Name: ________________
Section: ___
Homework 12
Due Thurs. 3/7

39.6 Prove Lemma 39.3 by induction (or Well-Ordering Principle) using Lemma 39.2.

Lemma 39.3 Suppose $p, q_1, q_2, ..., q_t$ are prime numbers. If $p|(q_1q_2...q_t)$ then $p = q_i$ for some $1 \leq i \leq t$.

Lemma 39.2 Suppose $a, b, p \in \mathbb{Z}$ and p is a prime. If $p|ab$, then $p|a$ or $p|b$.

39.17 *Euler's totient, continued* Suppose that p and q are unequal primes. Prove the following:

a. $\varphi(p) = p - 1$.
b. $\varphi(p^2) = p^2 - p$.
c. $\varphi(p^n) = p^n - p^{n-1}$ where n is a positive integer.
d. $\varphi(pq) = pq - q - p - 1 = (p - 1)(q - 1)$.

39.19 *Again with Euler's totient.* Now suppose n is any positive integer. Factor n into primes as $n = p_1^{a_1} p_2^{a_2} ... p_t^{a_t}$ where the p_is are distinct primes and the exponents a_i are all positive integers. Prove that the formulas from the previous problem are valid for this general n.

Formula from the previous problem: If n is a positive integer which can be represented as the product of distinct primes, $p_1 p_2 ... p_t$, then $\varphi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})...(1 - \frac{1}{p_t})$.

39.20 Rewrite the second proof of Proposition 39.6 to show the following:

Let n be an integer. If $\sqrt{n}$ is not an integer, then there is no rational number x such that $x^2 = n$.