Name: __________________
MATH55 Section ___
Homework 14
Due Tue. 4/9

49.10. Let G be a graph. Prove that G or $\overline{G}$ (or both) must be connected.

49.11. Let G be a graph with $n \geq 2$ vertices. Prove that if $\delta(G) \geq \frac{1}{2}n$, then G is connected.

Reminder: The maximum degree of a vertex in G is denoted $\Delta(G)$ and the minimum degree of a vertex in G is denoted $\delta(G)$.

49.12. Let G be a graph with $n \geq 2$ vertices.

a. Prove that if G has at least $\binom{n-1}{2} + 1$ edges, then G is connected.

b. Show that the result in (a) is best possible: that is, for each $n \geq 2$ prove that there is a graph with $\binom{n-1}{2}$ edges that is not connected.

49.14. Let A b the adjacency matrix of a graph G. That is, we label the vertices of G as $v_1, v_2, ...v_n$. The matrix A is an $n \times n$ matrix whose $i, j-$entry is 1 if $v_i v_j \in E(G)$ and is 0 otherwise.

Let $k \in N$. Prove that the $i, j-$entry of A^k is the number of walks of length k from v_i to v_j.