Name: ________________
MATH55 Section ___
Homework 16 (Trees)
Due Thu. 4/11

50.4 Let $d_1, d_2, ..., d_n$ be $n \geq 2$ positive integers. (not necessarily distinct). Prove that $d_1, ..., d_n$ are the degrees of the vertices of a tree on n vertices if and only if $\sum_{i=1}^{n} d_i = 2n - 2$.

50.5 Let e be an edge of a graph G. Prove that e is not a cut edge if and only if e is in a cycle of G.

50.6 Complete the proof of Theorem 50.4. That is, prove that if G is a graph in which any two vertices are joined by a unique path, then G must be a tree.

50.11 In this problem, you will develop a new proof that every tree with two or more vertices has a leaf. Here is an outline for your proof.

a. First prove, using strong induction and the fact that every edge of a tree is a cut edge (Theorem 50.5), that a tree with n vertices has exactly $n-1$ edges. Please note that our previous proof of this fact (Theorem 50.9) used the fact that trees have leaves; that is why we need an alternative proof.

b. Use (a) to prove that the average degree of a vertex in a tree is less than 2.

c. Use (b) to prove that every tree (with at least two vertices) has a leaf.