

Name: _____
MATH55 Section ____
Homework 14
Due Tue. 4/4

48.7 Suppose that G is a subgraph of H . Prove or disprove:

- a. $\alpha(G) \leq \alpha(H)$
- b. $\alpha(G) \geq \alpha(H)$
- c. $\omega(G) \leq \omega(H)$
- d. $\omega(G) \geq \omega(H)$

48.11(c) Recall the definition of graph isomorphism from Exercise 47.21. We call a graph G *self-complementary* if G is isomorphic to \overline{G} .

Definition of isomorphism from 47.21: We say that G is isomorphic to H provided there is a bijection $f : V(G) \rightarrow V(H)$ such that for all $a, b \in V(G)$ we have $a b$ (in G) if and only if $f(a) f(b)$ (in H). The function f is called an isomorphism of G to H .

c. Prove that if a self-complementary graph has n vertices, then $[n] = [0]$ or $[n] = [1]$ in $Z/4Z$.

48.12 Find a graph G on five vertices for which $\omega(G) < 3$ and $\omega(\overline{G}) < 3$. This shows that the number six in Proposition 48.13 is best possible.

48.14(g) Let $n, a, b \geq 2$ be integers. The notation $n \rightarrow (a, b)$ is an abbreviation for the following sentence:

Every graph G on n vertices has $\alpha(G) \geq a$ or $\omega(G) \geq b$.

For example, Proposition 48.13 says that if $n \geq 6$, then $n \rightarrow (3, 3)$ is true. However, Exercise 48.12 asserts that $5 \rightarrow (3, 3)$ is false.

g. Suppose $a, b \geq 3$. If $n \rightarrow (a - 1, b)$ and $m \rightarrow (a, b - 1)$, then $(n + m) \rightarrow (a, b)$.