Name: ________________
Section: ___
Homework 21 (Posets)
Due Tue. 4/30

54.6 Prove that *refines* is a partial order relation on the set of all partitions of a set A.

Let $\mathcal{P}$ and $\mathcal{Q}$ be partitions of a set A. We say that $\mathcal{P}$ *refines* $\mathcal{Q}$ if every part in $\mathcal{P}$ is a subset of some part in $\mathcal{Q}$. We also say that $\mathcal{P}$ is *finer* than $\mathcal{Q}$.

55.8 Let P be a finite, nonempty poset. We know that P must have a minimal and a maximal element. Prove the following stronger statement:

Let P be a finite, nonempty poset. Prove that P must contain a minimal element x and a maximal element y with $x \leq y$.

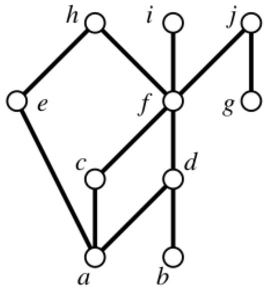
57.4 Let $P = (X, \leq)$ be a finite poset that is not a total order. Prove that P contains incomparable elemnets x and y such that

$$\leq' = \leq \cup \{(x, y)\}$$

is a partial order relation.

Such a pair of elements is called a *critical pair*.

58.1 Let P be the poset in the following figure:



a. Find $d = \dim P$.

b. Find a realizer of P containing d linear extensions.

c. Give an embedding of P in $\mathbb{R}^d$ (either via a picture or by specifying coordinates).