

Name: \_\_\_\_\_  
MATH55 Section \_\_\_\_  
Homework 2  
Due Thursday 1/31

**14.16** Give an example of a relation on a set that is both symmetric and transitive but not reflexive. Explain what is wrong with the following proof.

*Statement:* If  $R$  is symmetric and transitive, then  $R$  is reflexive.

*Proof:* Suppose  $R$  is symmetric and transitive. Symmetric means that  $x R y$  implies  $y R x$ . We apply transitivity to  $x R y$  and  $y R x$  to give  $x R x$ . Therefore  $R$  is reflexive.

**15.16** There is only one possible equivalence relation on a one-element set: If  $A = \{1\}$ , then  $R = \{(1, 1)\}$  is the only possible equivalence relation. There are exactly two possible equivalence relations on a two-element set: If  $A = \{1, 2\}$ , then  $R_1 = \{(1, 1), (2, 2)\}$  and  $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$  are the only equivalence relations on  $A$ .

How many different equivalence relations are possible on a three-element set? ... on a four-element set?

**16.8** Continued from the previous problem. Suppose six of these people are men, and the other six are women. In how many ways can they join hands for a circle dance, assuming they alternate in gender around the circle?

(Last problem: Twelve people join hands for a circle dance. In how many ways can they do this?)

**16.13** One hundred people are to be divided into ten discussion groups with ten people in each group. In how many ways can this be done?