Name: ________________
MATH55 Section ___
Homework 3
Due Tuesday 2/5

12.22 Let A be a set. The complement of A, denoted $\overline{A}$, is the set of all objects that are not in A. STOP! This definition needs some amending. Taken literally, the complement of the set $\{1, 2, 3\}$ includes the number -5, the ordered pair (3, 4), and the sun, moon, and stars! After all, it says . . . all objects that are not in A. This is not what is intended.

When mathematicians speak of set complements, they usually have some overarching set in mind. For example, during a given proof or discussion about the integers, if A is a set containing just integers, A stands for the set containing all integers not in A.

If U (for universe) is the set of all objects under consideration and $A \subseteq U$, then the complement of A is the set of all objects in U that are not in A. In other words, $\overline{A} = U - A$. Thus $\overline{\emptyset} = U$.

Prove the following about set complements. Here the letters A, B, and C denote subsets of a universe set U.

a. $A = B$ if and only if $\overline{A} = \overline{B}$.
b. $\overline{\overline{A}} = A$.
c. $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$.

The notation $\overline{A}$ is handy, but it can be ambiguous. Unless it is perfectly clear what the universe set U should be, it is better to use the set difference notation rather than complement notation.

13.1 Give an alternative proof of Proposition 13.1 in which you use list counting instead of subset counting.

Proposition 13.1: Let n be a positive integer. Then

$$2^0 + 2^1 + ... + 2^{n-1} = 2^n - 1$$

13.3 Substituting $x = 3$ into your expression in the previous problem yields

$$2 \times 3^0 + 2 \times 3^1 + 2 \times 3^2 + ... + 2 \times 3^{n-1} = 3^n - 1$$

Prove this equation combinatorially. Next, substitute $x = 10$ and illustrate the result using ordinary base-10 numbers.

For reference, the expression that is being referred to is $(x - 1)(1 + x + x^2 + ... + x^{n-1})$.

13.5 Let n be a positive integer. Give a combinatorial proof that $n^2 = n(n-1) + n$.