

Name: _____
MATH55 Section ____
Homework 11
Due Tue. 3/5

37.3 Solve the following equations for $[x]$ in the Z/nZ specified. Note: These are quite different from the previous set of problems. Why? Be sure you find all solutions.

- a. $[2][x] = [4]$ in $Z/10Z$.
- b. $[2][x] = [3]$ in $Z/10Z$.
- c. $[9][x] = [4]$ in $Z/12Z$.
- d. $[9][x] = [6]$ in $Z/12Z$.

37.4 Here are a few more equations for you to solve in the \mathbb{Z}_n specified. Be sure to find all solutions.

a. $[x][x] = [1]$ in $\mathbb{Z}/13\mathbb{Z}$.

b. $[x][x] = [11]$ in $\mathbb{Z}/13\mathbb{Z}$.

c. $[x][x] = [12]$ in $\mathbb{Z}/13\mathbb{Z}$.

d. $[x][x] = [4]$ in $\mathbb{Z}/15\mathbb{Z}$.

e. $[x][x] = [10]$ in $\mathbb{Z}/15\mathbb{Z}$.

f. $[x][x] = [14]$ in $\mathbb{Z}/15\mathbb{Z}$.

37.10 For ordinary integers, the following is true. If $ab = 0$, then $a = 0$ or $b = 0$. The analogous statement for Z/nZ is not necessarily true. For example, in $Z/10Z$, $[2][5] = [0]$ but $[2] \neq [0]$ and $[5] \neq [0]$. However, for some values of n (e.g., $n = 5$) it is true that $[a][b] = 0$ implies $[a] = 0$ or $[b] = 0$.

For which values of $n \geq 2$ does the implication

$$[a][b] = 0 \iff [a] = [0] \text{ or } [b] = [0]$$

hold in Z/nZ ? Prove your answer.

37.12 Let n be a positive integer and suppose $[a], [b] \in \mathbb{Z}/n\mathbb{Z}$ are both invertible. Prove or disprove each of the following statements.

- a. $[a] + [b]$ is invertible.
- b. $[a] - [b]$ is invertible.
- c. $[a][b]$ is invertible.
- d. $[a]/[b]$ is invertible.