Name: Santi Santichaivekin
MATH55 Section 3
Homework 4
Due Thu 2/7

17.16 Consider the following formula:

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

Give two different proofs. One proof should use the factorial formula for $\binom{n}{k}$ (Theorem 17.12). The other proof should be combinatorial; develop a question that both sides of the equation answer.

Factorial formula proof:

$$\begin{aligned}
k\binom{n}{k} &= k\left(\frac{n!}{k!(n-k)!}\right) && \text{: via factorial formula} \\
&= \frac{n(n-1)!}{(k-1)!(n-k)!} \\
&= \frac{n(n-1)!}{(k-1)!((n-1)-(k-1))!} \\
&= n\binom{n-1}{k-1} && \text{: via factorial formula}
\end{aligned}$$

17.19 Let n be natural number. Give a combinatorial proof of the following:

$$\binom{2n+2}{n+1} = \binom{2n}{n+1} + 2\binom{2n}{n} + \binom{2n}{n-1}$$

17.21 Use Stirling's formula (see Exercise 9.7) to develop an approximation formula for $\binom{2n}{n}$. Without using Stirling's formula, give a direct proof that $\binom{2n}{n} \leq 4^n$.

Reference from Exercise 9.7: The Scottish mathematician James Stirling found an approximation formula for $n!$. Stirlings formula is

$$n! \approx \sqrt{2\pi n} n^n e^{-n}$$

where $\pi = 3.13159...$ and $e = 2.71828....$ (Scientific calculators have a key that computes e^x; this key might be labeled "exp x".)

17.26 Prove: $\binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \binom{n}{2}\binom{n}{n-2} + ... + \binom{n}{n-1}\binom{n}{1} + \binom{n}{n}\binom{n}{0} = \binom{2n}{n}$.