

Name: _____
MATH55 Section ____
Homework 8
Due Thu. 2/21

22.9 A group of people stand in line to purchase movie tickets. The first person in line is a woman and the last person in line is a man. Use proof by induction to show that somewhere in the line a woman is directly in front of a man.

22.14 Let A_1, A_2, \dots, A_n be sets (where $n \geq 2$). Suppose for any two sets A_i and A_j either $A_i \subseteq A_j$ or $A_j \subseteq A_i$.

Prove by induction that one of these n sets is a subset of all of them.

22.17 A flagpole is n feet tall. On this pole we display flags of the following types: red flags that are 1 foot tall, blue flags that are 2 feet tall, and green flags that are 2 feet tall. The sum of the heights of the flags is exactly n feet.

Prove that there are $\frac{2}{3}2^n + \frac{1}{3}(-1)^n$ ways to display the flags.

22.18 Prove that every positive integer can be expressed as the sum of distinct Fibonacci numbers.

For example, $20 = 2 + 5 + 13$ where 2, 5, 13 are, of course, Fibonacci numbers. Although we can write $20 = 2 + 5 + 5 + 8$, this does not illustrate the result because we have used 5 twice.