

Name: \_\_\_\_\_  
MATH55 Section \_\_\_\_  
Homework 14  
Due Tue. 4/4

**48.7** Suppose that  $G$  is a subgraph of  $H$ . Prove or disprove:

- a.  $\alpha(G) \leq \alpha(H)$
- b.  $\alpha(G) \geq \alpha(H)$
- c.  $\omega(G) \leq \omega(H)$
- d.  $\omega(G) \geq \omega(H)$

**48.11(c)** Recall the definition of graph isomorphism from Exercise 47.21. We call a graph  $G$  *self-complementary* if  $G$  is isomorphic to  $\overline{G}$ .

**c.** Prove that if a self-complementary graph has  $n$  vertices, then  $[n] = [0]$  or  $[n] = [1]$  in  $Z/4Z$ .

Definition of isomorphism from exercise 47.21: We say that  $G$  is isomorphic to  $H$  provided there is a bijection  $f : V(G) \rightarrow V(H)$  such that for all  $a, b \in V(G)$  we have  $a \sim b$  (in  $G$ ) if and only if  $f(a) \sim f(b)$  (in  $H$ ). The function  $f$  is called an isomorphism of  $G$  to  $H$ .

**48.12** Find a graph  $G$  on five vertices for which  $\omega(G) < 3$  and  $\omega(\overline{G}) < 3$ . This shows that the number six in Proposition 48.13 is best possible.

**48.14(g)** Let  $n, a, b \geq 2$  be integers. The notation  $n \rightarrow (a, b)$  is an abbreviation for the following sentence: Every graph  $G$  on  $n$  vertices has  $\alpha(G) \geq a$  or  $\omega(G) \geq b$ .

For example, Proposition 48.13 says that if  $n \geq 6$ , then  $n \rightarrow (3, 3)$  is true. However, Exercise 48.12 asserts that  $5 \rightarrow (3, 3)$  is false.

**g.** Suppose  $a, b \geq 3$ . If  $n \rightarrow (a - 1, b)$  and  $m \rightarrow (a, b - 1)$ , then  $(n + m) \rightarrow (a, b)$ .