

17.33 A poker hand consists of 5 cards chosen from a standard deck of 52 cards. There are a variety of special hands that one can be dealt in poker. For each of the following types of hands, count the number of hands that have that type.

- a.** Four of a kind: The hand contains four cards of the same numerical value (e.g., four jacks) and another card.
- b.** Three of a kind: The hand contains three cards of the same numerical value and two other cards with two other numerical values.
- c.** Flush: The hand contains five cards all of the same suit.
- d.** Full house: The hand contains three cards of one value and two cards of another value.
- e.** Straight: The five cards have consecutive numerical values, such as 7-8-9-10-jack. Treat ace as being higher than king but not less than 2. The suits are irrelevant.
- f.** Straight flush: The hand is both a straight and a flush.

18.13 Prove:

$$\binom{n}{k} = \binom{k+1}{n-1}.$$

18.15 Let n, k be positive integers, prove:

$$\binom{n}{k} = \binom{n-1}{0} + \binom{n-1}{1} + \binom{n-1}{2} + \dots + \binom{n-1}{k}$$

18.16 Let n, k be positive integers, prove:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} + \dots + \binom{n-1}{n-k+1}$$