Name: ________________
MATH55 Section ___
Homework 9
Due Tue. 2/26

24.16 Let A and B be finite sets and let $f : A \to B$. Prove that any two of the following statements being true implies the third.

a. f is one-to-one.

b. f is onto.

c. $|A| = |B|$.

24.18 Suppose $f : A \to B$ is a bijection. Prove that $f^{-1} : B \to A$ is a bijection as well.

25.5 Let $(a_1, a_2, a_3, a_4, a_5)$ be a sequence of five distinct integers. We call such a sequence increasing if $a_1 < a_2 < a_3 < a_4 < a_5$ and decreasing if $a_1 > a_2 > a_3 > a_4 > a_5$. Other sequences may have a different pattern of $<$s and $>$s. For the sequence $(1, 5, 2, 3, 4)$ we have $1 < 5 > 2 < 3 < 4$. Different sequences may have the same pattern of $<$s and $>$s between their elements. For example, $(1, 5, 2, 3, 4)$ and $(0, 6, 1, 3, 7)$ have the same pattern of $<$s and $>$s as illustrated here:

$$1 < 5 > 2 < 3 < 4$$
$$0 < 6 > 1 < 3 < 7$$

Given a collection of 17 sequences of five distinct integers, prove that 2 of them have the same pattern of $<$s and $>$s.

25.7 Given a set of seven distinct positive integers, prove that there is a pair whose sum or whose difference is a multiple of 10.

You may use the fact that if the ones digit of an integer is 0, then that integer is a multiple of 10.