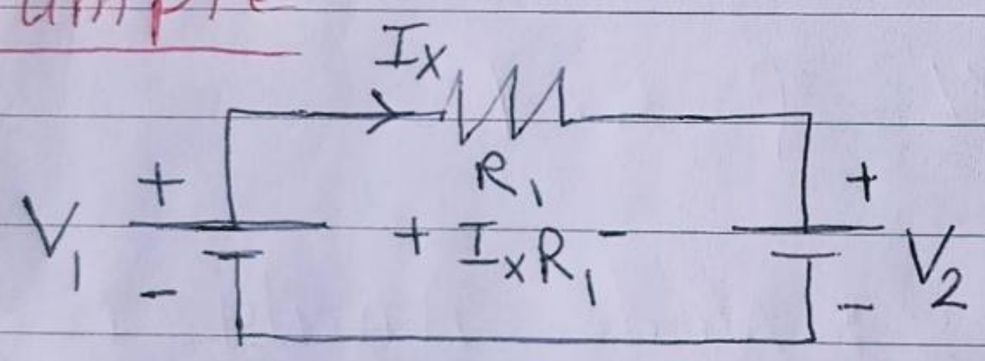
Example



$$-I_x R_1 - I_x R_2 + V_x = 0$$

$$V_x = I_x R_1 + I_x R_2$$

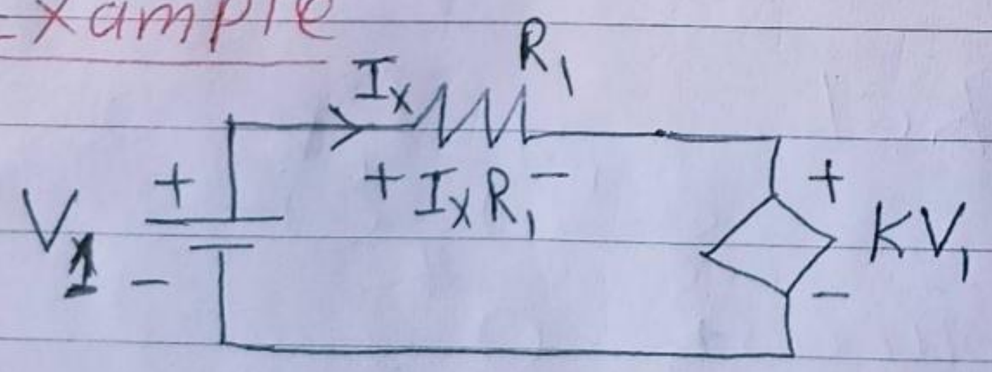
Example



$$+V_1 - I_x R_1 - V_2 = 0 \Rightarrow$$

$$I_x = \frac{V_1 - V_2}{R_1}$$

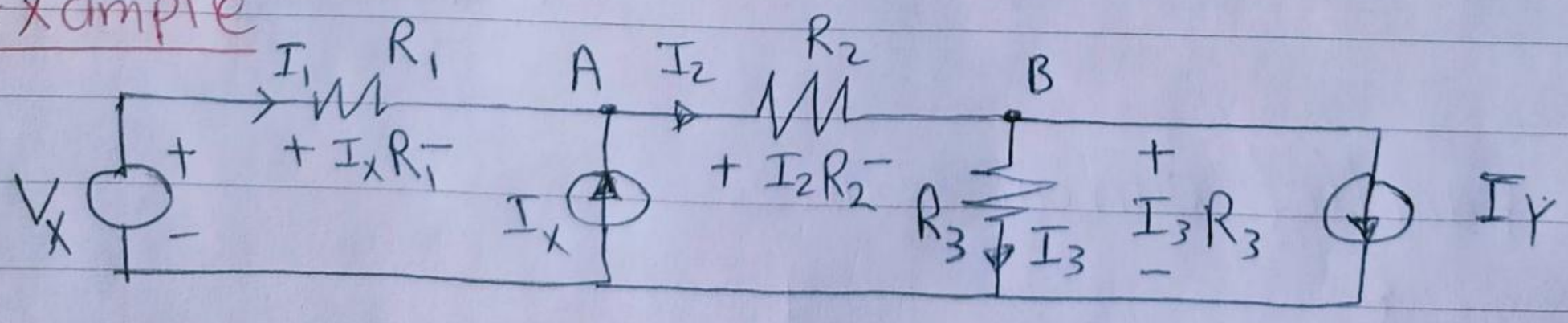
Example



$$+V_1 - I_x R_1 - KV_1 = 0$$

$$I_x = \frac{V_1 - KV_1}{R_1} = (1-K)\frac{V_1}{R_1}$$

Example



- $I_1 + I_x = I_2$
- $I_2 = I_3 + I_y$

- $+V_x - I_1 R_1 - I_2 R_2 - I_3 R_3 = 0$

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Lec 5: More on KVL/KCL; Node Analysis

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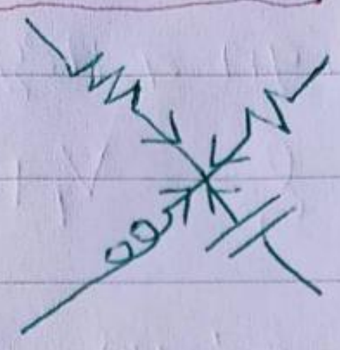
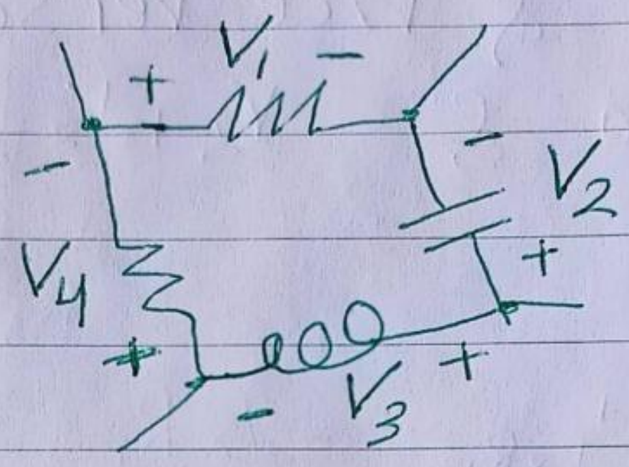
- Revie of lecture 4

- KCL



$\sum I_j = 0$

- KVL



$\sum V_j = 0$

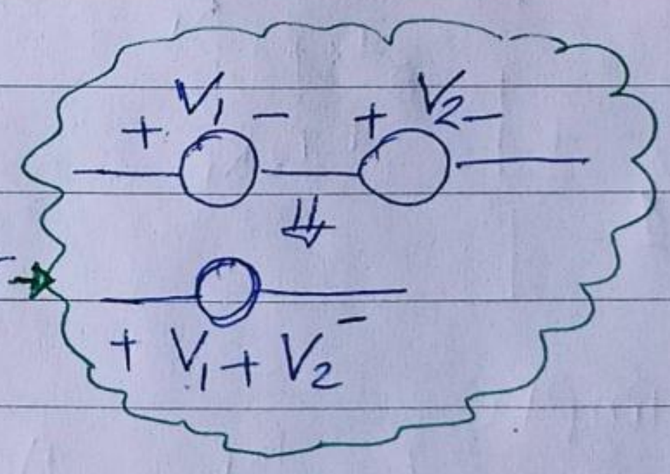
$-V_4 - V_1 + V_2 - V_3 = 0$

- Today's lecture

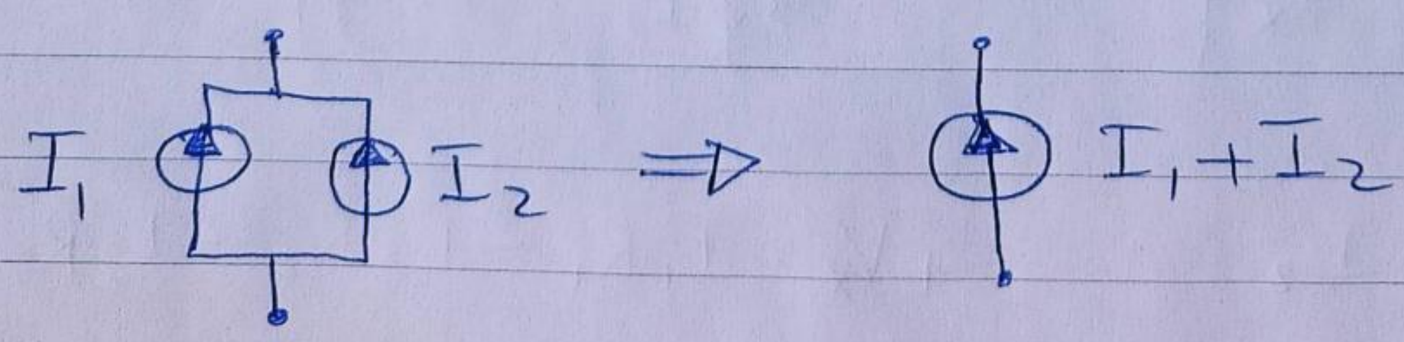
- mechanics of KVL & KCL.
- Examples.
- Analysis Techniques : Node Analysis.

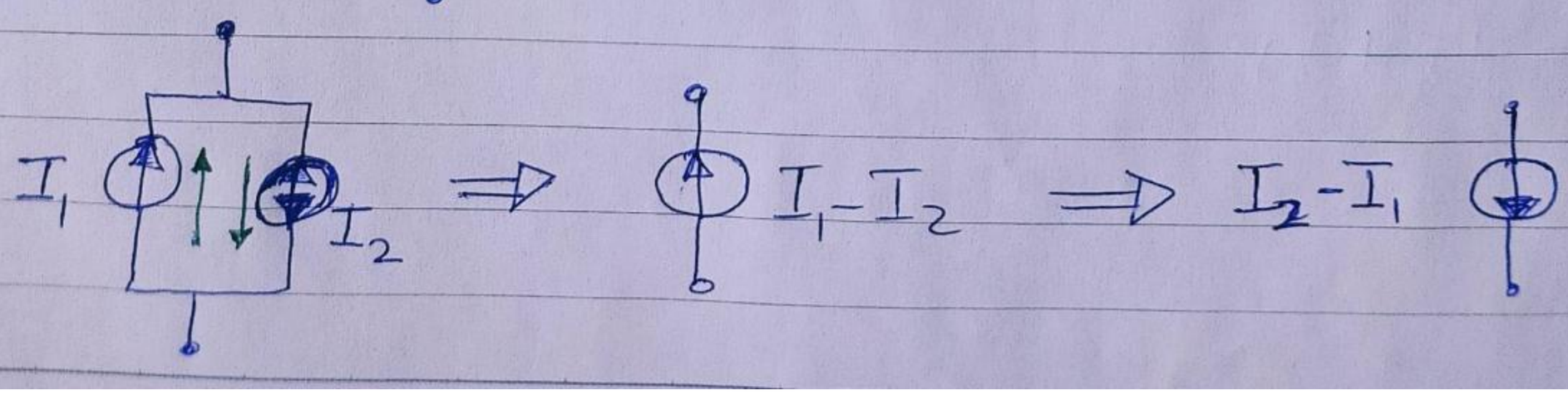
- Mechanics of KVL & KCL.

  - series Voltage sources



  - parallel current sources





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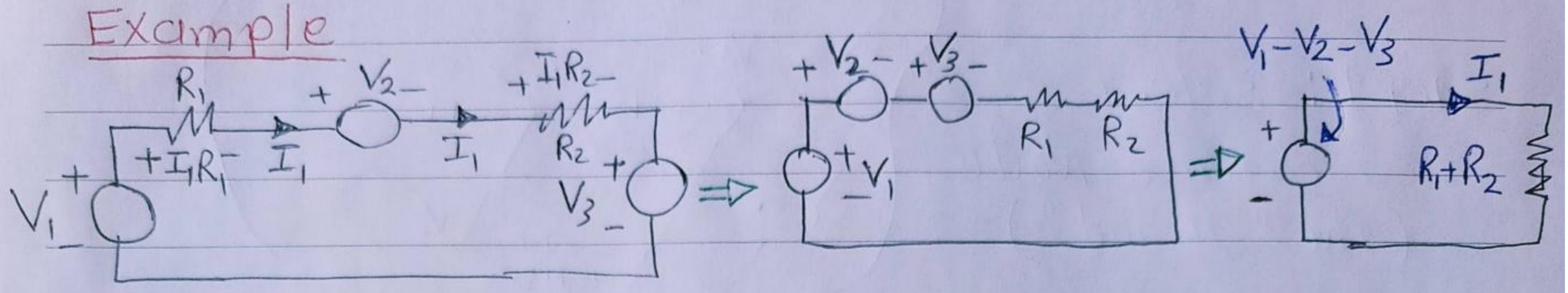
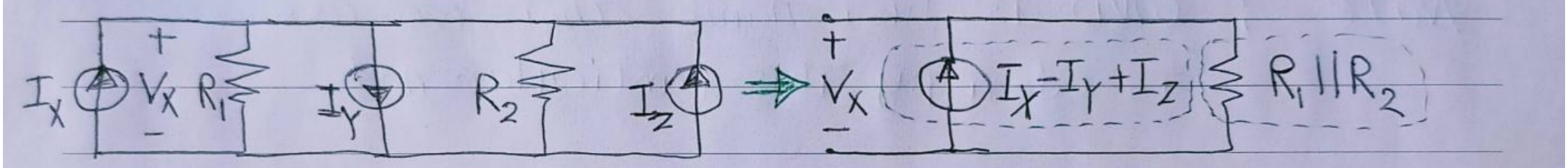
- Resistors in series or in parallel.

Example



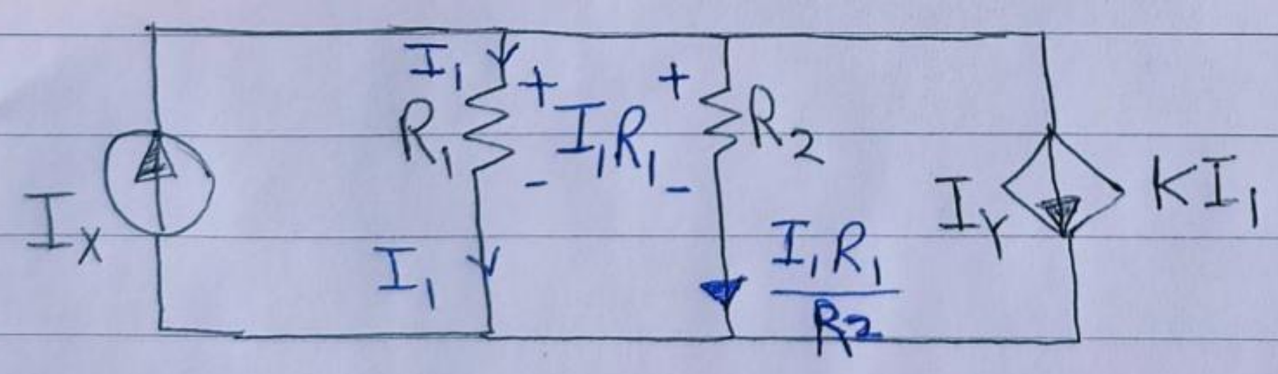
$$+V_1 - I_1R_1 - V_2 - I_1R_2 - V_3 = 0 \Rightarrow I_1 = \frac{V_1 - V_2 - V_3}{R_1 + R_2}$$

Example



$$V_X = (I_X - I_Y + I_Z)\, R_1 \| R_2 \Rightarrow V_X = (I_X - I_Y + I_Z)\frac{R_1 R_2}{R_1 + R_2}$$

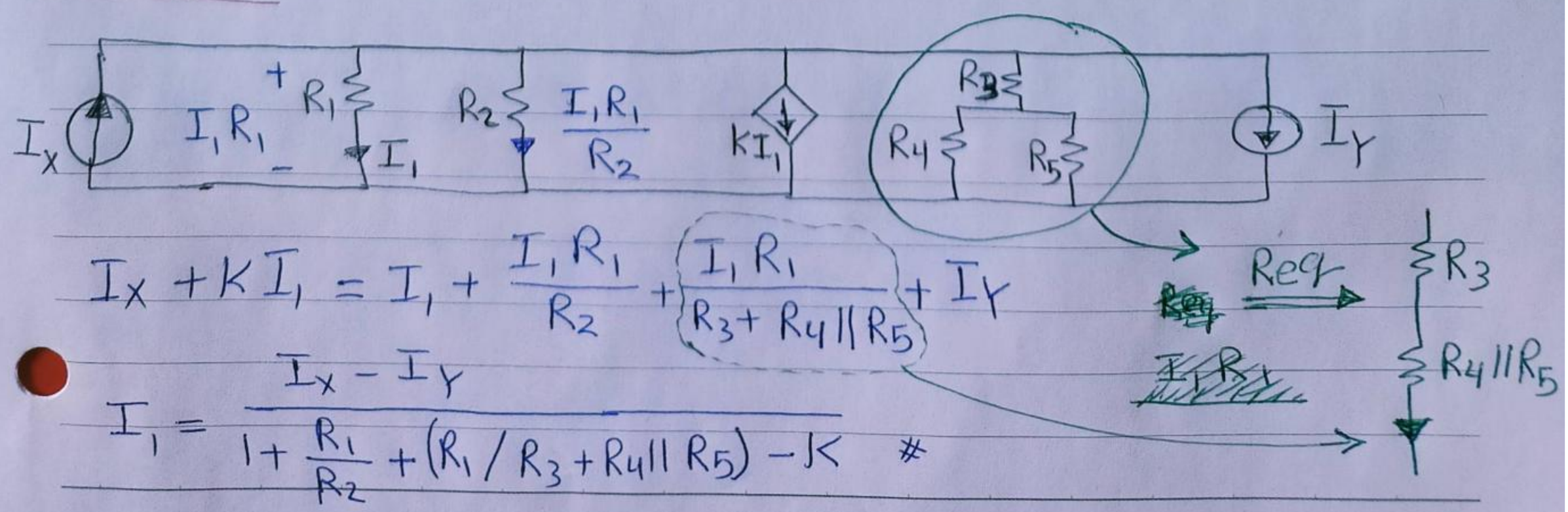
Example



$$I_X = I_1 + \frac{I_1R_1}{R_2} + KI_1$$

$$I_1 = \frac{I_X}{1 + R_1/R_2 + K}$$

Example



$$I_X + KI_1 = I_1 + \frac{I_1R_1}{R_2} + \frac{I_1R_1}{R_3 + R_4 \| R_5} + I_Y$$

$$I_1 = \frac{I_X - I_Y}{1 + \frac{R_1}{R_2} + (R_1 / R_3 + R_4 \| R_5) - K} \quad \#$$

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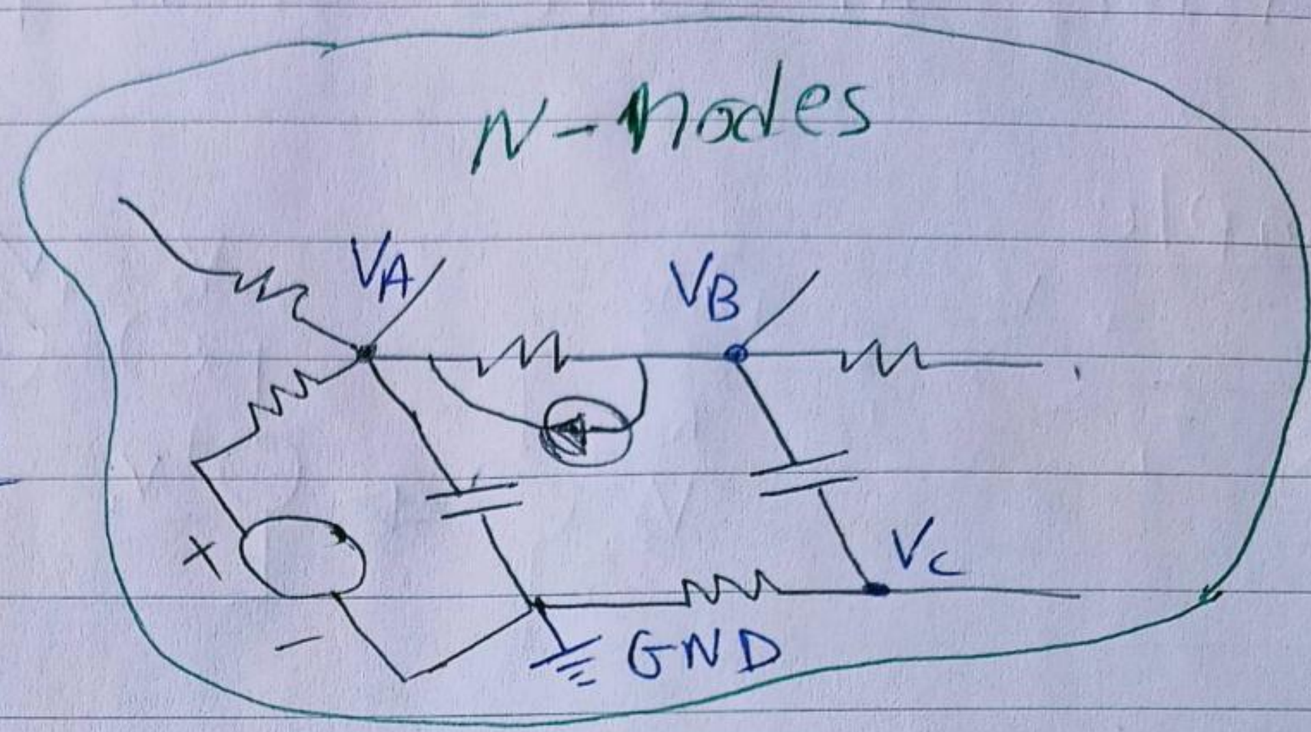
- Node Analysis



STEP 1: select a reference Node (The "ground")

STEP 2: Assign Voltages with respect To GND To all other nodes. ⇒ N-1 node Voltages

NOTE ; GND is just name That we use To Identify This particular node.

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Lec 6: Node Analysis, Supernodes

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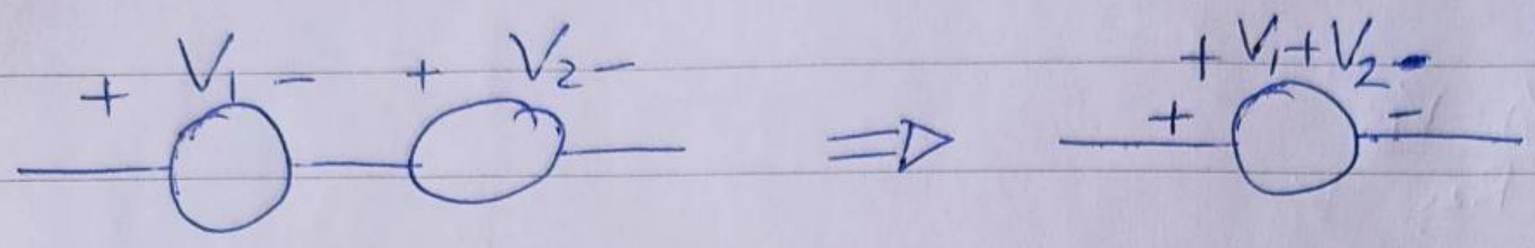
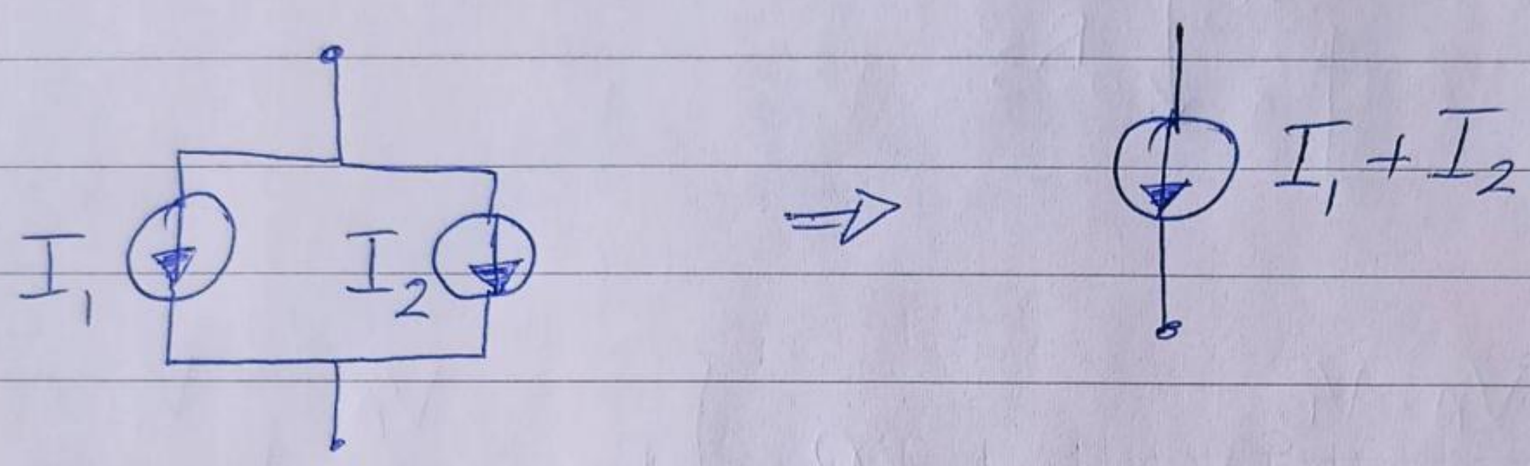
- Review of lecture 5

- Voltage sources in series.



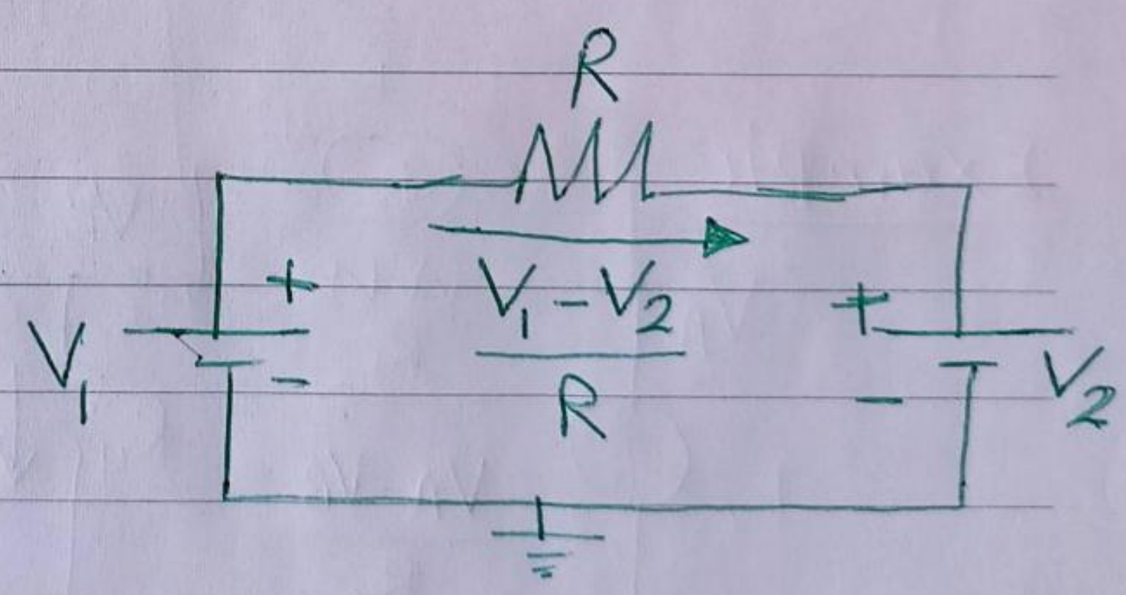
- Current sources in parallel



- Resistors in series or parallel
- Node Analysis; select a Ref. Node ("GND")

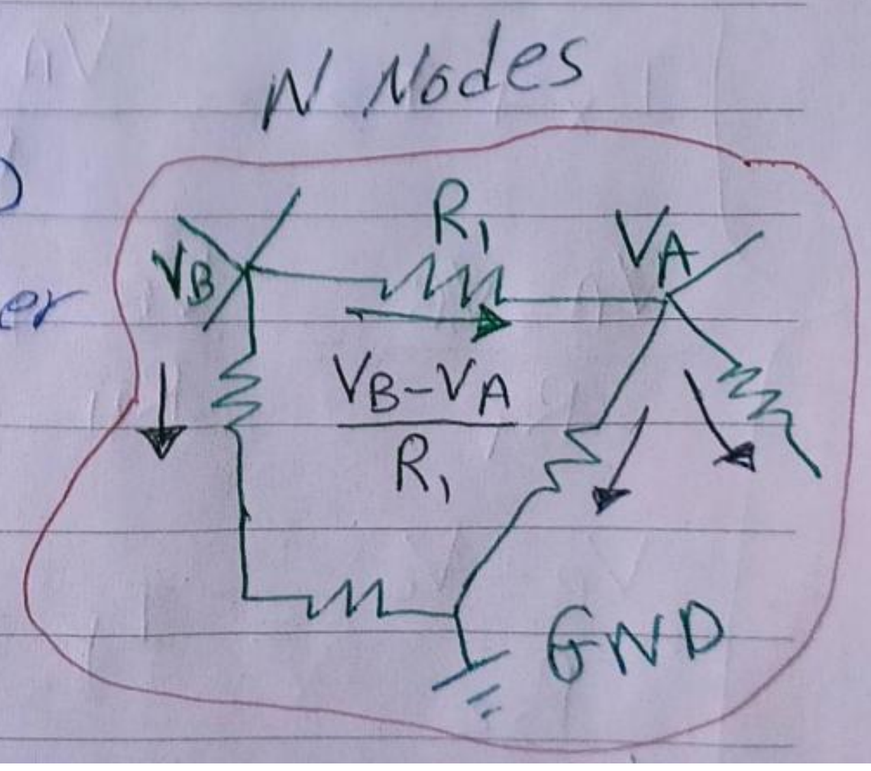
- Todays lecture

- Node Analysis
  - step by step procedure
  - Examples
  - concept of supernode



- Node Analysis

<u>Step 1</u>: select a reference Node, GND
<u>Step 2</u>: Assign Voltages to all other Nodes $\Longrightarrow$ N−1 Voltages.
<u>Step 3</u>: write the current through every branch.



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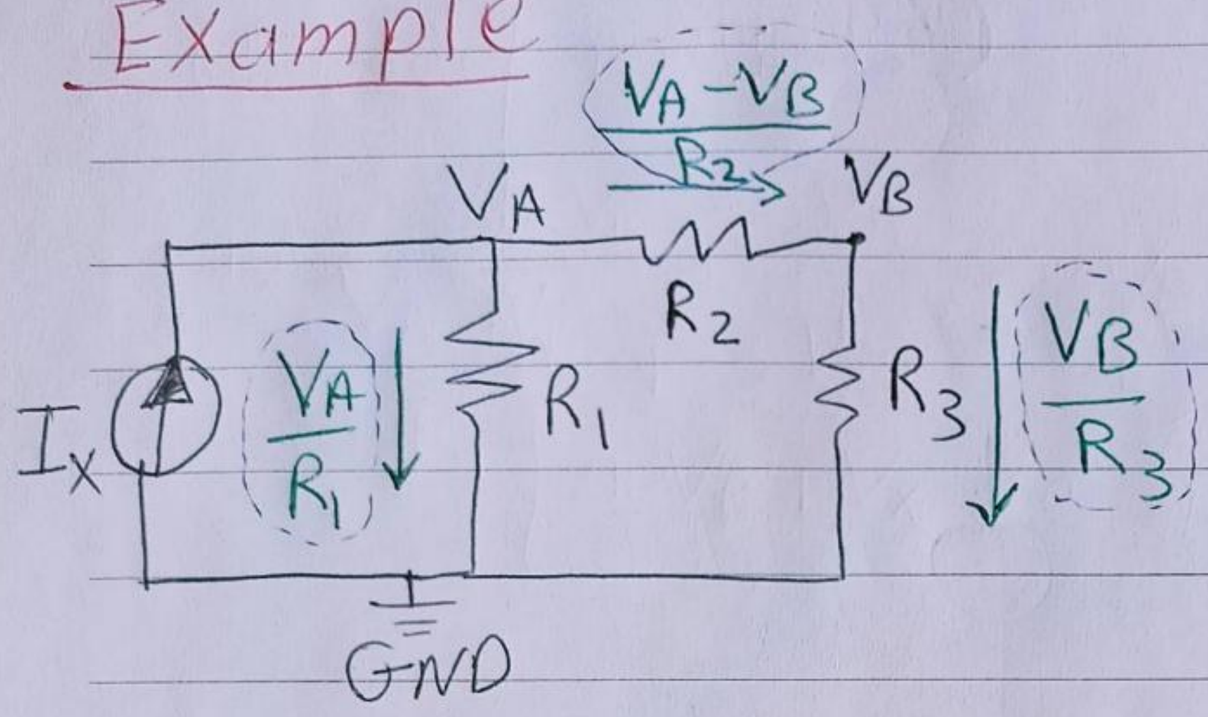
Step4: Write a KCL at all nodes (except at GND)
↳ N-1 equations ⇒ find the voltages.

Example

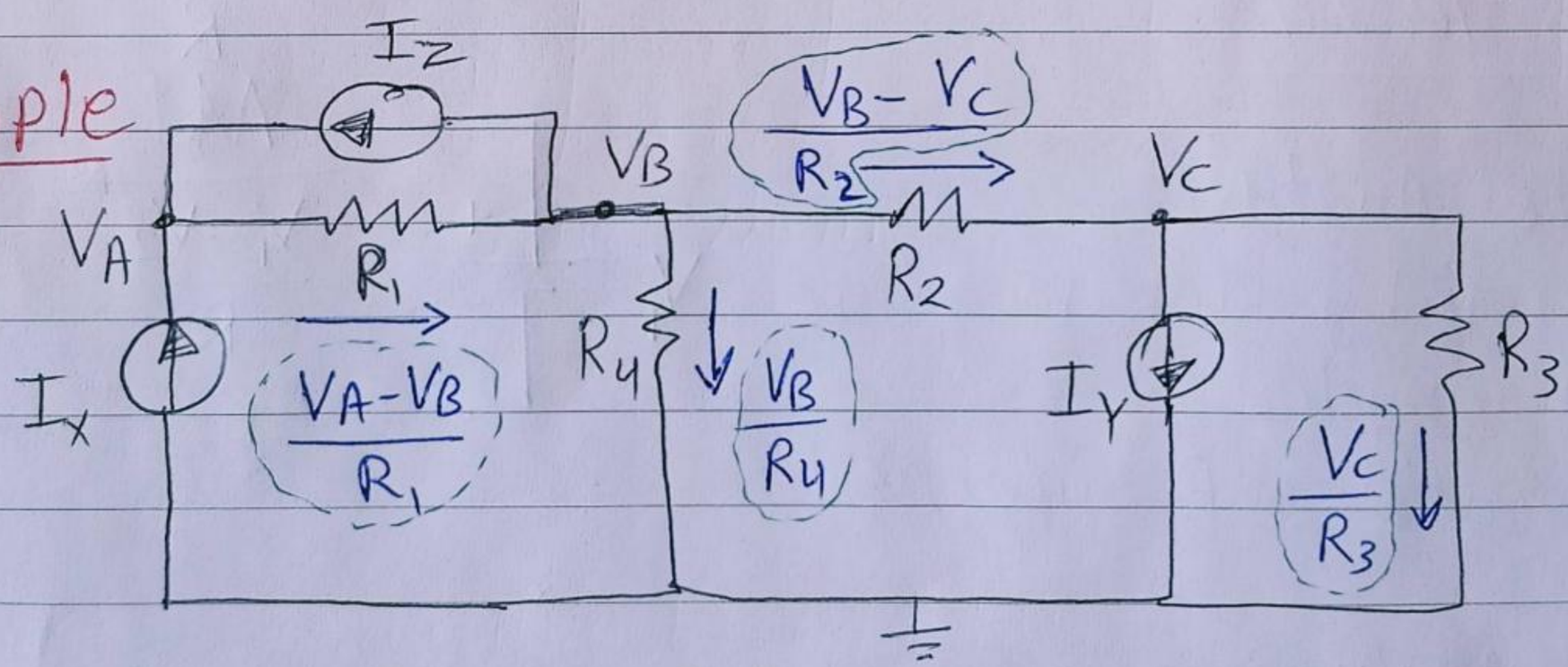


$$\begin{cases} I_x = \dfrac{V_A}{R_1} + \dfrac{V_A - V_B}{R_2} \\ \dfrac{V_A - V_B}{R_2} = \dfrac{V_B}{R_3} \Rightarrow V_A \ \& \ V_B \end{cases}$$

OR

$$\begin{cases} I_x = \dfrac{V_A}{R_1} + \dfrac{V_A}{R_2 + R_3} \\ \Rightarrow \text{Then finding } V_B \end{cases}$$

Example



$$\begin{cases} I_x + I_z = \dfrac{V_A - V_B}{R_1} \\ \dfrac{V_A - V_B}{R_1} = \dfrac{V_B}{R_4} + I_z + \dfrac{V_B - V_C}{R_2} \\ \dfrac{V_B - V_C}{R_2} = I_y + \dfrac{V_C}{R_3} \end{cases}$$

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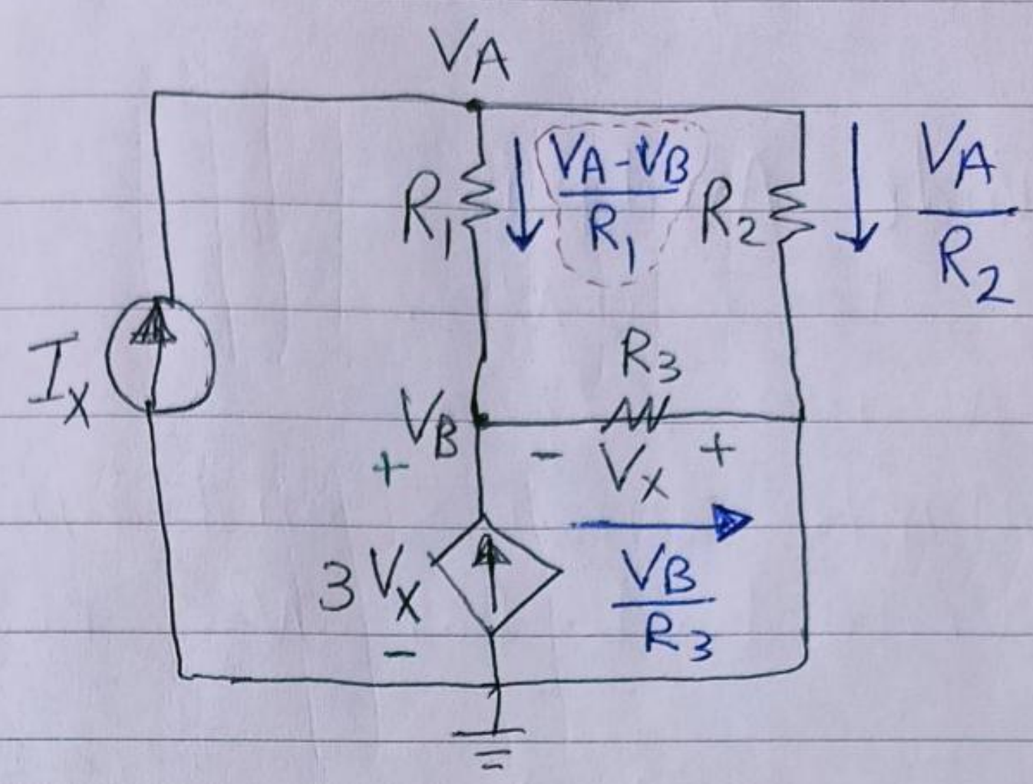
Example

- KCL at V_A, V_B Node ("Node Analysis")



$$\begin{cases} I_x = \dfrac{V_A - V_B}{R_1} + \dfrac{V_A}{R_2} \\ \dfrac{V_A - V_B}{R_1} + 3V_x = \dfrac{V_B}{R_3} \\ +V_B + V_x = 0 \Rightarrow V_B = -V_x \end{cases}$$

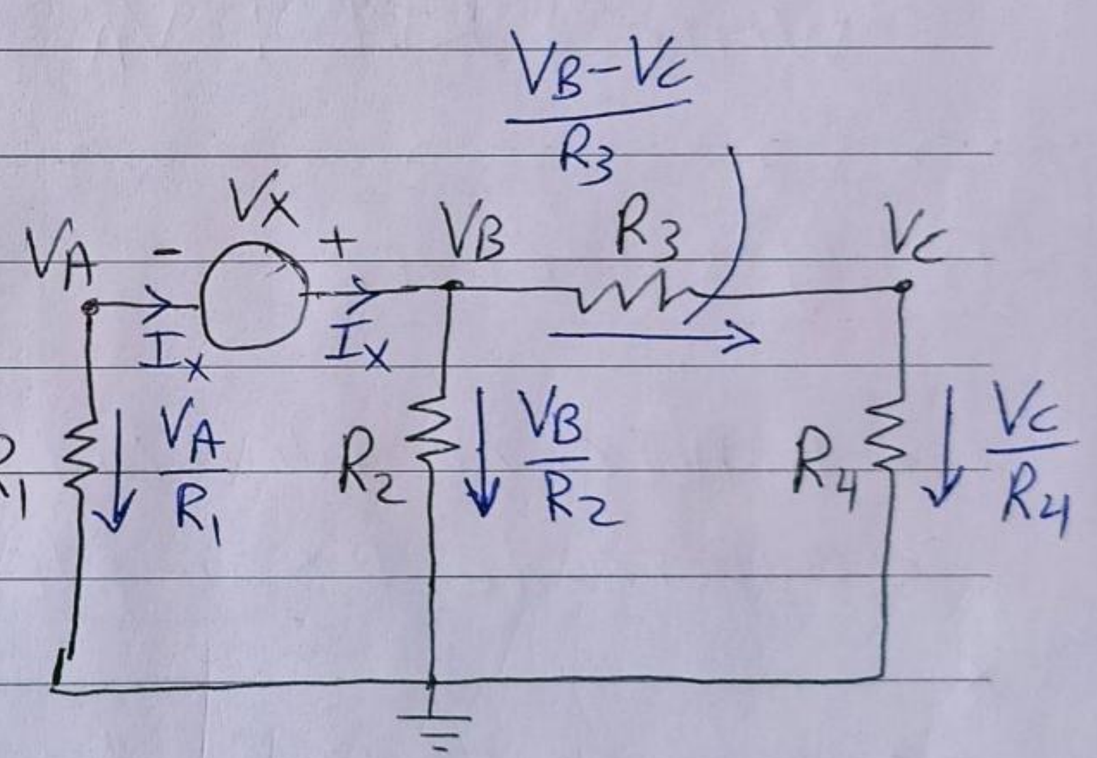
Example

$$\frac{V_B - V_C}{R_3} = \frac{V_C}{R_4} \rightarrow (1)$$

$$I_x = \frac{V_B - V_C}{R_3} + \frac{V_B}{R_2}$$

$$I_x + \frac{V_A}{R_1} = 0$$

- I_x is unknown
- Three equations in four unknowns so can not solve them



Take Two additional steps:

STEP 5: Form a "supernode" by "Merging" The nodes on Two sides of The Voltage Source.

$\Rightarrow$ Write KCL for The supernode

$$\frac{V_A}{R_1} + \frac{V_B}{R_2} + \frac{V_B - V_C}{R_3} = 0 \rightarrow (2)$$

STEP 6: Write a KVL involving The Voltage Source:

$$V_A + V_x = V_B \rightarrow (3)$$ "Note: V_x is known"

finally, Three unknown V_A, V_B, V_C in Three equations #

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Lec 7: Supernode Examples

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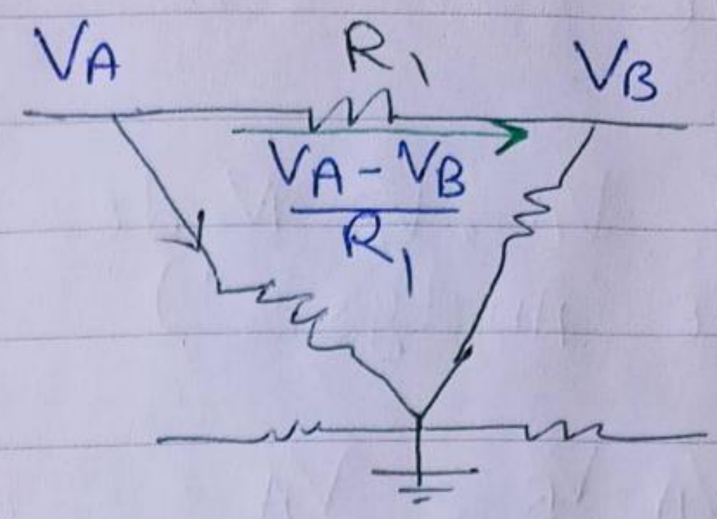
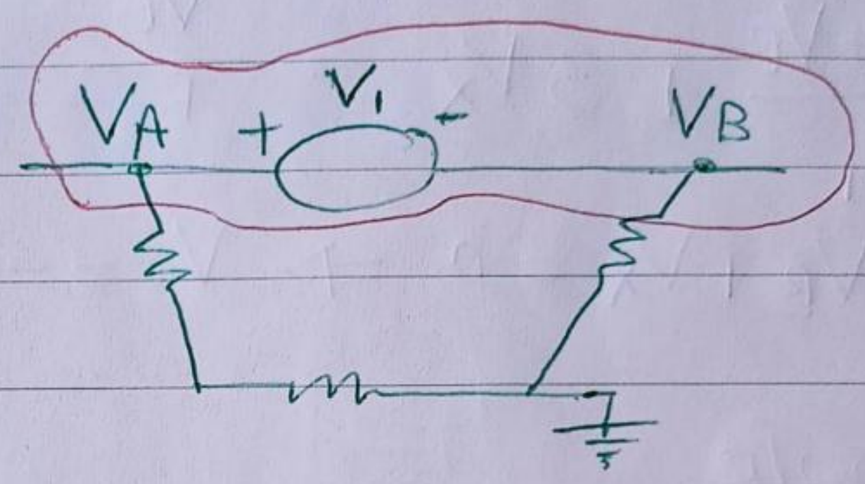
- Review of lecture 6

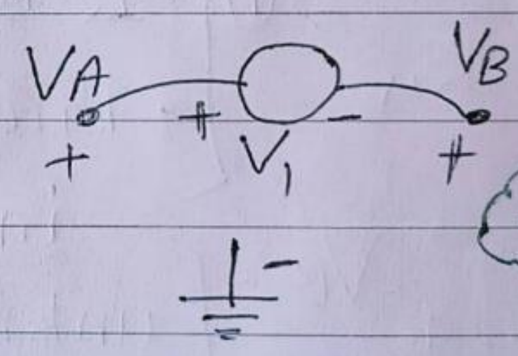


- Node Analysis.
  - select GND.
  - Assign Voltages.
  - Mark currents on diagram.
  - Write KCL for N-1 nodes.



- "Supernode"
  - Merge into a supernode & write a KCL.
  - Write a KVL: $V_A = V_1 + V_B$

- Today's lecture

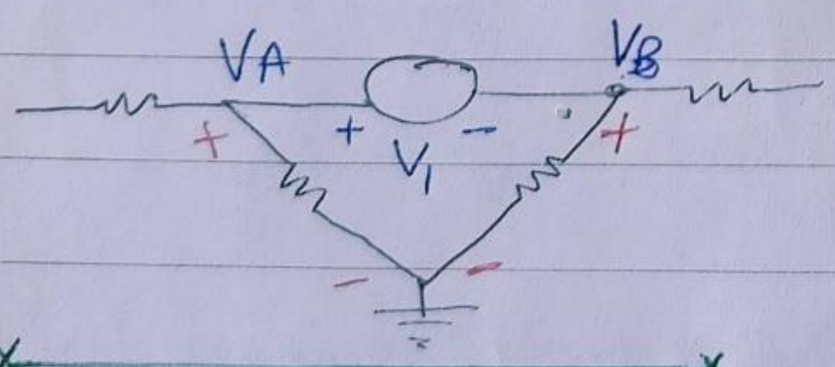


KVL:

$+V_A - V_1 - V_B = 0$

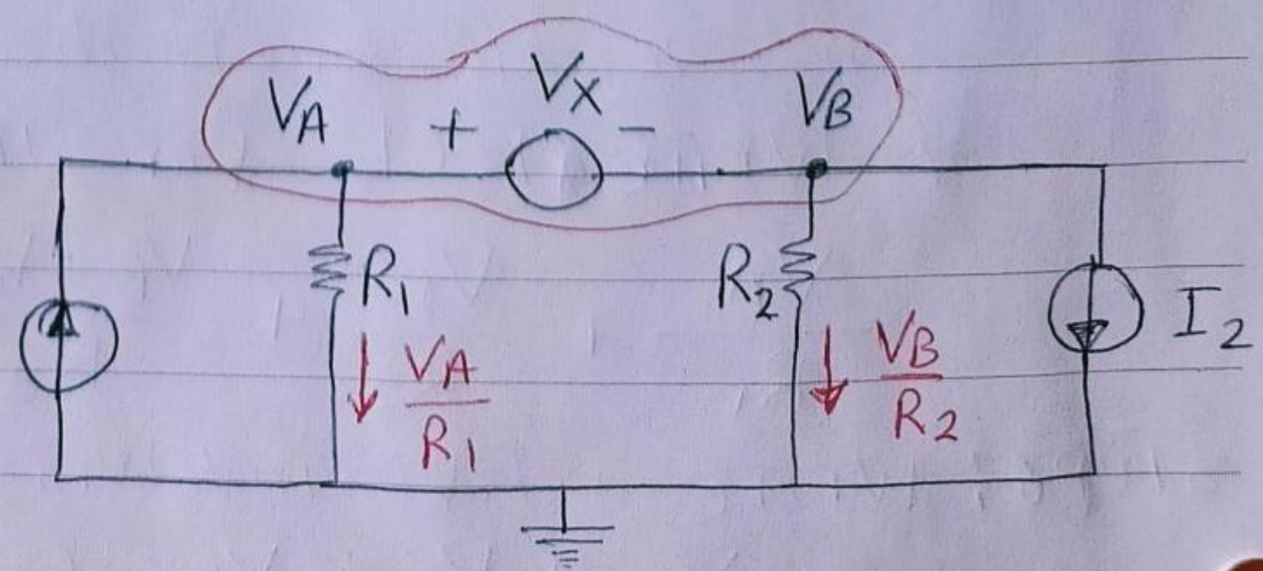
- A note on writing KVLs.
- Supernode Examples.

- A Note on Writing KVLs.



$+V_A - V_1 - V_B = 0$ (KVL)

Example



KCL:

$$I_1 = \frac{V_A}{R_1} + \frac{V_B}{R_2} + I_2$$

KVL:

$$+V_A - V_X - V_B = 0$$

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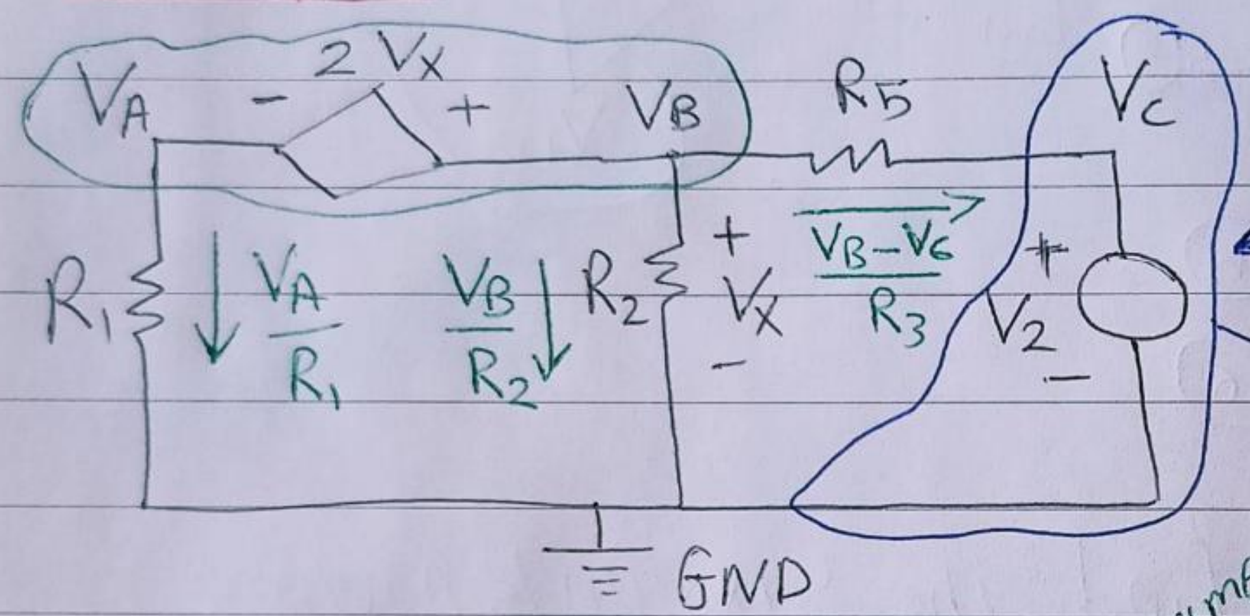
Important summary: ① every Voltage source reduces the number of KCLs by 1. ⇒ need to write a KVL.

② Do not write a KCL at either terminal of a Voltage source.

Example



note: NOT "supernode"

$$\frac{V_B - V_C}{R_3} + \cancel{\frac{V_A}{R_1}} + \frac{V_B}{R_2} = 0$$

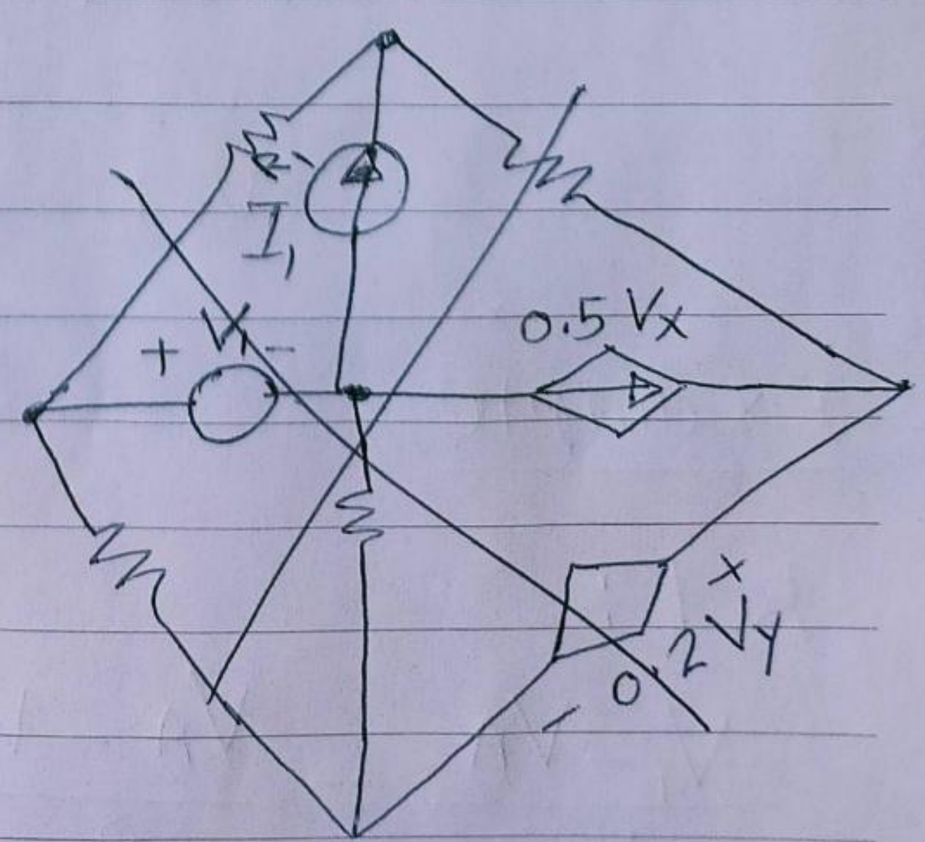
same eq

KCL:

$$\frac{V_A}{R_1} + \frac{V_B}{R_2} + \frac{V_B - V_C}{R_3} = 0$$

KVLs: $+V_A + 2V_x - V_B = 0$

$+V_2 - V_C = 0$ & $V_x = V_B$



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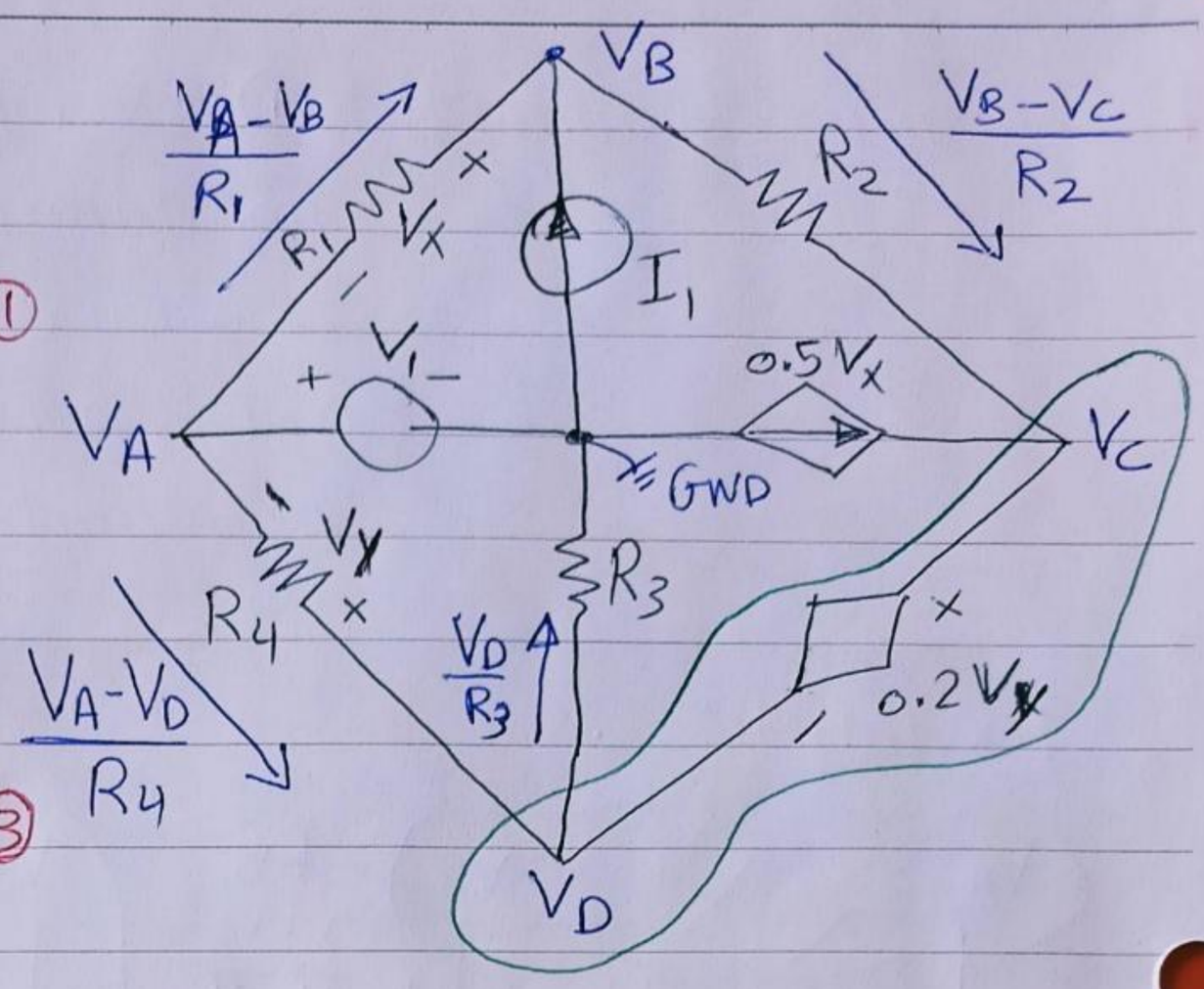
Example



KCLs: $\frac{V_A - V_B}{R_1} + I_1 = \frac{V_B - V_C}{R_2} \rightarrow (1)$

$(2) \rightarrow \frac{V_B - V_C}{R_2} + 0.5V_x + \frac{V_A - V_D}{R_4} = \frac{V_D}{R_3}$

KVLs: $+V_D + 0.2V_y - V_C = 0 \rightarrow (3)$

$V_A = V_1 \rightarrow (4)$

$V_A + V_x - V_B = 0 \rightarrow (5)$

$V_A + V_y - V_D = 0 \rightarrow (6)$

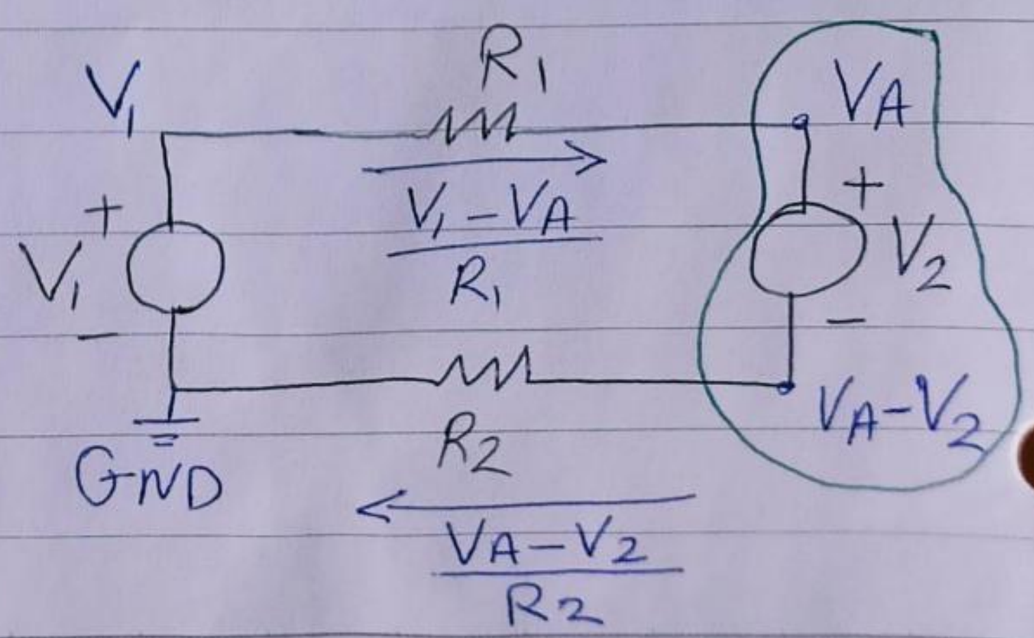
$\because$ six equations can find the six unknowns

i.e (1), (2), (3), (4), (5), (6) can find $V_A, V_B, V_C, V_D, V_x, V_y$ #

Summary:

- started with 5 nodes
- called one ground $\rightarrow$ 4 nodes
- Two voltage sources $\rightarrow$ lose 2 KCLs.
  $\Rightarrow$ 2 KCLs left
- write 2 KVLs involving the voltage sources
- write 2 more KVLs involving unknown voltages V_x and V_y.

Example



KCL:

$$\frac{V_1 - V_A}{R_1} = \frac{V_A - V_2}{R_2}$$

- one unknown in one eq. #

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Lec 8: Mesh Analysis, Linearity, Superposition

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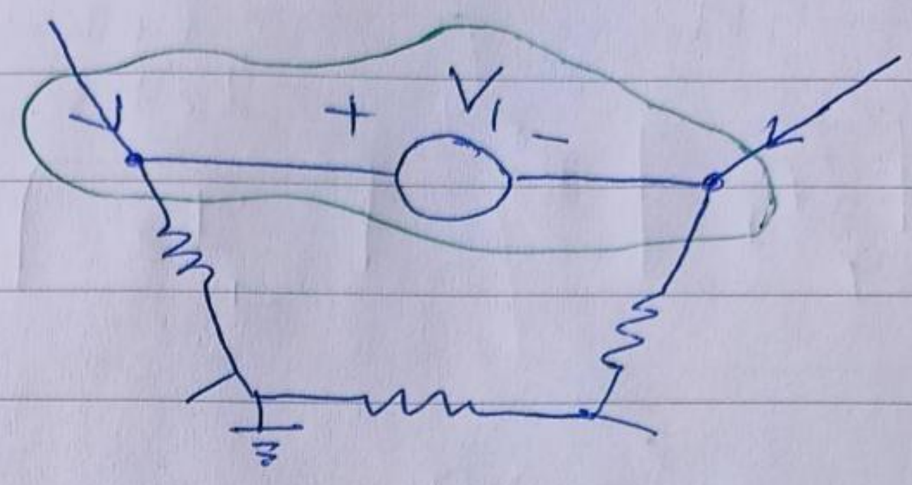
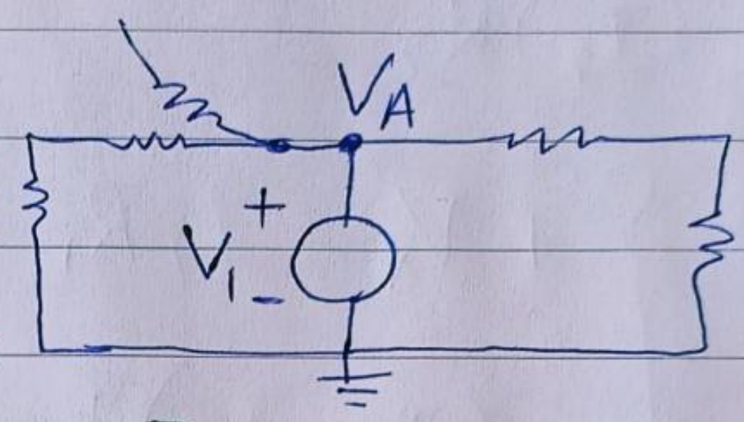
Review of lecture 7

- Supernode Analysis
  - each voltage source reduces the number of KCLs by 1.

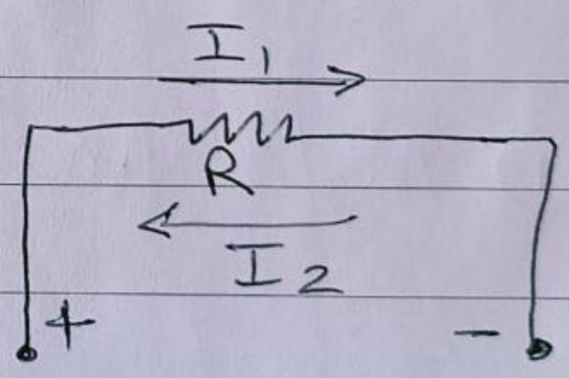


  - A voltage source tied to GND, does not lend itself to supernode KCL, i.e we does not try to create supernode due to No ~~ad~~ any additional information
    ⇒ Just write a KVL : $V_A = V_1$.



Today's lecture:

- Mesh Analysis.
- Excitation and response.
- Linearity and superposition.



$V = (I_1 - I_2) R$

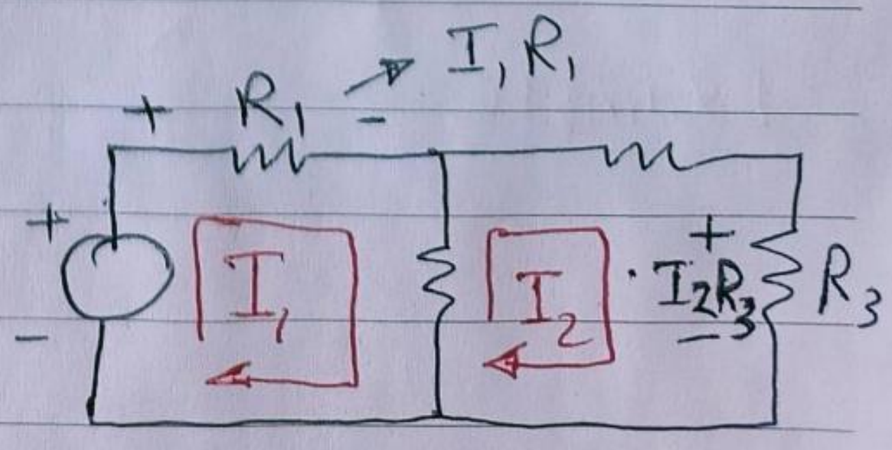
- Mesh Analysis.
  - A mesh is a loop that does not contain ~~any~~ any othe other loop within it.

Three steps:

① Identify all of the meshes and assign a current to each.

② write the voltage drops on the circuit.

③ write a KVL around every mesh.



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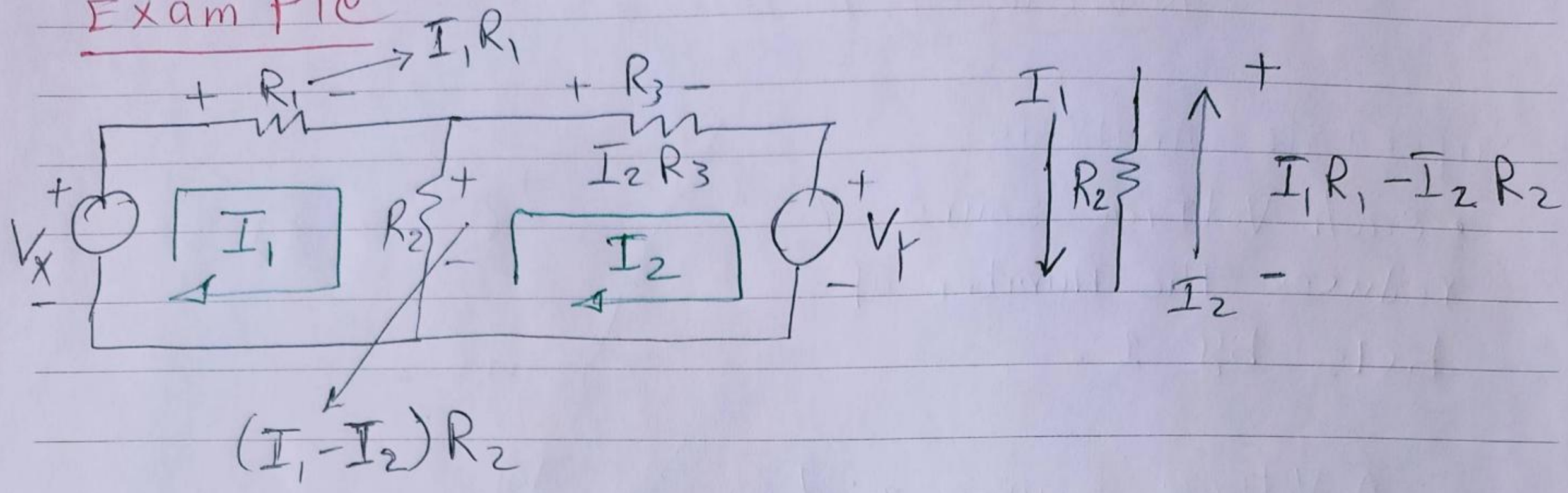
Example



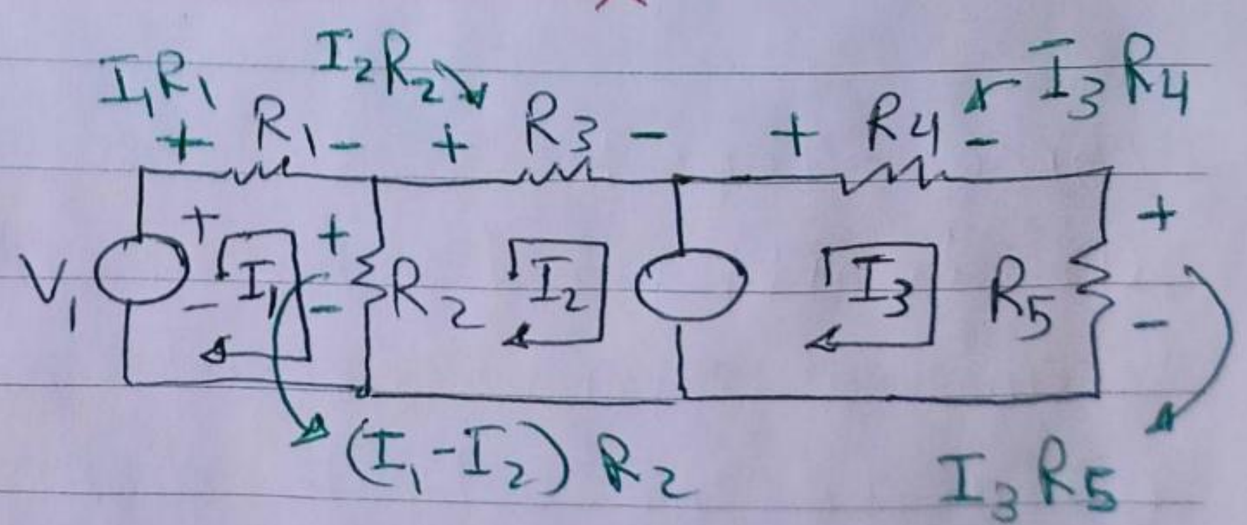
KVL:
$$\begin{cases} +V_x - I_1R_1 - (I_1-I_2)R_2 = 0 \\ +(I_1-I_2)R_2 - I_2R_3 - V_y = 0 \end{cases}$$

Two observations

① Node Analysis:
- KCL - centric.
- Handles current sources easily.
- Has difficulty with voltage sources. (supernode).

② Mesh Analysis:
- KVL - centric.
- Handles voltage sources easily.
- Has difficulty with current sources.

Example



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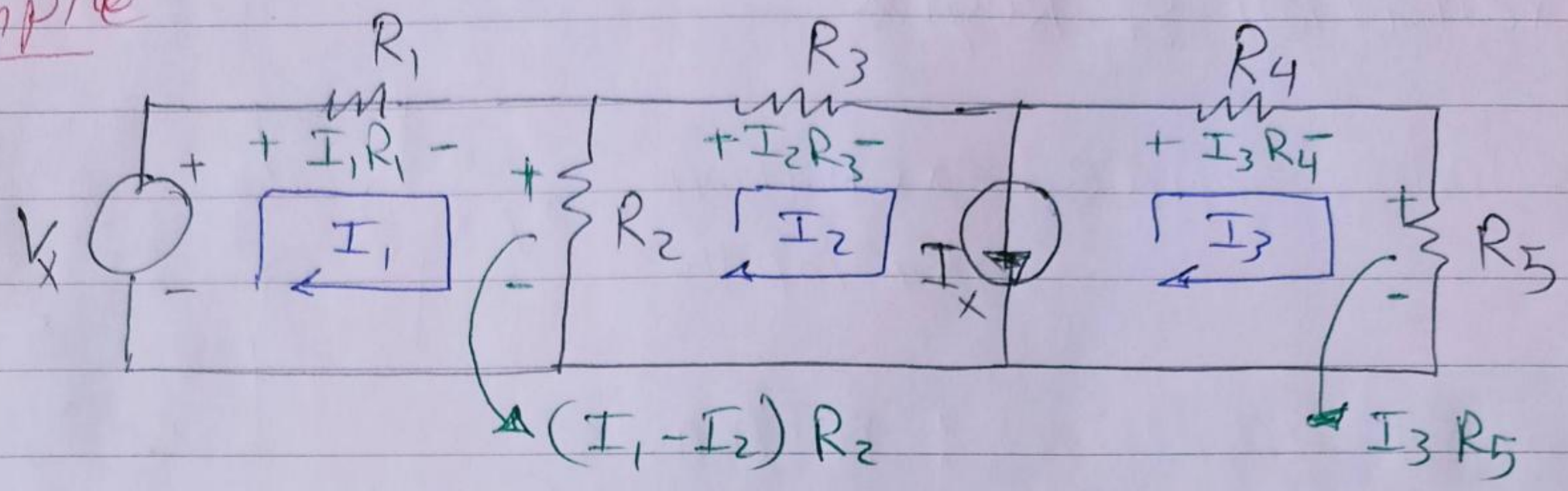
Example



KVLs:

$$+V_x - I_1R_1 - (I_1 - I_2)R_2 = 0 \rightarrow ①$$

$$+(I_1 - I_2)R \cancel{- I_2R_3} + ? = 0$$

- apply supermesh: we merge the two meshes that share the current source into a supermesh.

KVL of supemesh: $+(I_1 - I_2)R_2 - I_2R_3 - I_3R_4 - I_3R_5 = 0 \rightarrow ②$

- We lose one KVL, look for KCL involving the current source and the currents of the merging meshes.

KCL of current source: $I_2 = I_x + I_3 \rightarrow ③$

∴ Three equations ①, ②, ③ with three unknowns I_1, I_2, I_3

#

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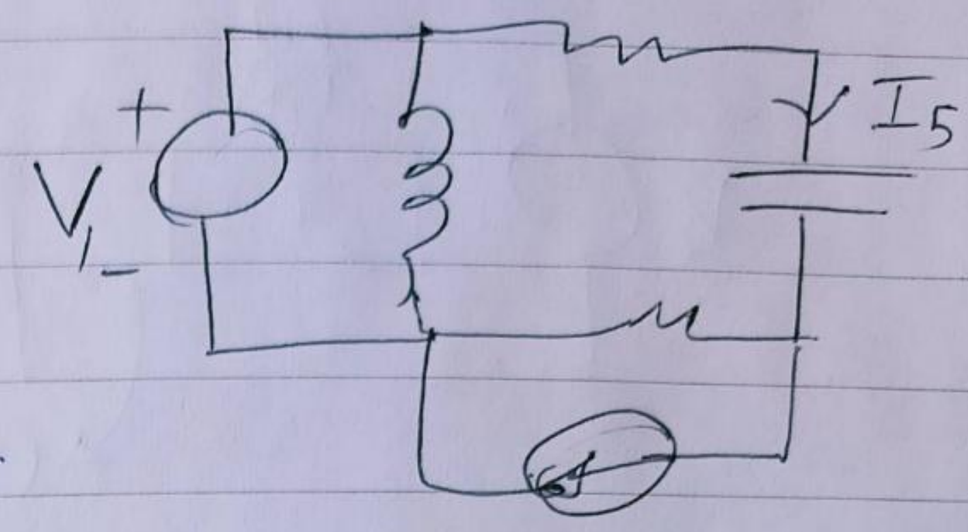
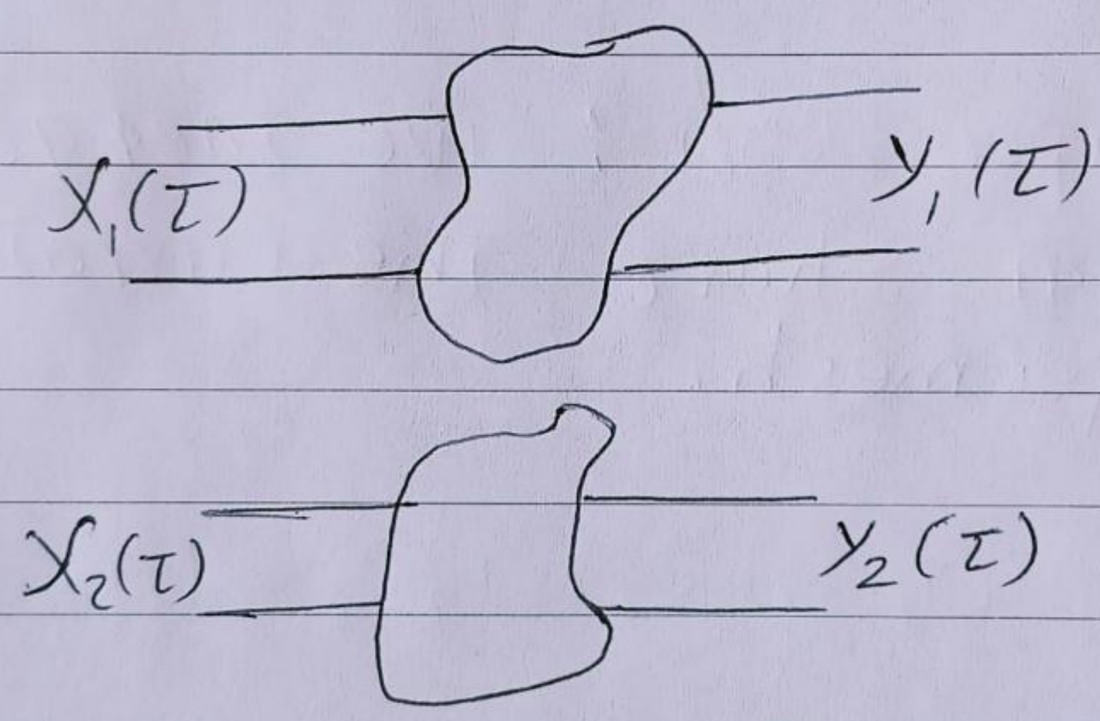
- Excitation & Response

  - indep. source : excitations
    , aka "inputs"



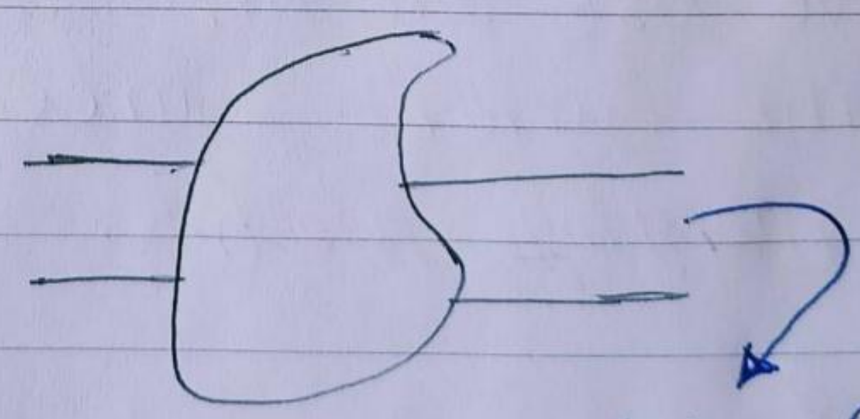
  - other quantities : responses
    , aka "outputs"

- Linearity



Linearity:

$\alpha x_1(t) + \beta x_2(t)$



$\alpha y_1(t) + \beta y_2(t)$

- Superposition

  — in a linear system, the total response to multiple excitations is equal to the sum of the corresponding responses.

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Lec 9: Superposition Examples

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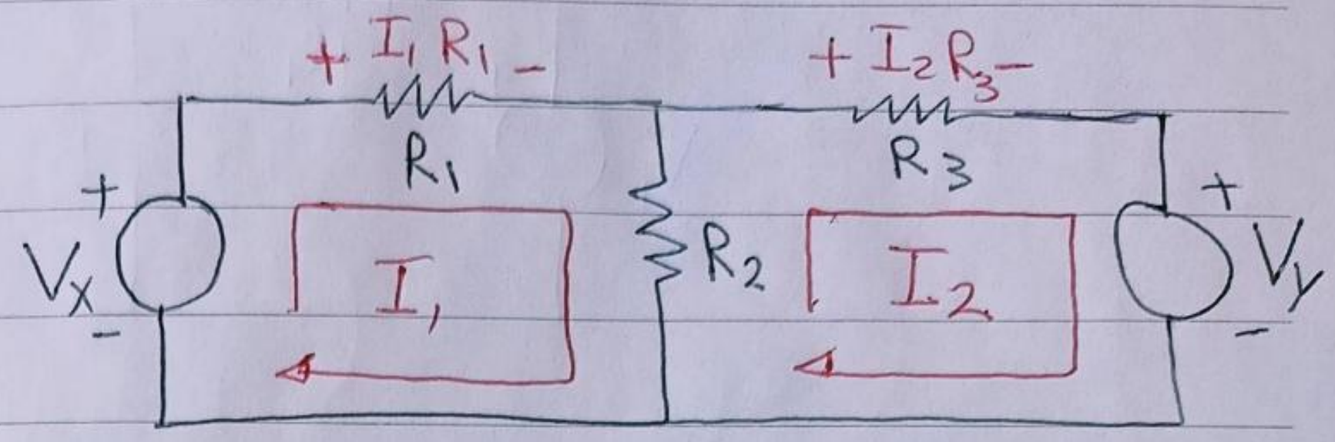
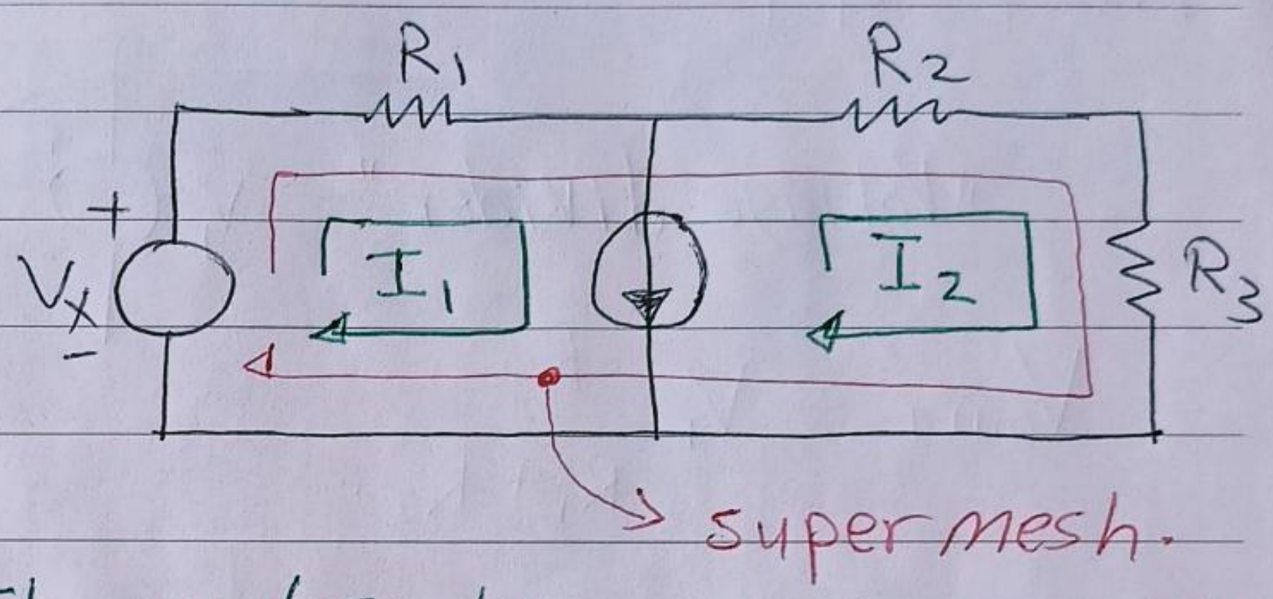
Review of lecture 8

- Mesh Analysis.
  - Identify meshes ....
  - write voltage drops.
  - write KVLs.



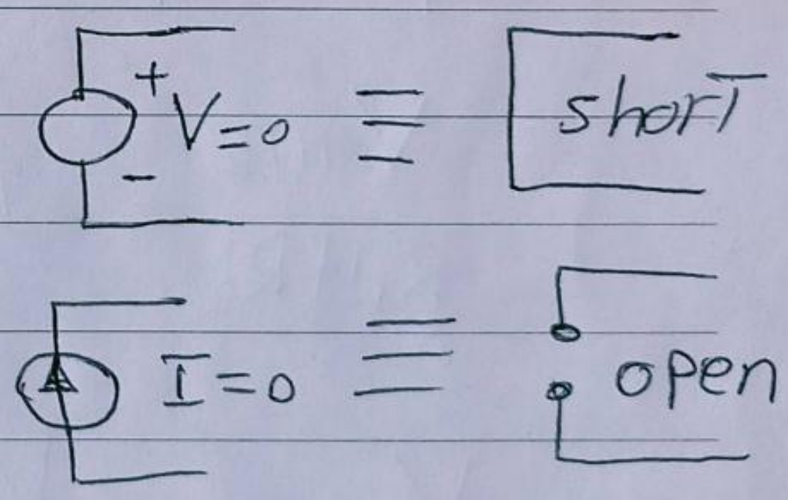
  - Find supermesh and write KVL.
  - write KCL.



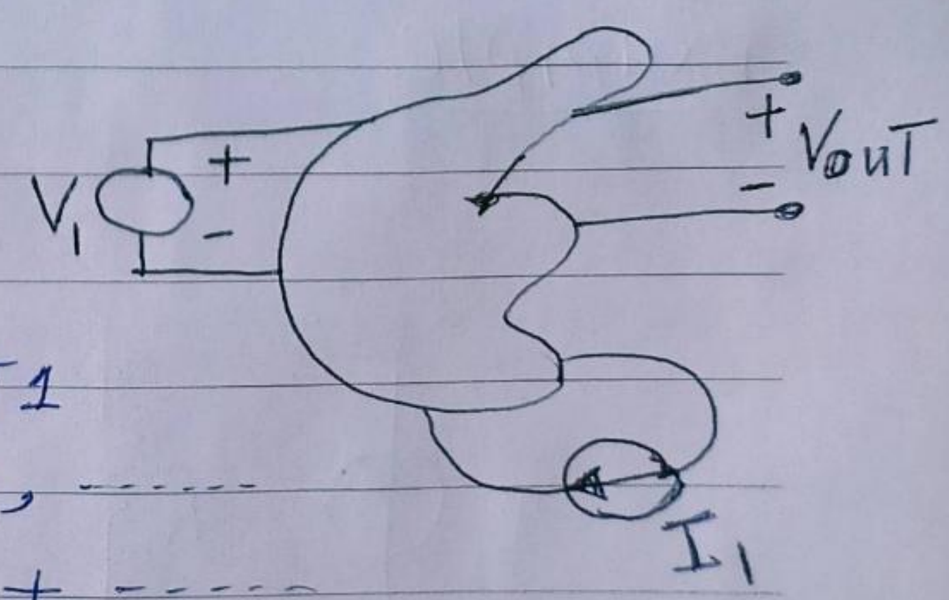
- Superposition: with multiple inputs; keep one input alive, find response; repeat; add responses.

⇒ Today's lecture

- Superposition Examples.
- Thevenin's Theorem.
- Norton's Theorem.



- Superposition procedure.
  - Keep input #1 and kill all other inputs.
  - Find response to input #1: V_{out1}
  - Repeat for each input: V_{out2}, V_{out3}, ...
  - Total response = $V_{out1} + V_{out2} + \cdots$



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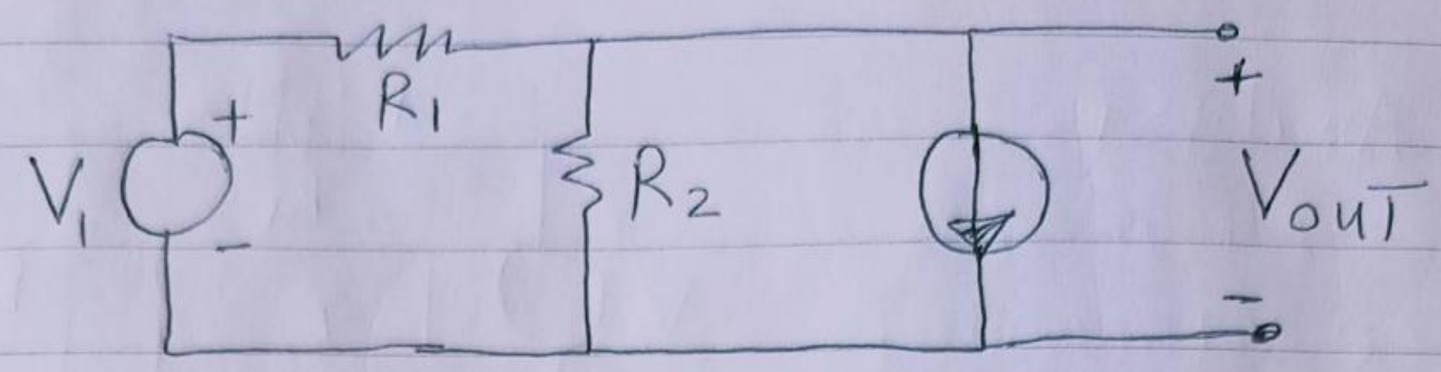
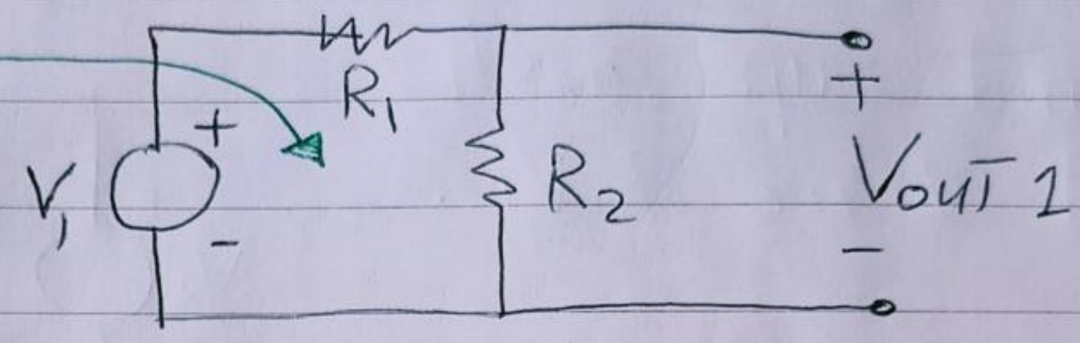
Example



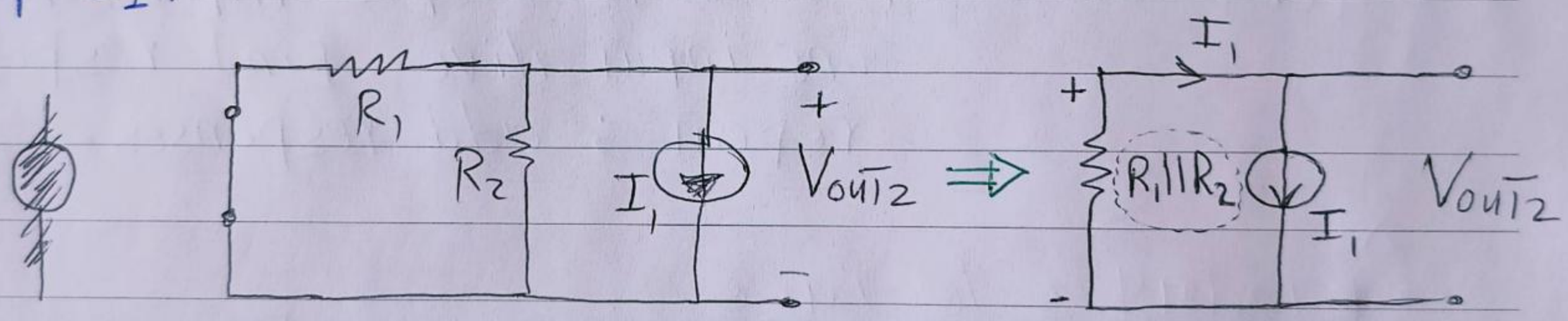
- Keep V_1:

"Voltage divider"



$$V_{OUT1} = V_1 \frac{R_2}{R_1 + R_2}$$

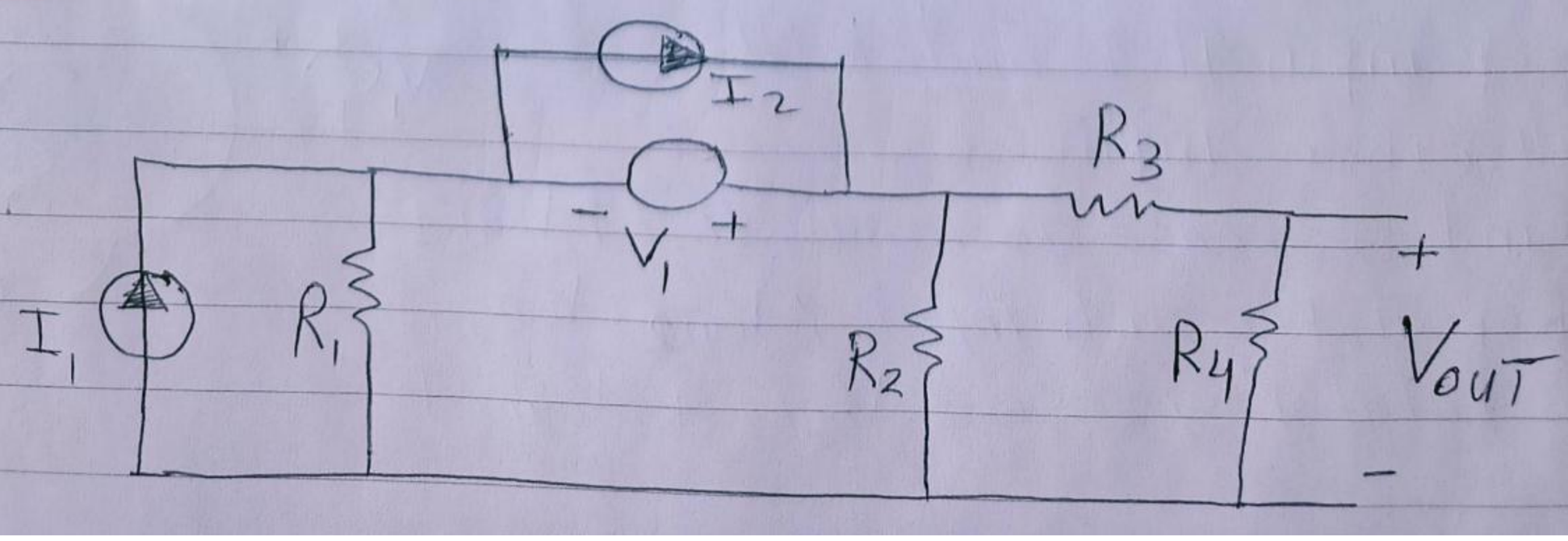
- Keep I_1:



$$\frac{V_{OUT2}}{R_1 \| R_2} = -I_1 \Rightarrow V_{OUT2} = -I_1 \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{OUT,TOT} = V_{OUT1} + V_{OUT2} = V_1 \frac{R_2}{R_1 + R_2} - I_1 \frac{R_1 R_2}{R_1 + R_2} \quad \#$$

Example



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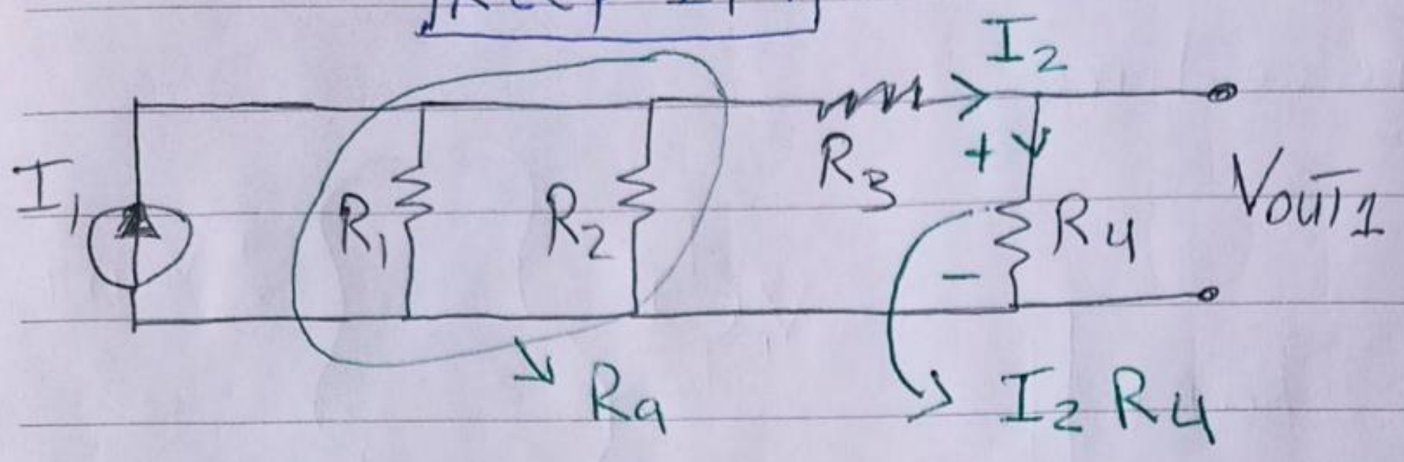
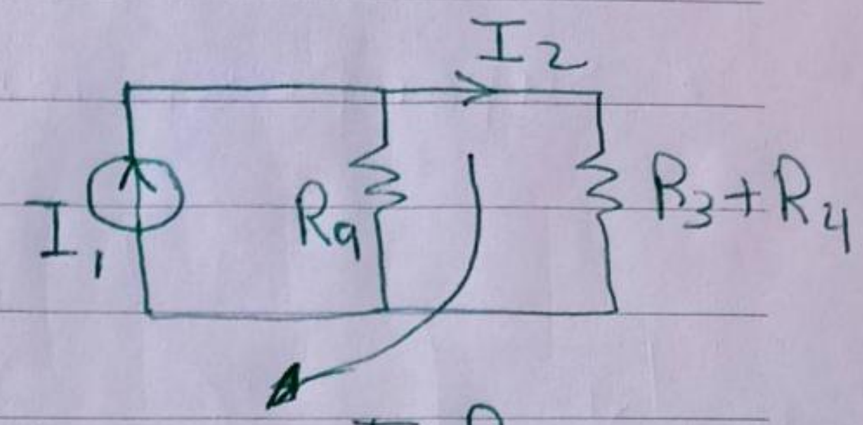
Solution:

Keep I_1:

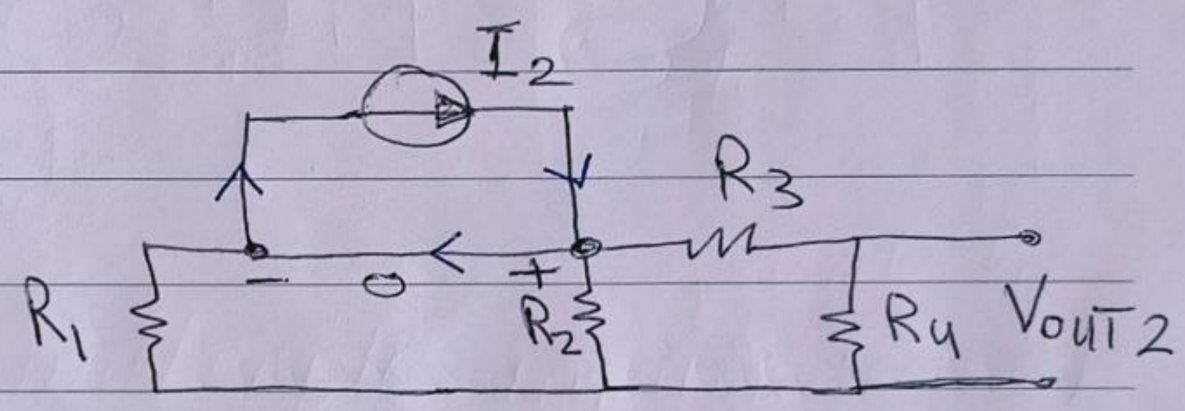




$$I_2 = \frac{I_1 R_a}{R_a + R_3 + R_4}$$

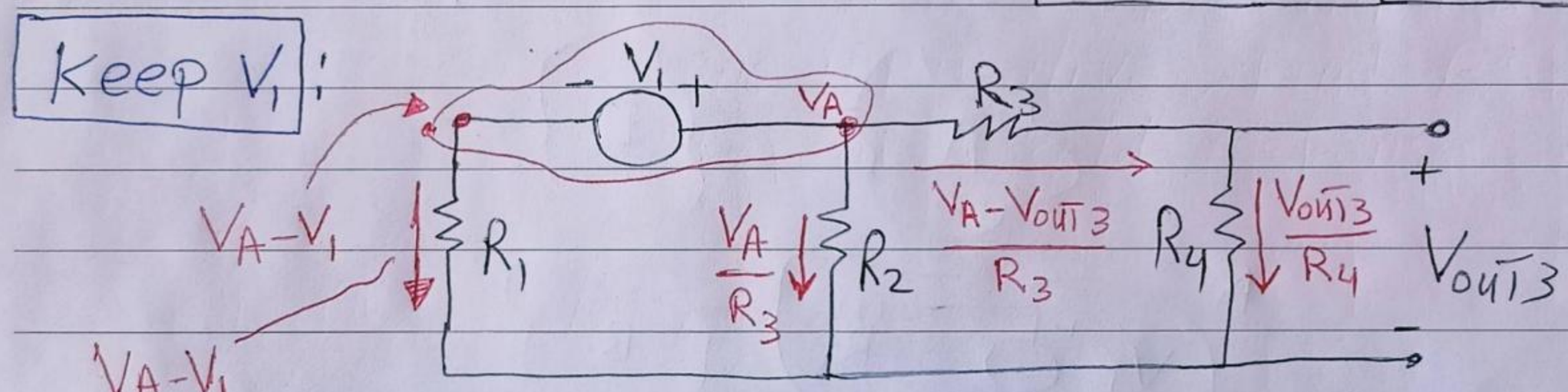
$$V_{OUT_1} = I_2 R_4 = \frac{I_1 R_a R_4}{R_a + R_3 + R_4}$$

Keep I_2:



$$V_{OUT2} = 0$$

Keep V_1:

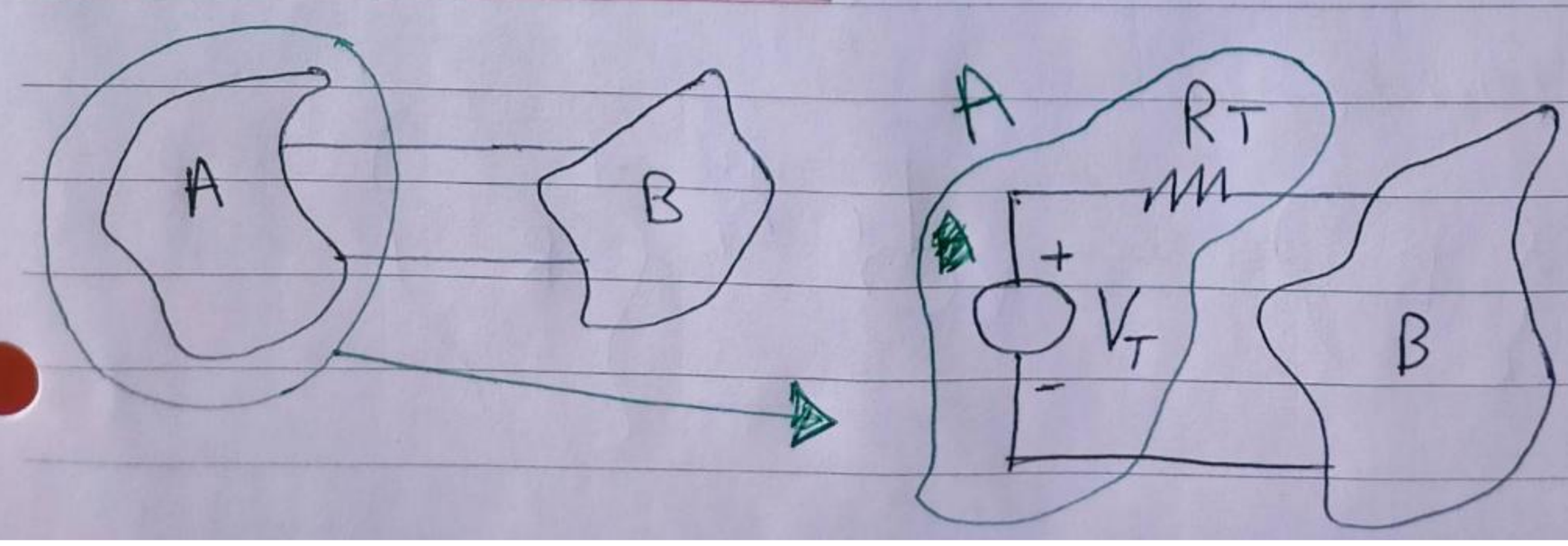


KCLs: $\frac{V_A - V_{OUT3}}{R_3} = \frac{V_{OUT3}}{R_4}$

$$\frac{V_A - V_1}{R_1} + \frac{V_A}{R_2} + \frac{V_A - V_{OUT3}}{R_3} = 0$$

$$\therefore V_{OUT,TOT} = V_{OUT1} + V_{OUT3} \quad \#$$

Thevenin's Theorem



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Lec 10: Thevenin's Theorem

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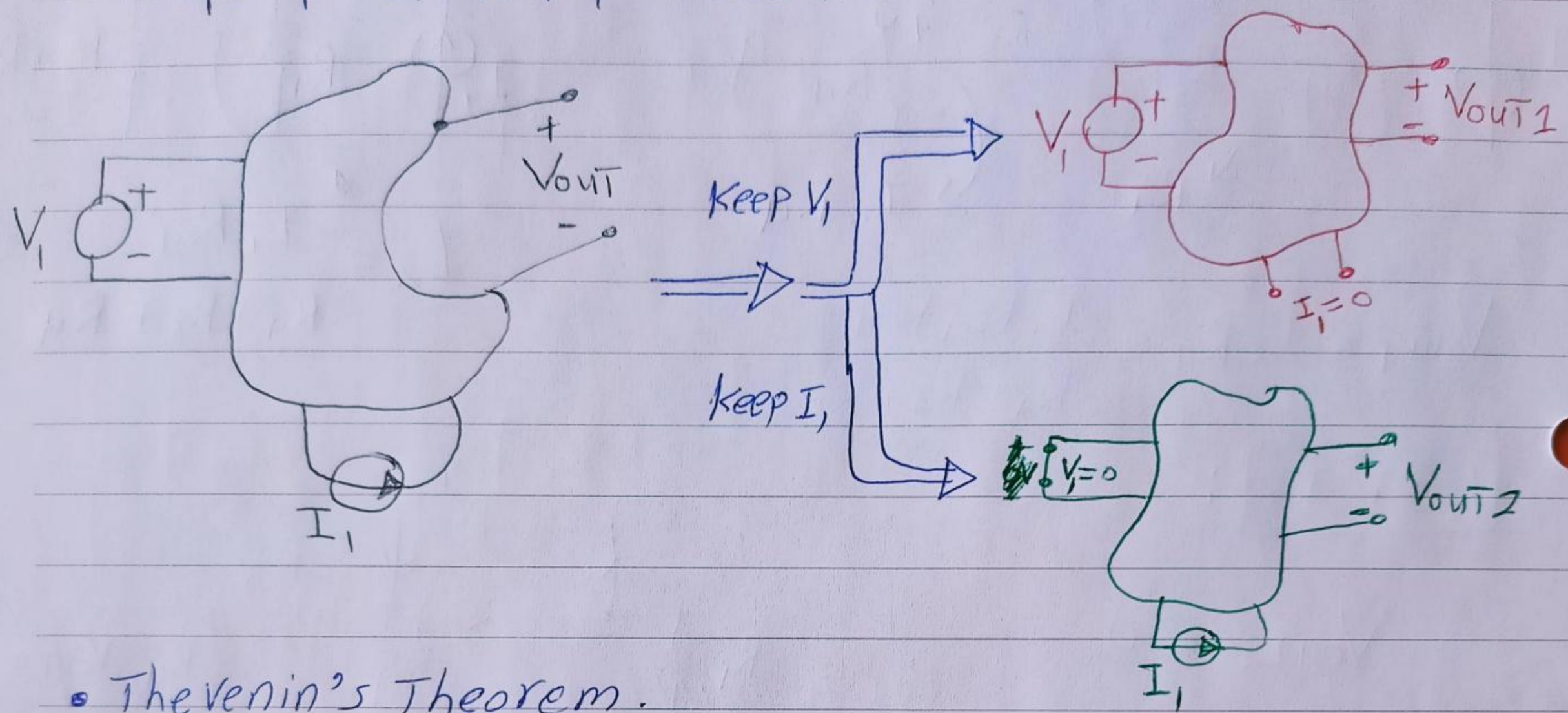
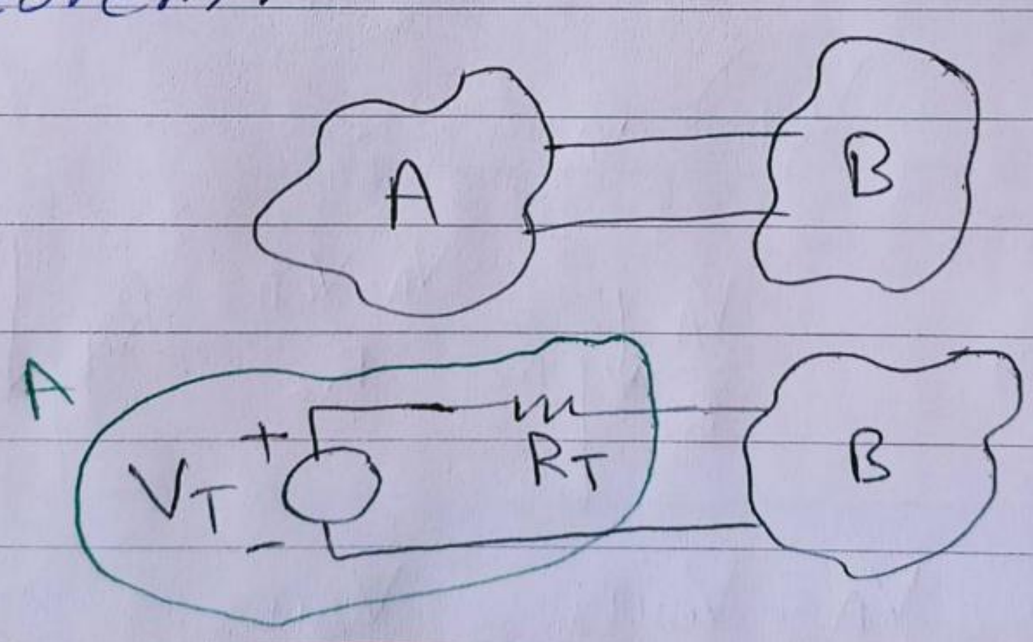
Review of Lecture 9

- Superposition procedure.



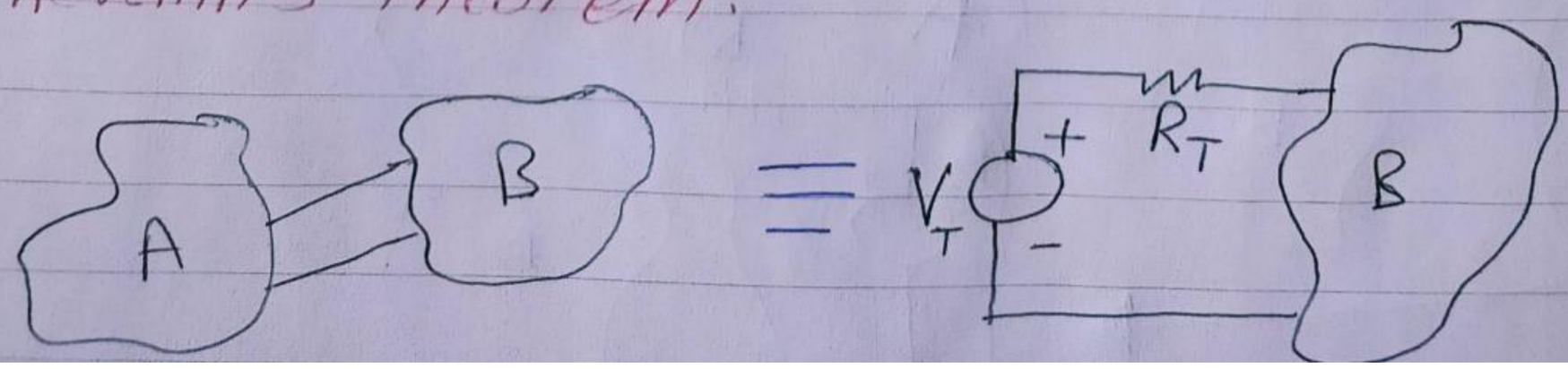
- Thevenin's Theorem.



Today's lecture

- Thevenin's Theorem.
- Norton's Theorem.
- Source Transformation.

- Thevenin's Theorem.



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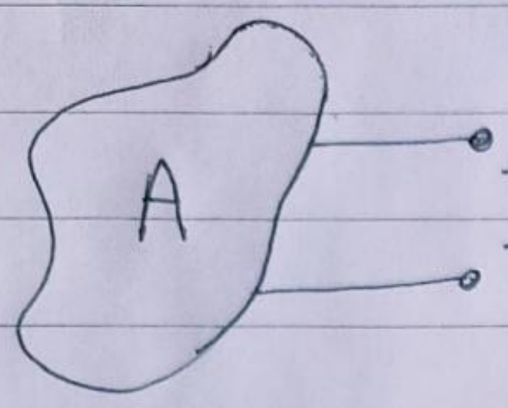
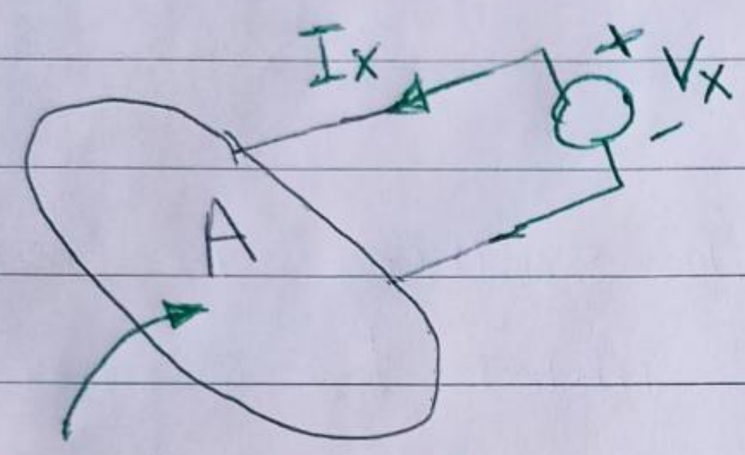
- calculation of V_T



V_T : "open-cicuit" Voltage.

- Calculation of R_T : STEP 1 : SET all independent sources in section A To Zero.

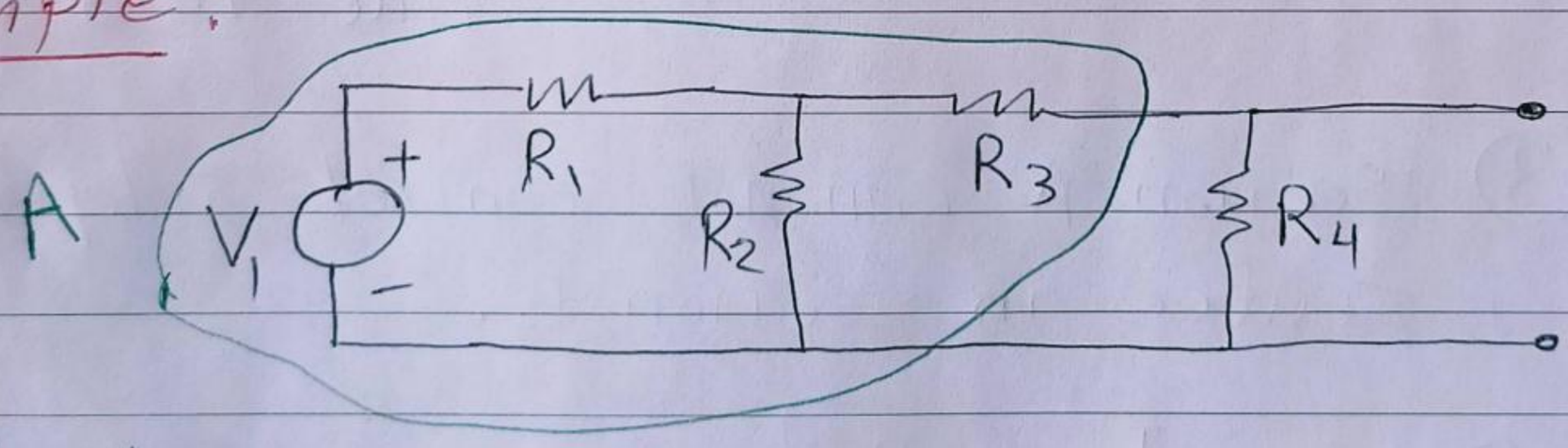


indep. sources = 0

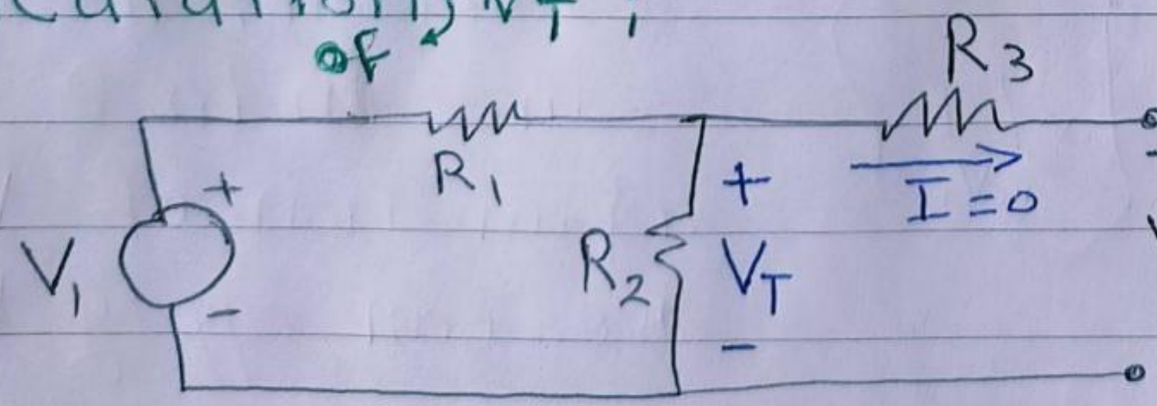
STEP 2 : Find The resistance between The output wires of section A.

$$R_T = \frac{V_x}{I_x}$$

Example :

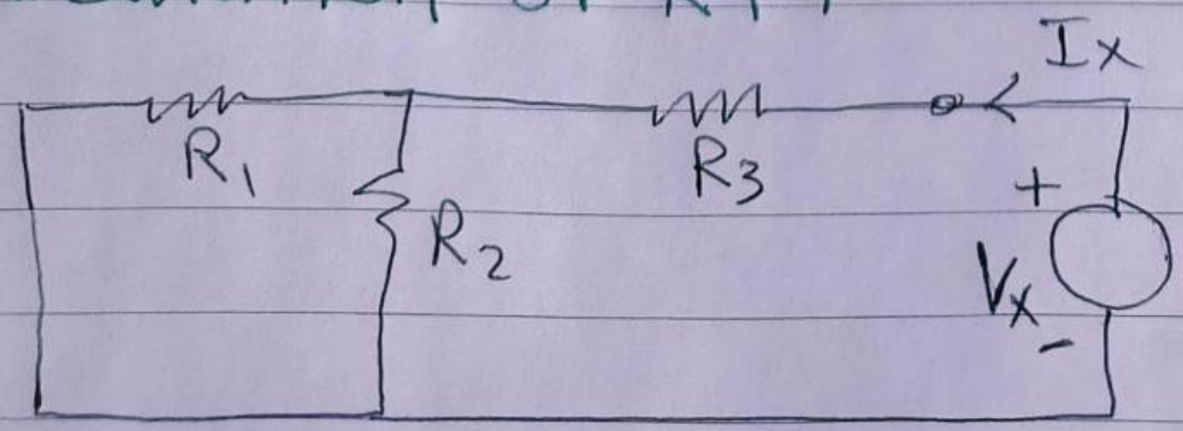


- Calculations of V_T :



$$V_T = \frac{R_2}{R_1 + R_2} V_1$$

- calculation of R_T :



$\therefore R_T = R_3 + R_1 \| R_2$

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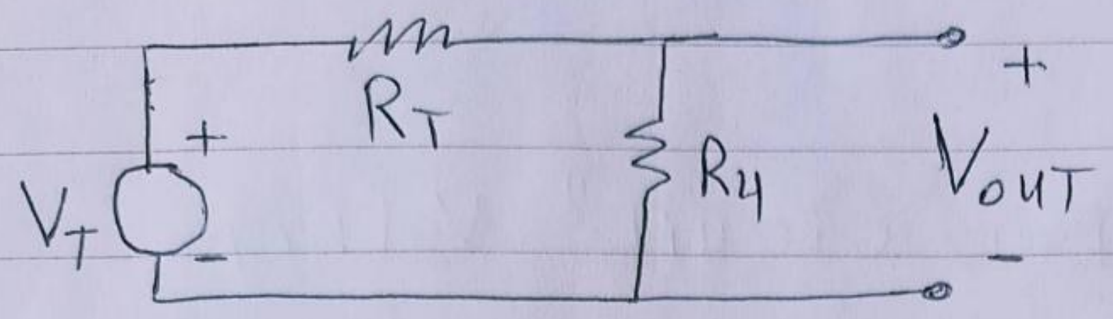
- finally circuit



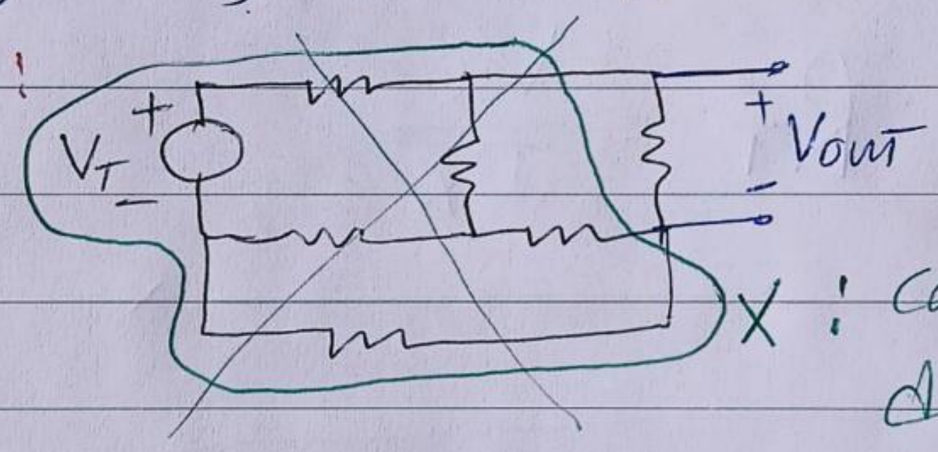
$$V_{OUT} = \frac{R_4}{R_4 + R_3 + R_1 \| R_2} \cdot \frac{R_2}{R_1 + R_2} V_1 \quad \#$$

NOTES:

① Do not set dependent sources to zero.

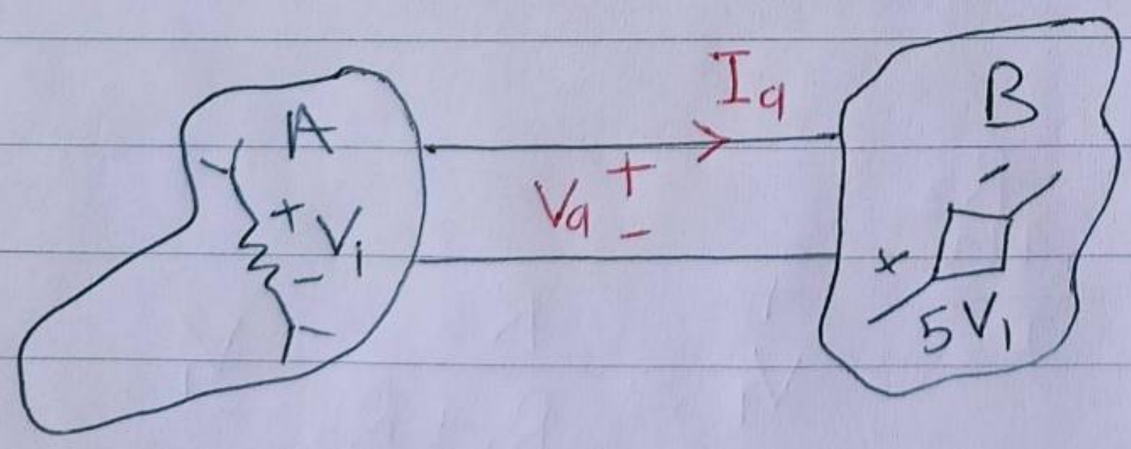
② sections A and B must be connected by only 2 wires.

Ex:

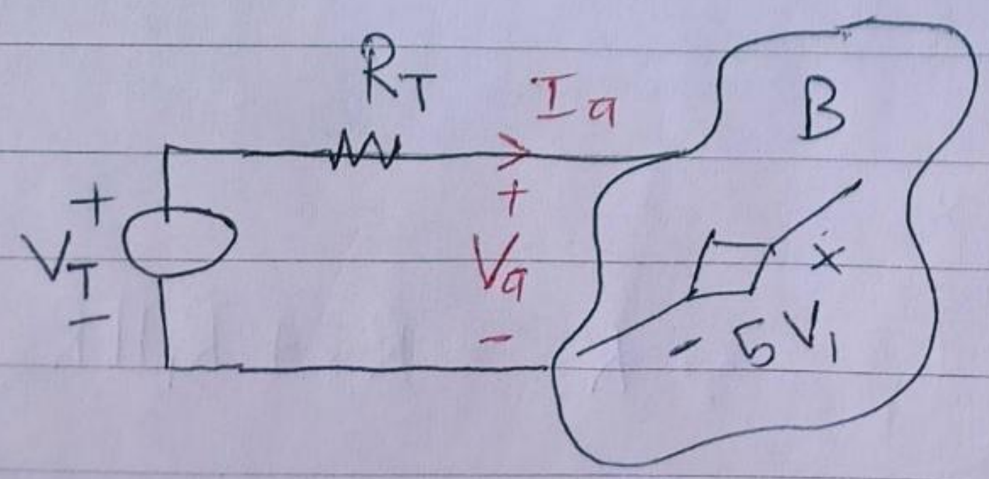


X : can not do that due to 3 wires

③ Section A can not control a dependent source in section B.



Exception: If a dep. source in B is a function of V_a, I_a we can apply V_T, R_T.



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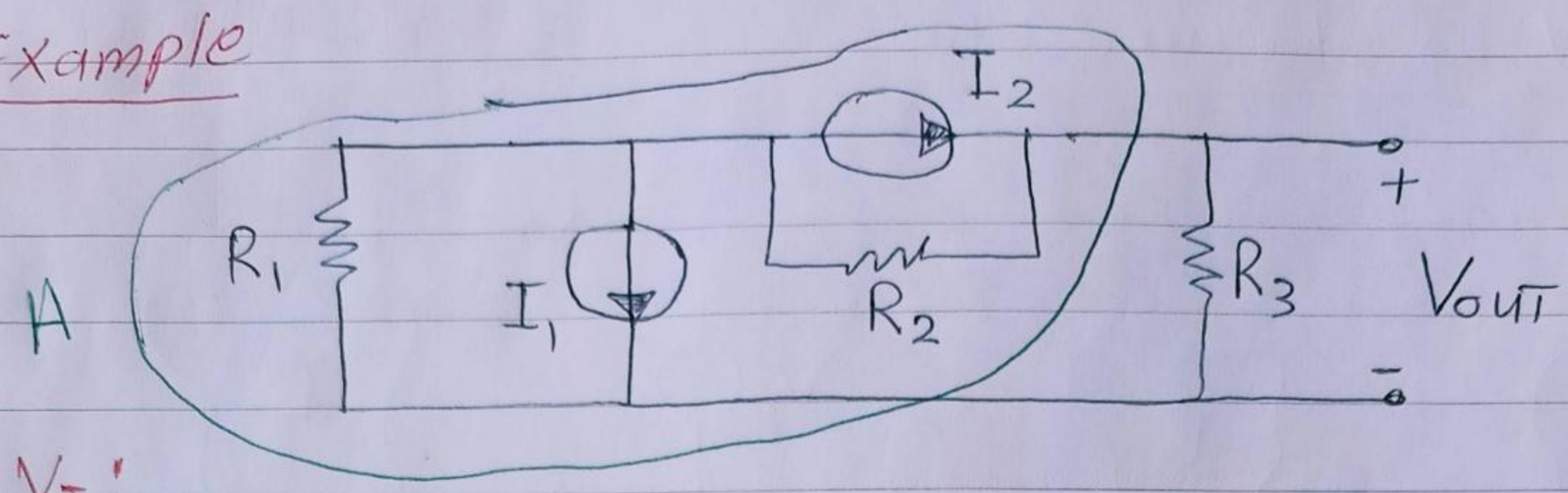
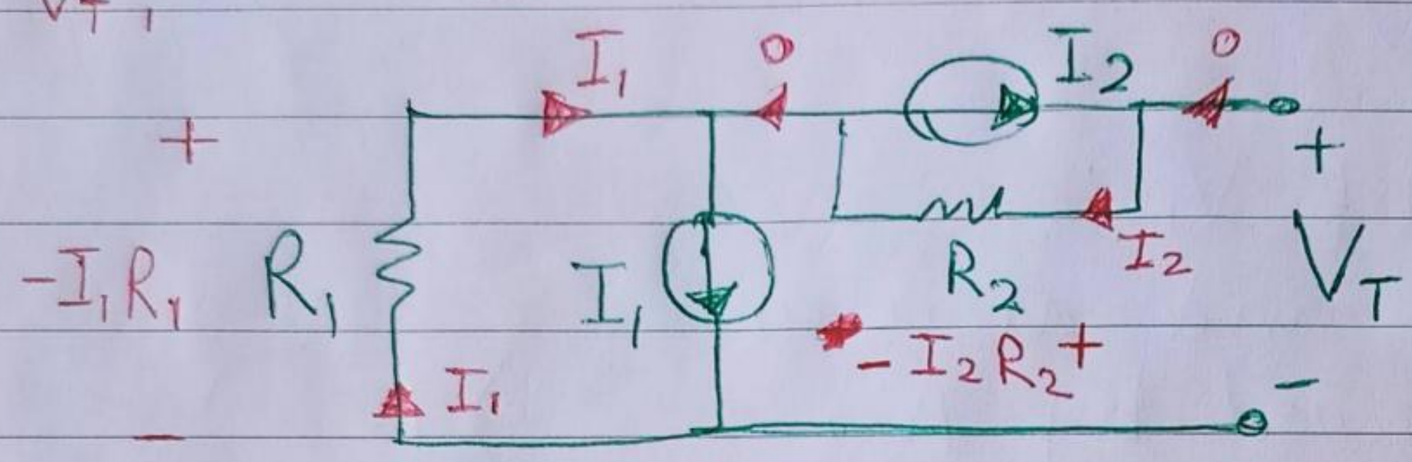
Example



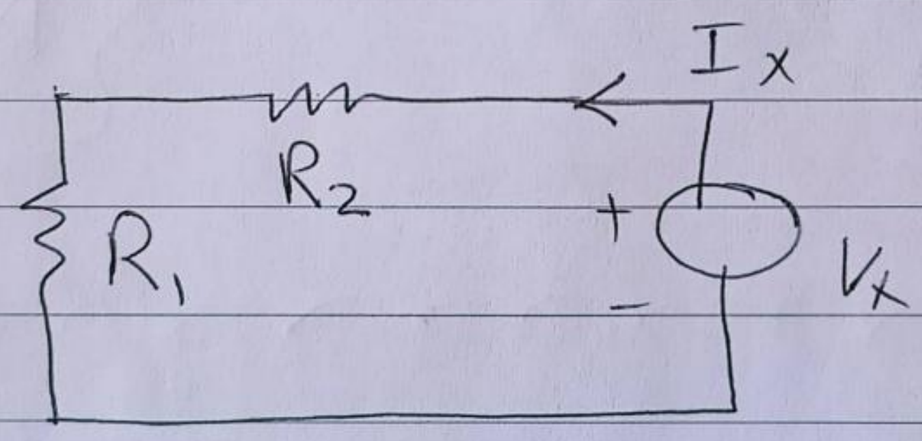
V_T:



$$-I_1R_1 + I_2R_2 - V_T = 0$$

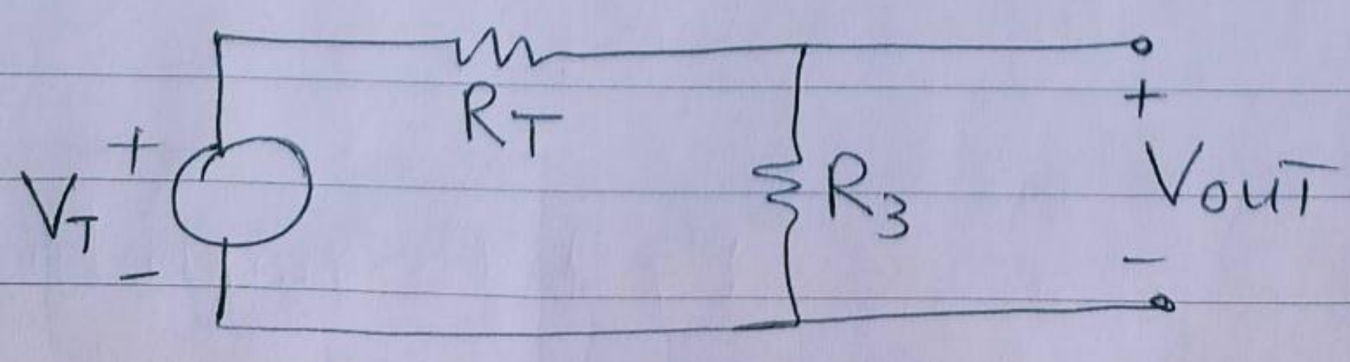
$$V_T = I_2R_2 - I_1R_1$$

R_T:



$$R_T = R_1 + R_2 \quad \#$$

- Finally circuit "Thevinin's equivelant"



$$V_{OUT} = \frac{R_3}{R_3 + R_T} V_T \quad \#$$

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Lec 11: Norton's Theorem

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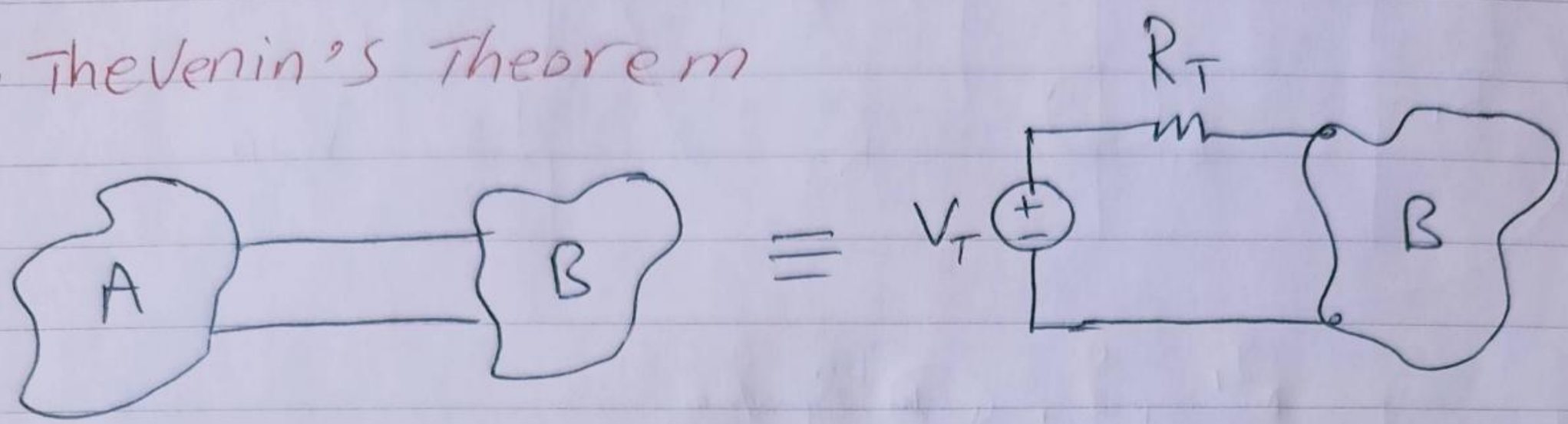
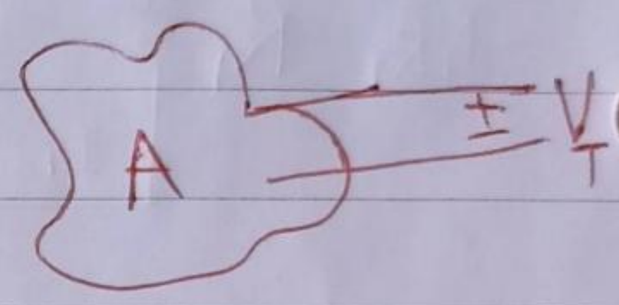
Review of lecture 10

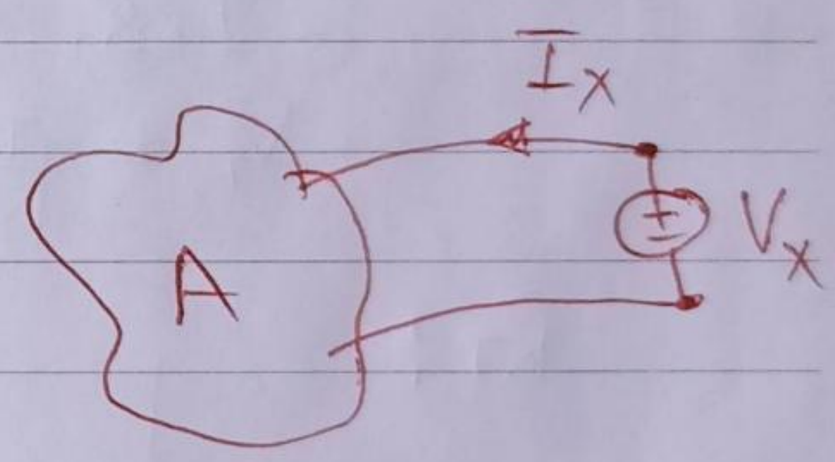
- Thevenin's Theorem



- Calculation of V_T:
  Disconnect B and measure Voltage generated by A.

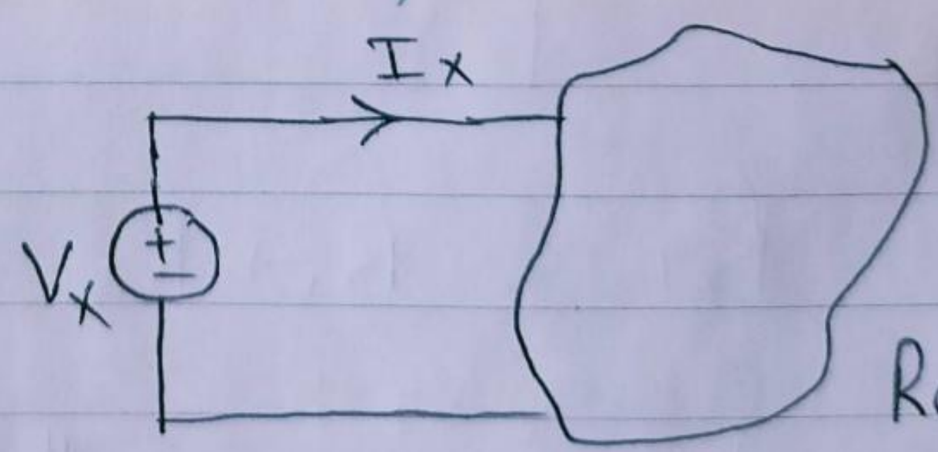


- Calculation of R_T:
  kill all indep. source in A and find the "output" resistance:



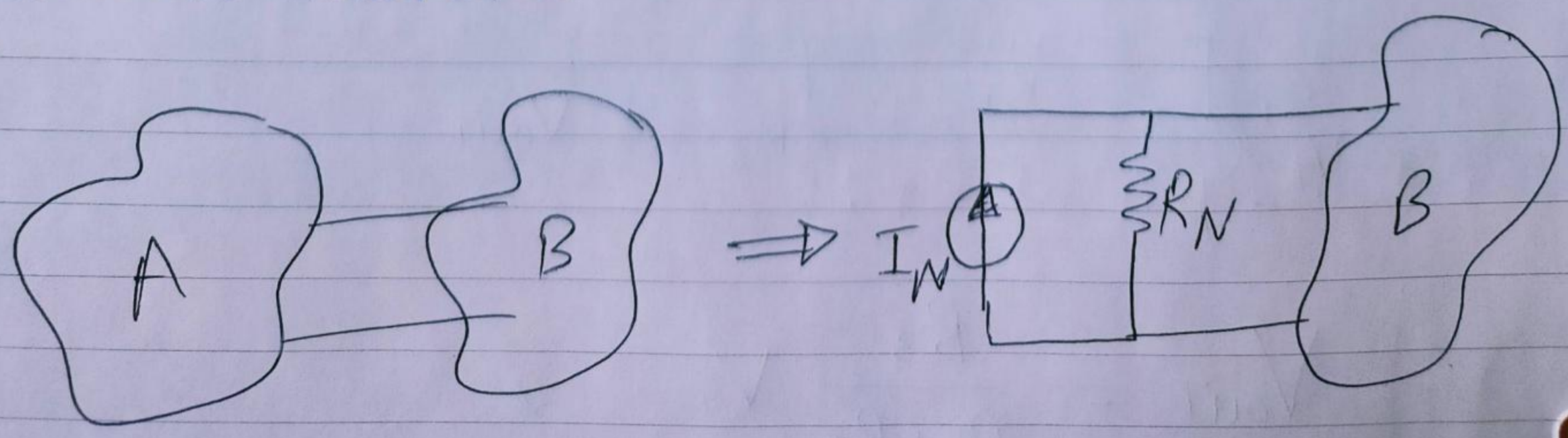
Today's lecture

- Norton's Theorem
- Examples



$$R_{eq} = \frac{V_x}{I_x}$$

- Norton's Theorem:



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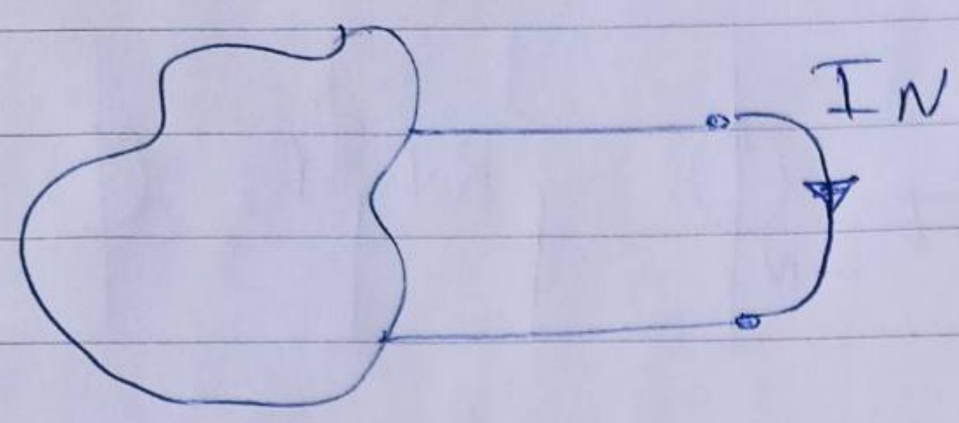
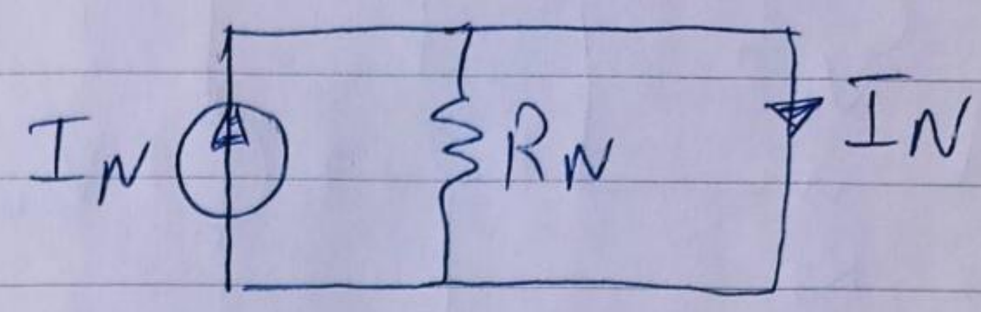
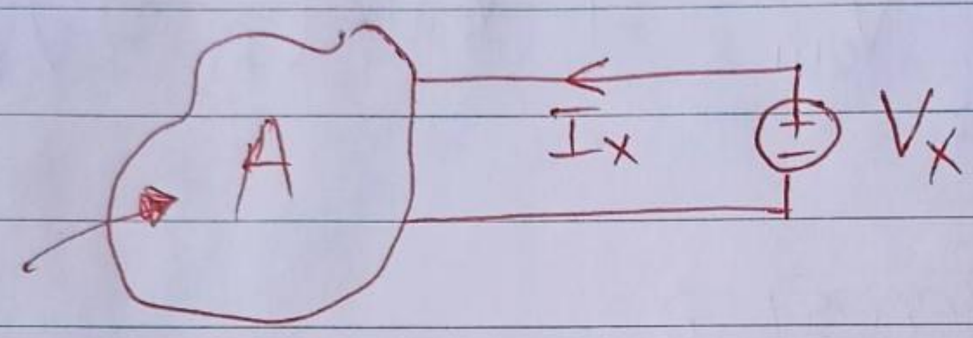
- calculation of I_N : "short circuit current"



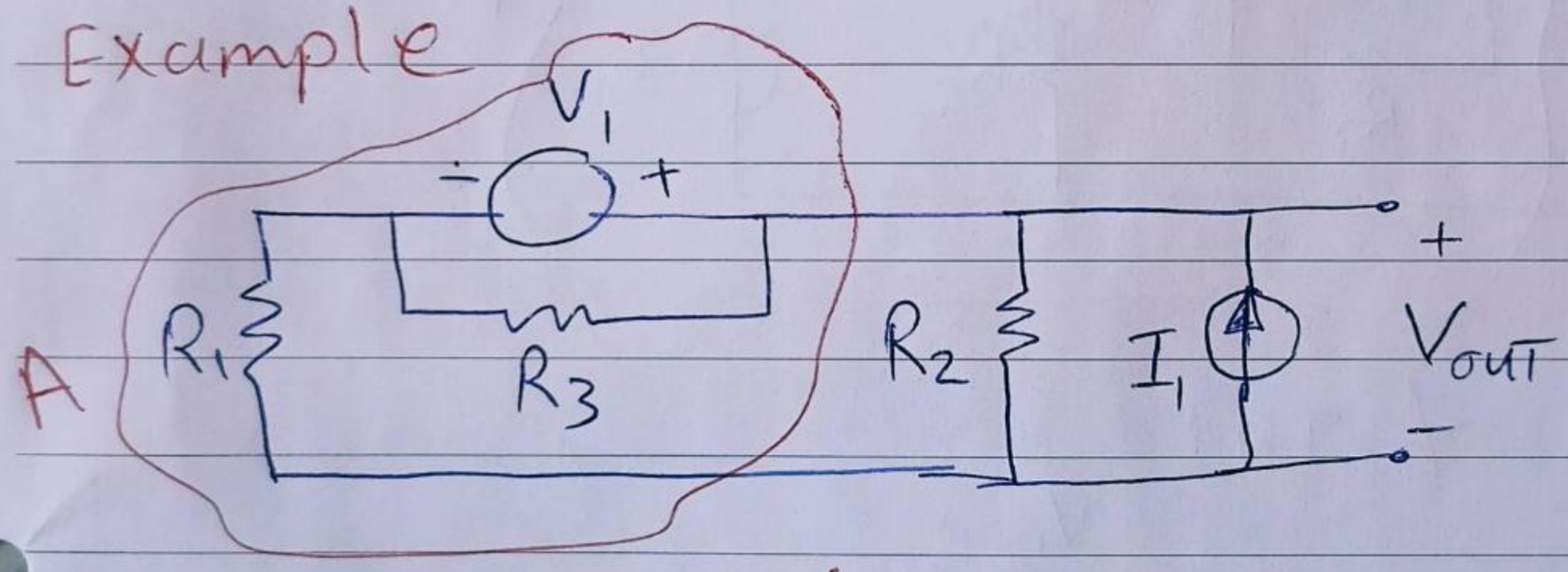


- calculation of R_N :

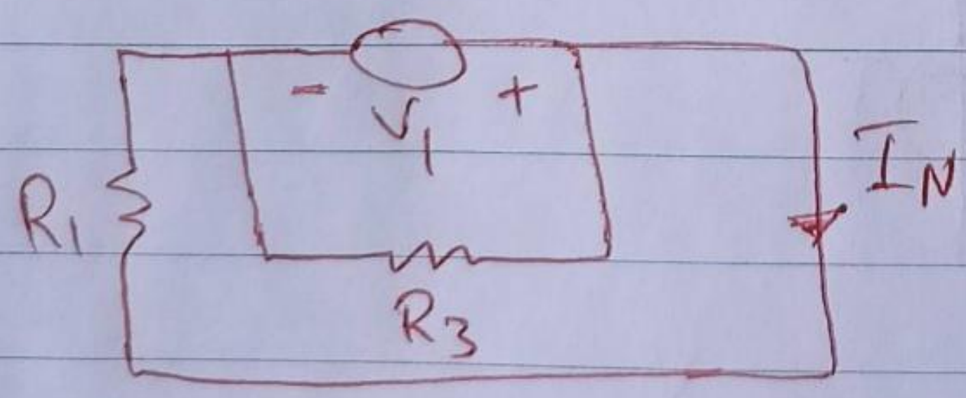


indep.
sources = 0

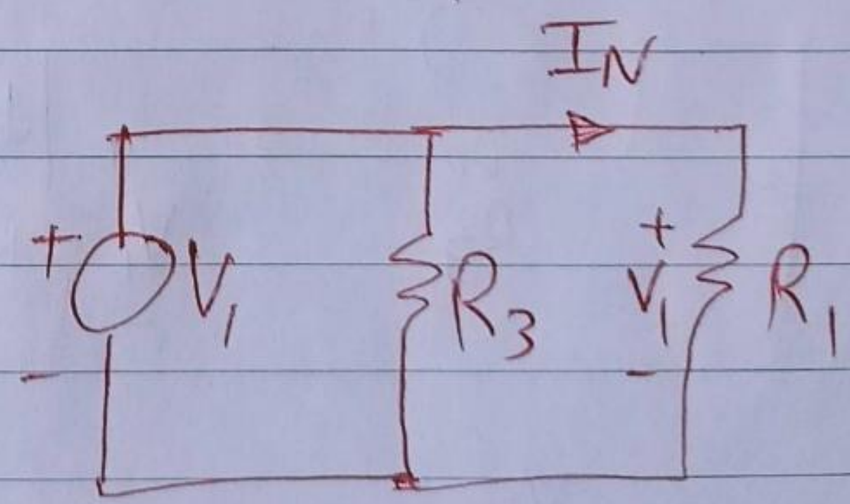
Example



- calculation of I_N :

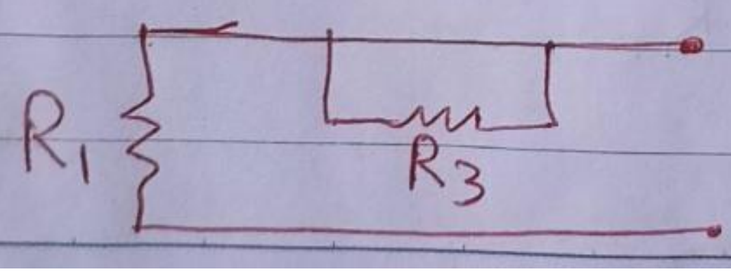


$\Rightarrow$



$$I_N = \frac{V_1}{R_1}$$

- calculation of R_N :



$R_N = R_1$

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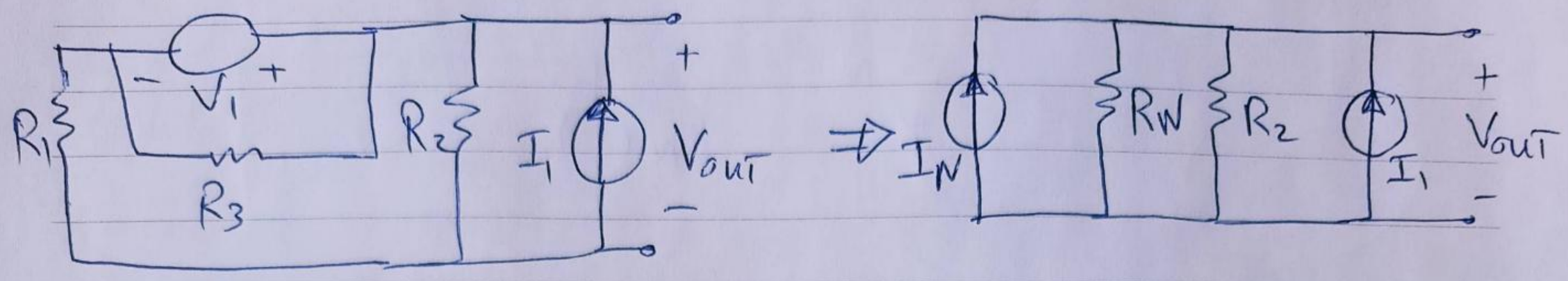
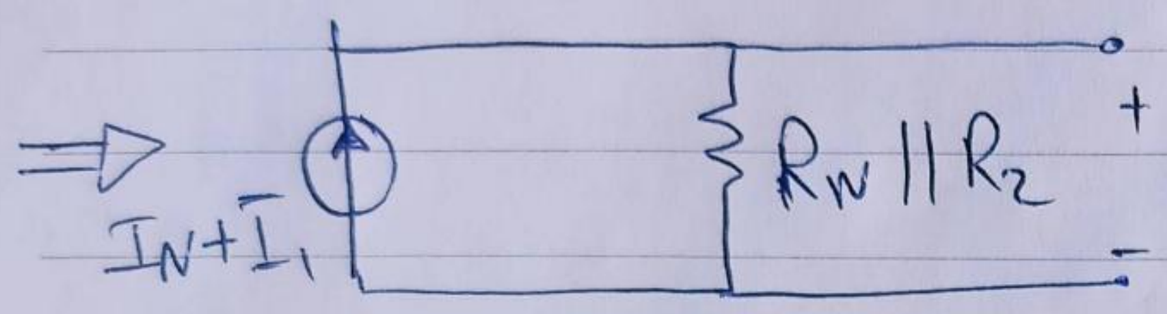
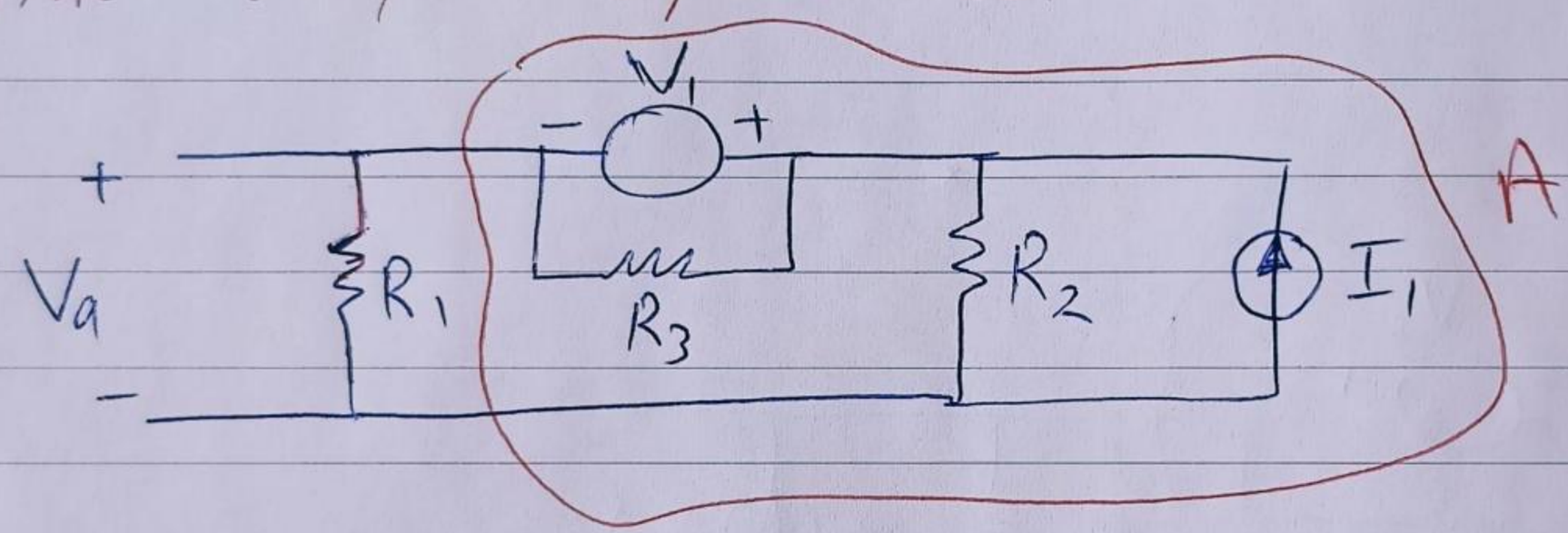
∴ equivelant circuit



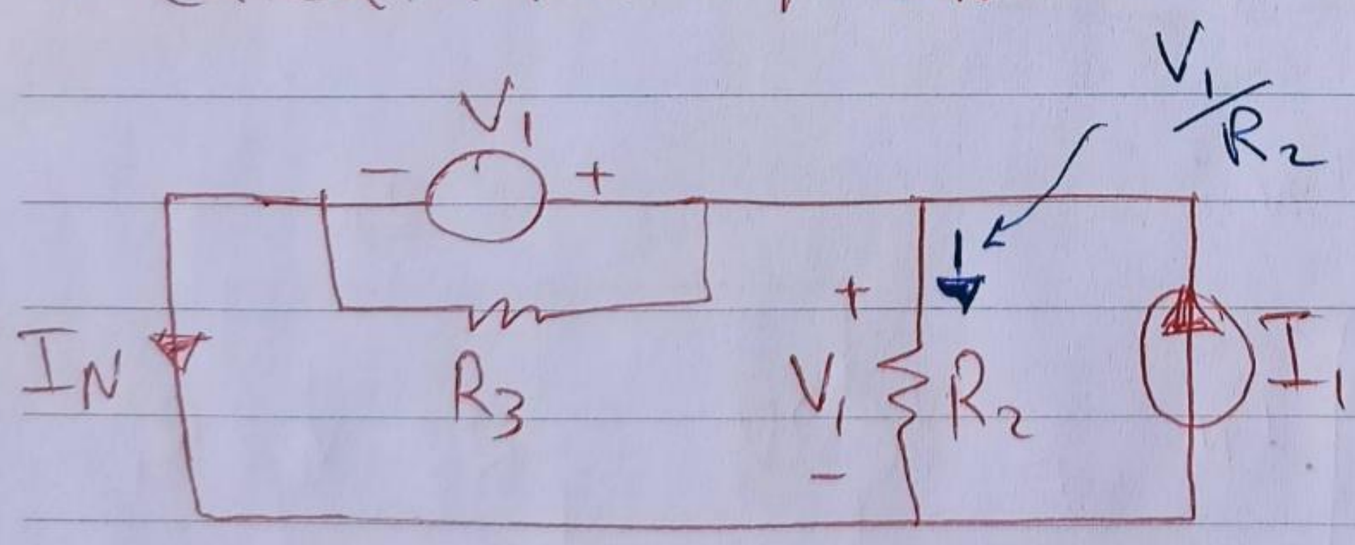


$$\Rightarrow V_{OUT} = \left(\frac{V_1}{R_1} + I_1\right)\frac{R_1 R_2}{R_1 + R_2}$$

another output of interest:

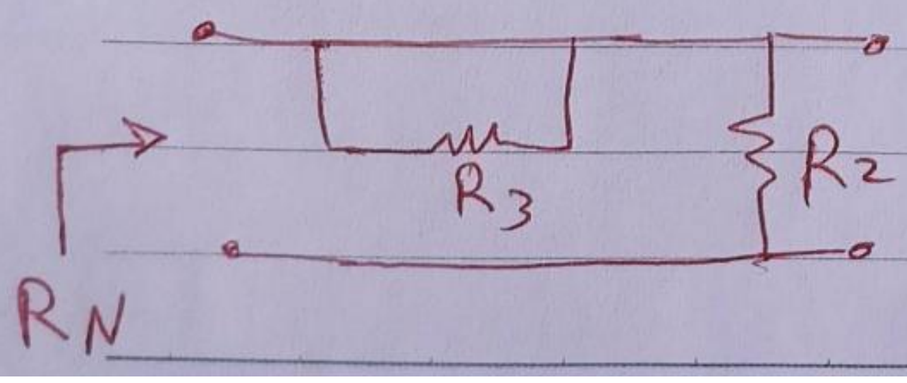


- Calculation of I_N:



$$I_N + \frac{V_1}{R_2} = I_1 \Rightarrow I_N = I_1 - \frac{V_1}{R_2}$$

- Calculation of R_N



$$\Rightarrow R_N = R_2$$

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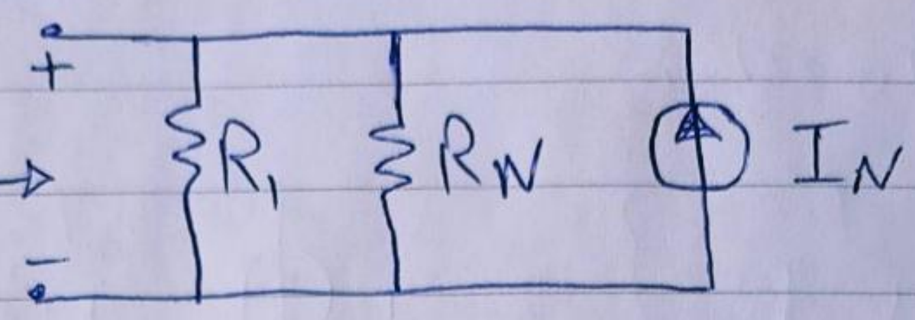
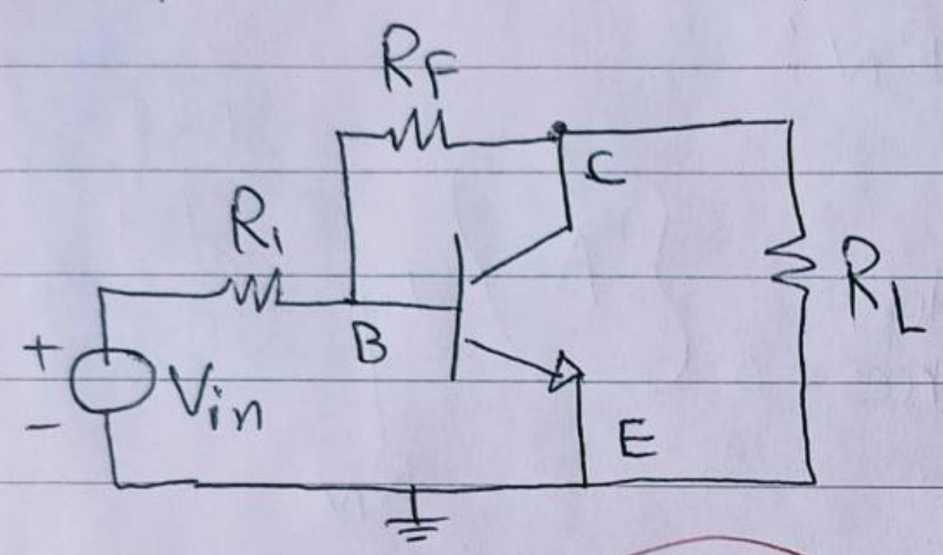
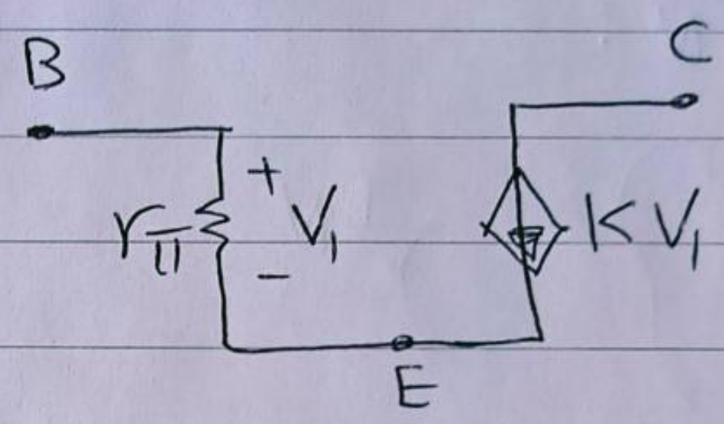
∴ equivelant circuit:

$V_a = I_N(R_1 \| R_N) \rightarrow$

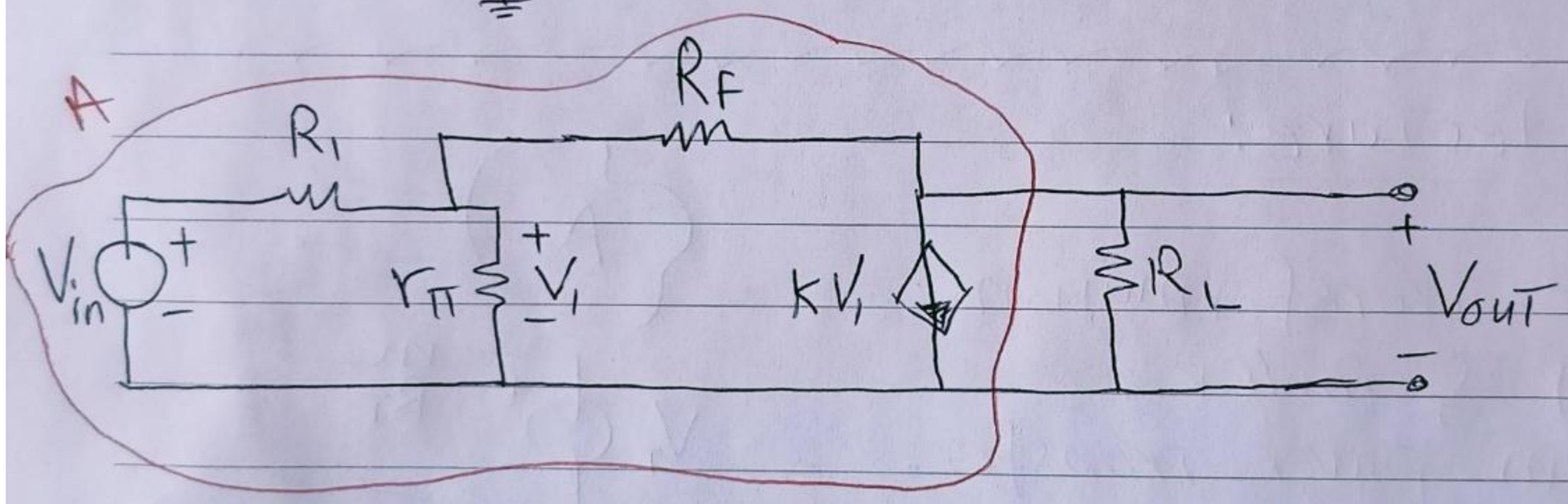


Amplifier Example:

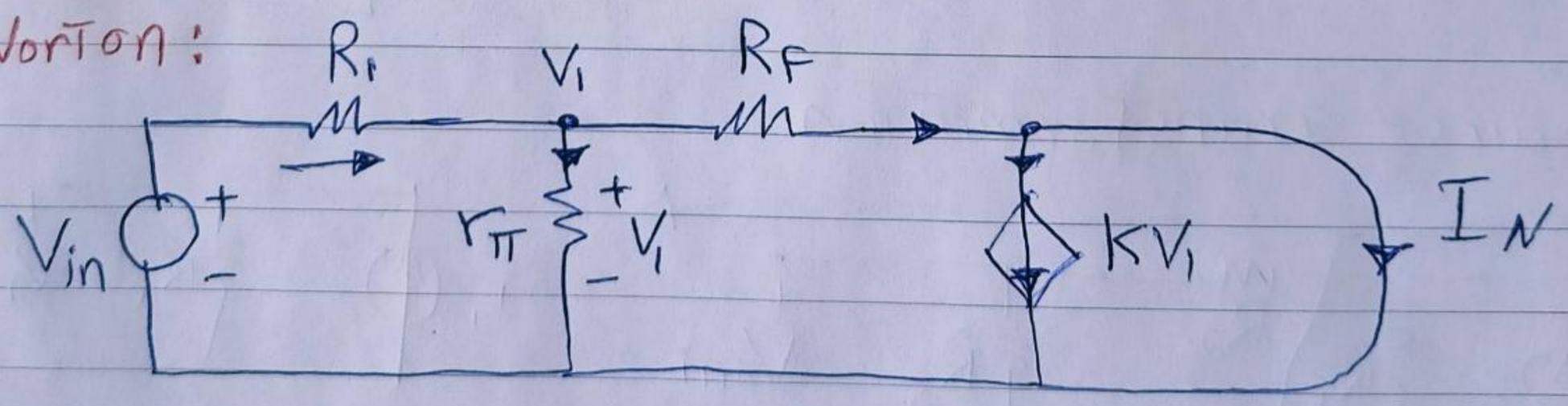




A

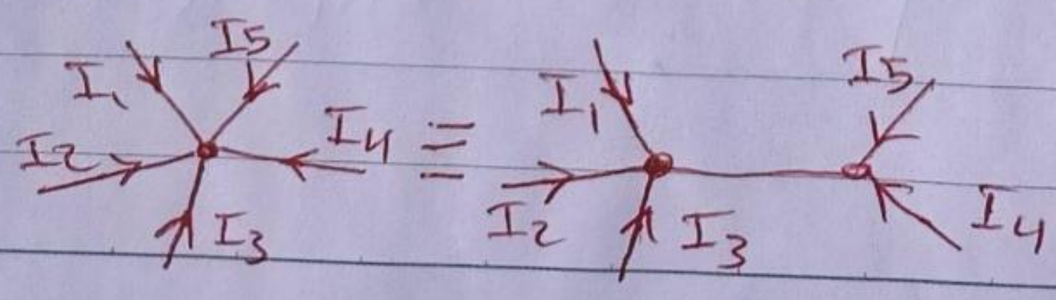


Norton:



$$\frac{V_{in}-V_1}{R_1} = \frac{V_1}{r_\pi} + \frac{V_1}{R_F} \Rightarrow V_1 = \frac{V_{in}/R_1}{1/R_1 + 1/r_\pi + 1/R_F}$$

$$\frac{V_1}{R_F} = KV_1 + I_N \Rightarrow I_N = V_1\left(\frac{1}{R_F} - K\right)$$



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Lec 12: Source Transformation, Power & Energy

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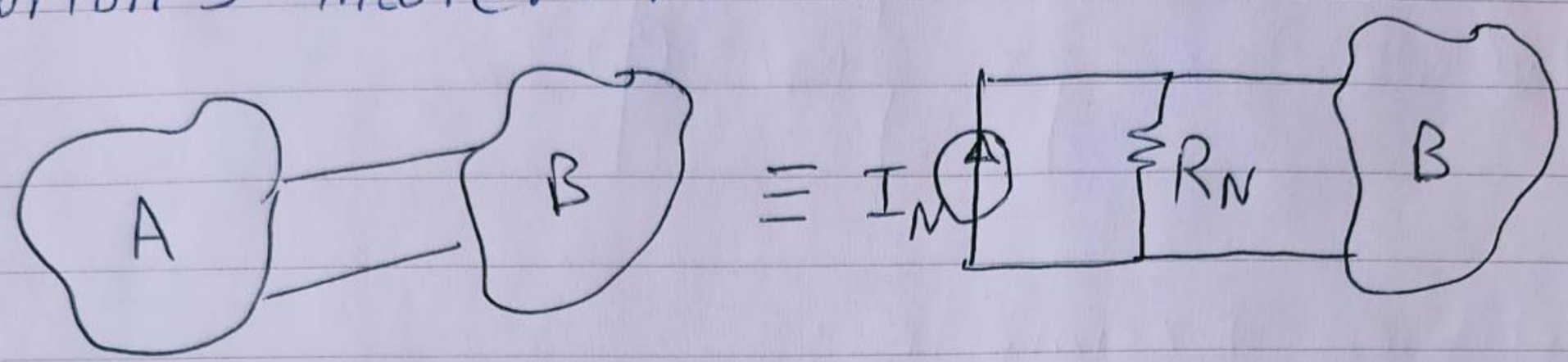
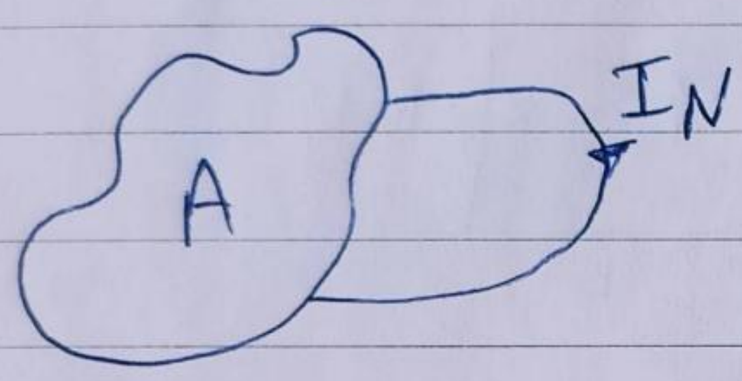
Review of lecture 11:

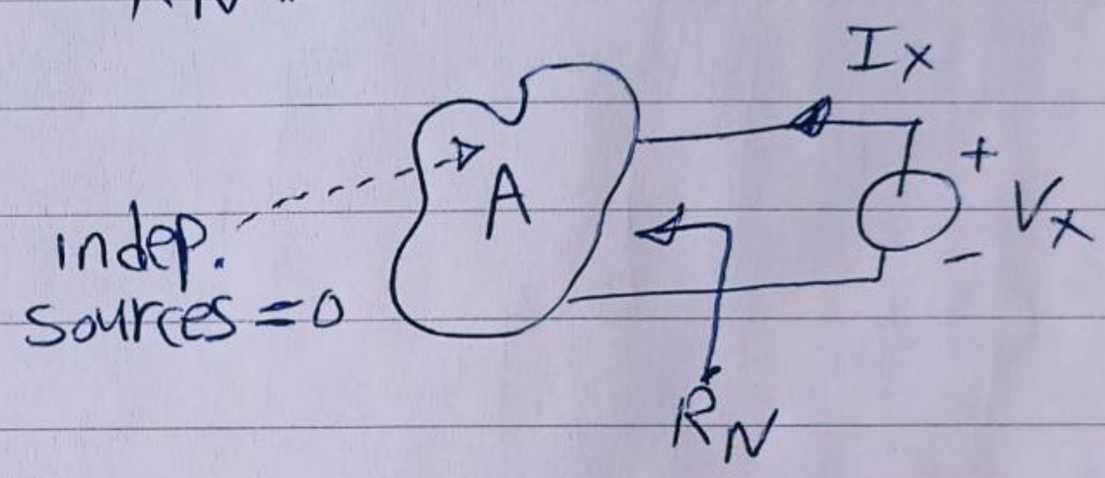
- Norton's Theorem:



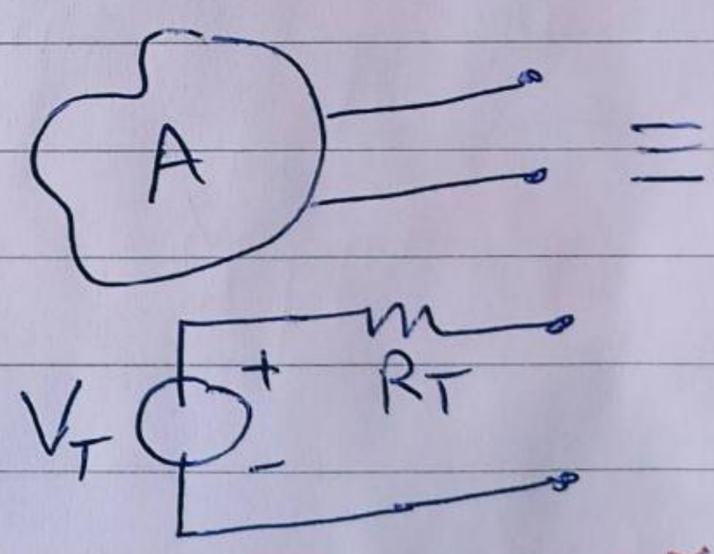
- I_N:



R_N:

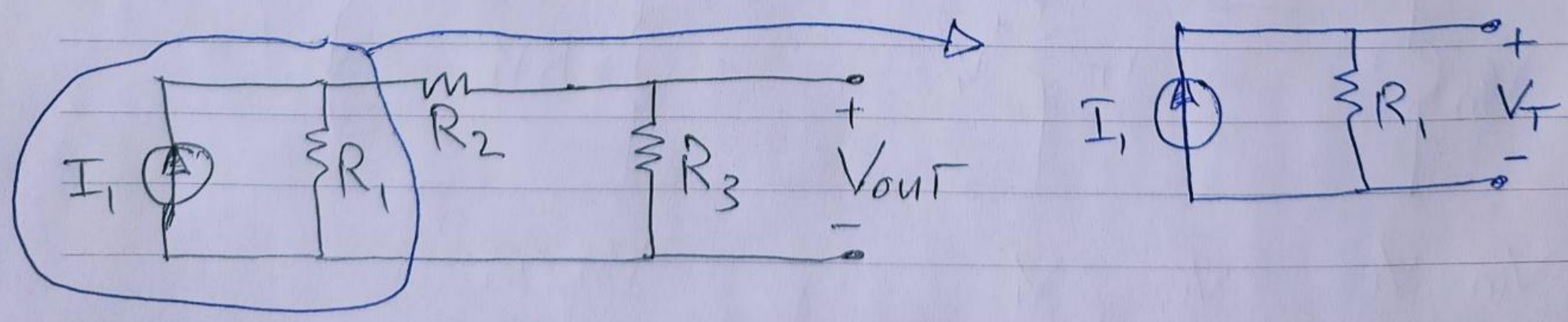


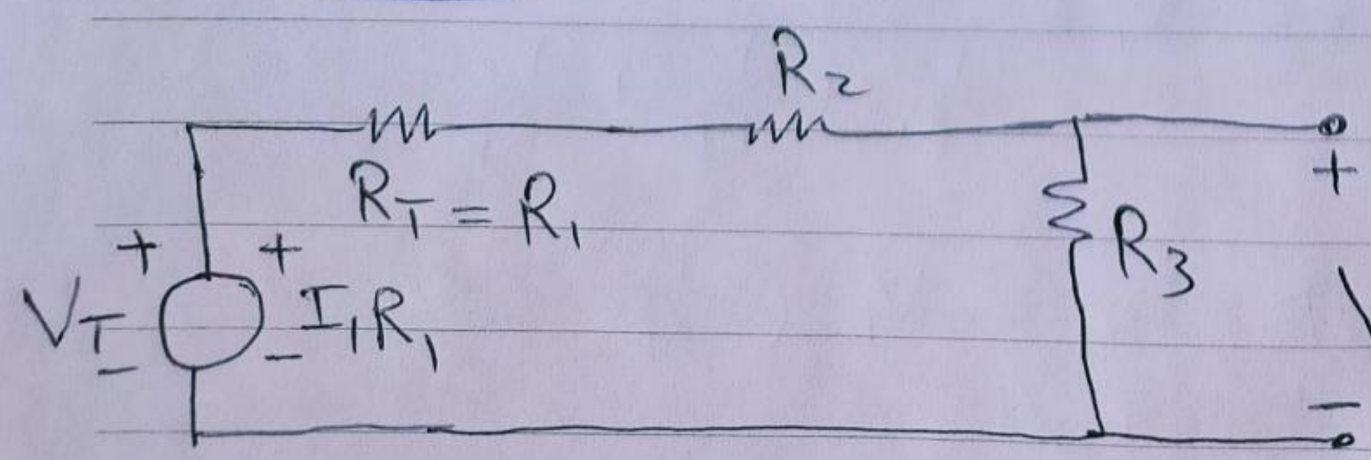
Today's lecture:



- Source Transformation.
- Energy & power.
- Maximum power transfer.

- Source Transformation





$$V_{OUT} = \frac{R_3}{R_1 + R_2 + R_3} \cdot I_1 R_1$$

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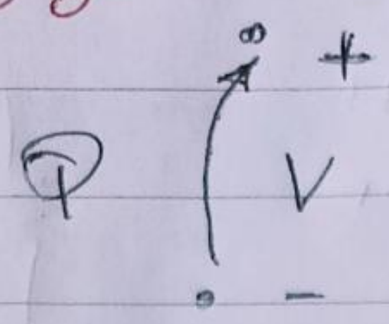
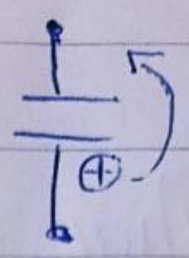
- Energy



$$E \overset{?}{=} Q \cdot V$$
$$dE = V\,dQ$$



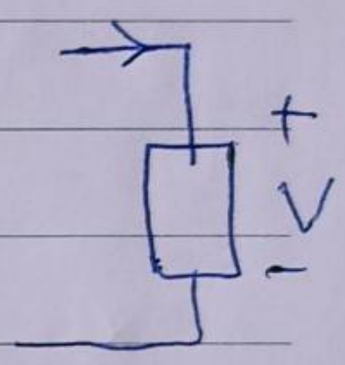
- Power:

$$P = \frac{dE}{d\tau} \quad \text{e.g, 20 watt bulb} \Rightarrow 20\,W = P$$

$$20W = \frac{20J}{1s}, \quad E = \int_0^T P\,d\tau$$

- Power in circuits:

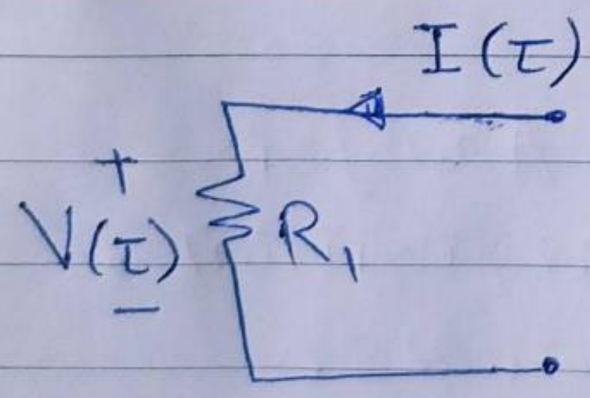
$$dE = V\,dQ \qquad P = \frac{dE}{d\tau} = \underbrace{\frac{dE}{dQ}}_{V} \cdot \underbrace{\frac{dQ}{d\tau}}_{I}$$



$$\boxed{P = V(\tau)\, I(\tau)}$$

→ instantaneous power

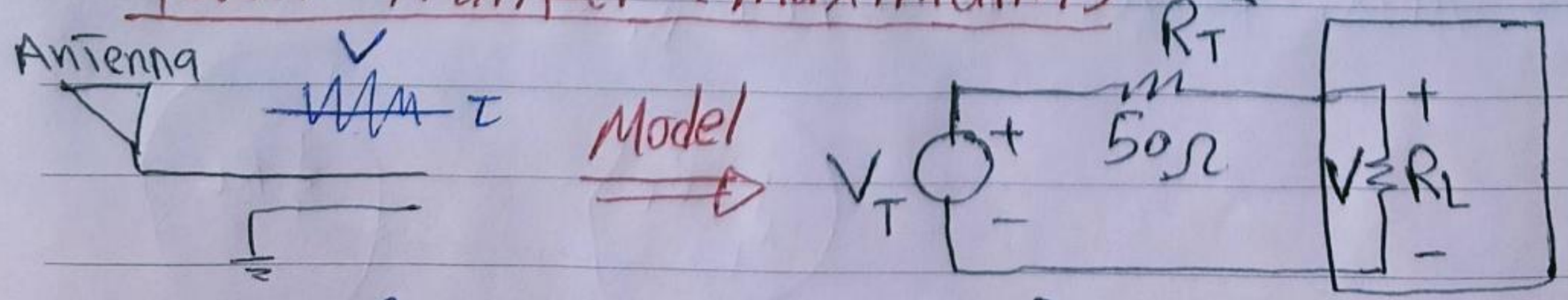
Example:



$$P = V(\tau) \cdot \frac{V(\tau)}{R_1} = \frac{V^2(\tau)}{R_1} = I^2(\tau)\, R_1$$

Power Tranfer (Maximuim)

Source Resistance



$$P_{R_L} = \frac{V^2}{R_L} = \frac{1}{R_L}\left(\frac{R_L}{R_L + R_T} V_T\right)^2 \Rightarrow \frac{\partial P_{R_L}}{\partial R_L} = 0 \Rightarrow \boxed{R_L = R_T}$$

Matched condition.

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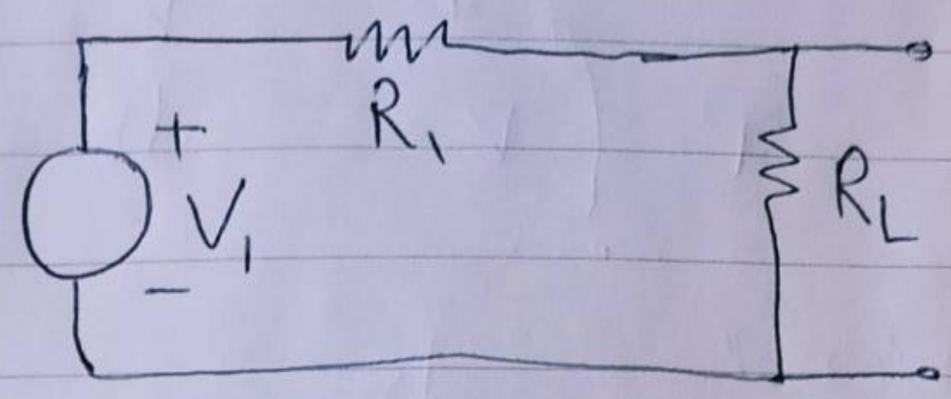
Caution

Q: under what condition is the power delivered to R_L maximuim?



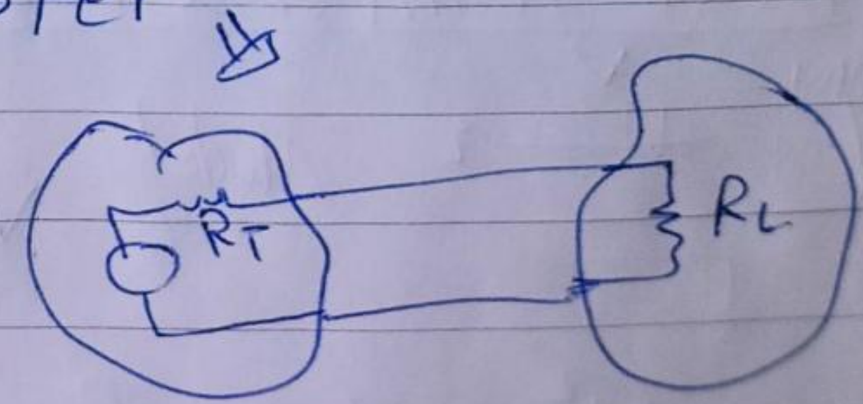
Ans): [1] $R_L = R_1$ → True only if R_L is variable but R_1 is NOT.

[2] R_1 is NOT fixed, but R_L is:

$$P_{R_L} = \frac{1}{R_L}\left(\frac{R_L}{R_L + R_1} V_1\right)^2 \Rightarrow R_1 = 0$$

Summary:

- Electric Quantities: charge, current, voltage, energy and power.
- Electric Devices: wire, switch, voltage sources, current sources, resistors
- KVL & KCL
- Node Analysis, mesh Analysis
- circuit Theorems: Superposition, Thevenin's & Norton's Theorems.
- Source Transformation
- Max. power Transfer



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Lec 13: Capacitors and Their Properties

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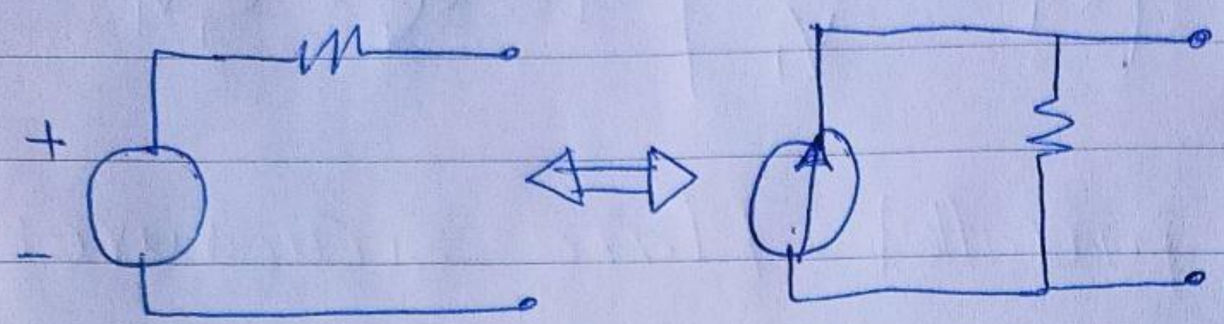
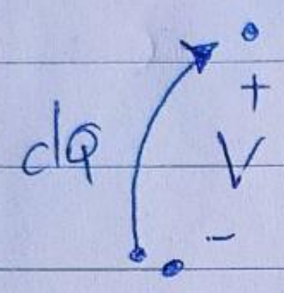
Review of lecture 12

- Source Transformation:



- Energy:



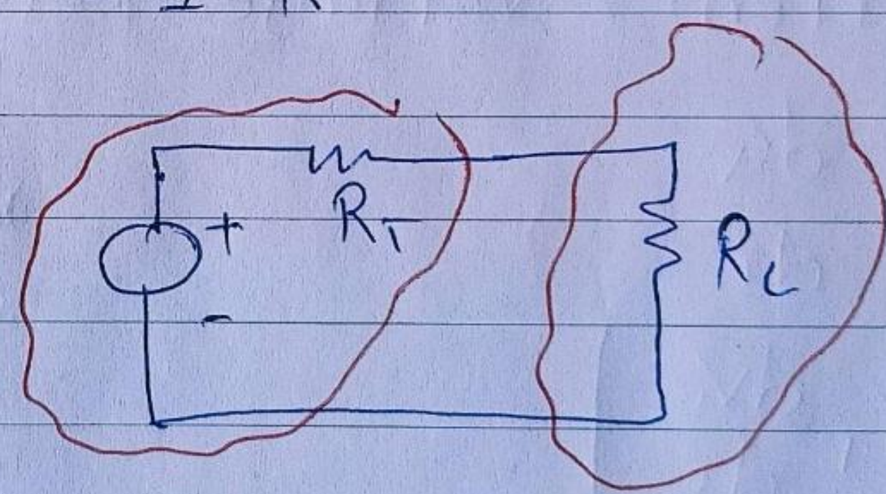
$$d\mathcal{E} = V\,dQ$$

- Power:

$$\frac{d\mathcal{E}}{dt} = V(t)\,I(t)$$

- for a resistor: $P = \frac{V^2}{R} = I^2 R$

- max. Power Transfer:

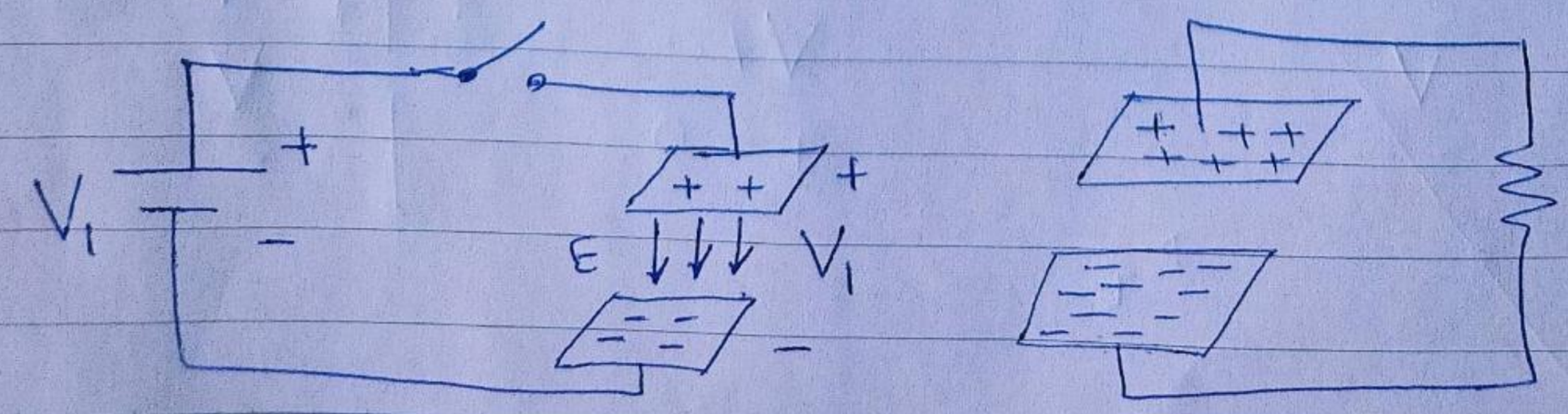


$$R_T = R_L$$

Today's lecture:

- Capacitors
  - Definition of capacitance.
  - Electric properties of caps.

- Simple Experiment



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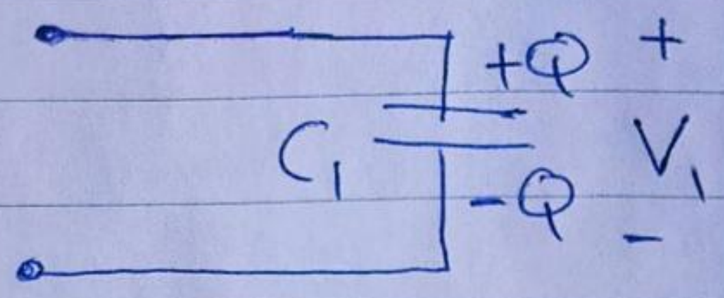
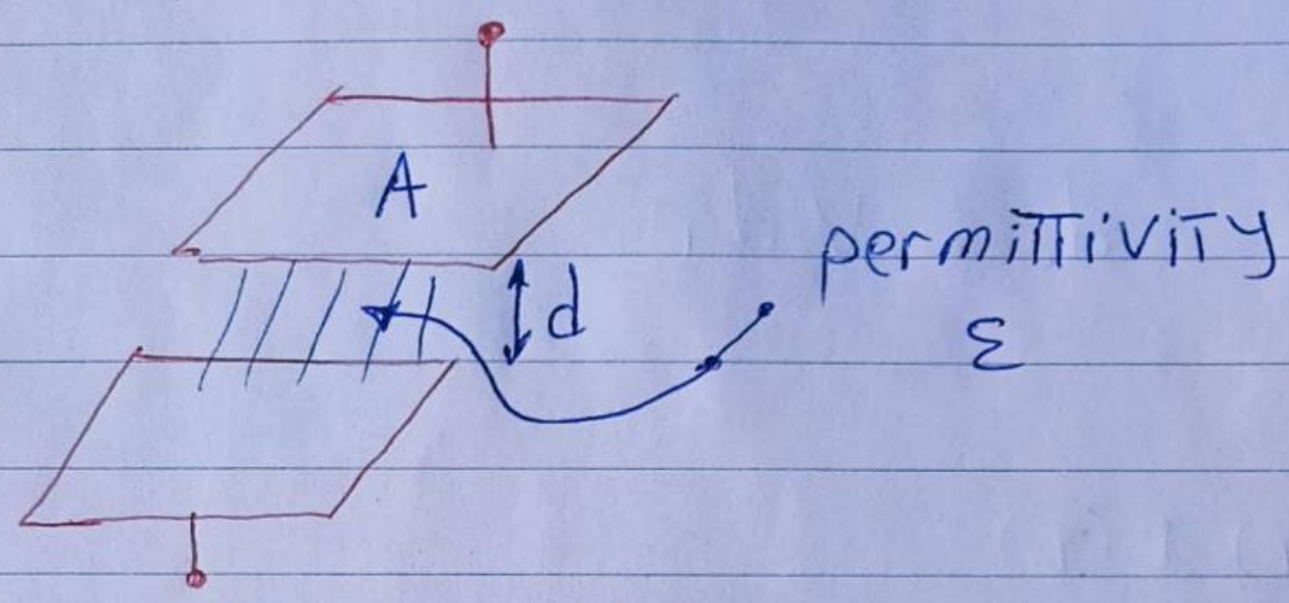
- Definition of capacitance:



$$\Rightarrow \boxed{C_1 = \frac{Q}{V_1}} \Rightarrow Q = C_1 V_1$$

- capacitance of a parallel-plate structure:

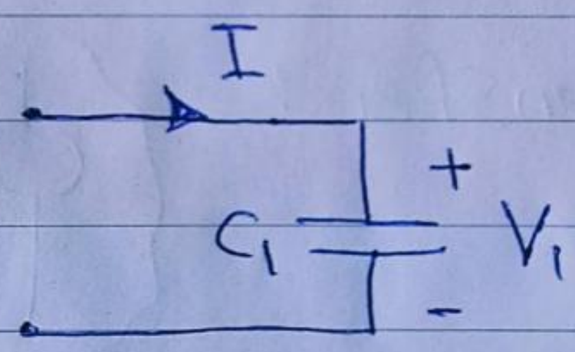


$$C = \varepsilon \frac{A}{d}$$

- Current - Voltage Relationships:

$$I = \frac{dQ}{dt}$$

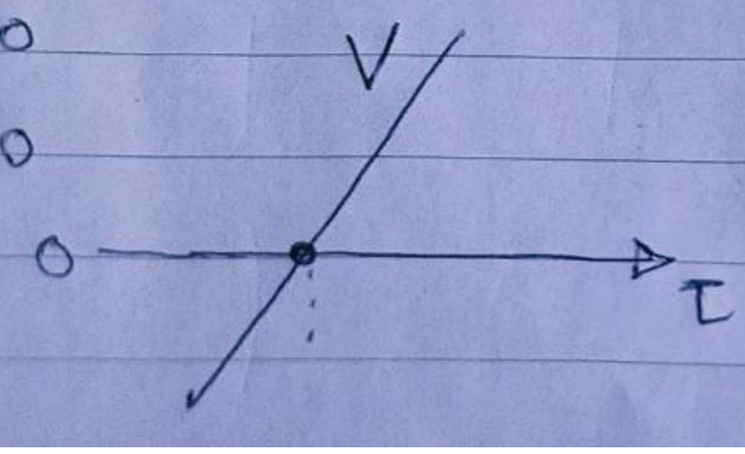
$$\boxed{I = C_1 \frac{dV_1}{dt}}$$



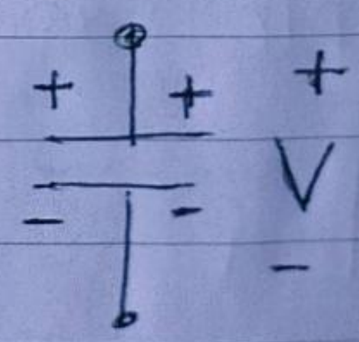
Some observations:

① unlike resistors (for which $V=0 \Rightarrow I=0$), capacitors can have $V=0$, $I \neq 0$ or $V \neq 0$, $I=0$)

$V=0$
$I \neq 0$



$V \neq 0$
$I = 0$



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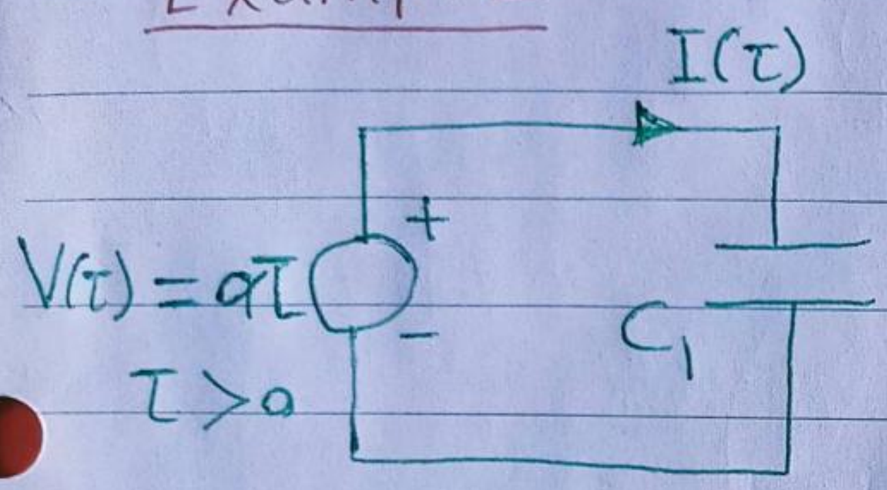
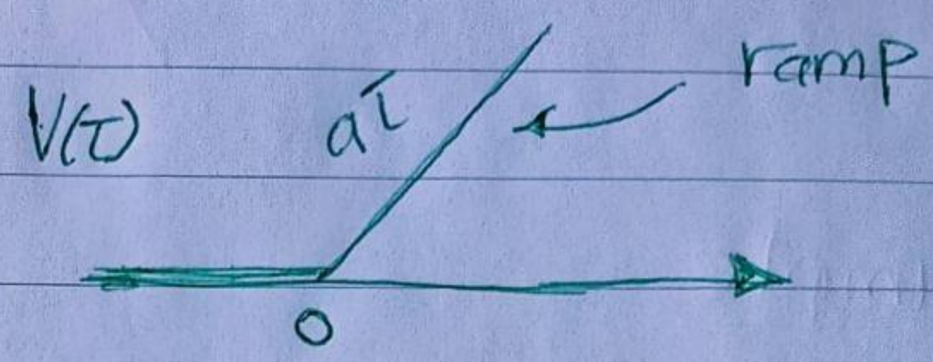
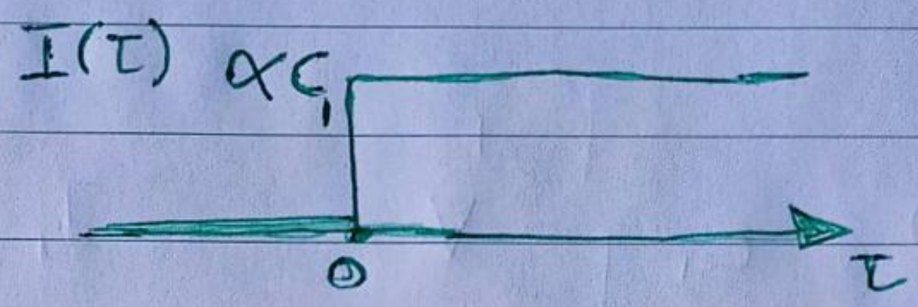
② The voltage can be found from the current

$$V = \frac{1}{C_1}\int_0^{t} I \, dt$$

Example



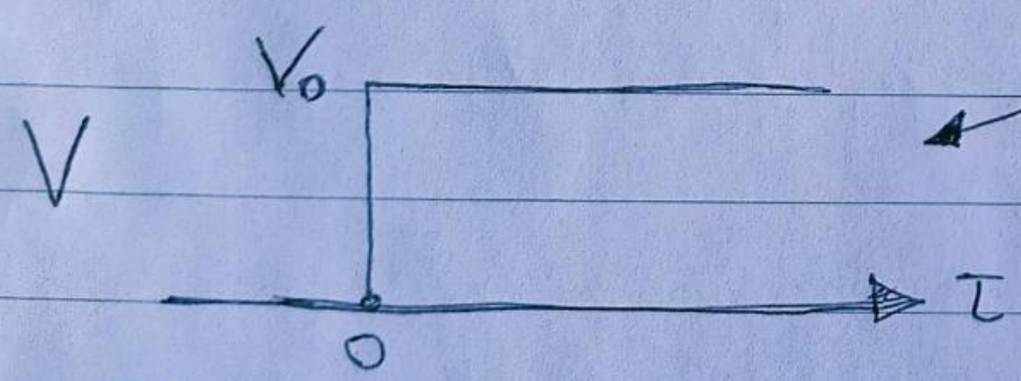


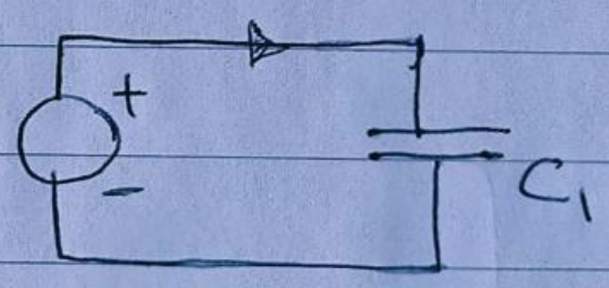


Important points:

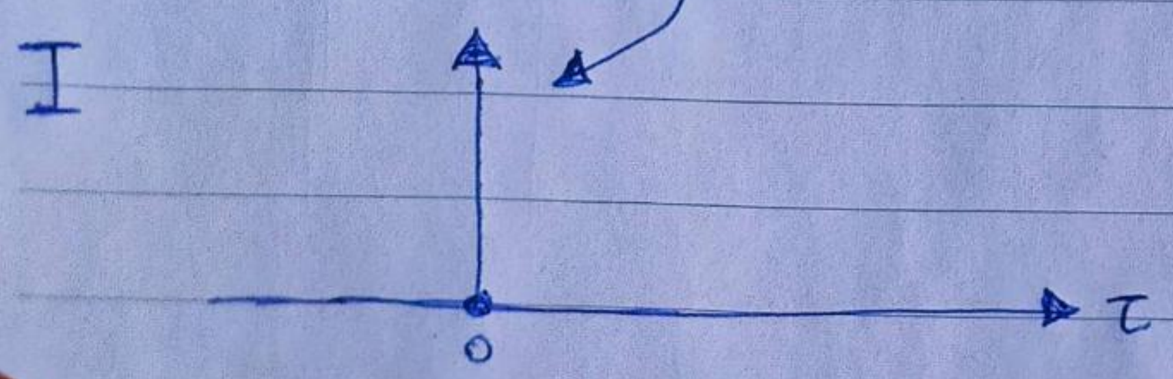
① If the voltage on cap is const $\Rightarrow I = C_1 \frac{dV}{dt} = 0$

② Very important: for the voltage on a cap to change instantaneously, the current must be ∞





$$I = C_1 \frac{dV}{dt} = C_1 V_0 \, \delta(t) \quad , \quad \delta(t) = \frac{d\,u(t)}{dt}$$



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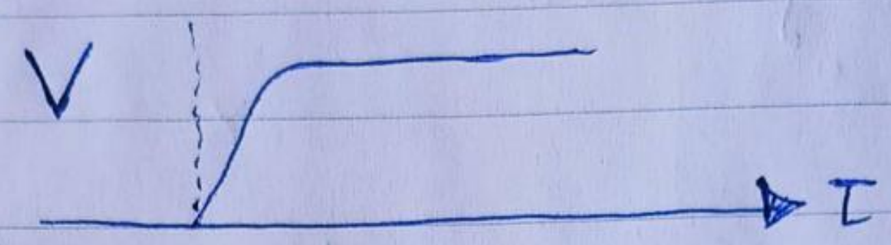
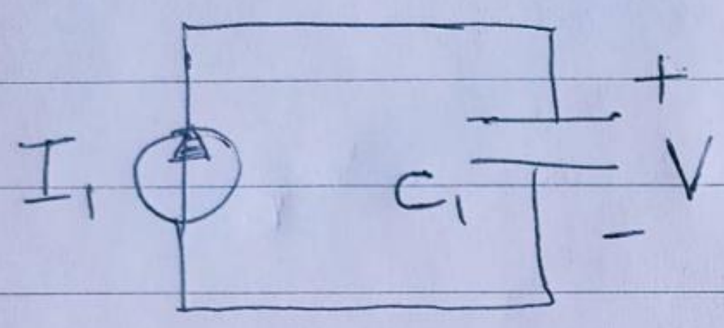
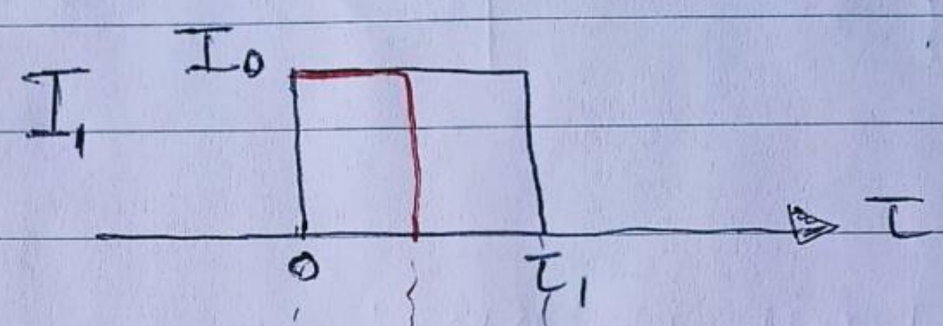
⇒ If The Current available To charge a cap is $< \infty$ ⇒ The cap Voltage cannot jump in zero Time:

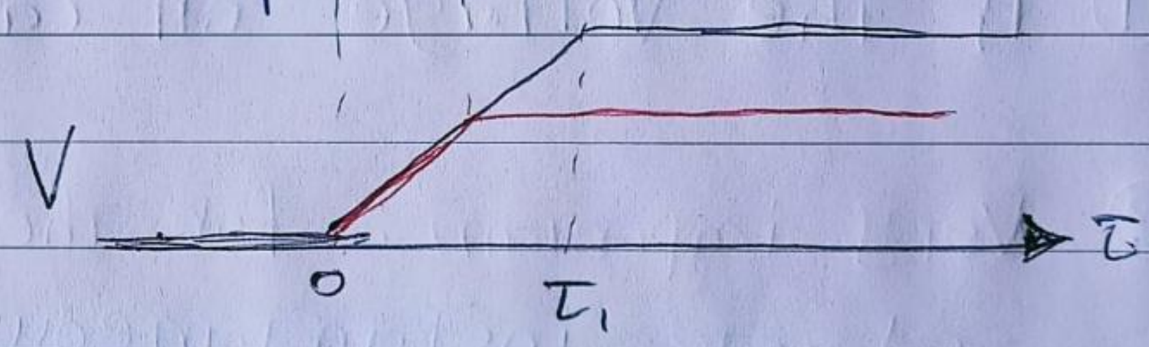


Example

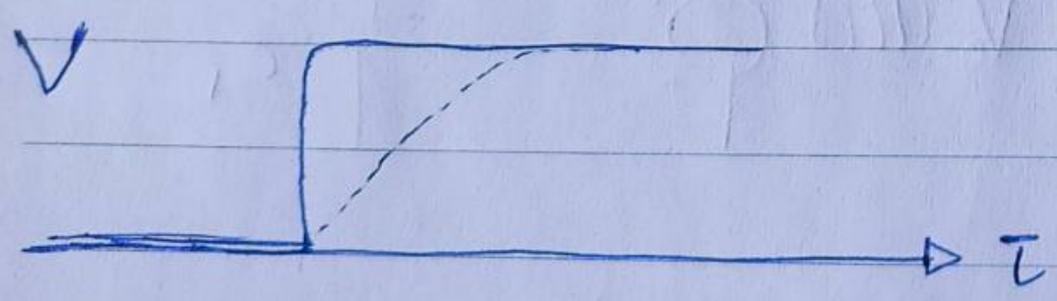




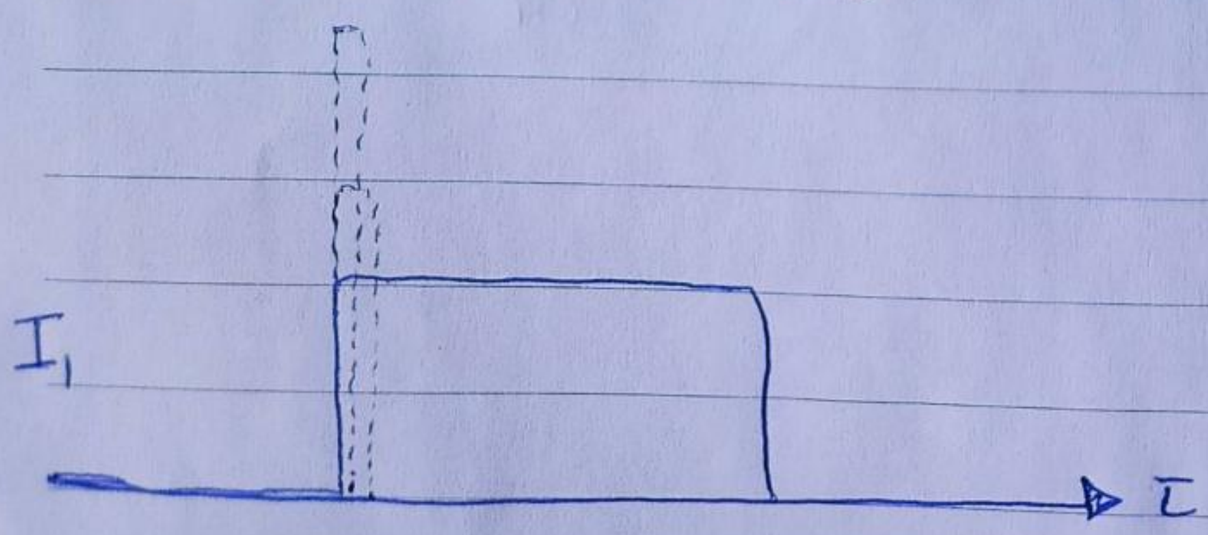
$$V = \frac{1}{C_1}\int_0^{t_1} I_0 \, dt \Rightarrow V = \frac{I_0}{C_1} t$$



- How do I Make The Charging process Faster?



We need increase I_0:



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Lec 14: Energy in Capacitors; Series & Parallel

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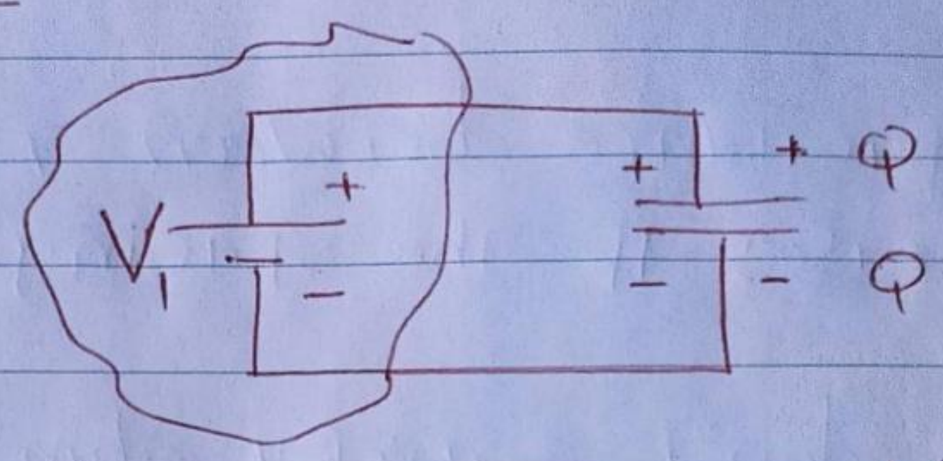
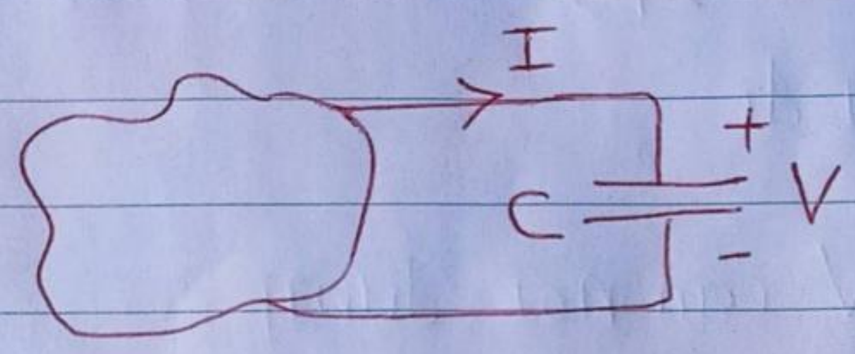
Review of lecture 13

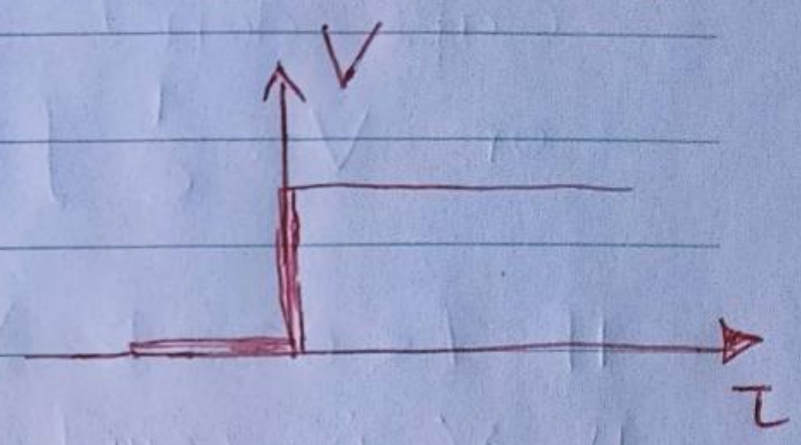
- Capacitance

$$C = \frac{Q}{V_1} \ (F)$$



- $I = C \dfrac{dV}{d\tau}$



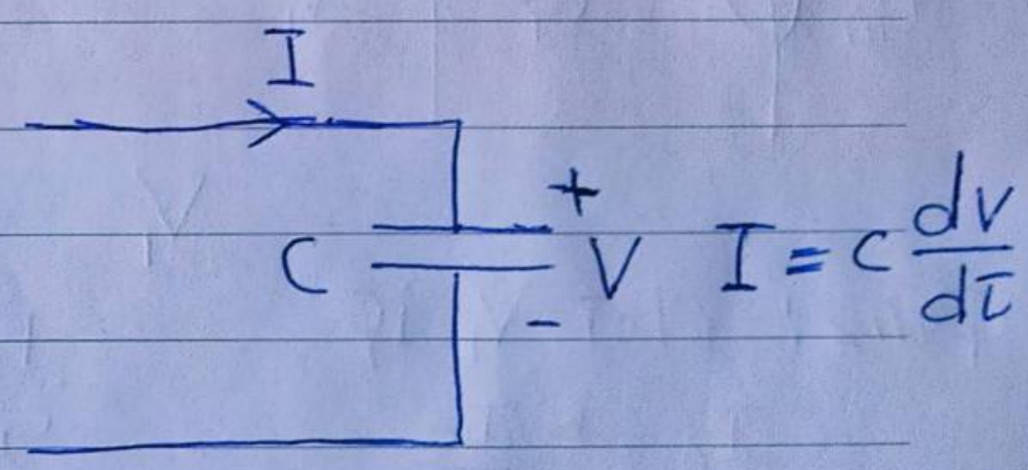


for V To jump , $I = \infty$

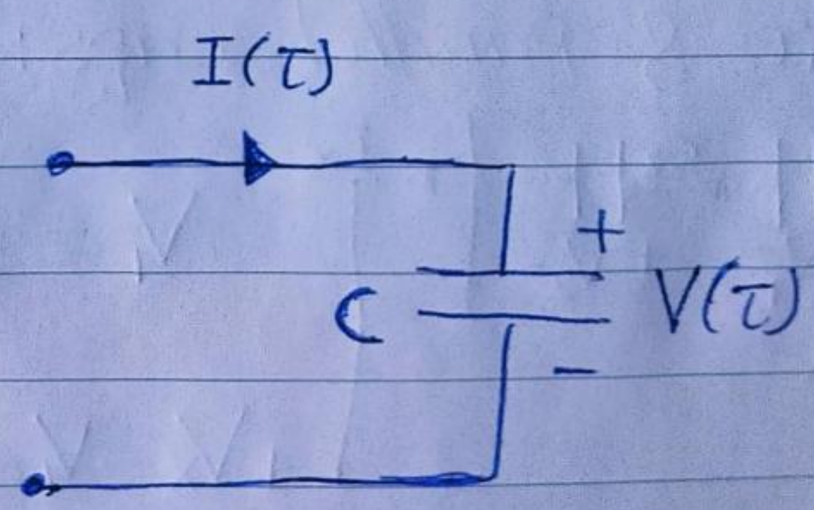
- $V = \dfrac{1}{C} \displaystyle\int_0^{\tau} I \, d\tau$
- $\mathcal{E} = \dfrac{1}{2} C V^2$

Today's lecture:

- Energy in Capacitors
- Summary
- series & parallel caps
- Good Cap , Bad cap



- Energy in Caps



$$P = V(\tau)\, I(\tau)$$

$$\mathcal{E} = \int P \, d\tau$$

$$= \int V(\tau)\, C \frac{dV}{d\tau} \, d\tau = \frac{1}{2} C V^2(\tau)$$

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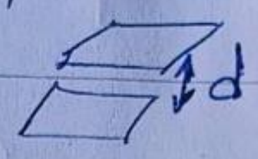
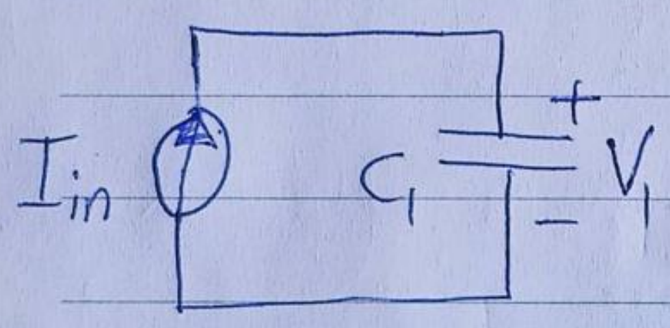
- Summary:

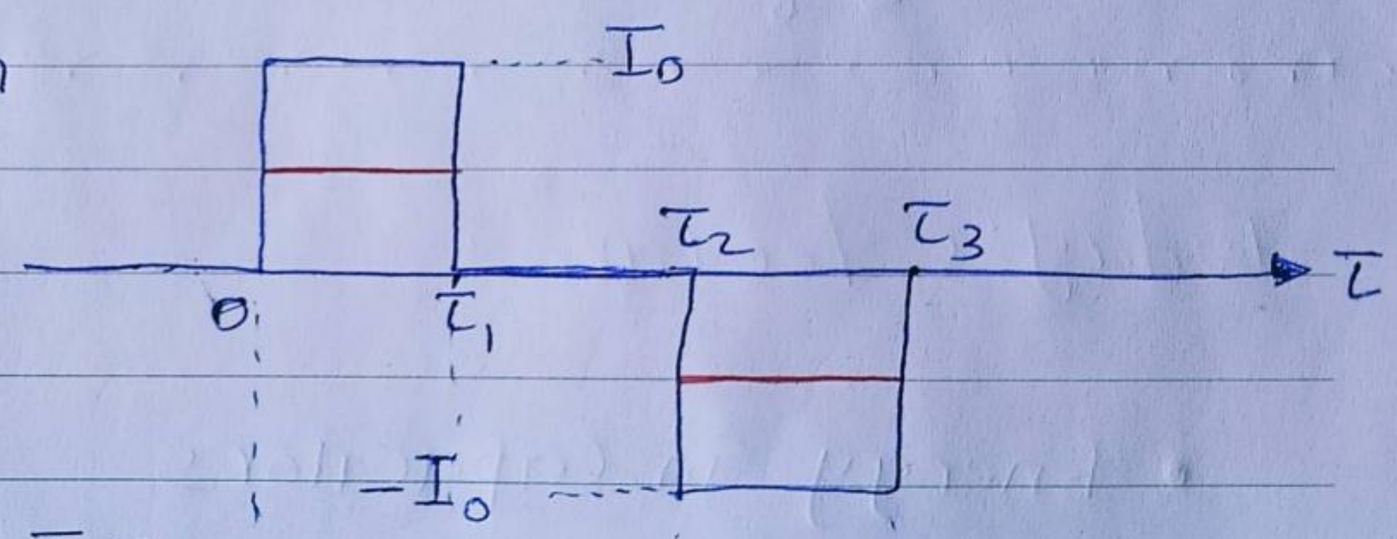
  - Capacitance between two parrallel plates increases if the spacing decreases 

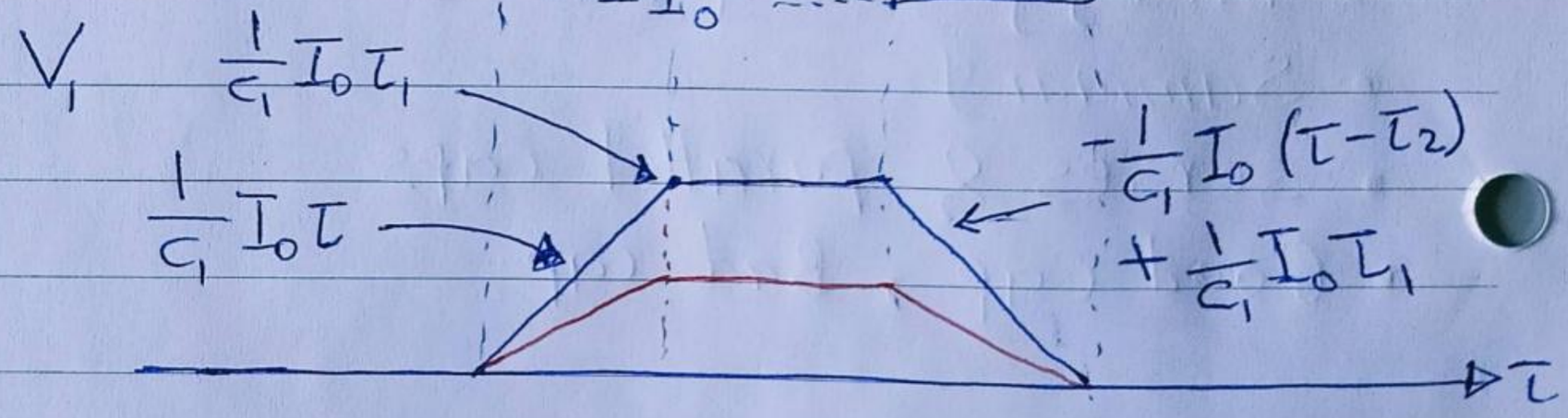
  - Caps don't satisfy ohm's law; $I = C\frac{dV}{d\tau}$ or $V = \frac{1}{C}\int I\, d\tau$

  - If $I < \infty \Rightarrow V$ can not jump
  - If V is const. $\Rightarrow I = 0 \Rightarrow$ cap act as an open circuit.

Example

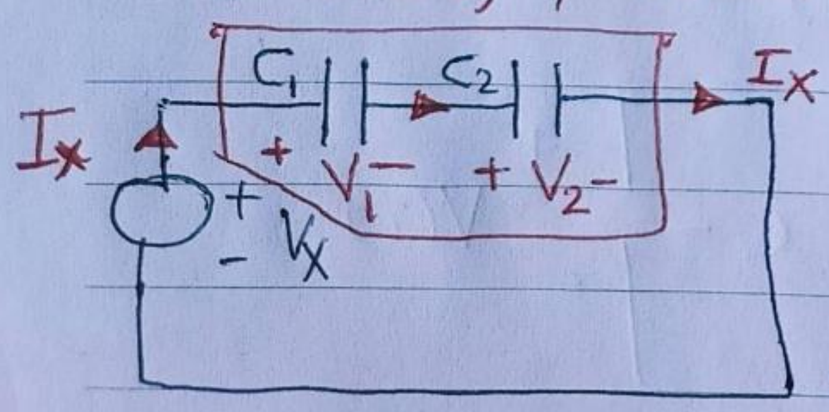






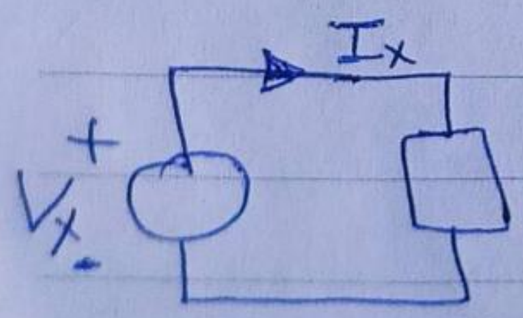
Quiz: plot V_1 if $I_0 \rightarrow \frac{I_0}{2}$

- Series & parallel Caps:



$$V_1 = \frac{1}{C_1}\int I_x\, d\tau \quad \& \quad V_2 = \frac{1}{C_2}\int I_x\, d\tau$$

$$+V_x - V_1 - V_2 = 0 \Rightarrow V_x = \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}\int I_x\, d\tau$$



$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$ "series caps"

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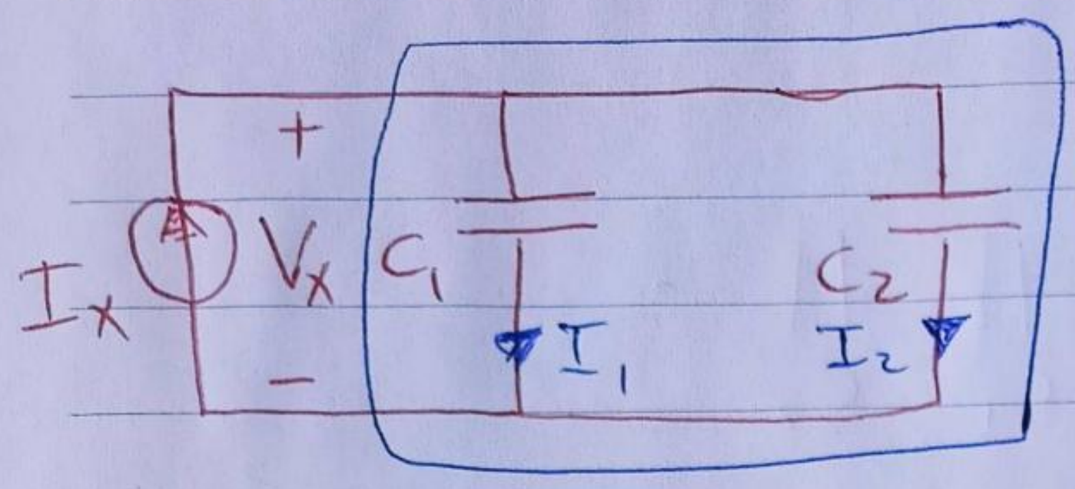
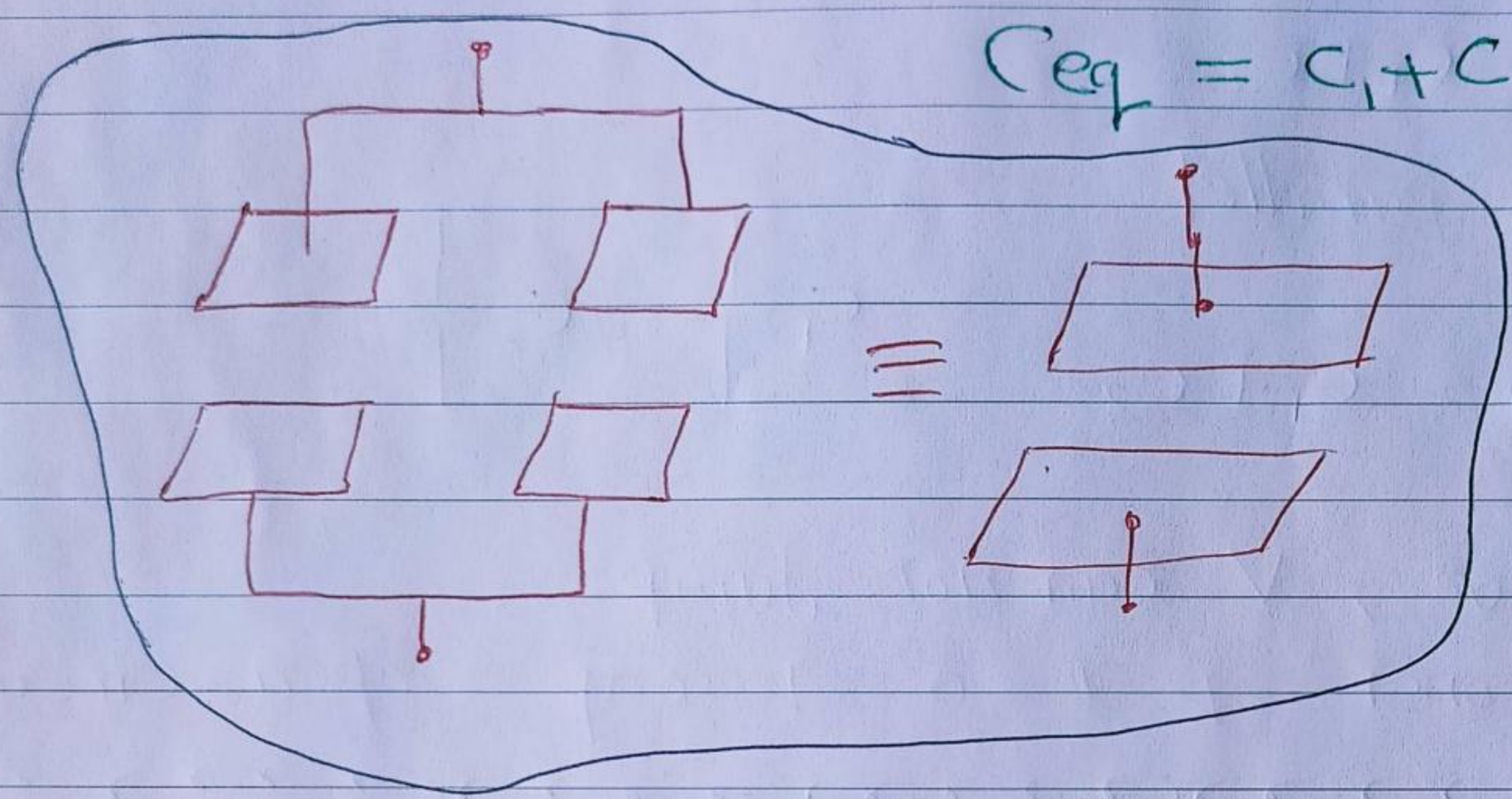


$$I_1 = C_1 \frac{dV_x}{dt}, \quad I_2 = C_2 \frac{dV_x}{dt}$$

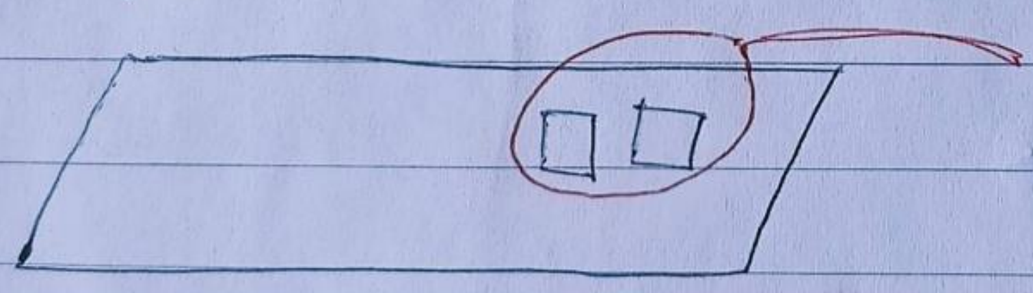
$$I_x = I_1 + I_2 = \underbrace{(C_1 + C_2)}\frac{dV}{dx}$$

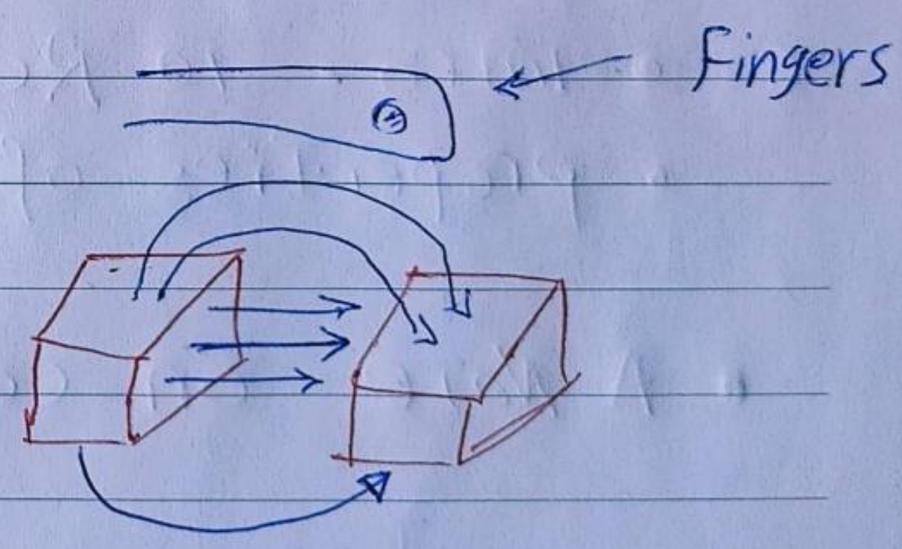
$C_{eq} = C_1 + C_2$ "parallel caps"



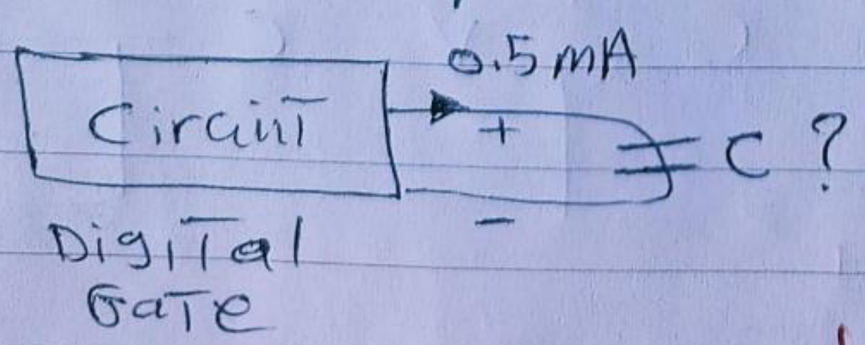
→ beauty of circuits "38:00 To 43:00"

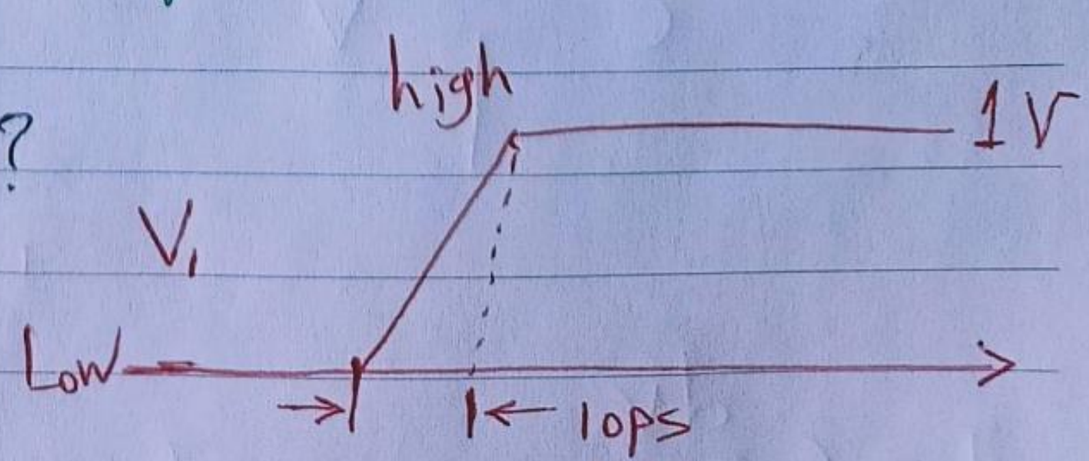
- Good cap, Bad cap
  - Capacitive sensors





  - parasitic caps "Bad cap"





$$I = C\frac{dV}{dt} \Rightarrow 0.5\,mA = C\frac{1V}{10ps} \Rightarrow C = 5\times10^{-15}\,F = 5\,fF$$

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Lec 15: Introduction to RC Circuits

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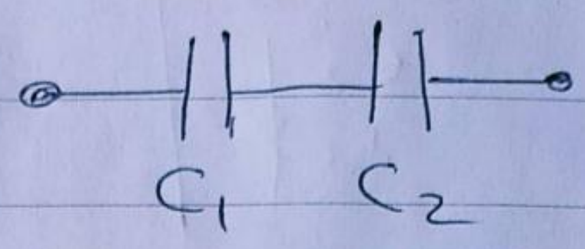
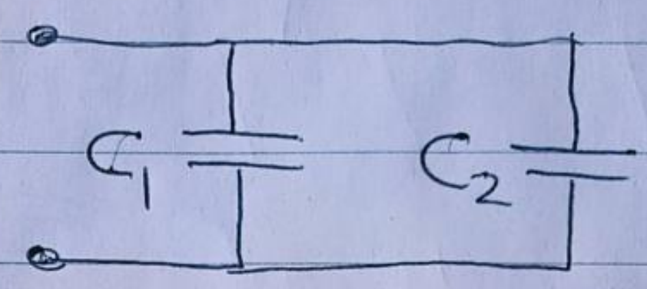
Review of lecture 14

- Caps in series



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

- Caps in parallel



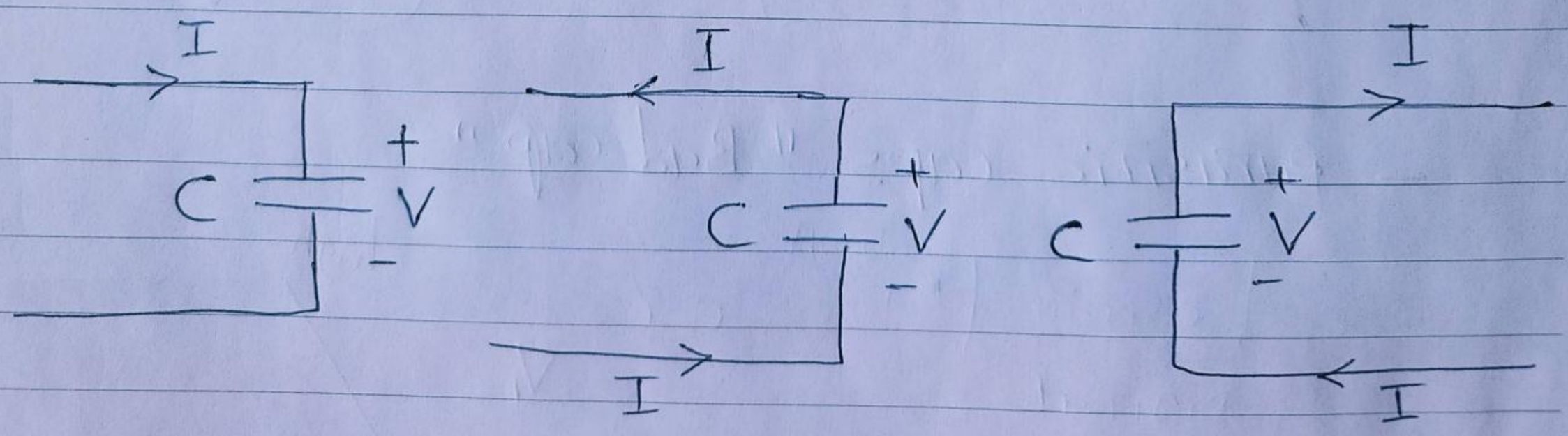
$$C_{eq} = C_1 + C_2$$

- If $I < \infty$, V can not jump
- If $V = \text{const} \Rightarrow I = 0 \Rightarrow$ cap = open-circuit

Today's lecture:

- a Note on caps
- intro. to RC circuits
- properties of Exponential Decay

- A Note on caps



$$I = C\frac{dV}{dt} \qquad I = -C\frac{dV}{dt} \qquad I = -C\frac{dV}{dt}$$

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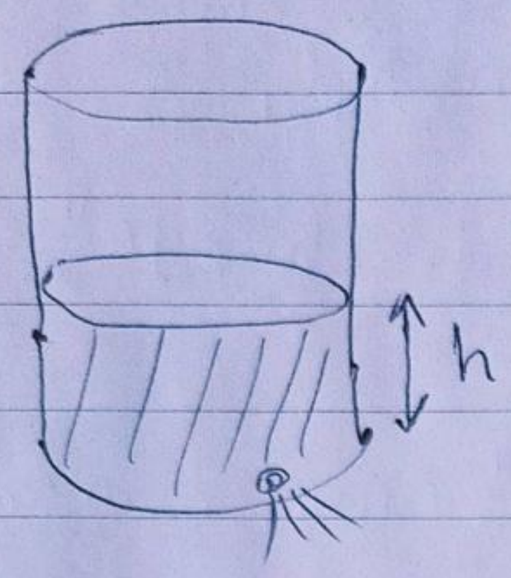
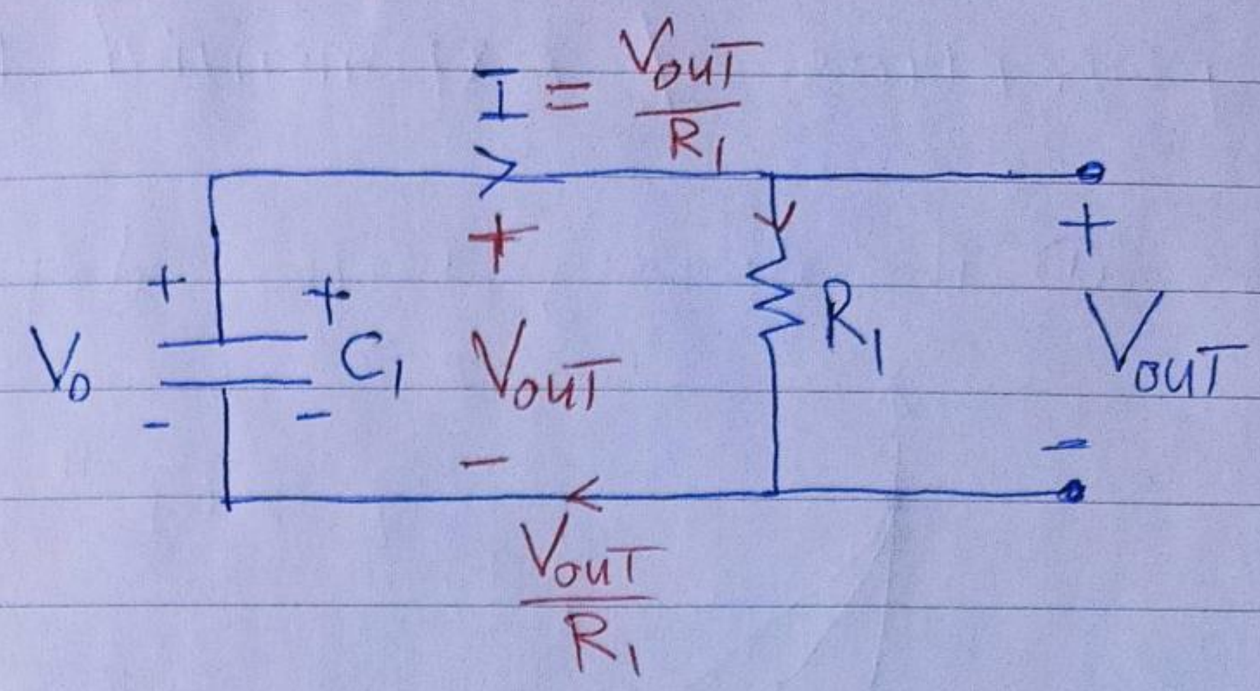
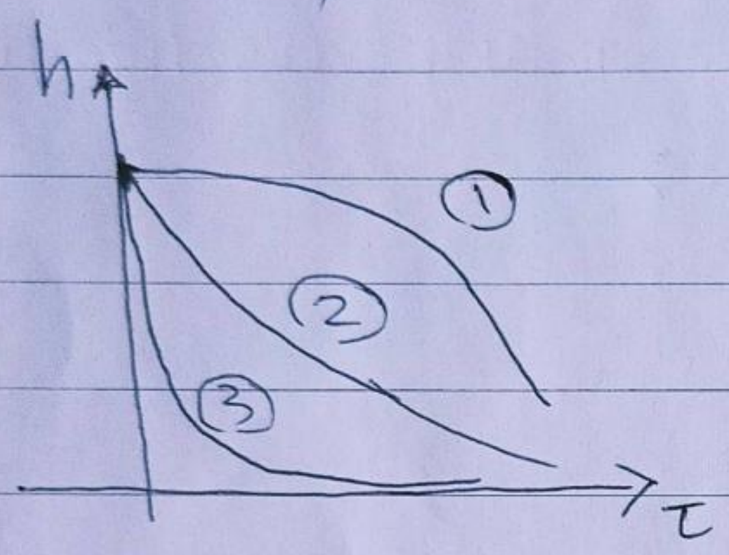
- Simple RC circuit







$$\begin{cases} \dfrac{V_{OUT}}{R_1} = -C_1 \dfrac{dV_{OUT}}{d\tau} \rightarrow (1) \\ V_{OUT}(0) = V_0 \quad \text{"initial condition"} \end{cases}$$

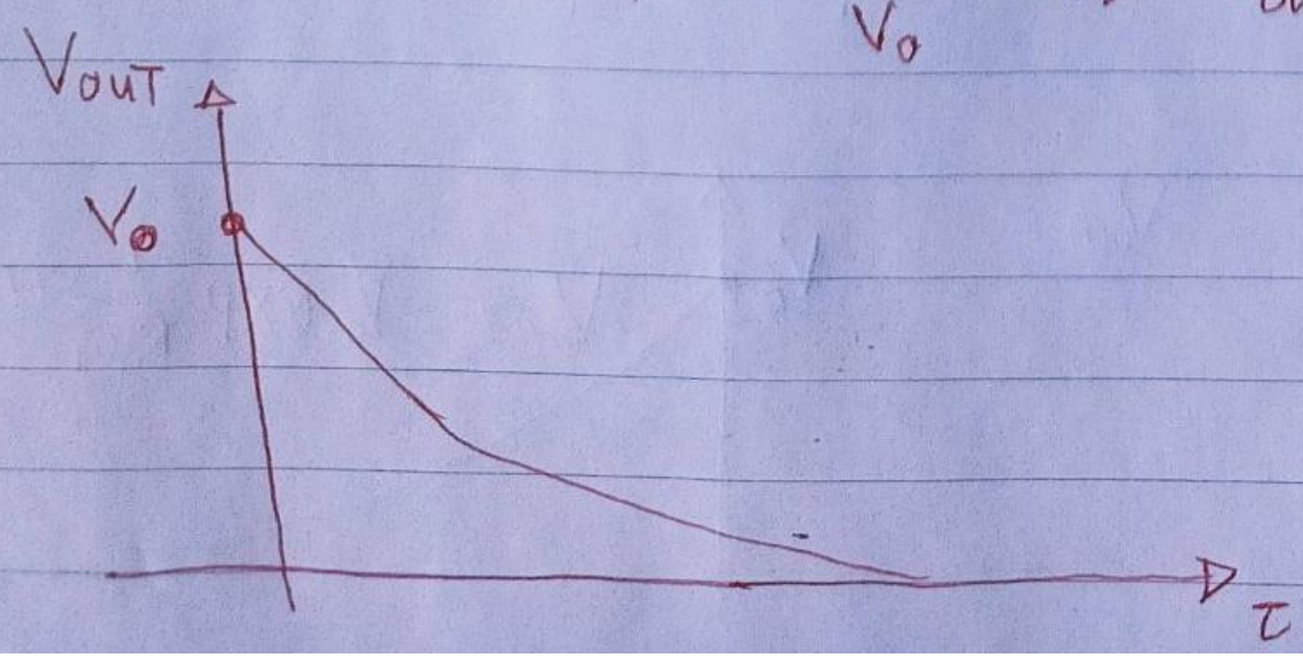
NOTE: "first order differential equation require one initial condition"

separation of variables for eq (1)

$$d\tau = -R_1 C_1 \frac{dV_{out}}{V_{out}} \Rightarrow \int_0^{\tau} d\tau = \int_{V_0}^{V_{out}} -R_1 C_1 \frac{dV_{out}}{V_{out}}$$

$$\Rightarrow \tau = -R_1 C_1 \ln V_{out} \Big|_{V_0}^{V_{out}} = -R_1 C_1 \left[\ln V_{out} - \ln V_0\right]$$

$$= -R_1 C_1 \ln \frac{V_{out}}{V_0} \Rightarrow V_{out} = V_0 \exp \frac{-\tau}{R_1 C_1}$$



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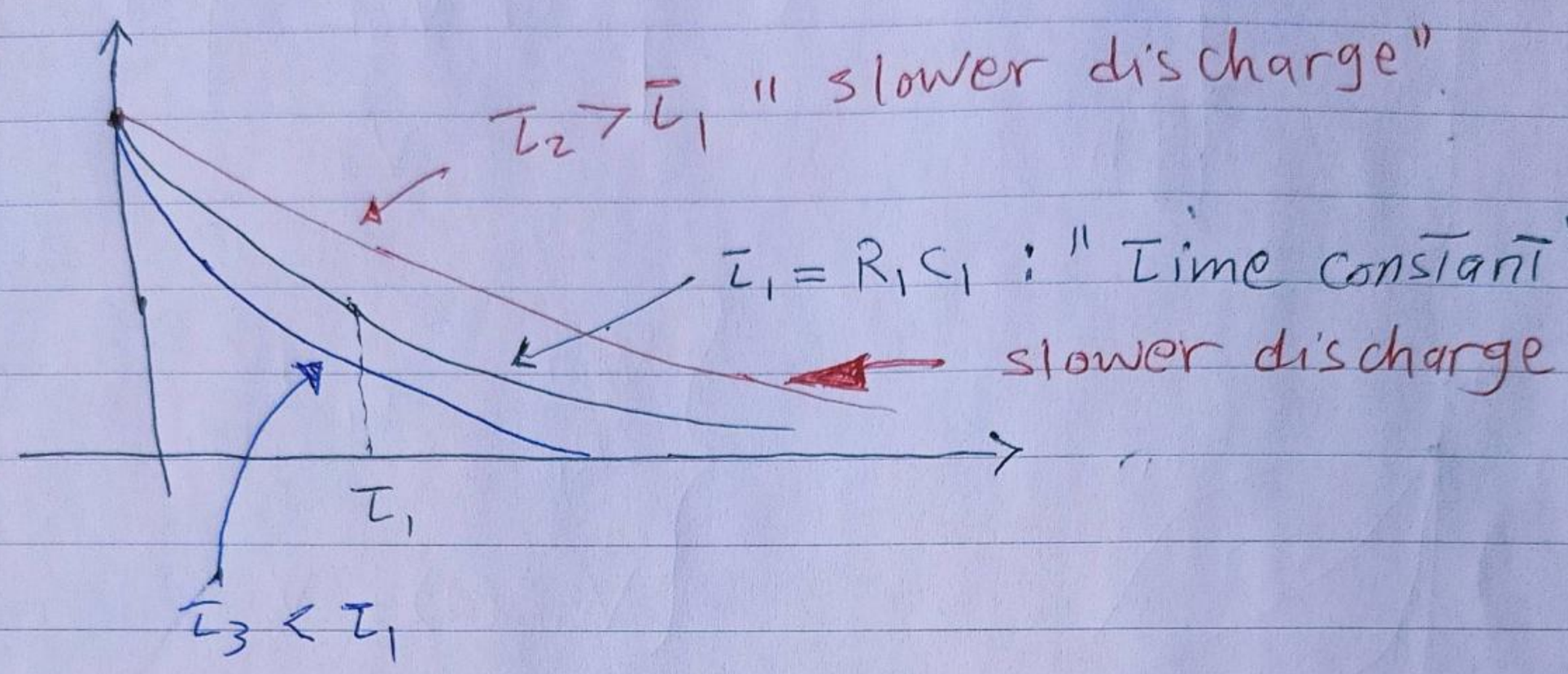
- properties of exponential Decay

① The decay rate depends on R_1C_1



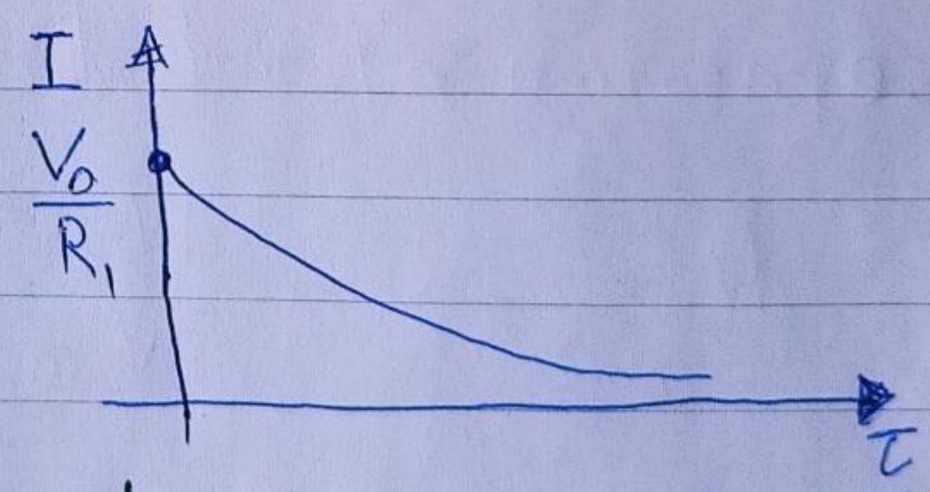
② $V_{out}(\tau = \tau) = V_0 e^{-1} = 0.37\, V_0$

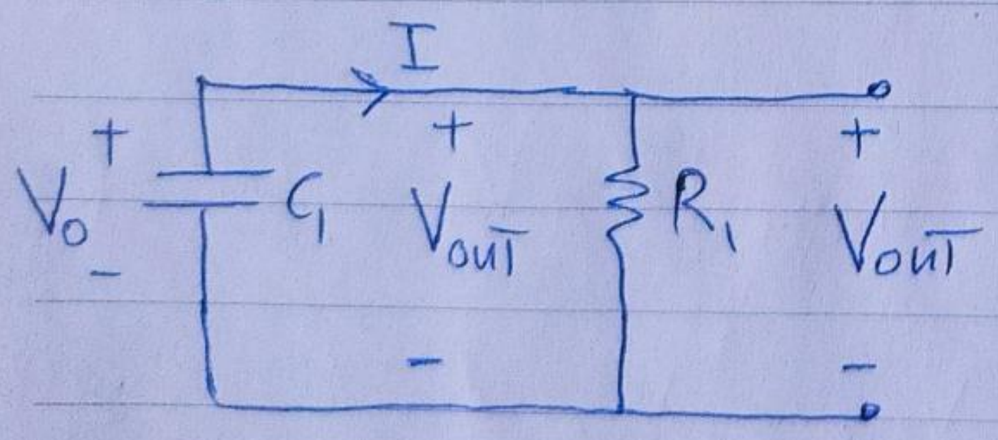
$V_{out}(\tau = 2\tau) = V_0 e^{-2} = 0.14\, V_0$

$V_{out}(\tau = 3\tau) = V_0 e^{-3} = 0.05\, V_0$

Current:

$$I = \frac{V_{out}}{R_1} = \frac{V_0}{R_1} \exp \frac{-\tau}{R_1C_1}$$

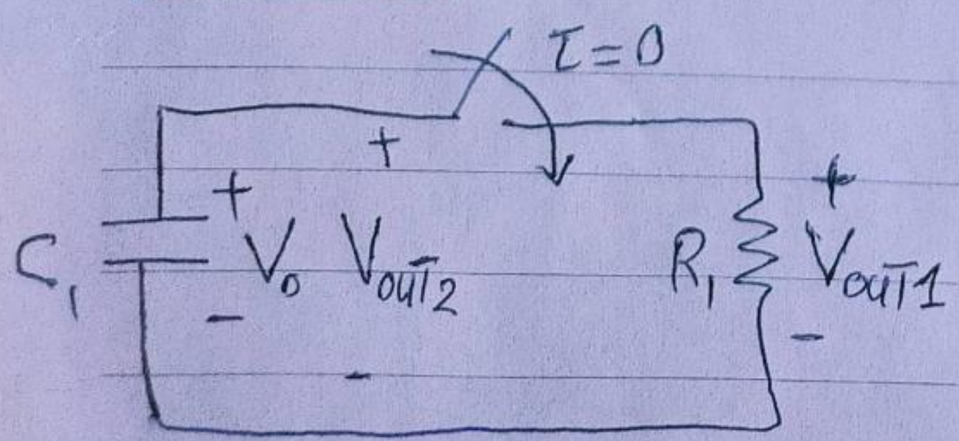


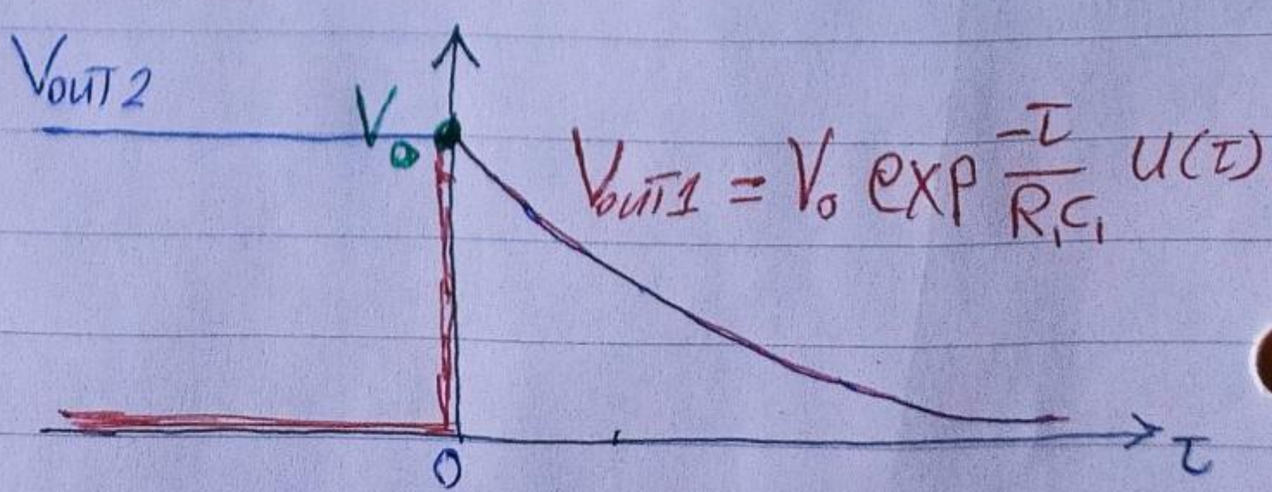


$$I = -C_1 \frac{dV_{out}}{d\tau} = -C_1 \left(\frac{-1}{R_1C_1}\right) V_0\, e^{-\tau/R_1C_1}$$

$$= \frac{V_0}{R_1} \exp \frac{-\tau}{R_1C_1}$$

Example





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Lec 16: RC Circuit Examples

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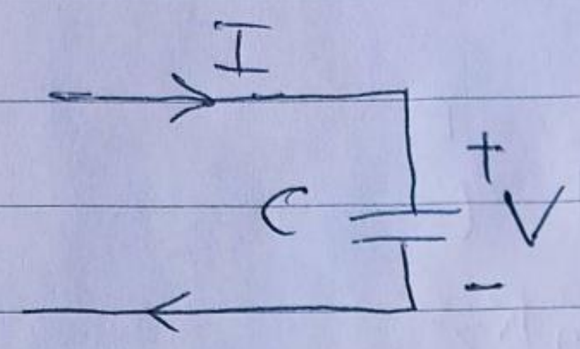
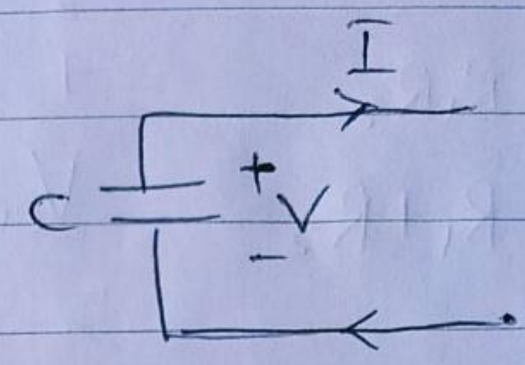
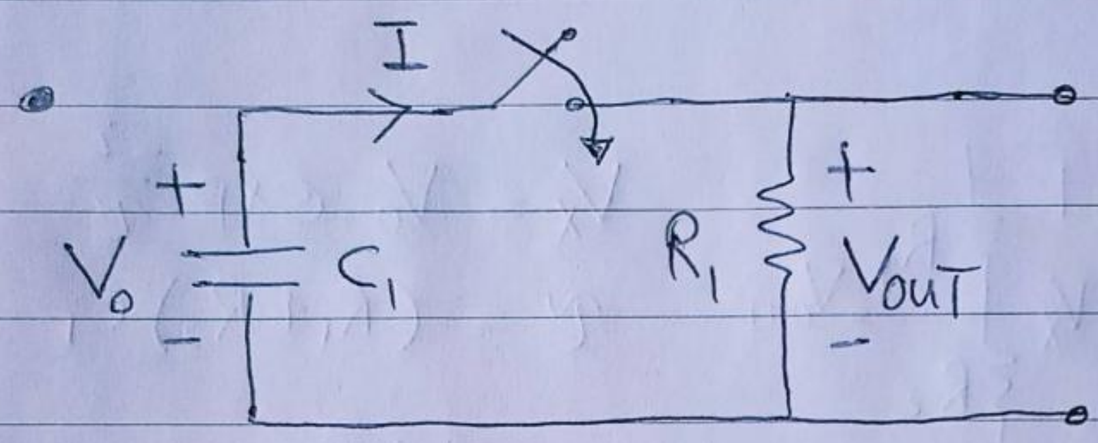
Review of lecture 15





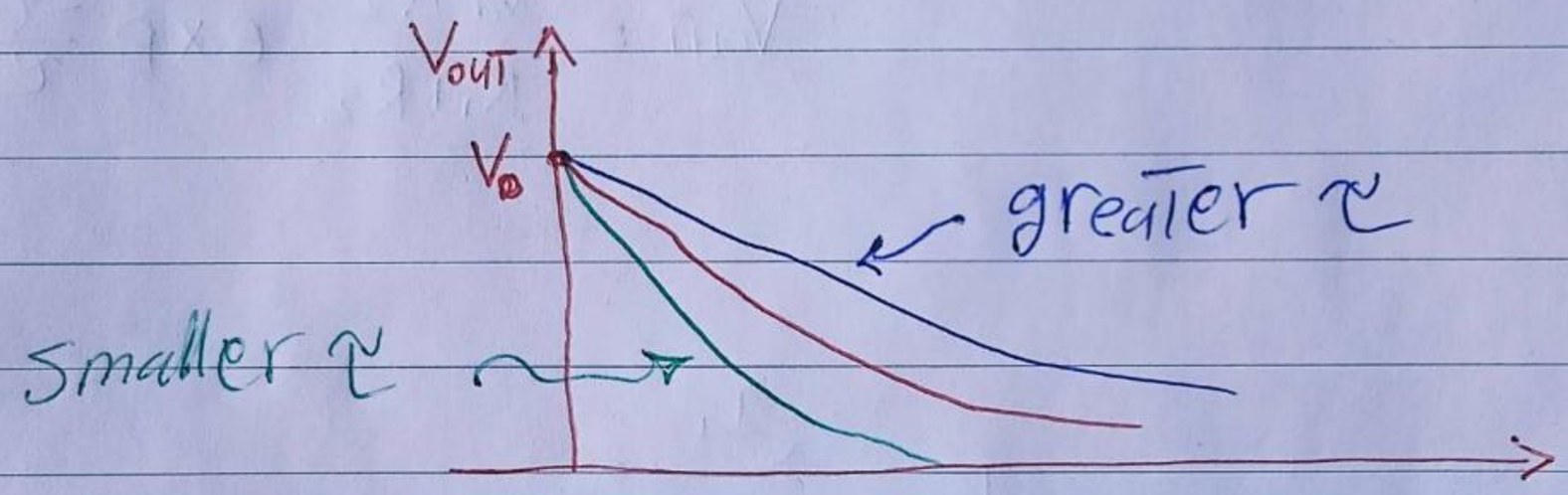
$$I = C\frac{dV}{d\tau}$$

$$I = -C\frac{dV}{d\tau}$$



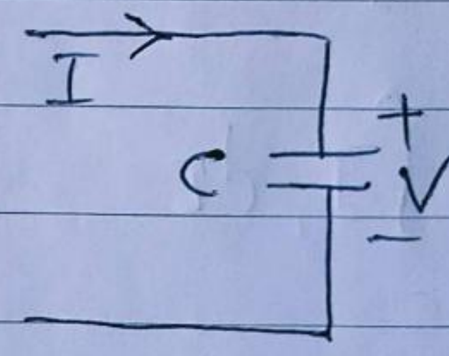
$$V_{OUT} = V_0 \exp\frac{-\tau}{R_1C_1}\,u(\tau)$$

$$I = \frac{V_0}{R_1}\exp\frac{-\tau}{R_1C_1}\,u(\tau)$$



Today's lecture

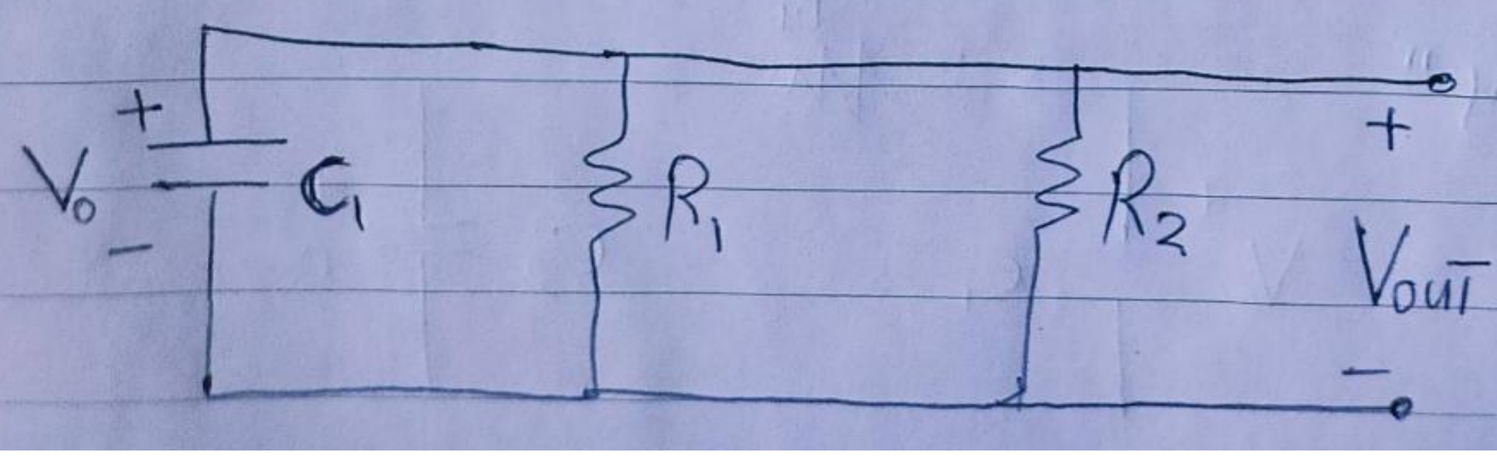
- Examples of RC circuits
- Notes on Caps
- The Two-capacitor problem



$$I = C\frac{dV}{d\tau}$$

$$Q = CV$$

Example



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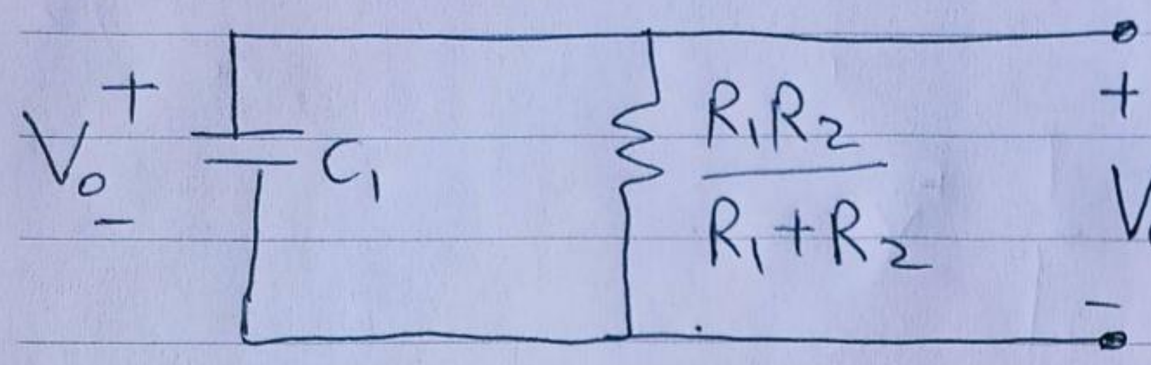
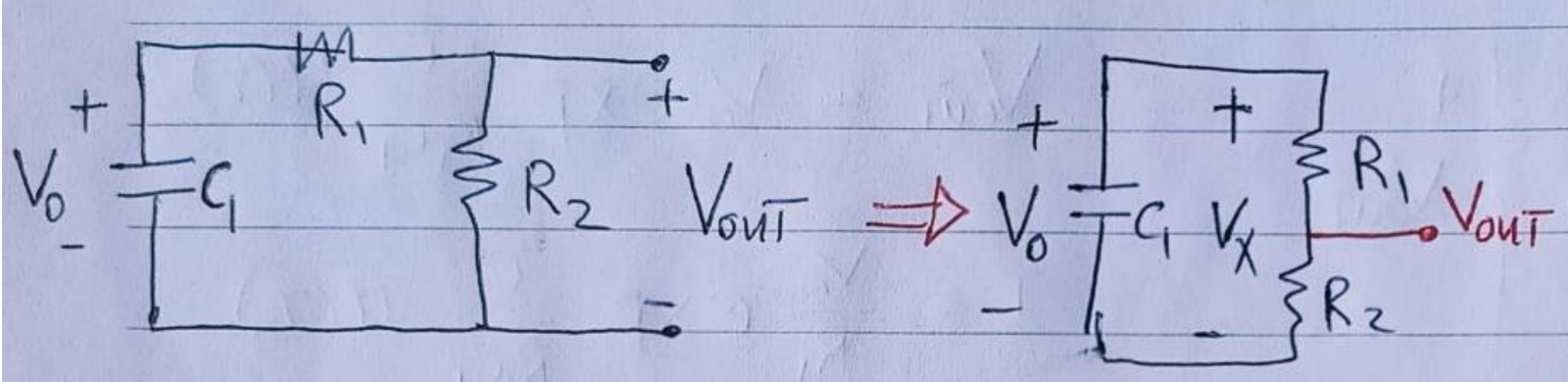
solution:



$$V_{OUT} = V_0 \exp \frac{-t}{\tau}$$

$$\therefore \tau = \frac{R_1 R_2}{R_1+R_2} C_1$$

Example

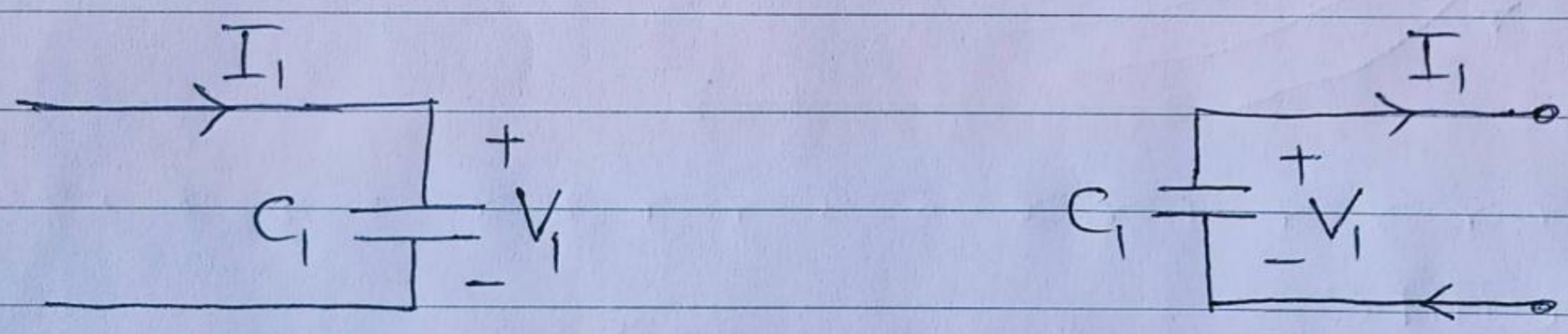


$$V_X = V_0 \exp \frac{-t}{\tau}$$

$$\tau = (R_1+R_2) C_1$$

$$V_{OUT} = \frac{V_0 R_2}{R_1+R_2} \exp \frac{-t}{\tau}$$

• Notes on Caps

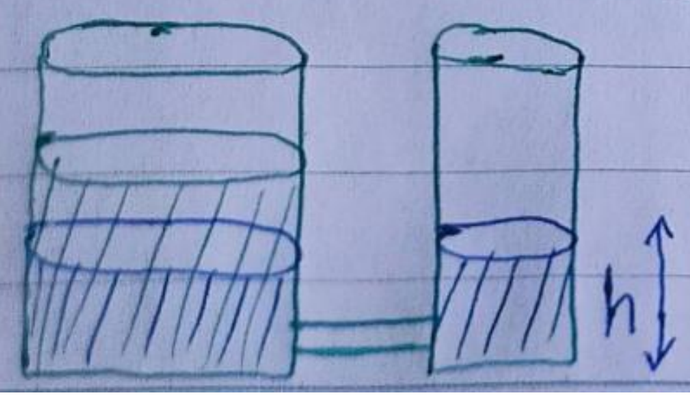


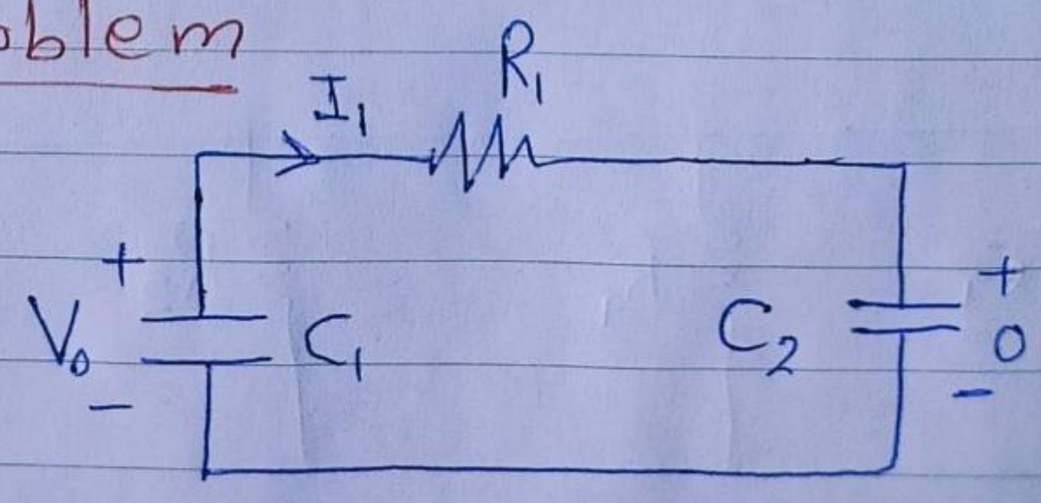
$$V_1 = \frac{1}{C_1} \int I_1 \, d\tau \qquad V_1 = -\frac{1}{C_1} \int I_1 \, d\tau$$

After integration, we must include a constant and, based on the initial condition(s), find its value.

• Two - Capacitor problem

"Water Analogy"





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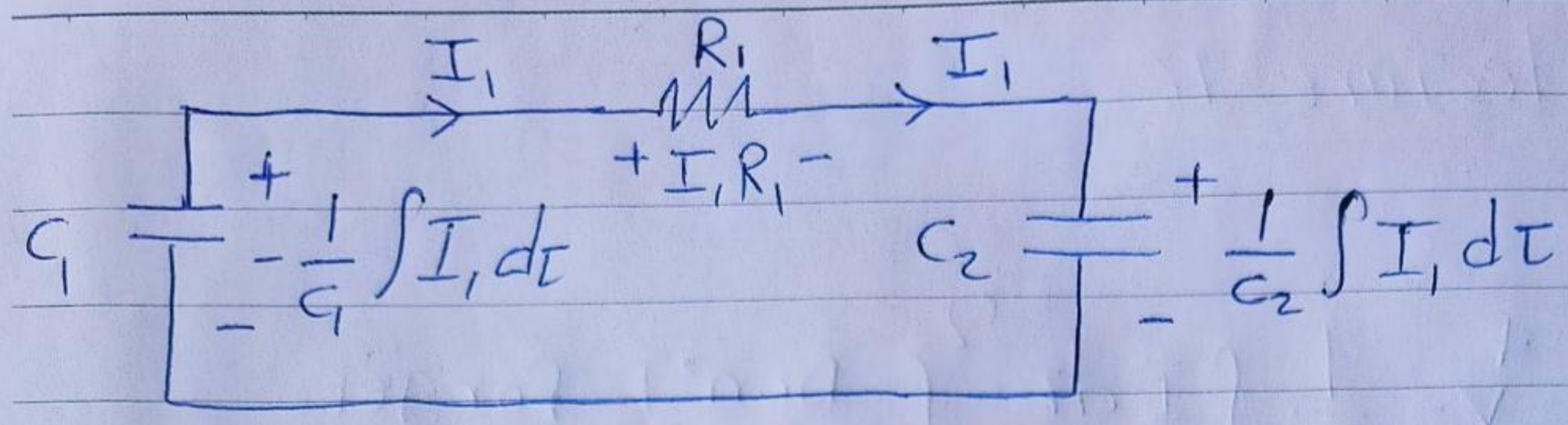
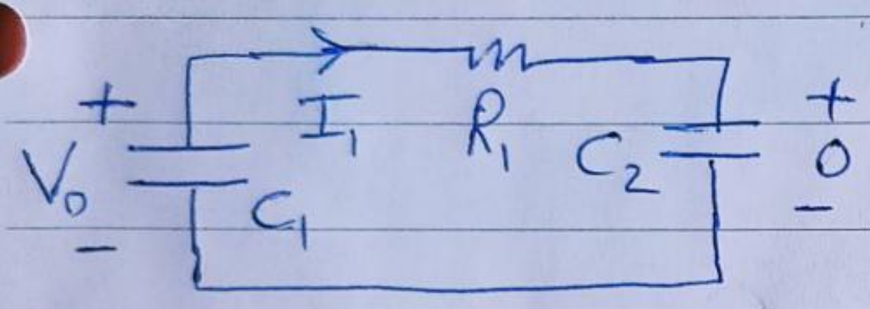


KVL: $-\frac{1}{C_1}\int I_1 d\tau = I_1 R_1 + \frac{1}{C_2}\int I_1 d\tau$

$$\Rightarrow -\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\int I_1 d\tau = R_1 I_1 \Rightarrow \begin{cases} -\left(\frac{1}{C_1}+\frac{1}{C_2}\right) I_1 = R_1 \frac{dI_1}{d\tau} \\ I_1(\tau=0) = V_0/R_1 \end{cases}$$



$$-d\tau = \frac{R_1}{\frac{C_1+C_2}{C_1C_2}} \cdot \frac{dI_1}{d\tau}$$

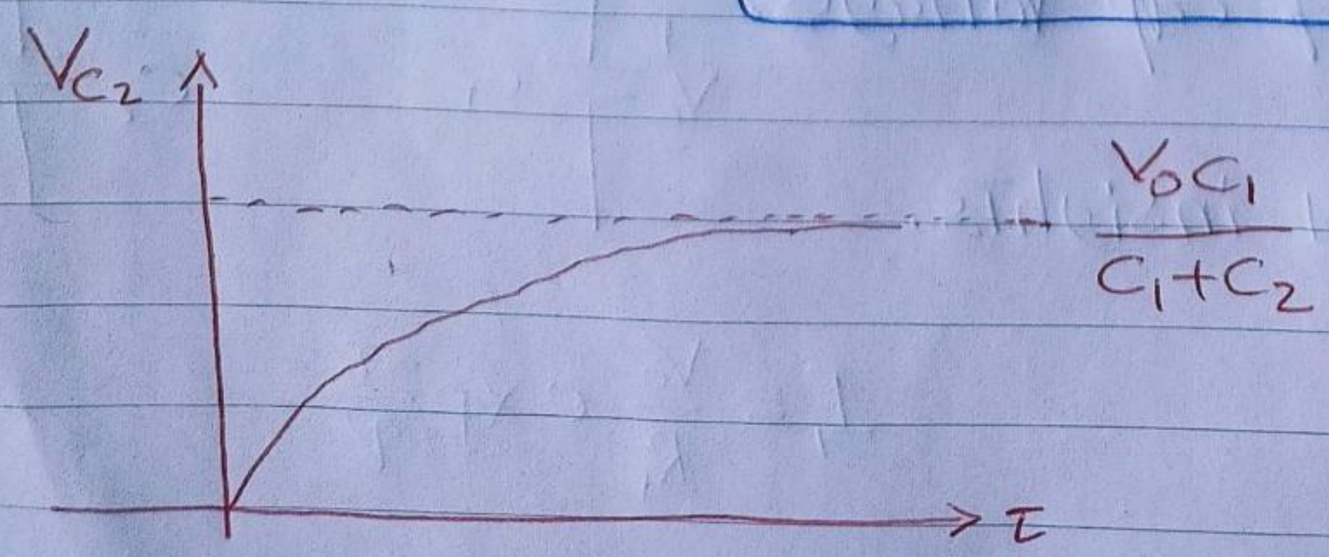
$$\Rightarrow -\int_0^{\tau} d\tau = \frac{R_1 C_1 C_2}{C_1+C_2} \ln I_1 \Big|_{V_0/R_1}^{I_1}, \quad \tau = \frac{R_1 C_1 C_2}{C_1+C_2}$$

$$\frac{-\tau}{\tau} = \ln \frac{I_1}{V_0/R_1} \Rightarrow \boxed{I_1(\tau) = \frac{V_0}{R_1}\exp\frac{-\tau}{\tau}}$$

$$V_{C_2} = \frac{1}{C_2}\int I_1 d\tau = \frac{1}{C_2}\frac{V_0}{R_1}(-\tau)\exp\frac{-\tau}{\tau} + K$$

$$= \frac{-V_0 C_1}{C_1+C_2}\exp\frac{-\tau}{\tau} + K, \quad V_{C_2}(\tau=0) = 0 = \frac{-V_0 C_1}{C_1+C_2} + K$$

$$\Rightarrow K = \frac{V_0 C_1}{C_1+C_2} \Rightarrow \boxed{V_{C_2}(\tau) = \frac{V_0 C_1}{C_1+C_2}\left(1-\exp\frac{-\tau}{\tau}\right)}$$



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Lec 17: More RC Circuit Examples

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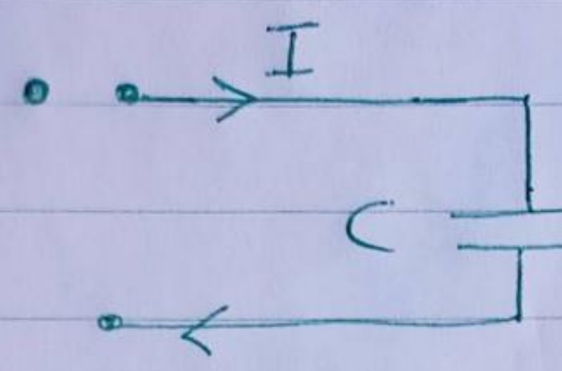
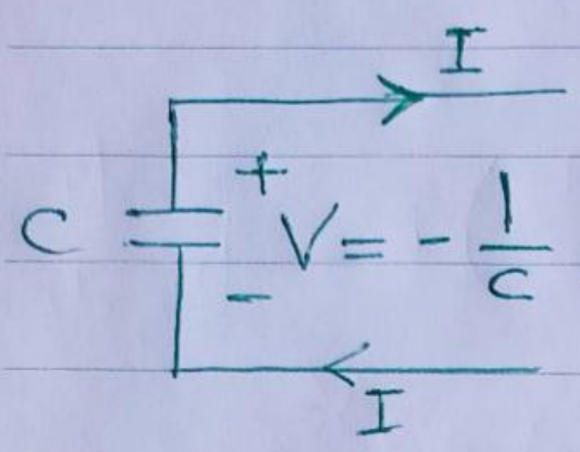
Review of lecture 16

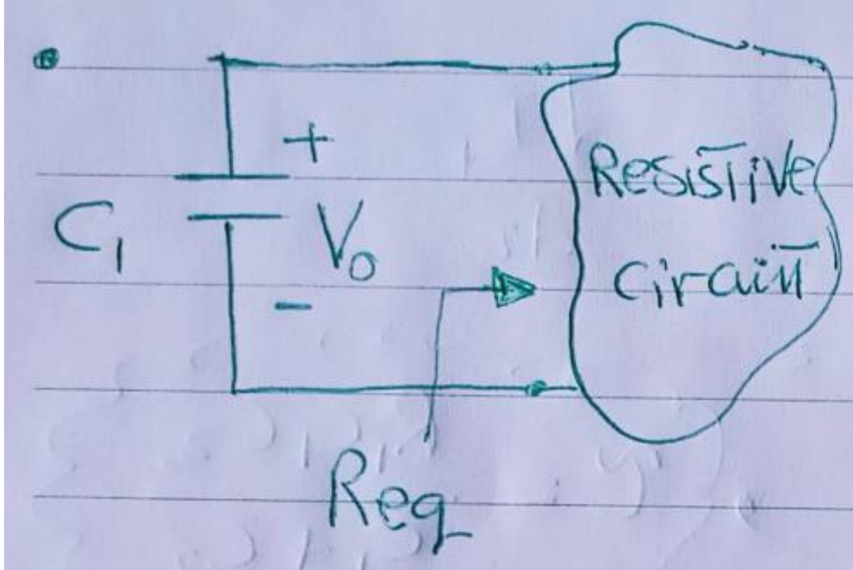


$$V = \frac{1}{C}\int I\,d\tau$$

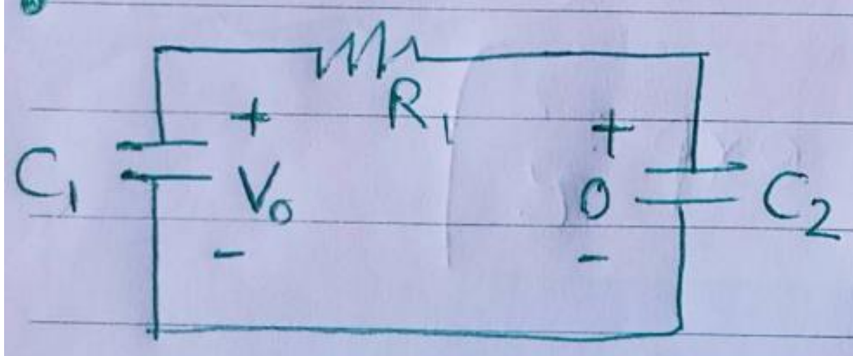
Don'T forget The integration const.



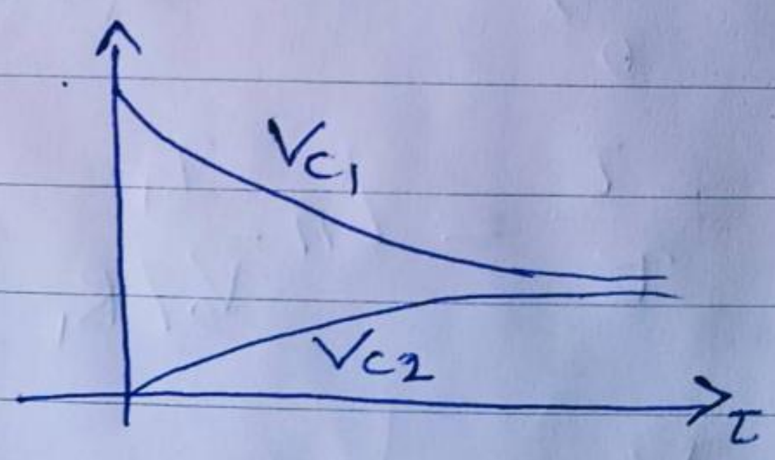
$$V = -\frac{1}{C}\int I\,d\tau$$



$$\tau = R_{eq} C_1$$

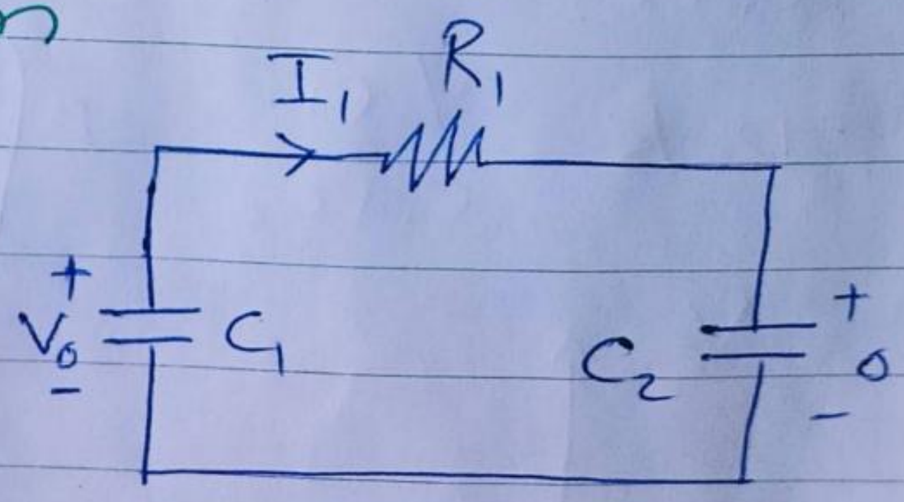


$$\tau = \frac{R_1 C_1 C_2}{C_1 + C_2}$$



Today's lecture

- Two - Capacitor problem
  - Behavior of cap Voltages.
  - other interesting insights.



$$I_1 = \frac{V_0}{R_1}\exp\frac{-t}{\tau}$$

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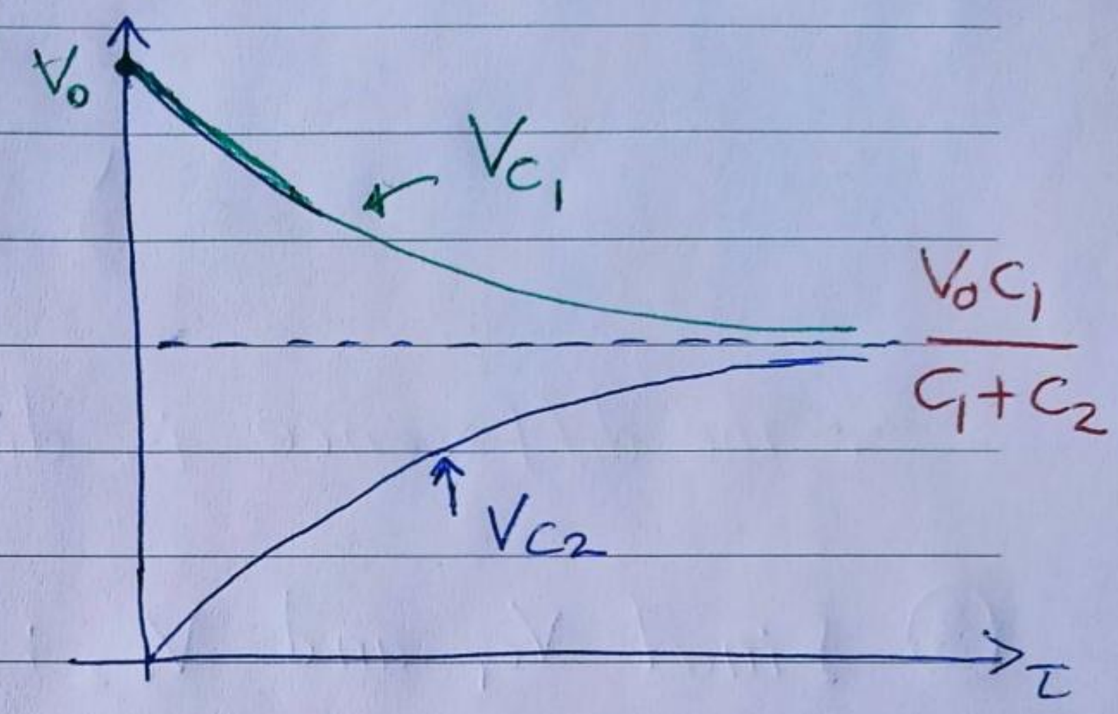
- Note on τ $\quad \tau = R_1 \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C_1$ and C_2 are in series.

- $V_{C_2} = \frac{1}{C_2} \int I_1 \, d\tau = \frac{1}{C_2} \frac{V_0}{R_1} (-\tau) \exp \frac{-\tau}{\tau} + K$

$= \frac{-V_0 C_1}{C_1 + C_2} \exp \frac{-\tau}{\tau} + K \quad , \quad V_{C_2}(\tau = 0) = 0 = \frac{-V_0 C_1}{C_1 + C_2} + K$

$$\boxed{\therefore V_{C_2} = \frac{V_0 C_1}{C_1 + C_2} \left(1 - \exp \frac{-\tau}{\tau}\right)}$$



- $V_{C_1} = -\frac{1}{C_1} \int I_1 \, d\tau$

$= -\frac{1}{C_1} \frac{V_0}{R_1} (-\tau) \exp \frac{-\tau}{\tau} + K$

$= +\frac{V_0 C_2}{C_1 + C_2} \exp \frac{-\tau}{\tau} + K \quad , \quad V_{C_1}(\tau = 0) = V_0 = \frac{V_0 C_2}{C_1 + C_2} + K$

$\Rightarrow V_0 \left(1 - \frac{C_2}{C_1 + C_2}\right) = K = V_0 \frac{C_1}{C_1 + C_2}$

$$\boxed{\therefore V_{C_1} = V_0 \frac{C_2}{C_1 + C_2} \exp \frac{-\tau}{\tau} + V_0 \frac{C_1}{C_1 + C_2}}$$

<u>observations</u>

① is charge conserved ?

at $\tau = 0$, $Q = C_1 V_0$

at $\tau = \infty$, $Q = C_1 \frac{V_0 C_1}{C_1 + C_2} + C_2 \frac{V_0 C_1}{C_1 + C_2}$

$= \frac{V_0 C_1}{C_1 + C_2} (C_1 + C_2) = V_0 C_1$

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② is energy conserved ?

at $t=0$, $\mathcal{E} = \frac{1}{2} C_1 V_0^2$

at $t=\infty$, $\mathcal{E} = \frac{1}{2} C_1 \left(\frac{V_0 C_1}{C_1+C_2}\right)^2 + \frac{1}{2} C_2 \left(\frac{V_0 C_1}{C_1+C_2}\right)^2$

$$= \frac{1}{2}\left(\frac{V_0 C_1}{C_1+C_2}\right)^2 (C_1+C_2)$$

$$= \frac{1}{2}\frac{V_0^2 C_1^2}{C_1+C_2} = \frac{1}{2} C_1 V_0^2 \frac{C_1}{C_1+C_2}$$

$\therefore$ The difference is dissipated in R_1 (The energy loss)

③ Final V_{C_1} and V_{C_2} : (conservation of charge)

at $t=0$, $Q = C_1 V_0$

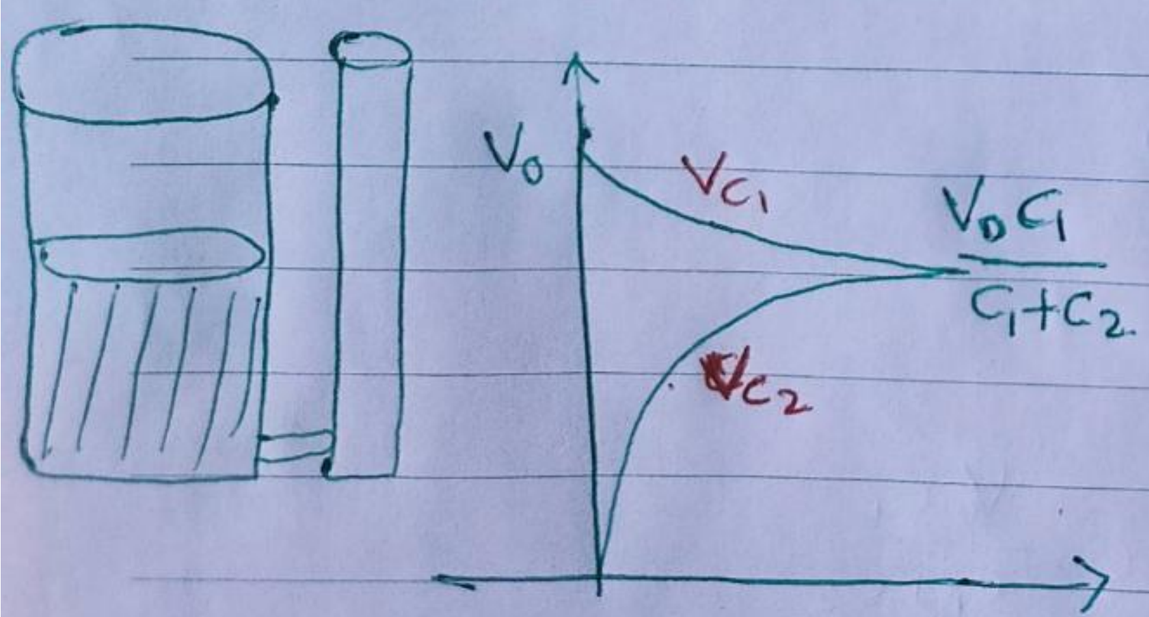
at $t=\infty$, $Q = C_1 V_\infty + C_2 V_\infty$, C_1 & C_2 have same voltage at ∞

$Q(t=0) = Q(t=\infty)$

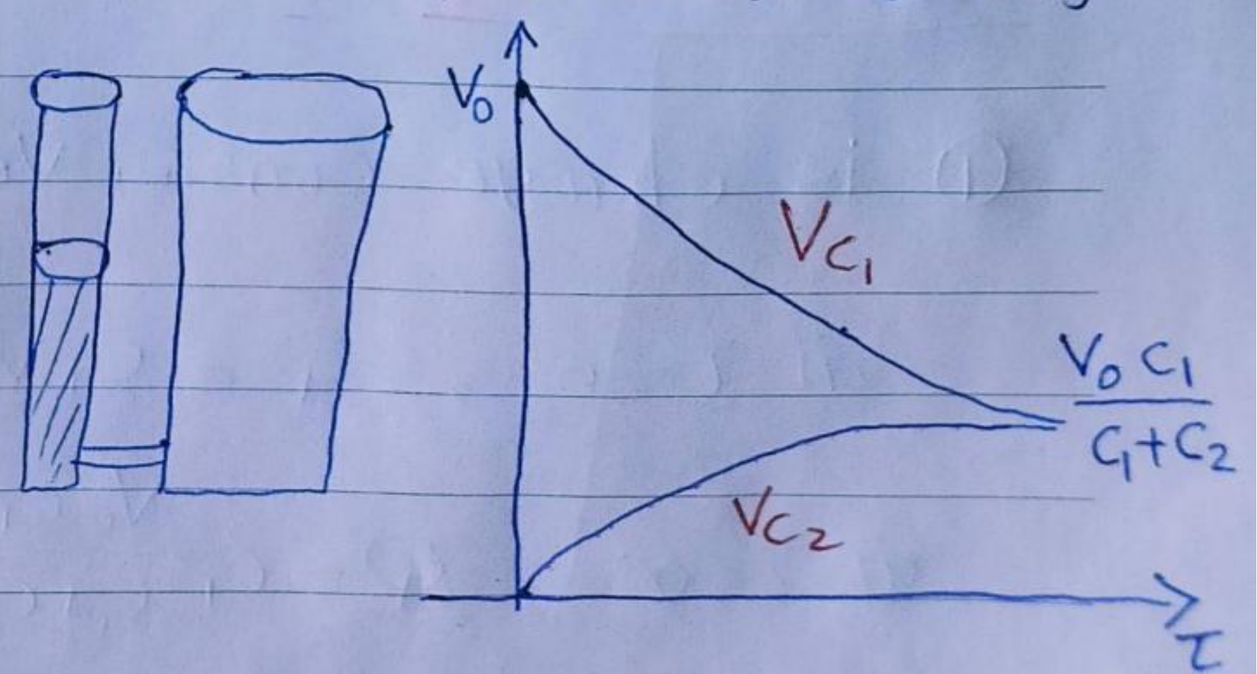
$C_1 V_0 = C_1 V_\infty + C_2 V_\infty \Rightarrow \boxed{V_\infty = V_0 \frac{C_1}{C_1+C_2}}$

④ Effect of Capacitor Values:

- C_1 : large , C_2 : small



- C_1 : small , C_2 : large



This is called "charge sharing"

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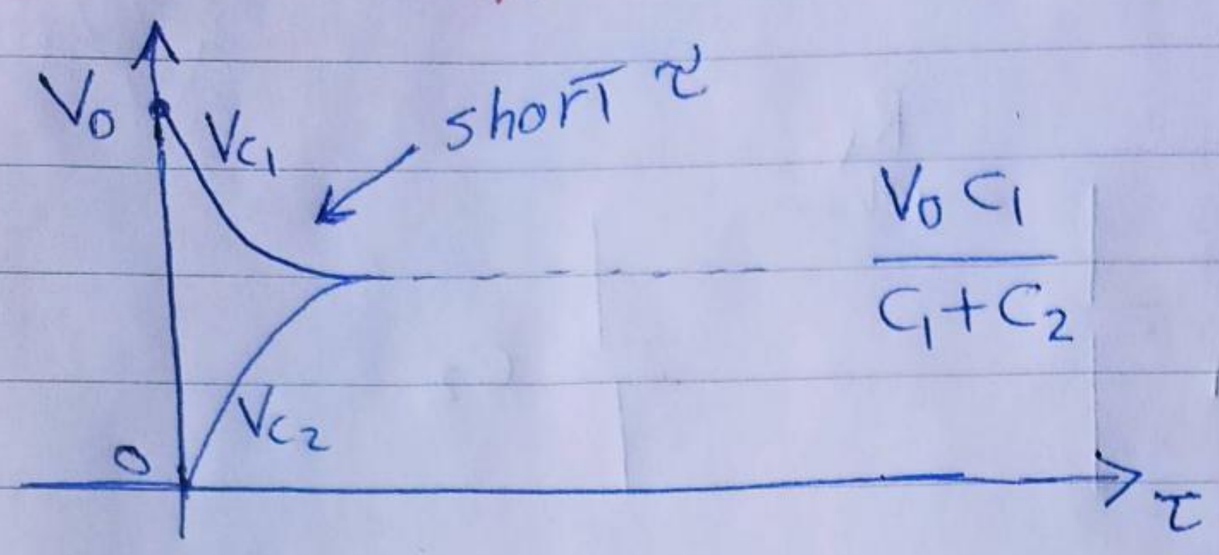
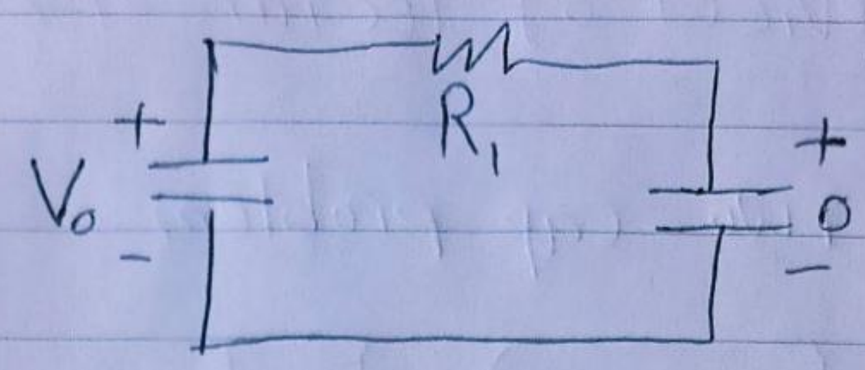
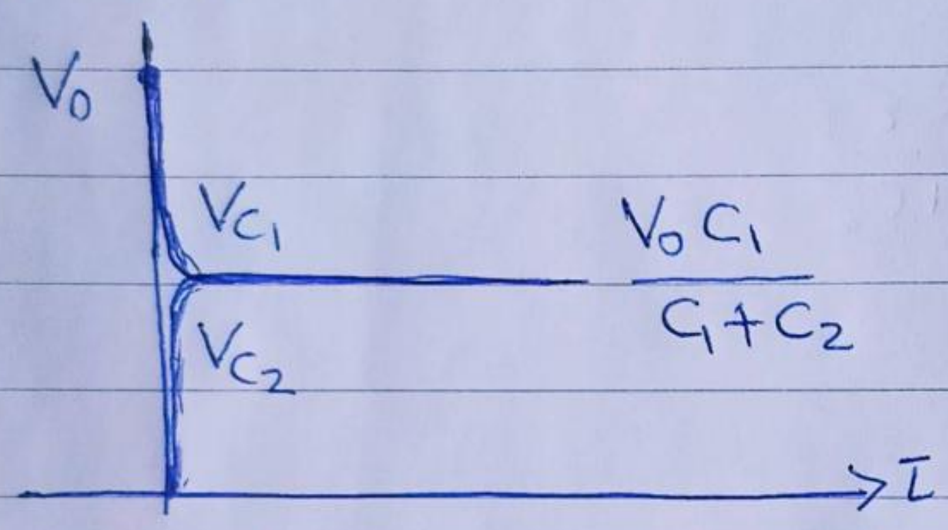
⑤ What happens as $R_1 \downarrow$?



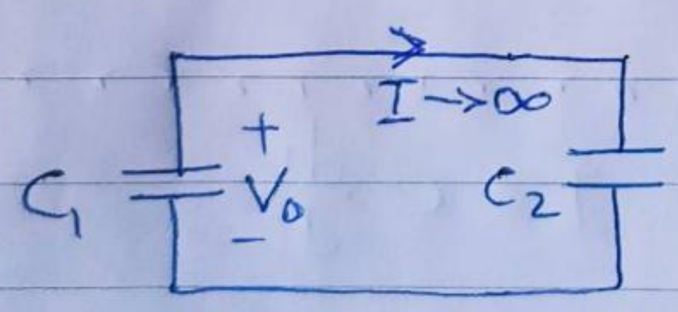


$R_1 \longrightarrow 0$

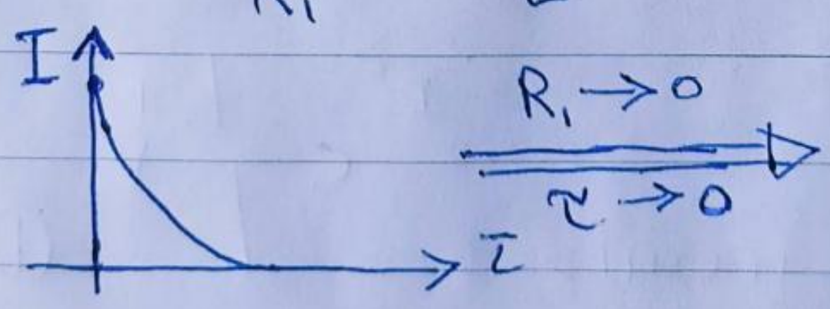


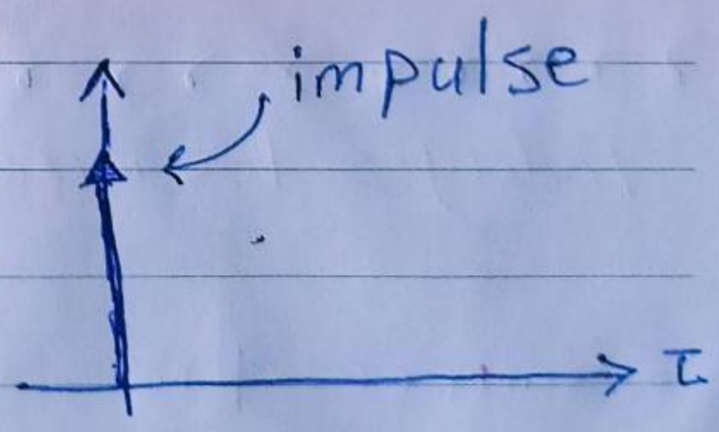
$\therefore$ initial current : $\frac{V_0}{R_1} \longrightarrow \infty$ as $R_1 \longrightarrow 0$

$\Rightarrow$ instantaneous change in V_{C_1} and V_{C_2}.

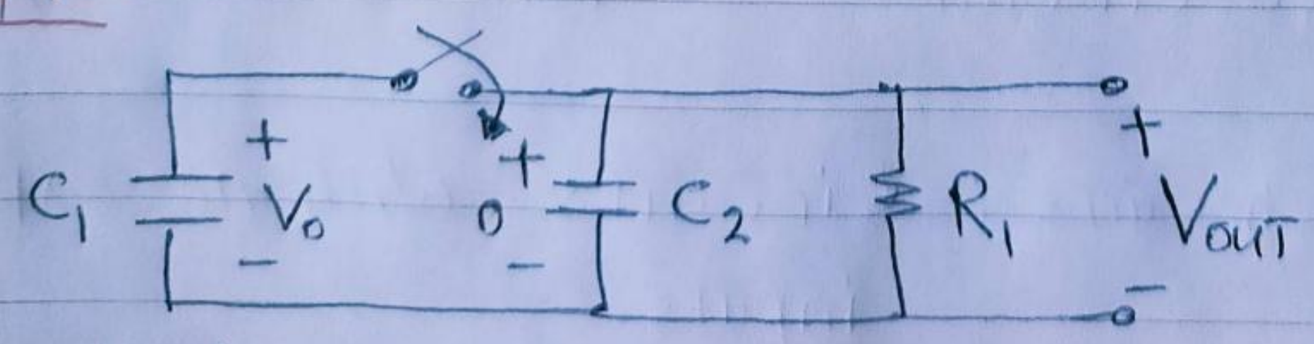


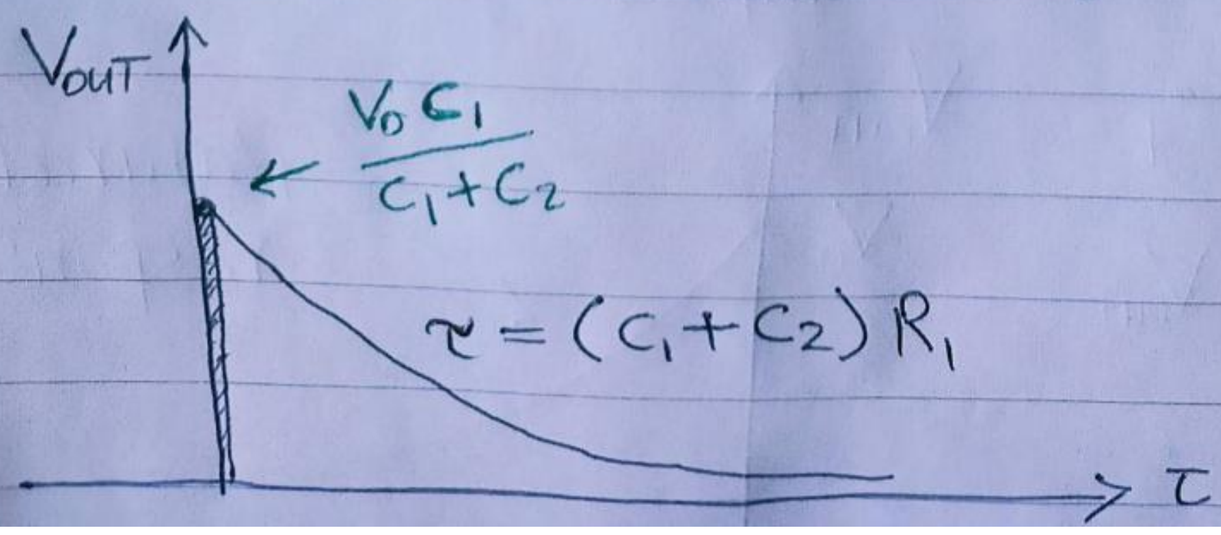
$$I = \frac{V_0}{R_1} \exp \frac{-\tau}{\tau}$$





Example





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Lec 18: Source-Free and Driven Circuits; Sampling by Impulses

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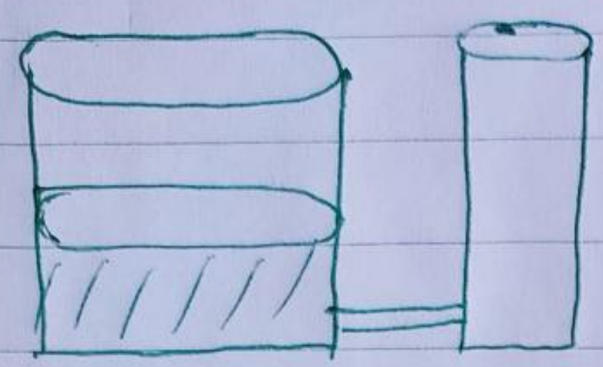
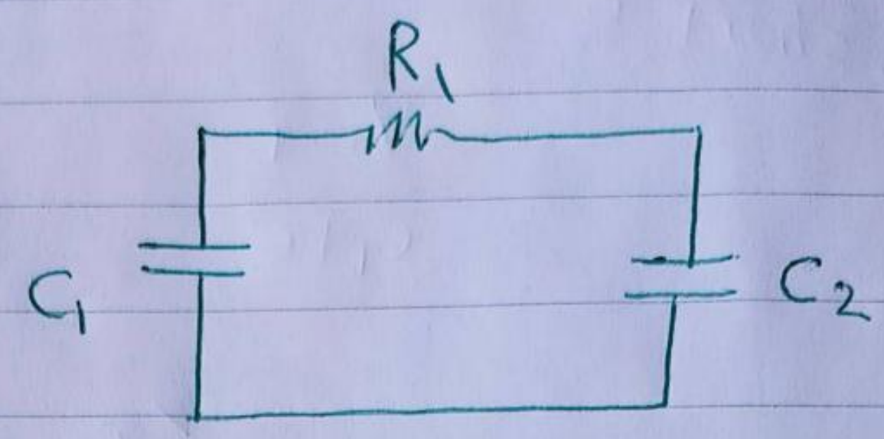
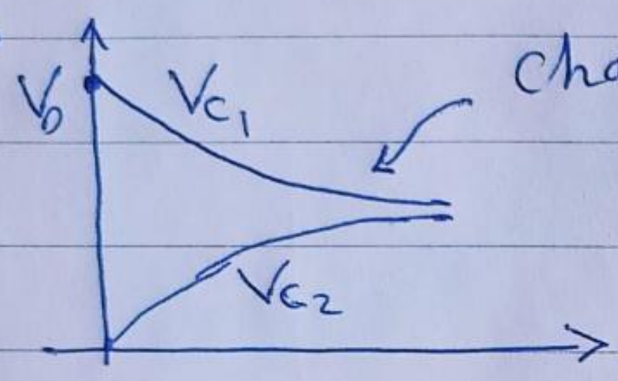
Review of lecture 17

- Two-cap problem

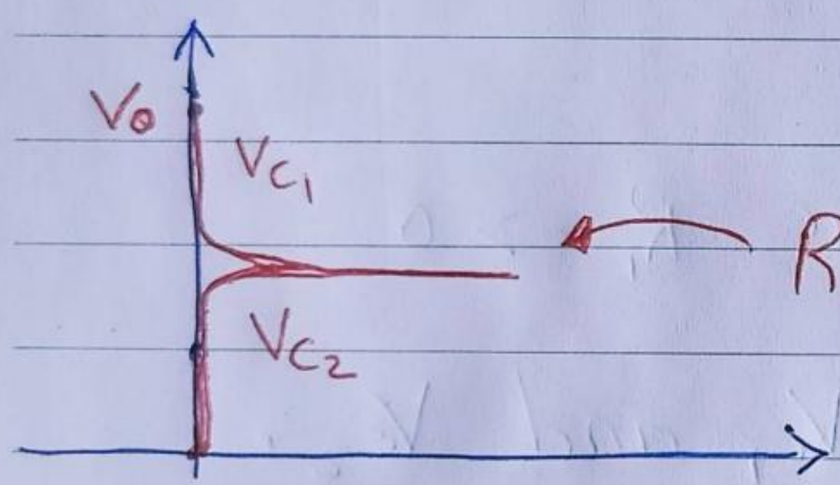




- 

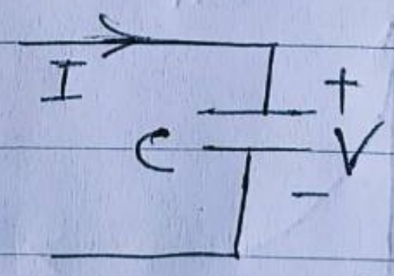
charge sharing

$$\tau = R_1 \frac{C_1 C_2}{C_1 + C_2}$$



R_1 small

Today's lecture



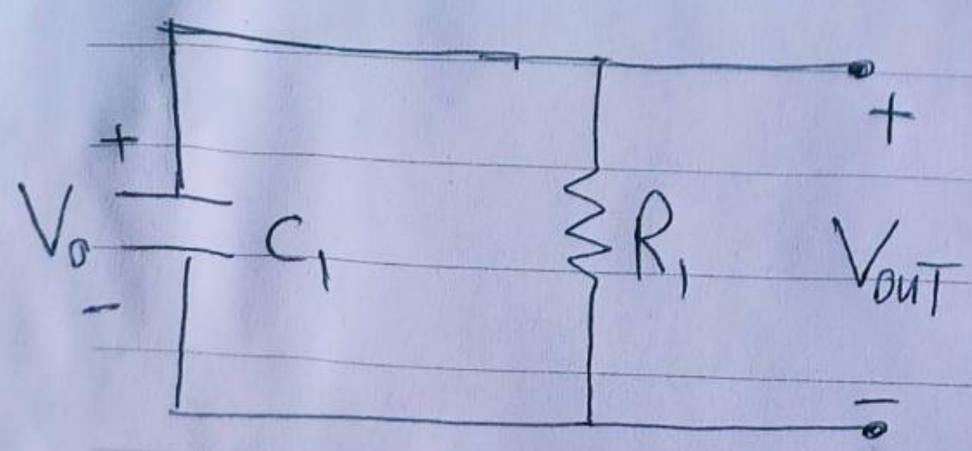
$$I = C \frac{dV}{d\tau}$$

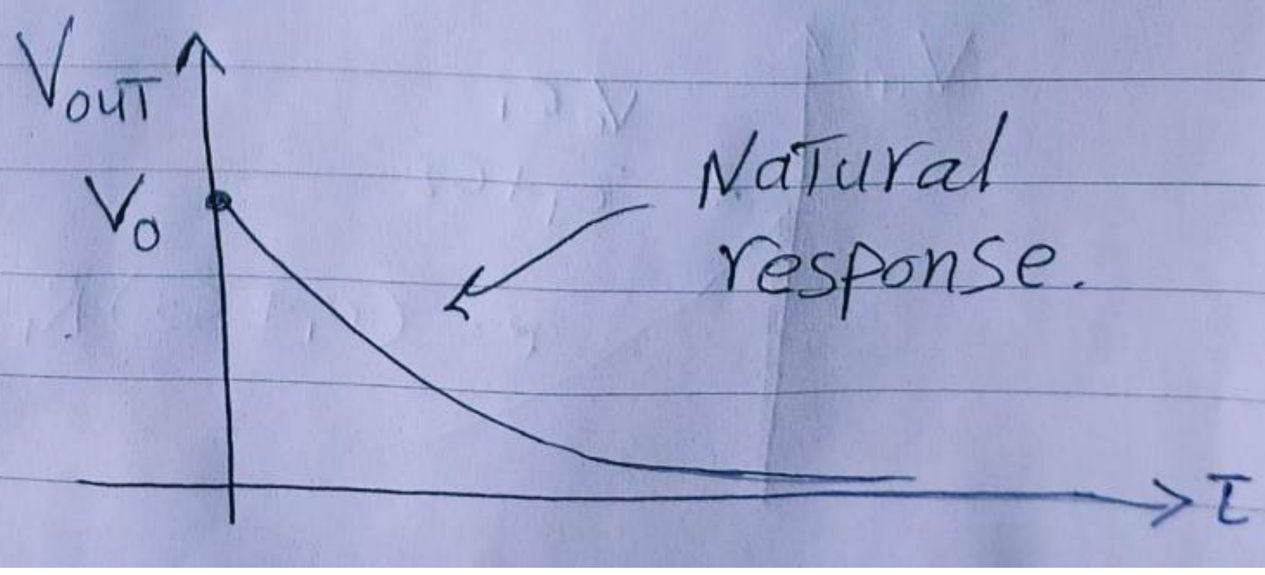
- Source-free circuits
- Driven circuits
- Simple driven RC circuit

① V can't jump if $I < \infty$

② if $dV/d\tau = 0 \Rightarrow$ cap looks like an open circuit

- Source-free circuits: circuits with no explicit inputs.





Natural response.

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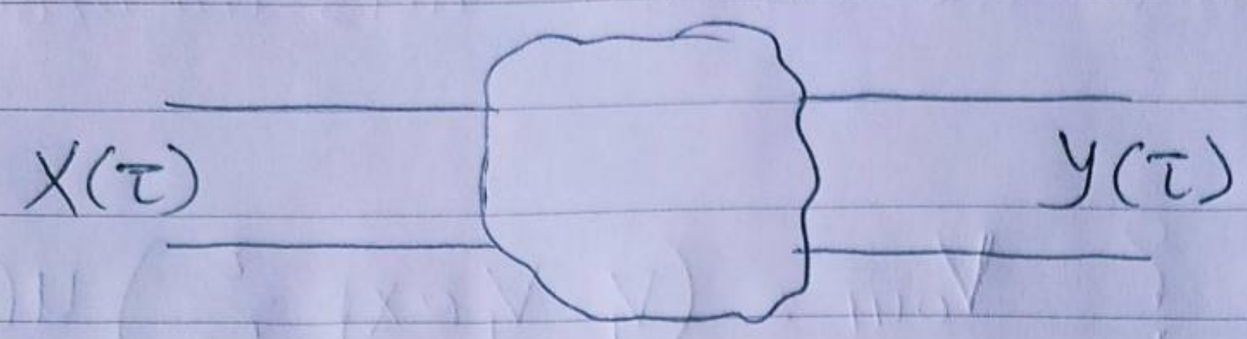
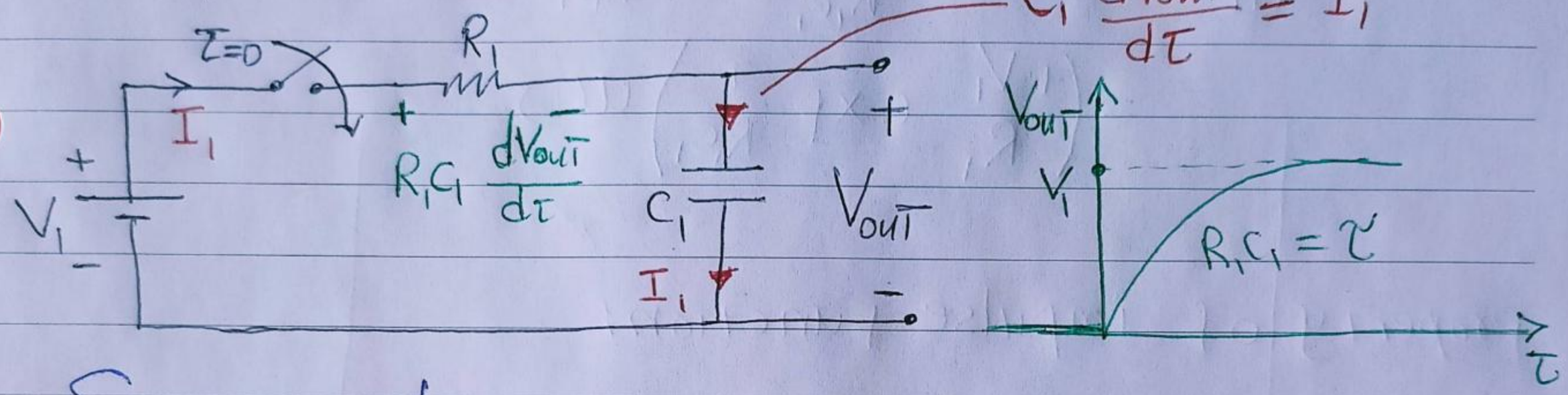
- Driven circuits : circuit with explicit inputs.



- Simple driven RC circuit



$$\begin{cases} V_1 = R_1C_1 \dfrac{dV_{out}}{d\tau} + V_{out} \\ V_{out}(0^+) = ? = 0 \end{cases}$$

- "Cap voltage cannot jump because available current < ∞"

$$R_1C_1 \frac{dV_{out}}{d\tau} = V_1 - V_{out} = -(V_{out} - V_1)$$

$$\Rightarrow \int_0^{V_{out}} R_1C_1 \frac{dV_{out}}{V_{out} - V_1} = -\int_0^{\tau} d\tau \Rightarrow R_1C_1 \ln(V_{out} - V_1)\Big|_0^{V_{out}} = -\tau$$

$$\Rightarrow R_1C_1 \ln \frac{V_{out} - V_1}{-V_1} = -\tau \Rightarrow \frac{V_{out} - V_1}{-V_1} = \exp\frac{-\tau}{R_1C_1}$$

$$V_{out} = \Big(\underbrace{V_1}_{\text{Forced response}} - \underbrace{V_1 \exp\frac{-\tau}{R_1C_1}}_{\text{Natural response}}\Big)u(\tau) = V_1\Big(1 - \exp\frac{-\tau}{R_1C_1}\Big)u(\tau)$$

Forced response : response that is left at $\tau = \infty$.

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- "order" of a circuit = order of diff. equation.
  ⇒ simple RC circuit has an order of 1.

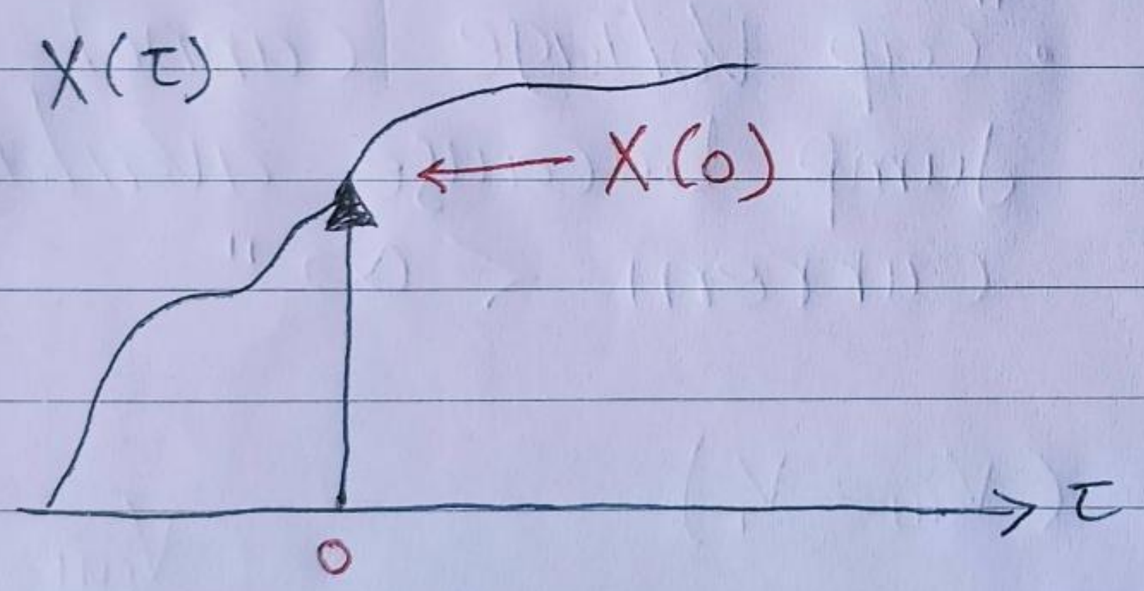
Example

Plot the current? $V_{out} = \left(V_1 - V_1 \exp\frac{-t}{R_1C_1}\right) u(t)$

$$I_1 = C_1 \frac{dV_{out}}{dt} = -\cancel{C_1} V_1 \frac{-1}{R_1\cancel{C_1}} \exp\frac{-t}{R_1C_1} u(t)$$

$$= \left(\frac{+V_1}{R_1} \exp\frac{-t}{R_1C_1}\right) u(t)$$

property of impulse function:



$$X(t)\,\delta(t) = X(0)\,\delta(t)$$

$$V_{out} = V_1\left(1 - \exp\frac{-t}{\tau}\right) u(t)$$

$$I_1(t) = \frac{V_1}{R_1} \exp\frac{-t}{\tau} u(t) + \underbrace{V_1\left(1 - \exp\frac{-t}{\tau}\right)}_{0} \delta(t)$$

impulse with zero Area

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Lec 19: Shortcut Method for First-Order Systems

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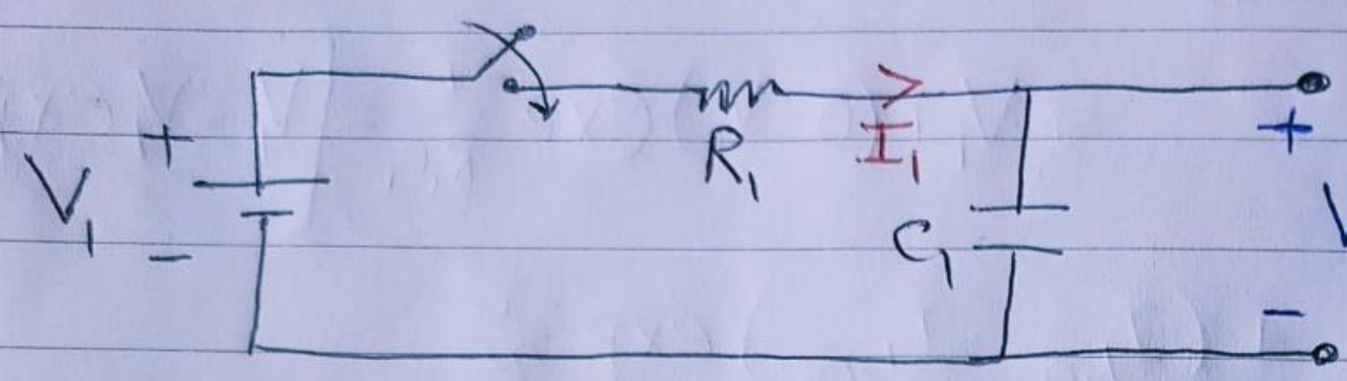
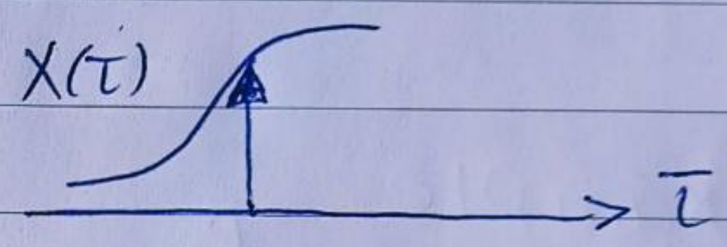
Review of lecture 18

- Source free & Driven circuits
- Natural & Forced Responses



$$V_{OUT} = \left(\underbrace{V_1}_{\text{Forced resp.}} - \underbrace{V_1 \exp\frac{-t}{\tau}}_{\text{Natural response}}\right) u(t)$$

- $I_1 = C_1 \dfrac{dV_{OUT}}{dt} = \dfrac{V_1}{R_1} \exp\dfrac{-t}{\tau}\, u(t)$

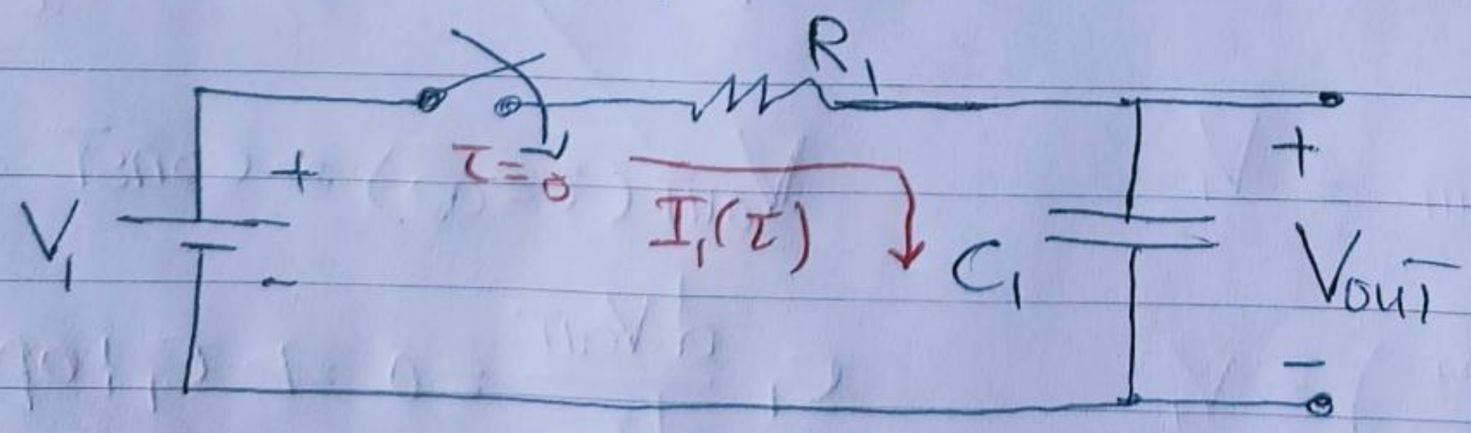


$$X(t)\,\delta(t) = X(0)\,\delta(t)$$

Today's lecture

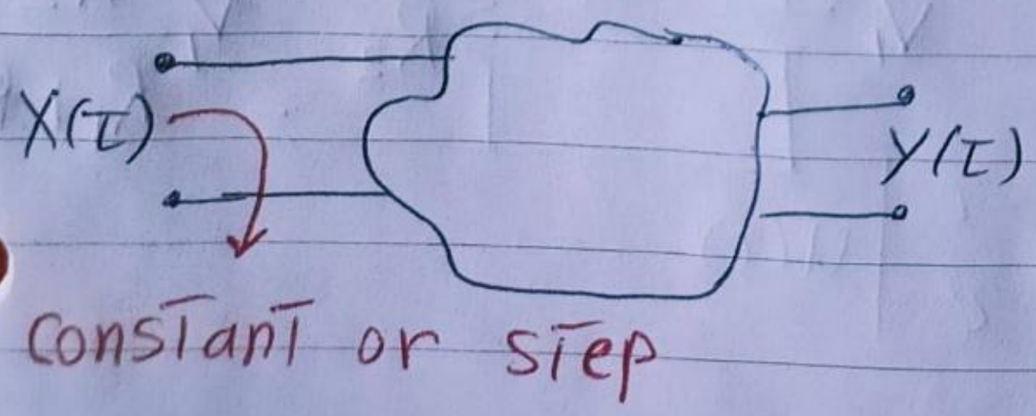
- Shortcut Method for solving first-order circuits
- Examples

- Shortcut Method for first-order circuits.



$$\begin{cases} R_1 C_1 \dfrac{dV_{OUT}}{dt} + V_{OUT} = V_1 \\ V_{OUT}(0^+) = 0 \end{cases}$$

- General first order system: $\alpha \dfrac{dy}{dt} + y = X(t)$



$$\because\ y(t) = \underbrace{y_\infty}_{y(t=\infty)} + \underbrace{(y_0 - y_\infty)}_{y(t=0)} \exp\frac{-t}{\underbrace{\tau}_{\tau=\alpha}}$$

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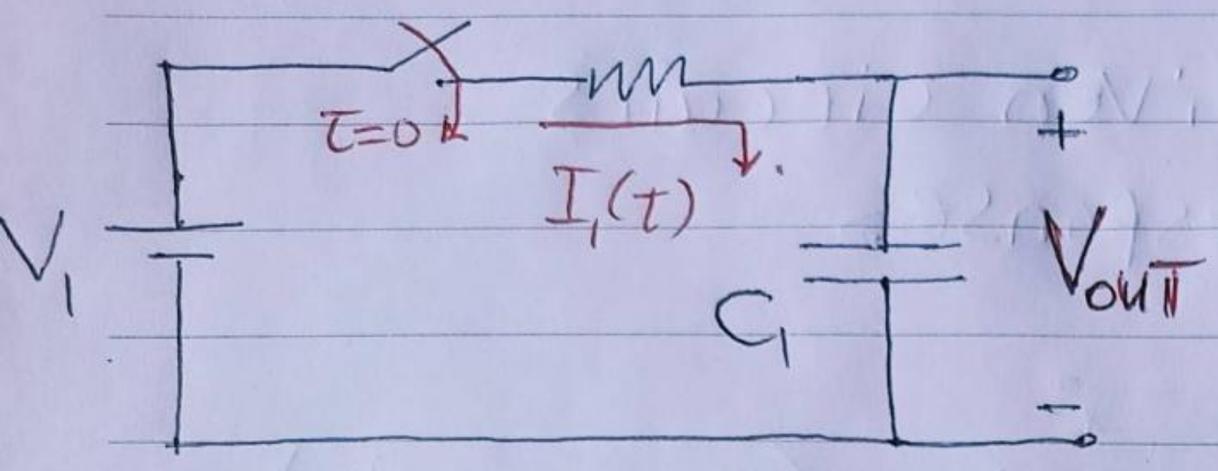
Example



Find $V_{OUT}(t)$:

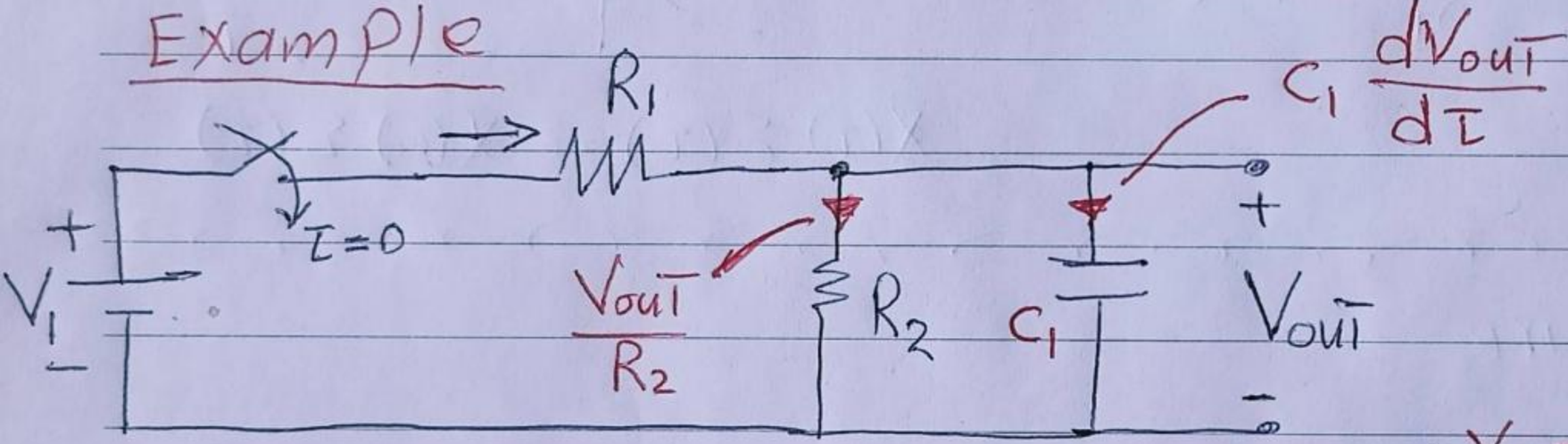
$$V_{OUT} = V_{OUT,\infty} + (V_{OUT,0} - V_{OUT,\infty}) \exp\frac{-t}{\tau}$$

$$= V_1 + (0 - V_1)\exp\frac{-t}{R_1C_1} = \left[V_1 - V_1 \exp\frac{-t}{R_1C_1}\right]u(t)$$

Find $I_1(t)$: $I_1(t) = I_{1,\infty} + (I_{1,0} - I_{1,\infty})\exp\frac{-t}{\tau}$

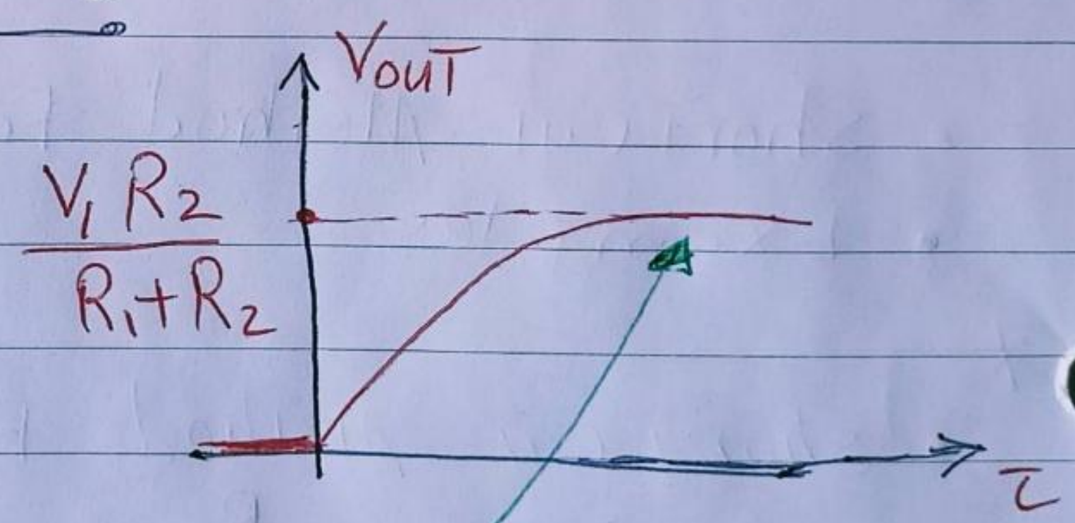
$$= 0 + \left(\frac{V_1}{R_1} - 0\right)\exp\frac{-t}{\tau} = \frac{V_1}{R_1}\exp\frac{-t}{\tau}\,u(t)$$

Example



$V_{OUT}(t=0^+) = 0$

$V_{OUT}(t=\infty) = V_1 \frac{R_2}{R_1+R_2}$



To find τ:

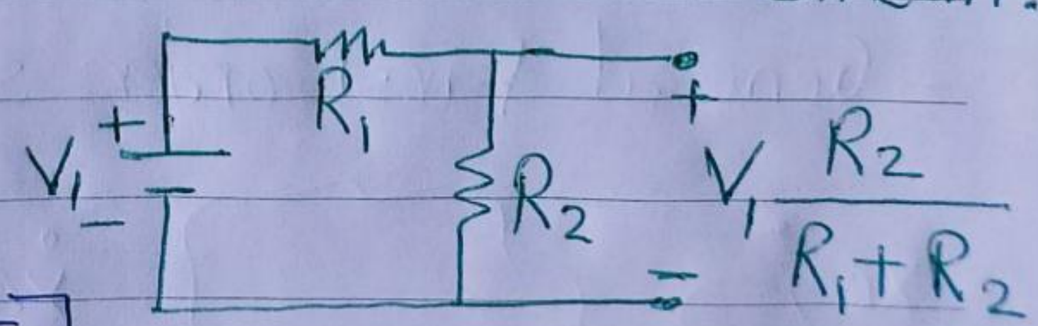
$$\left(\frac{V_{OUT}}{R_2} + C_1\frac{dV_{OUT}}{dt}\right)R_1 + V_{OUT} = V_1$$

$V_{OUT}(t=\infty) = \text{const}$

$$R_1C_1\frac{dV_{OUT}}{dt} + V_{OUT}\left(1 + \frac{R_1}{R_2}\right) = V_1$$

$C_1 \frac{dV_{OUT}}{dt} = 0 \Rightarrow C_1$: open circuit.

$$\tau = \frac{R_1C_1}{1 + \frac{R_1}{R_2}} = \frac{R_1R_2}{R_1+R_2}C_1$$



$$\therefore V_{OUT}(t) = \left[\frac{V_1R_2}{R_1+R_2} + \left(0 - \frac{V_1R_2}{R_1+R_2}\right)\exp\frac{-t}{\tau}\right]u(t)$$

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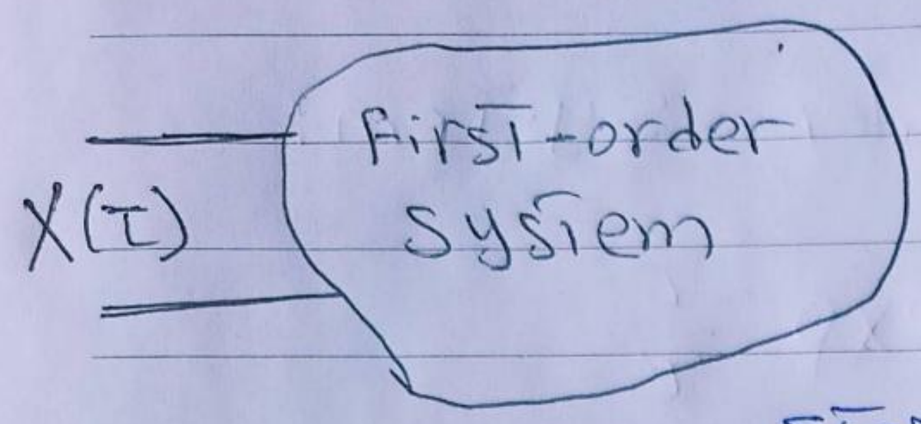
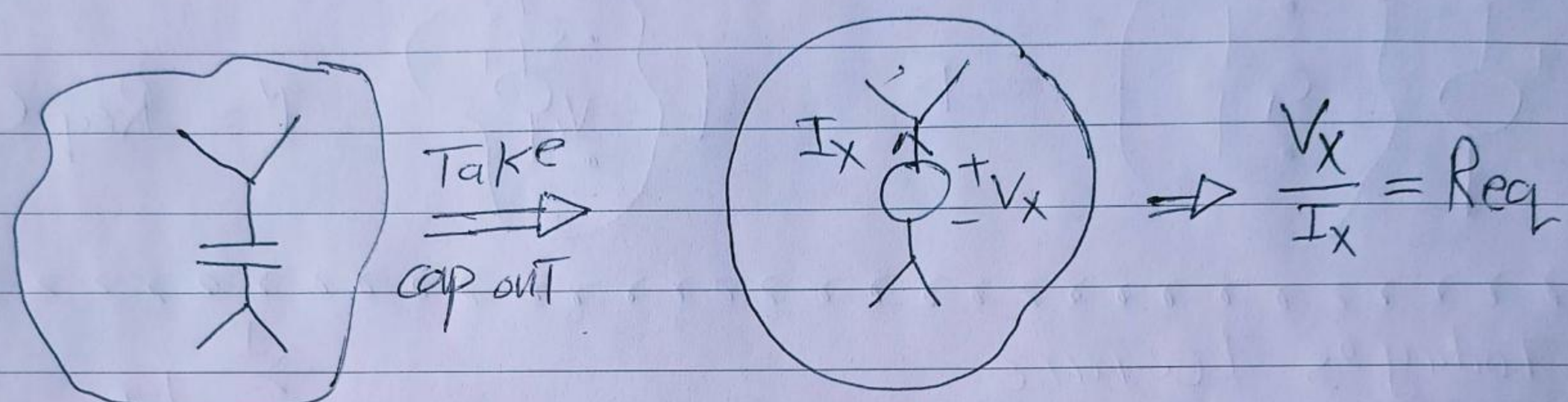
Finding The Time constant by inspection.



step 1: set all independent sources and intial conditions To zero.

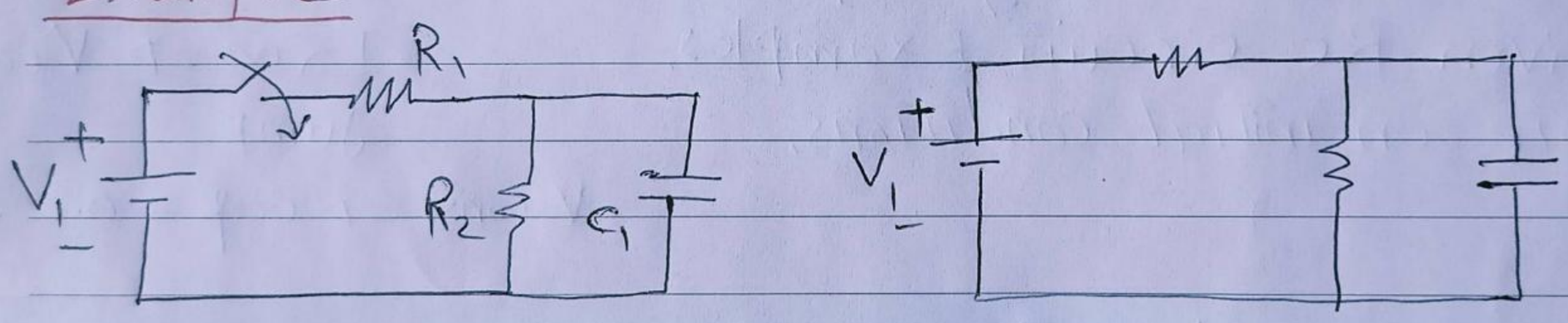
step 2: Find The resistance "seen" by The cap.



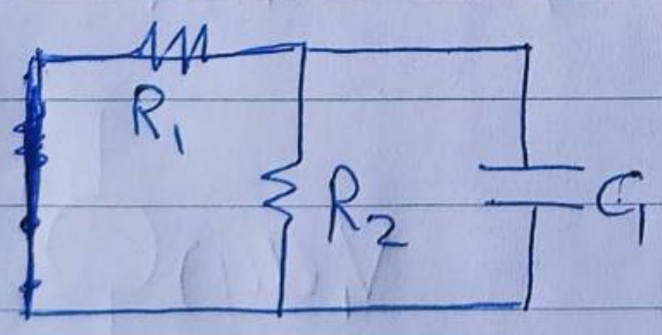
$$\Rightarrow \frac{V_x}{I_x} = R_{eq}$$

step3 : $\tau = C_1 R_{eq}$

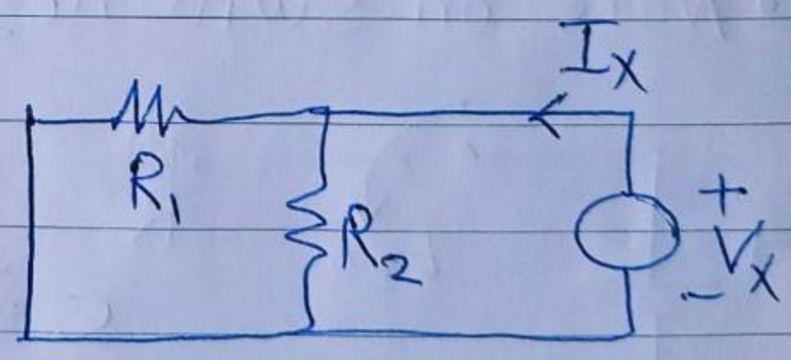
Example



Step 1:

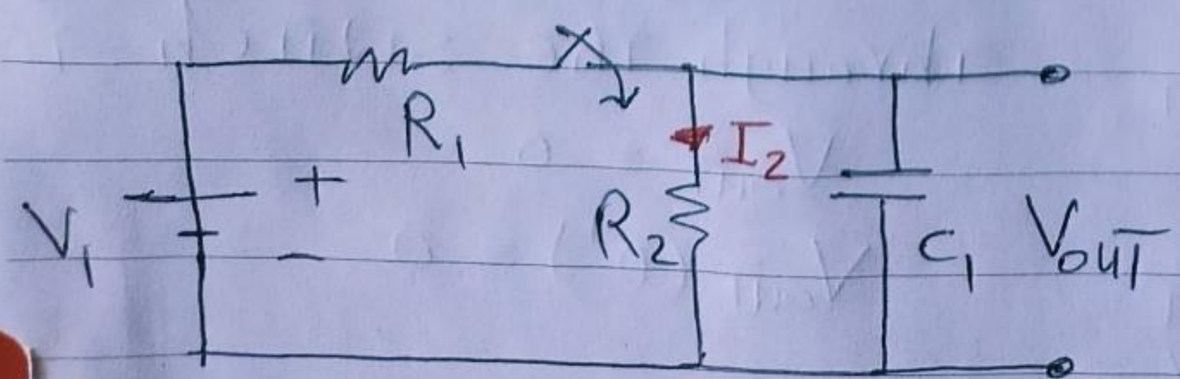


Step 2:



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \tau = C_1 \frac{R_1 R_2}{R_1 + R_2}$$

Example



V_{out} is The same as in previous Example.

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Lec 20: Driven RC Circuit Examples

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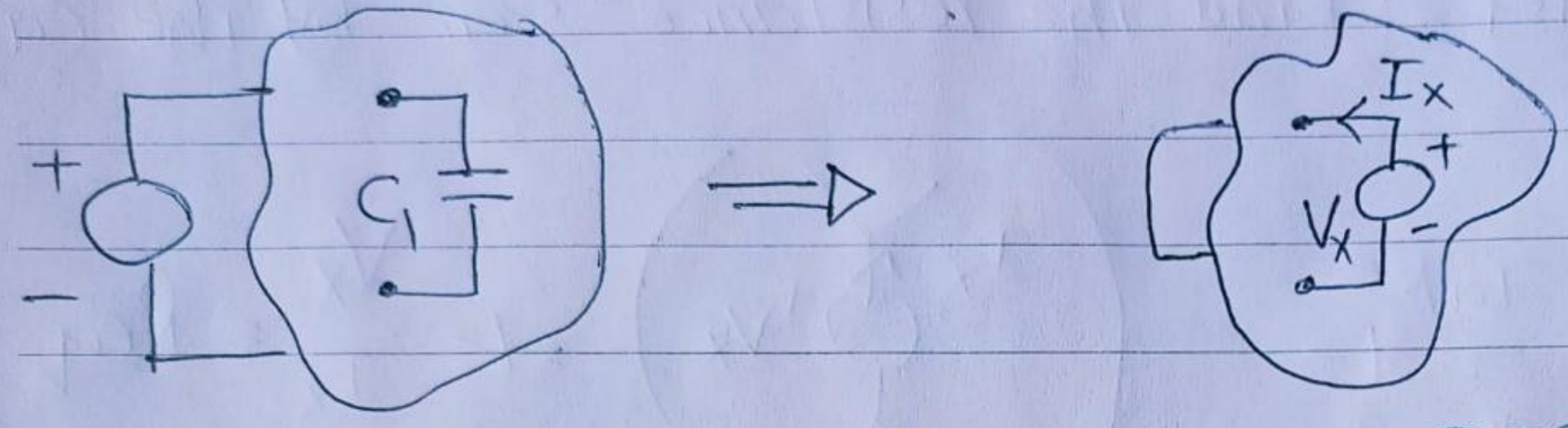
Review of lecture 19

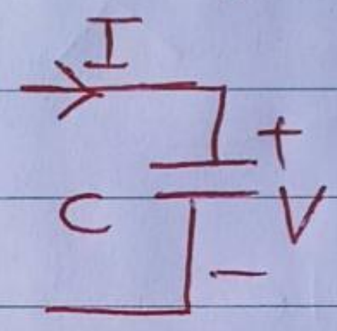
- General Response of First-order system:

$$y(t) = y_\infty + (y_0 - y_\infty)\exp\frac{-t}{\tau}$$



$\tau = R_{eq} C_1$

Today's Lecture

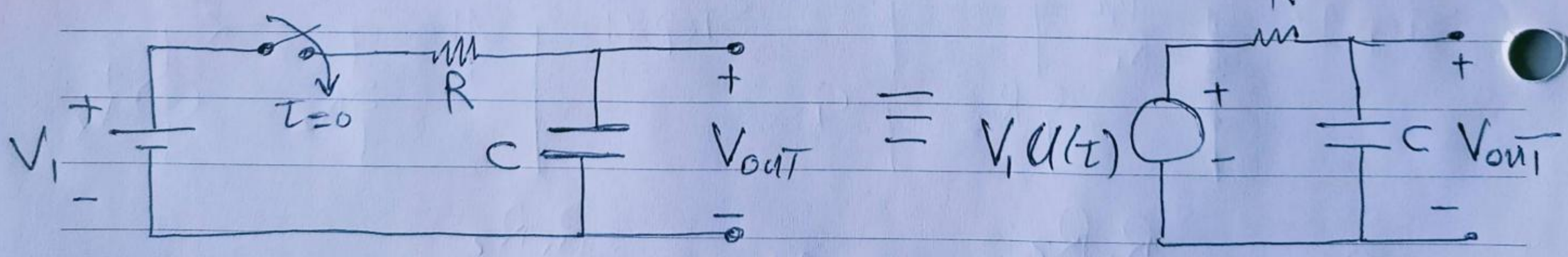


$I = C\frac{dV}{dt}$

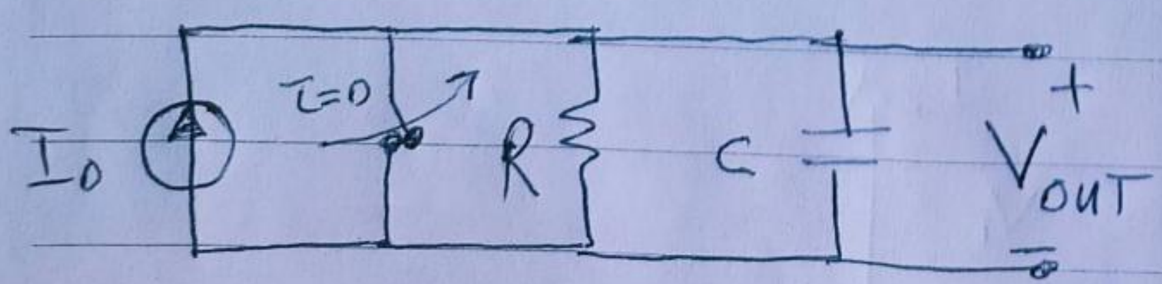
- Note on switches & step functions.
- Driven RC circuit Examples.
- Note on initial conditions.

- $I < \infty \Rightarrow V$ can't jump
- V const $\Rightarrow$ cap = o.c.

- Note on switches & step functions.



Example

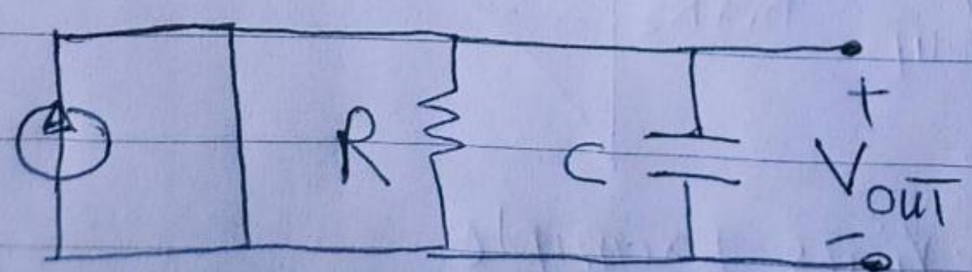


$t<0$: I_0 flows entirely through the switch $\Rightarrow$

$V_{OUT} = 0$

$V_{OUT}(0^+) = 0$

$t<0$



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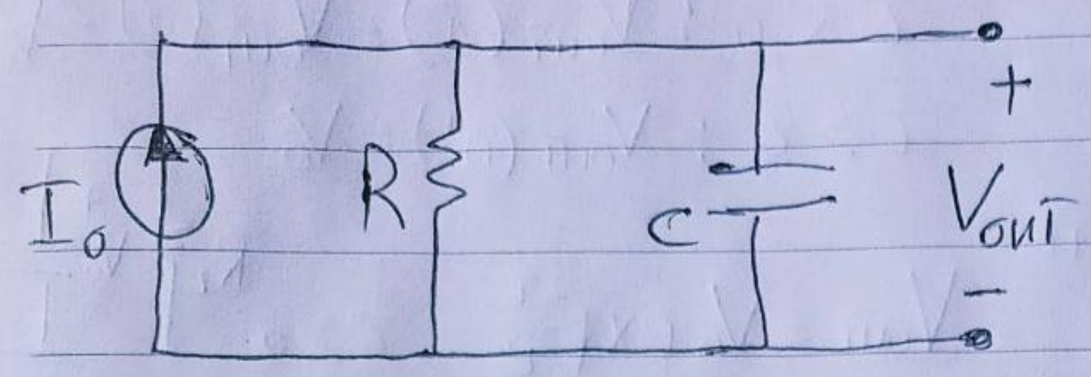
$t > 0$:



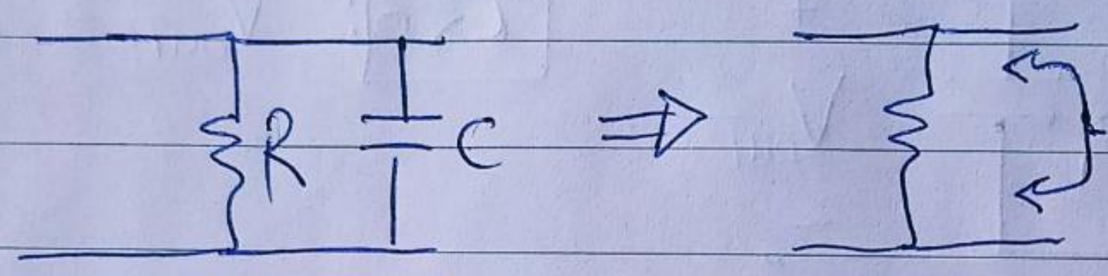
$V_{out}(t=\infty) = I_0 R$

↳ constant → cap is o.c

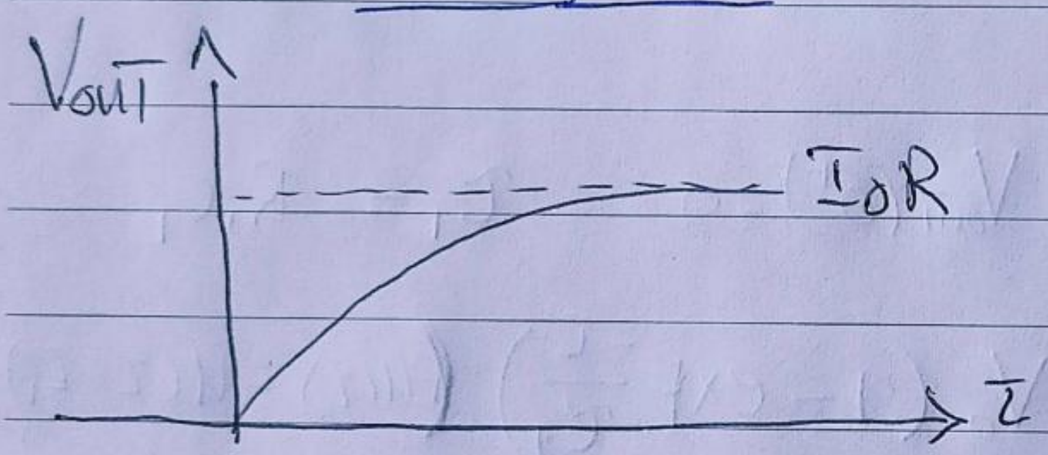
$\therefore V_{out}(t) = V_\infty + (V_0 - V_\infty)\exp\frac{-t}{\tau}$

$\therefore V_{out} = \left[I_0 R + (0 - I_0 R)\exp\frac{-t}{\tau}\right] u(t)$

τ calculation:



$\Rightarrow \tau = RC$

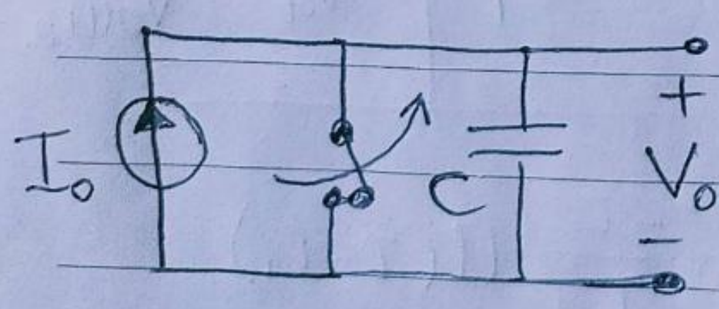


$\Rightarrow V_{out} = \left[I_0 R - I_0 R \exp\frac{-t}{RC}\right] u(t)$

Example

what happens in the previous Eample if $R \to \infty$?

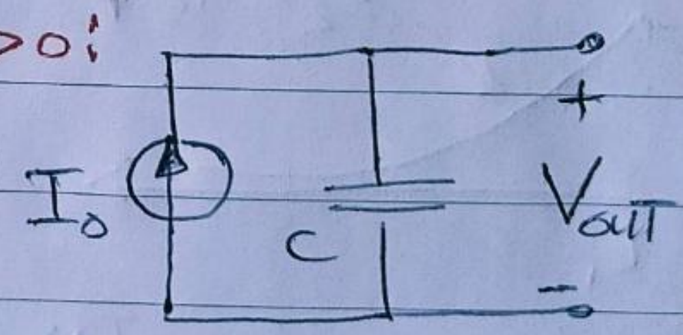
$t < 0$:



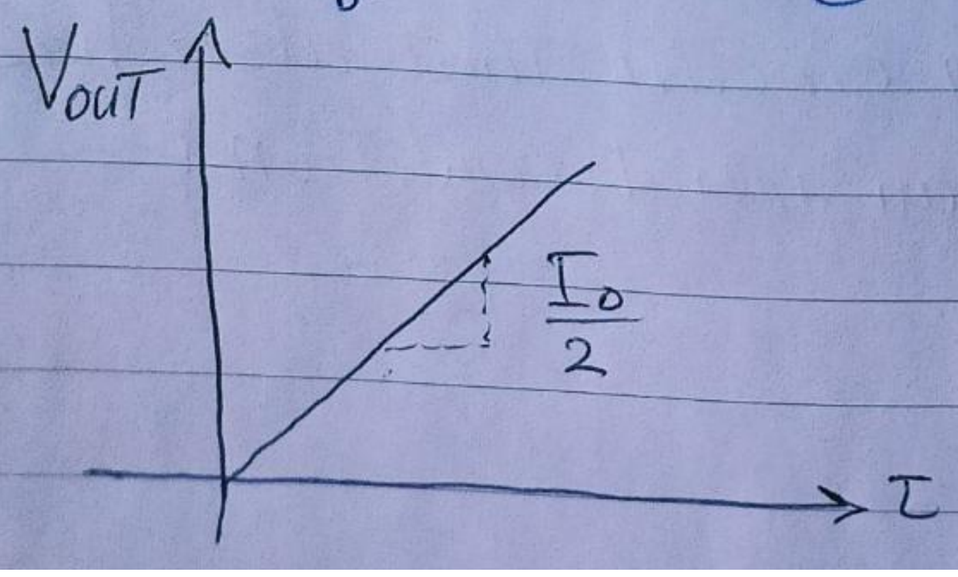
$V_{out}(0^-) = 0$

$V_{out}(0^+) = 0$

$t > 0$:



$\therefore V_{out} = \frac{1}{C}\int_0^t I_0\, dt = \frac{I_0}{C} t\, u(t)$



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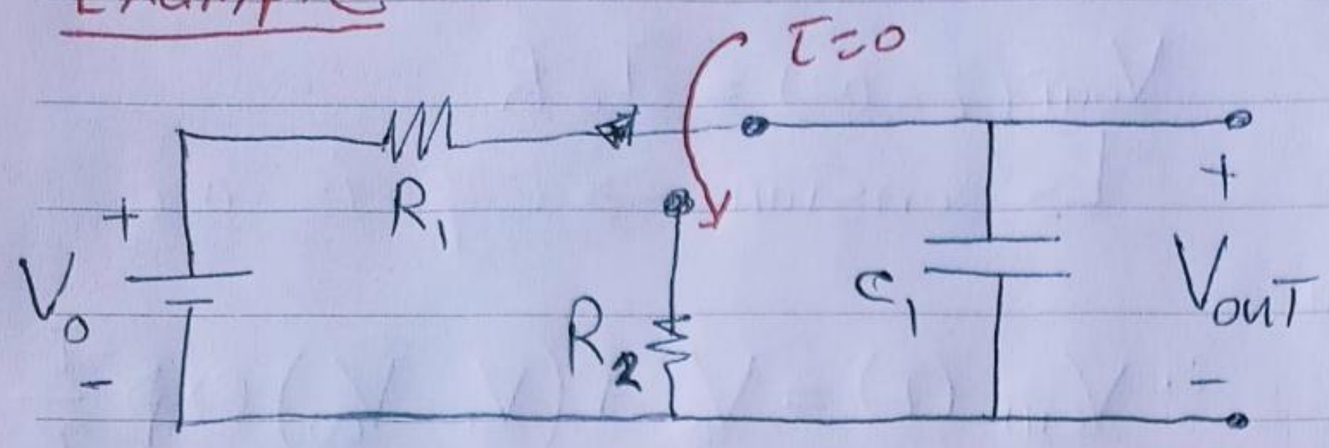
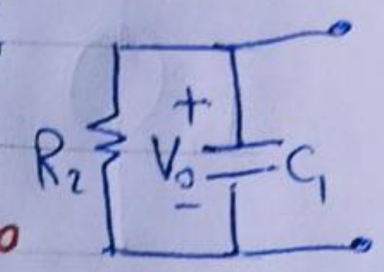
Example



$t<0$: $V_{out}(0^-) = V_{out}(0^+) = V_0$

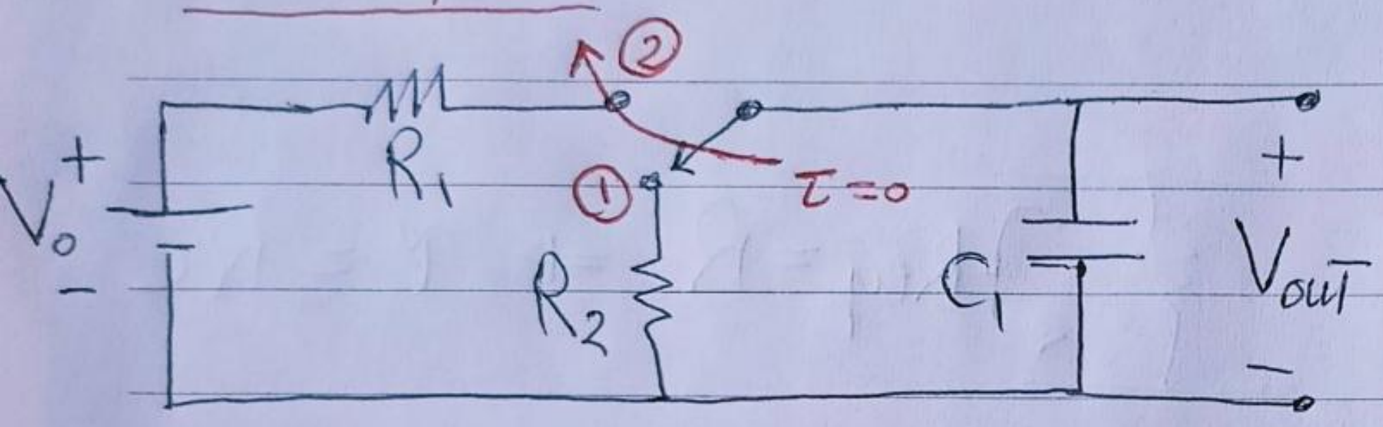
$t>0$: $V_{out}(0^+) = V_0$



$V_{out} = V_0 \exp\frac{-t}{\tau}$, $t>0$

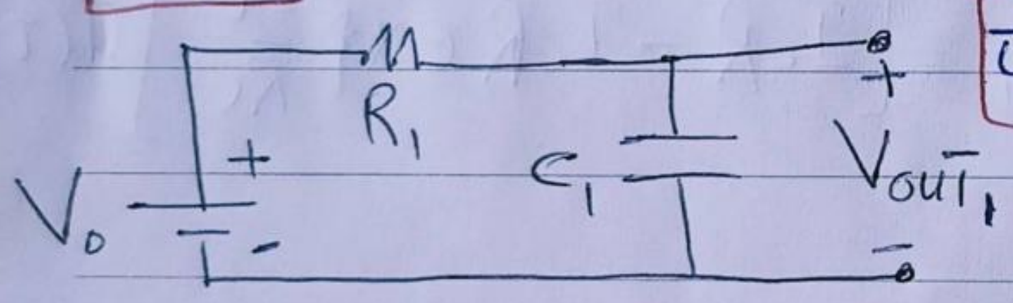
, $\tau = R_2 C_1$ *

Example



$\boxed{t<0}$: $V_{out} = 0$

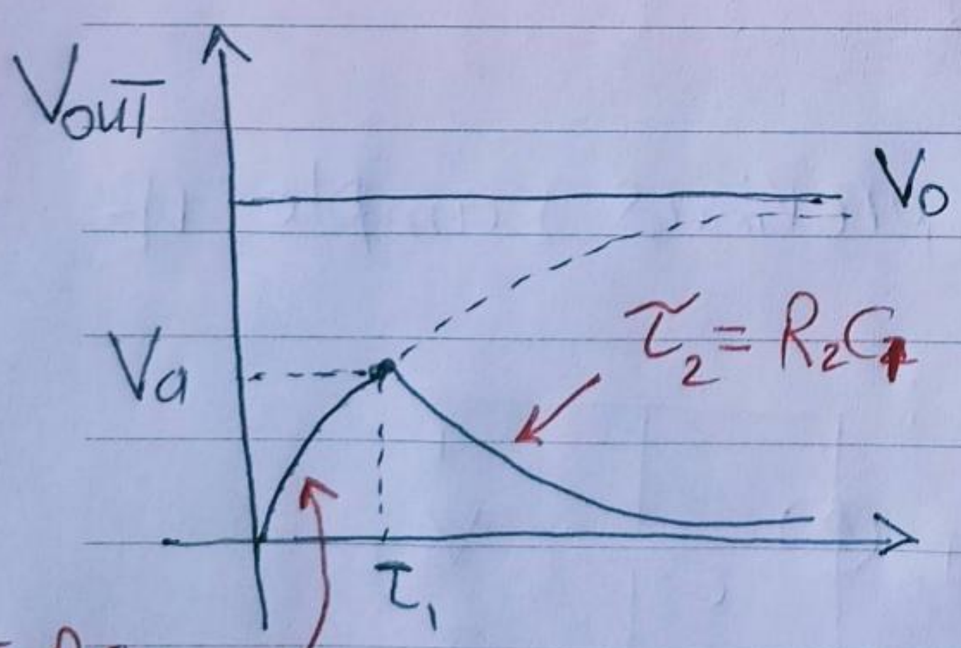
$\boxed{t>0}$:



$\boxed{t_1 > t > 0}$: $V_{out}(0^+) = 0$ $\tau_1 = R_1 C_1$

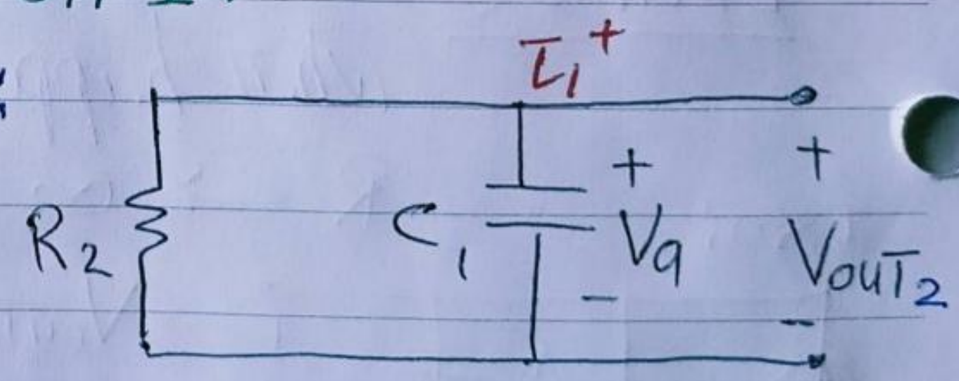
$V_{out_1} = V_0 \left(1 - \exp\frac{-t}{\tau_1}\right)\left(u(t) - u(t-t_1)\right)$

$\Rightarrow$ at $t = t_1$, switch goes back to position 1:



$V_a = V_0 \left(1 - \exp\frac{-t_1}{\tau}\right)$

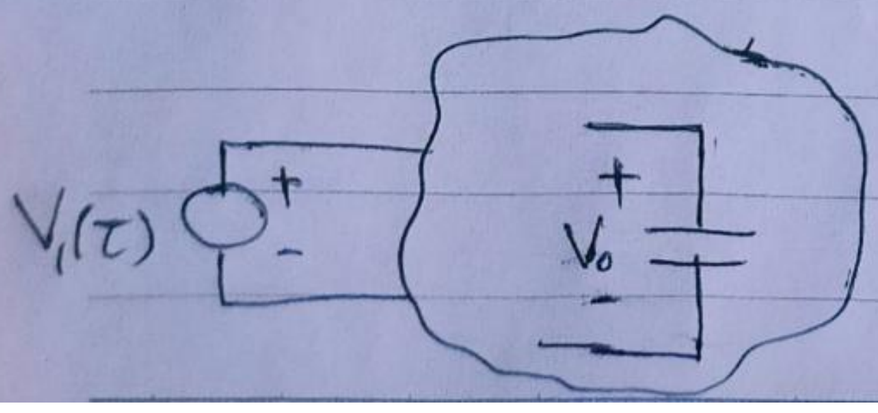
$\boxed{t > t_1}$:



$V_{out_2} = V_a \exp\frac{-t}{\tau_2} u(t-t_1)$

$\tau_2 = R_2 C_1$

- How do we analyze a driven circuit that also had an intial condition?



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Lec 21: Driven Circuits with Initial Conditions

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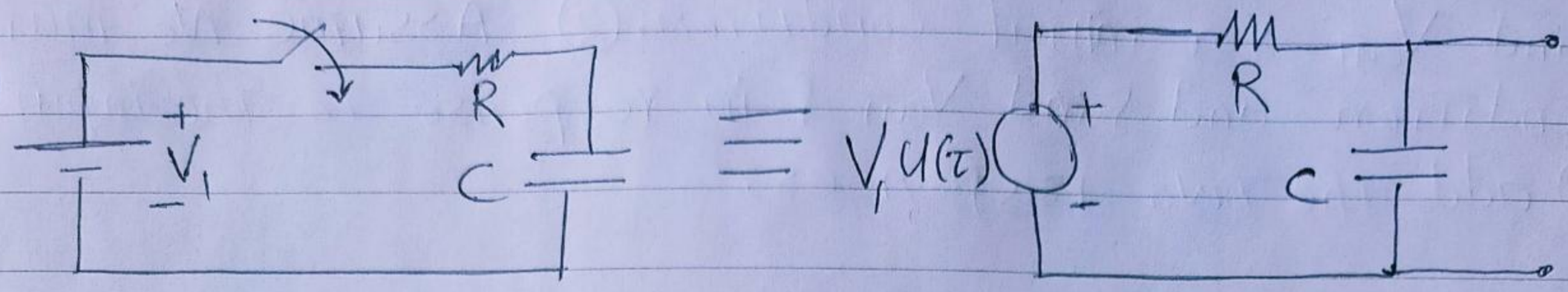
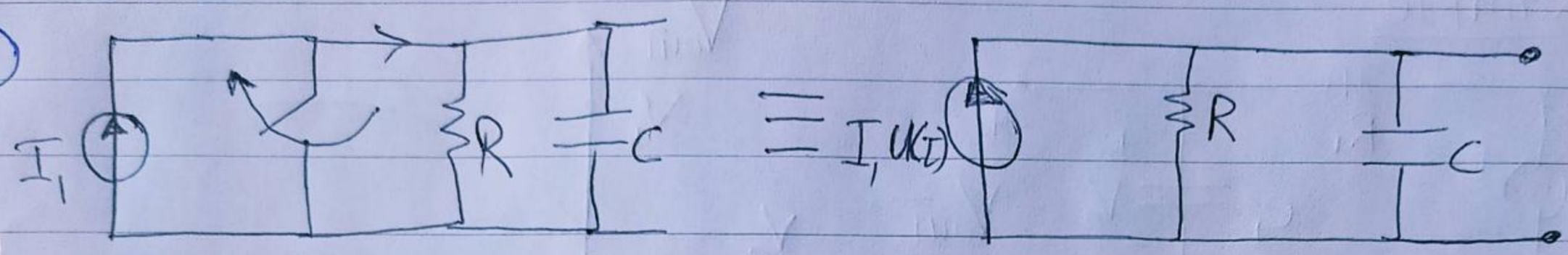
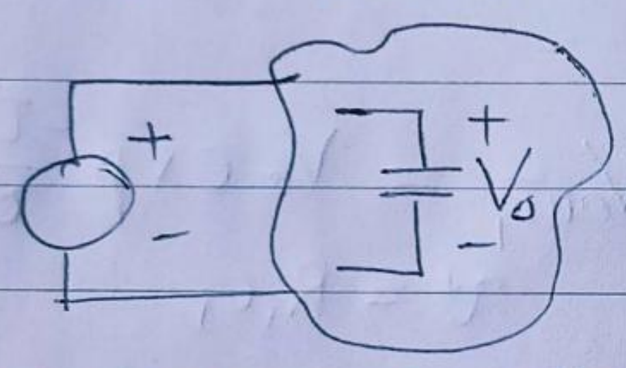
Review of lecture 20

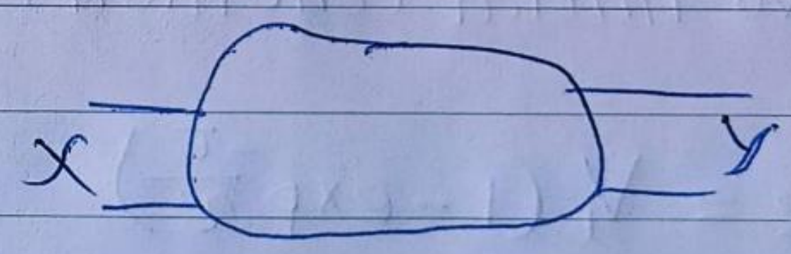
① 

② 

③ priven circuit with initial condition



Todayos lecture



- Driven circuit with initial conditions
- more Examples

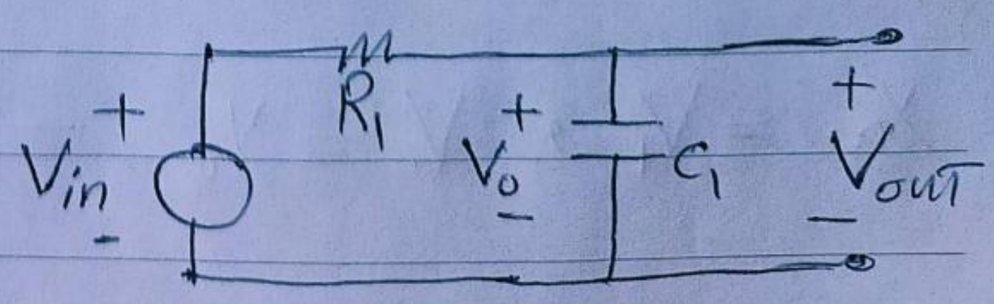
$X_1 \to Y_1$

$X_2 \to Y_2$ $\Rightarrow \alpha X_1 + \beta Y_2 + \beta X_2 + \alpha Y_1$

- Driven circuits with initial conditions:

Method 1: Solve differential equation with initial condition(s).

$$\begin{cases} R_1 C_1 \dfrac{dV_{out}}{dt} + V_{out} = V_{in} \\ V_{out}(0^+) = V_0 \end{cases}$$



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