Name: ________________
MATH55 Section ___
Homework 10
Due Thu. 2/28

35.6 Explain why Theorem 35.1 does not make sense with $b = 0$ or with $b < 0$. The case $b = 0$ is hopeless. Develop (and prove) an alternative to Theorem 35.1 that allows $b < 0$.

Theorem 35.1: (Division) Let $a, b \in Z$ with $b > 0$. There exist integers q and r such that

$$a = qb + r \quad \text{and} \quad 0 \leq r < b.$$

Moreover, there is only one such pair of integers (q, r) that satisfies these conditions. The integer q is called the quotient and the integer r is called the remainder.

36.2 For each pair of integers (a, b) in the previous problem, find integers x and y such that $ax + by = \gcd(a, b)$.

a. $\gcd(20, 25)$
b. $\gcd(0, 10)$
c. $\gcd 123, 123)$
d. $\gcd(89, 98)$
e. $\gcd(54321, 50)$
f. $\gcd(1739, 29341)$

36.14 Suppose n and m are relatively prime integers. Prove that n and $m + jn$ are relatively prime for any integer j.

Conclude that if n and m are relatively prime, and $m' = m \mod n$, then n and m' are relatively prime.

36.15 Suppose that a and b are relatively prime integers and that $a|c$ and $b|c$. Prove that $(ab)|c$.