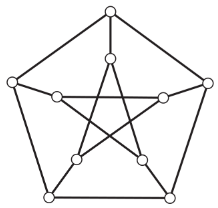
Name: ________________
MATH55 Section ___
Homework 18 (Planar Graphs)
Due Thurs. 4/18

53.7 For which values of n is the n-cube Q_n planar? Prove your answer.

Definition of Q_n:
Let n be a positive integer. The *n-cube* is a graph, denoted Q_n, whose vertices are the 2^n possible length-n lists of 0s and 1s. For example, the vertices of Q_3 are 000, 001, 010, 011, 100, 101, 110, and 111.
Two vertices of Q_n are adjacent if their lists differ in exactly one position. For example, in Q_4, vertices 1101 and 1100 are adjacent (they differ only in their fourth element), but 1100 and 0110 are not adjacent (they differ in positions 1 and 3).

53.8 The graph in the figure is known as *Petersen's Graph.* Prove that it is nonplanar by finding either a subdivision of K_5 or a subdivision of $K_{3,3}$ as a subgraph.



53.12 A *Platonic* graph is a connected planar graph in which all vertices have the samee degree r (with $3 \le r \le 5$) and in whose crossing-free embedding all faces have the same degree s (with $3 \le s \le 5$). Let G be a platonic graph with v vertices, e edges, and f faces.

a. Prove that $vr = fs$. How is this quantity related to e?

b. Prove that if $r = s = 3$, then $v = f = 4$. Conclude that K_4 is the only Platonic graph with $r = s = 3$.

c. Prove that

$$e = \frac{2}{\frac{2}{r} + \frac{2}{s} - 1}$$

d. In all, there are nine ordered pairs (r, s) with $3 \le r, s \le 5$. Use the equation in part (c) to rule out the existence of Platonic graphs with some of these values.

e. For the pairs (r, s) that were not ruled out in part (d), find a Platonic graph with vertex degree r and face degree s.

53.13 A soccer ball is formed by stitching together pieces of material that are regular pentagons and regular hexagons. The lengths of the sides of these polygons are all the same, so the edges match up exactly. Each corner of a polygon is the meeting place for exactly three polygons.

Prove that there must be exactly 12 pentagons.