Name: _____
MATH55 Section ___
Homework 6
Due Thu. 2/14

19.4 Let A, B, and C be finite sets. Prove or disprove: If $|A \cup B \cup C| = |A| + |B| + |C|$, then A, B, and C must be pairwise disjoint.

19.5 How many five-letter words can you make in which no two consecutive letters are the same? A word may be any list of the standard 26 letters, so WENJW is a word you would count, but NUTTY is not.

Here is an easy solution: By the list-counting methods of Section 8, the answer is $26 \times 25 \times 25 \times 25 \times 25 = 26 \times 25^4$.

Give a hard solution using inclusion-exclusion, and then show that the two answers are the same.

19.6 This problem asks you to give two proofs for

$$9^n = \sum_{k=0}^{n} (-1)^k \binom{n}{k} 10^{n-k}$$

a. The first proof should use the binomial theorem (see Theorem 17.8).
b. The second should be a combinatorial proof using inclusion-exclusion.

19.7 How many six-digit numbers do not have three consecutive digits the same? (For this problem, you may consider six-digit numbers whose initial digits might be 0. Thus you should count 012345 and 001122, but not 000987 or 122234.)