

# BDI-Learning Discussion Paper: Towards Improving *Stable* Performance

Dhirendra Singh  
dhirendra.singh@rmit.edu.au

30 June 2009

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## 1 Motivating example

### 1.1 The scenario

Continuing on with the investigation of learning context conditions in worlds with multiple variables, I created another test (referred to as *testImpactvars* in Figure 1 and herein) with a G/P tree that handles multiple variables and has the following properties.

- The tree handles  $2^3$  worlds described by the variables set  $[a, b, c]$ .
- All worlds have a unique solution in the G/P tree.

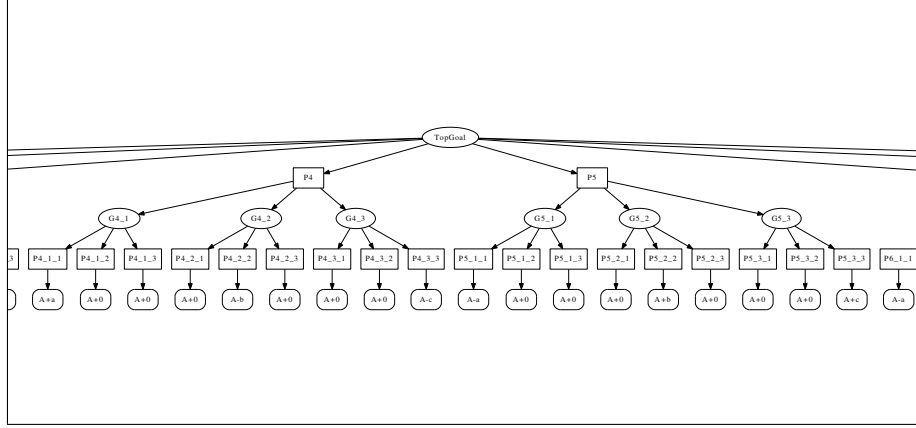


Figure 1: *testImpactvars* input tree (sample portion only)

- At level one, *TopGoal* is handled by 8 plans  $[P1 \dots P8]$  such that the worlds space is evenly distributed among these sub-plans and there is no overlap.
- The plans  $P1..P8$  have 3 sub-goals each, such that the sequence required to succeed is of length 3.
- At level two, each sub-goal of  $[P1 \dots P8]$  is handled by 3 leaf plans, only one of which will ever succeed. So the probability of selecting a successful sequence  $p_{success}$  is given by the product of the probability of selecting a correct plan at level one and the probability of selecting 3 correct sub-plans at level two. Therefore,  $p_{success} = p_{level1} * p_{level2}^3 = \frac{1}{8} * \frac{1}{3}^3 = \frac{1}{216}$ .
- The G/P tree itself is evenly balanced i.e. the G/P hierarchy is of uniform breadth and depth.
- Finally, the distribution of the worlds within the tree is also evenly balanced i.e. each sub-tree handled the same proportion of all possible worlds  $\frac{1}{8}$ .

In Figure 1, the leaf nodes represent actions. Here actions with suffix  $+0$  always fail. Actions with suffix  $+a$  succeed when  $a$  is *true* while those with  $-a$  succeed when  $a$  is *false*. Similarly for  $\pm b$  and  $\pm c$ . Looking at the sub-tree of plan  $P4$  for instance, we can see that it will succeed only in the world  $a\bar{b}\bar{c}$ . In this way, the level one plans  $[P1 \dots P8]$  uniquely handle the worlds  $[abc, ab\bar{c}, a\bar{b}c, a\bar{b}\bar{c}, \bar{a}bc, \bar{a}b\bar{c}, \bar{a}\bar{b}c, \bar{a}\bar{b}\bar{c}]$  respectively.

Figure 2 shows the performance of our *Concurrent* and *Stable* approaches in this scenario. Notice that *Stable* performance is almost four fold worse than *Concurrent* in this case.

The reason why this result is relevant is that *testImpactvars* was not purposely crafted to favour one approach over the other. Furthermore, *testImpactvars* is relatively shallow and has a low branching factor compared to tests we have performed in the past. All in all, the test is simpler in hierarchy and is arguably a better representation of a *typical* BDI tree than previously. The primary difference between this and previous tests (bar Stéphane's tree) of course is that we are experimenting here with multiple worlds.

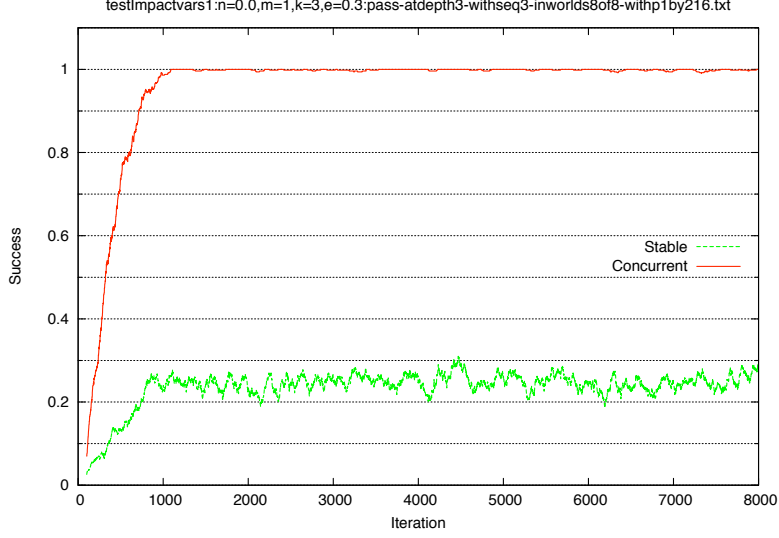


Figure 2: Performance comparison for *testImpactvars*

## 1.2 Why *Stable* performs poorly for *testImpactvars*

So what causes *Stable* to perform so poorly in this case? We can start with the obvious differences between the two approaches and see if we can eliminate this disparity. Intuitively, we know that *Concurrent* is an aggressive or *optimistic* approach compared to *Stable* that is controlled and relatively *pessimistic*.

The parameters that fine-tune *Stable* behaviour are  $k$  (the minimum number of instances of a given world required for a decision tree to be considered stable) and  $\epsilon$  (the maximum change in probability between two instances before a decision tree can be considered stable). For our experiments we use the default values of  $k = 3$  and  $\epsilon = 0.3$ .

Already we can appreciate that  $k = 3$  imposes a strong constraint at our leaf level before failure updates can be propagated to the top. For instance in Figure 1, goal  $G4.1$  will be considered stable for say world  $abc$  when all its children  $[P4.1.1 \dots P4.1.3]$  are stable too. For  $k = 3$  that will take  $3 + 3 + 3 = 9$  instances. For  $P4$  to be updated, all its children  $[G4.1 \dots G4.3]$  must be stable. That in turn will take  $9 + 9 + 9 = 27$  instances. Then for  $P4$  to be stable for all possible worlds will take  $27 * 8 = 216$  samples. For the entire tree to be stable will require a *minimum* of  $216 * 8 = 1728$  samples. The actual number will be more than that because samples are chosen randomly and will naturally result in duplicates.

Figure 2 shows a simulation of 8000 samples. Even considering the randomisation, shouldn't *Stable* be performing optimally by then? Note that  $k = 3$  determines the lower bound on the number of samples. The actual number of samples required for stability of a node also depends on  $\epsilon$ , and to some extent on the *noise*  $n$  in the environment.

To reduce the disparity then, we run the experiment again with  $k = 1$ ,  $\epsilon = 1.0$ , and  $n = 0.0$ . This would make *Stable* performance almost the same as *Concurrent*.

Note that *Stable* still requires the stability of each child and the entire tree still requires at least  $(1 + 1 + 1) * 3 * 8 * 8 = 576$  samples. While this number is the same for *Concurrent*, the difference is that the timing of *Concurrent* updates will be different to that of *Stable*.

On conducting the experiment again with these lenient parameters the result is unexpected. Instead of *Stable* performance converging towards *Concurrent*, there is *no change* to *Stable* performance when compared to Figure 2.

This result suggests that other factors are at play here than those determined above. Debugging the implementation at length shows that some core decisions in the system introduce subtleties that eventually lead to performance degradation.

## 2 Understanding the Problem

### 2.1 When is it ok to start using a decision tree?

The absolute minimum number of instances required to build a decision tree is 1. Currently this is the number we use to decide when to build and start using a tree, as determined by the runtime parameter  $m = 1$ . This decision causes several problems.

At it's core, the problem is that we are constructing a decision tree with a single sample of *one* world and then using this tree to determine the probabilities for *all* worlds. This is not reasonable.

This problem manifests itself in various symptoms, some of which are listed here.

- Consider three leaf nodes  $[P_i, P_j, P_k]$ . At the start, none of the nodes have trees and always return a probability of 1 by default. We are interested in world  $W1$  where we know that  $P_j$  will succeed. We may see the following possible sequence of events:
  1. The starting probabilities for selection in  $W1$  are  $[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$ . Let's say  $P_i$  is randomly selected, executed, and fails in  $W1$ . It then builds a decision tree from this sample and will use it thereon.
  2. Second time around in  $W1$  the selection probabilities are  $[\approx 0, \frac{1}{2}, \frac{1}{2}]$ . This time  $P_k$  is randomly selected, fails, and builds a decision tree.
  3. Third time around in  $W1$ , the selection probabilities should be  $[\approx 0, \frac{1}{1}, \approx 0]$  and  $P_j$  should inevitably be selected. However the probabilities have somehow changed to  $[\approx 0, \approx 0, \approx 0]$ . Why? Because sometime between the second and third instances of  $W1$ ,  $P_j$  was selected in *some other world* and failed. It then constructed a single-sampled decision tree in that world that it is now using in world  $W1$  and returning a probability of  $\approx 0$ .

That's the power of interpolation! The result is that the selection probabilities are now equal and back to the original value of  $\frac{1}{3}$  each (but with each absolute probability  $\approx 0$  instead of 1 as at the start). The impact is that hereon the probability of selection of  $P_j$  will not improve doesn't matter how many times  $P_i$  and  $P_k$  may fail in between.

The problem worsens as the branching factor increases. Consider a set of 20 plans, only one of which is setup to succeed. If it's probability incorrectly reduces to  $\approx 0$  thanks to a misinformed decision tree, then it's chances of selection will never improve beyond  $\frac{1}{20}$  *even though every other siblings may have been*

*tried and failed numerous times.* For correct operation, the probability of this plan should gradually increase  $\rightarrow 1$  as other siblings are tried and fail. Note that here the environment is not stochastic so if a plan fails then it is a true failure and there is no chance of it passing in that given world in the future; furthermore the single plan wired for success will succeed in *all* worlds not just a small portion of the worlds space. Even in this lenient case the problem is significant.

However, while the problem worsens as the branching factor increases, the probability of the problem state occurring in the first place decreases. In fact for  $M$  applicable plans, the probability of recovering from the problem is  $1/M$  and the probability of witnessing the problem state is also  $1/M$ .

- A similar symptom is where the initial probabilities for  $W1$  are all  $[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$ , but change to say  $[\frac{1}{2}, \approx 0, \frac{1}{2}]$  because a decision tree was constructed for  $P_j$  in some other world that is returning a misguided probability for  $W1$ . This even before  $W1$  was ever encountered.

As a result the exploration of *Stable* is unfairly biased leading to a slower convergence than expected. This is what I think we are seeing in *testImpactvars*. Note that this problem is evident in experiments with multiple worlds, hence why we haven't come across it earlier.

## 2.2 When no decision tree exists, what should the default $p$ be?

This question impacts the performance of both *Concurrent* and *Stable*, and must be addressed carefully. So let us first ensure we understand the question clearly. Currently, the following (pseudo) code in every plan determines the likelihood of success in a given world.

```
probability = useDT(planID) ? probabilityDT(planID) : 1
```

The decision is to use a probability of success of 1 for any given world when the decision tree for the plan is not ready for use (note that *useDT* returns *false* when we haven't encountered the minimum number of instance i.e.  $m = 1$ ), otherwise use the probability as determined by the decision tree. We have already seen in Section 2.1 what happens when the decision tree being used is ill-informed and returns misleading probabilities for the world in question. But what about the other part of the equation? Does it matter what we use as the default probability when we have no decision tree available? Turns out it does.

Table 1 shows the impact of the default probability  $p$  on plan selection from a set of applicable plans  $[P_i, P_j, P_k]$  in a given world  $W$ . It highlights the case when the choice of returning a default probability of 1 does not work in our favour for plan selection. This poses the question if the default  $p = 1$  is the right choice and if not then what is? A default of  $p \approx 0$  does not work either for the following reasons:

- If the default is  $p \approx 0$  and some applicable plan is using a decision tree that returns a probability  $\rightarrow 1$  for the given world, then that plan will almost always be selected. This may cause other applicable plans to never be selected and tried. As a result the parent node will *almost never* become stable (requirement for stable is that all children be stable so should have been tried in the given world at least  $k$  times).

useDT	$p$ Used	Outcome	Comment
[F F F]	[1 1 1]	Select $P_j$ and fail	The event is recorded for $P_j$ .
[F T F]	[1 $\approx 0$ 1]	Select $P_i$ and fail	This time around in $W$ , a decision tree was created for $P_j$ and used. The decision tree returned $p \approx 0$ for $W$ . Subsequently $P_i$ was randomly selected and failed. The event was recorded for $P_i$ .
[T T F]	[ $\approx 0 \approx 0$ 1]	Select $P_k$ and pass	This time around in $W$ , a decision tree was created for $P_i$ and used along with the existing decision tree for $P_j$ . Both returned $p \approx 0$ for $W$ . Subsequently $P_k$ was inevitably selected and succeeded. The event was recorded for $P_k$ .
[T T T]	[ $\approx 0 \approx 0$ 1]	Select $P_k$ and pass	All decision trees are in use. Hereon $P_k$ will inevitably be selected most of the time which is what we expect.
[F F F]	[1 1 1]	Select $P_k$ and pass	The event is recorded for $P_k$ .
[F F T]	[1 1 $\approx 1$ ]	...	This time around in $W$ , a decision tree was created for $P_k$ and used. The decision tree returned $p \approx 1$ for $W$ which is what we expect. However since the default $p$ for $P_i$ and $P_j$ is also 1, then the selection probabilities have not changed at all. So even though we have witnessed previously that $P_k$ succeeds in $W$ , the probabilities used do not reflect this. This is not optimal.

Table 1: Impact of  $p$  on plan selection in  $W$  for a set of applicable plans  $[P_i, P_j, P_k]$

- If the default is  $p \approx 0$  then failing in a given world will not change the probabilities. The impact is that the probability of selection of the good plan will not improve doesn't matter how many times other siblings have been tried and have failed. So even though we may witness numerous times that every other applicable plan has failed in the given world, the selection probability of the good plan (that has never been tried before and has the default  $p \approx 0$ ) will not improve.

### 3 Improving *Stable* Performance I

#### 3.1 When to use a decision tree?

Section 2.1 shows how the choice  $m = 1$  leads to ill-informed decision trees that distort plan selection probabilities. An obvious remedy is to increase  $m$  to a suitable number that guarantees prediction within tolerance from the newly formed decision tree. However, one cannot determine this optimal number since instances are generated randomly and include duplicates. Furthermore, the higher the number the longer we have to wait to use the power of decision trees, which is also not ideal.

The function *useDT* currently determines when we are ready to start using a deci-

sion tree as follows:

```
if(sub-treeOK && instances>=m){
```

The code first checks to see that all children have their decision trees built and then confirms that the number of instances *of any world* seen so far is greater than  $m$ .

I recommend we change this as follows:

```
if(sub-treeOK && instances>=m && (doStable?haveSeen(W):1)){
```

The recommended change is that when deciding if we are ready to use a decision tree, we include one additional check that we must have witnessed the world  $W$  in question at least once before. In effect, we are saying that we are not confident in the tree for the given world unless we have seen that world at least once before, regardless of the number of total instances  $m$  seen so far. The change applies only to *Stable* and not *Concurrent* (determined by the *doStable?* check).

At this point one could argue that the additional check is too restrictive because you lose the case where  $m$  is large enough that the resulting tree would still give a good estimate of the probability in  $W$  even though we have never seen  $W$  before (that's the power of decision trees remember). That is true, and we could form a more complex condition as follows:

```
if(...?(haveSeen(W) || instances>=newM):...){
```

This would allow us to start using the tree even when we have not seen  $W$  but have seen enough instances to be confident that the decision tree prediction will be meaningful. While  $m$  is a static requirement,  $newM$  could be calculated dynamically based on a number of factors one of which would be the total number of worlds. For instance we could say that we are confident in a decision tree *if we have seen the world  $W$  before OR we have seen at least half (or any other fraction) of all possible worlds*. This decision is open for discussion, but for now I recommend only introducing the *haveSeen* check.

### 3.2 When no decision tree exists, default $p$ should be 0.5

Section 2.2 explains how the default values of  $p = 1$  or  $p = 0$  (for when no decision tree exists for a given plan) can distort plan selection probabilities. I recommend we change the default probability to  $p = 0.5$  for the following reasons:

- Using a default  $p = 0.5$ , when a plan finally switches to using the decision tree  $p$  will start to converge towards either 0 or 1 which is the true probability for that plan in the given world. We can say that the value  $p = 0.5$  is *neutral* towards the true probability of 0 or 1.
- When no other information is considered, and we have to *estimate* (i.e. by setting a default) at design time what the chances of success of a plan are, then the logical choice is 50/50, so a  $p = 0.5$  makes rational sense.

#### 3.2.1 Test Results

The test results from applying the recommended changes are included in the Appendix A.1.

- Page "Prior to applying suggested changes (Repository Revision 49)" shows the benchmark results before any changes were applied. Notice the problematic *testImpactvars* result on that page. These tests all used  $k = 3$ .
- Page "After applying suggested changes using  $k=3$  (Repository Revision 49+)" shows the results after applying the changes. The change does not break any previous tests but shows a slight improvement in *testImpactvars* convergence from  $\approx 0.25$  to  $\approx 0.4$ . These tests all used  $k = 3$ .
- Page "After applying suggested changes using  $k=1$  (Repository Revision 49+)" shows the results after applying the changes but using  $k = 1$ . What's the point? Well we want to verify that our changes make a difference to *testImpactvars*. This time, we get the result we expected in Section 1.2.

## 4 Improving *Stable* Performance II

### 4.1 When to use a decision tree: The *confidence* measure

The core issue discussed in Section 2 is that of mis-classification by decision trees due to incorrect generalisation, that in turn results from the paucity of samples during the early stages of online learning. Our discussions highlight the necessity to consider two key elements in the use of decision trees in this case:

1. The current *probability* of success in a given world as predicted by the decision tree; and
2. Our *confidence* in the current prediction.

For measuring confidence we presently use a crude criterion, *that the change in probabilities be small between successive queries*. Section 2 shows that this criterion is insufficient when dealing with multiple worlds as it does not consider the specific world in question. An improvement using a *haveSeen* check suggested in Section 3.1 is also not appropriate since it forces the strict constraint that the world be witnessed before the decision tree may be used to classify it, thereby defeating the purpose (i.e. interpolation) of using a decision tree in the first place.

In defining the characteristics of a *confidence* measure we identify the following properties:

- It must consider the world  $W$  being witnessed. Since we have not seen  $W$  before, then this becomes a function of how times have we seen *similar* worlds before. For a decision tree, we might frame this as - given a decision tree leaf node  $L$  that will classify  $W$ , how many instances of other worlds  $W' \neq W$  are being classified by the same node  $L$ .
- Over time, the measure must monotonically tend from  $0 \rightarrow 1$ .

For instance, consider the issue described in Section 2.1 where we have 20 applicable plans such that 19 bad plans report a correct  $p = 0$  while the single good plan reports an incorrect  $p = 0$  (generalisation error). Here  $\frac{19}{20}$  times a bad plan will be executed but the choice will be *wasted* because the result it will not change the relative probabilities (they are already all at zero). Ideally, one would expect that as the 19 other plans are getting selected and failing, that the relative probability of selection of



the good plan (not selected yet) should keep improving as a consequence. And when the good plan is finally selected and succeeds, its relative probability should increase further  $\rightarrow 1$ . Currently, the former does not happen - we gain no information from the failure of the other 19 plans when we should.

A well formed confidence measure would resolve this issue. In this case as each bad plan is selected and fails, this information is recorded in the confidence measure that consequently tends  $\rightarrow 1$ , even though the probability of success (already at zero) does not change with each failure.

Given this confidence measure, we can then use a threshold value to determine when it makes sense to use the probability given by the decision tree. For situations where the confidence measure is lower than the threshold, the default probability to use is 0.5 as described in Section 3.2.

The formulation of the confidence measure is yet to be completed and is open for discussion.

## 4.2 Guided exploration based on confidence

Currently, the probabilistic plan selection mechanism only considers the probabilities of success of the applicable plans and biases selection towards those that are relatively higher. Again, the earlier discussion in Section 2 shows that this criterion alone is not sufficient when the relative probabilities are the same.

The plan selection mechanism must therefore be extended to also include our confidence in the given probabilities of success. An informed exploration strategy might bias selection in order to maximise the *information gain* from the choice. Stéphane has proposed the following preliminary ideas for this extension:

- Given information about the number of instances  $n$  classified by leaf node  $L$  (where  $L$  is the node that also classifies  $W$ ), a first simple idea would be to select the plan with the lowest  $n$ . Equally we could select a plan with a probability inversely proportional to  $n$ .
- One possible refinement to this would be to take into account the size of the input space represented by the leaf node. For example, if the leaf nodes for two plans correspond to worlds  $[a]$  and  $[a \cdot \bar{b} \cdot c]$  respectively and each contains the same number of instances, then the selection could favour the plan that expresses the larger portion of the worlds space, in this case  $[a]$ .
- Another refinement is to use *entropy*. This would take into account the number of failures/successes instead of a simple count of the instances contained in the node of the decision tree. We can compute the change in entropy if the plan succeeds and if the plan fails, and maybe go with the plan with the biggest entropy drop. (Stéphane says: I would need more time to check whether that makes sense, if observing say a failure is more promising than observing a success in terms of information gain for one DT/plan, and the opposite for another DT/plan, the decision is not that simple).
- Another option is to *simulate* the possible updates of the decision tree for each plan and base the decision on the outcome. For instance, it is possible that for the decision tree of a plan  $P1$  that regardless of whether  $P1$  succeeds or fails, the decision tree structure will not change (the probability in the corresponding leaf node of the decision tree only will change). For another plan  $P2$  however,

the new instance may trigger a change in the decision tree structure. In this case, we would favor  $P2$  over  $P1$ .

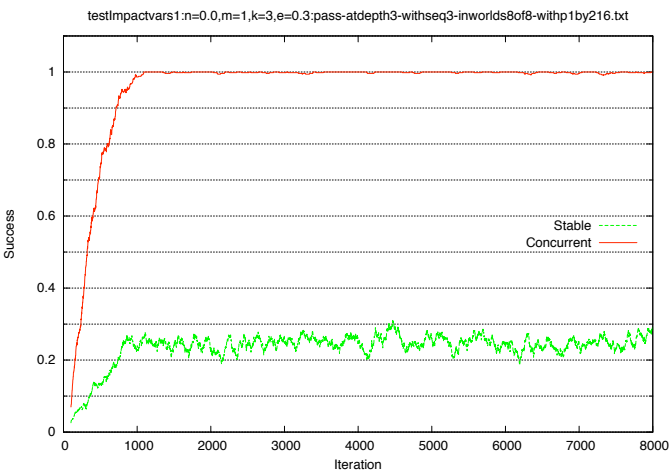
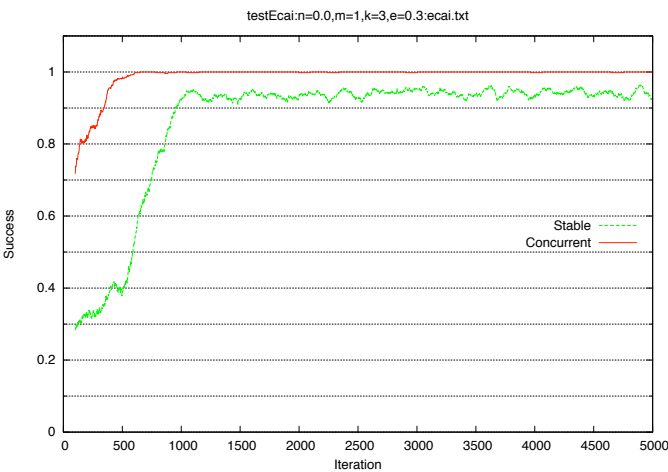
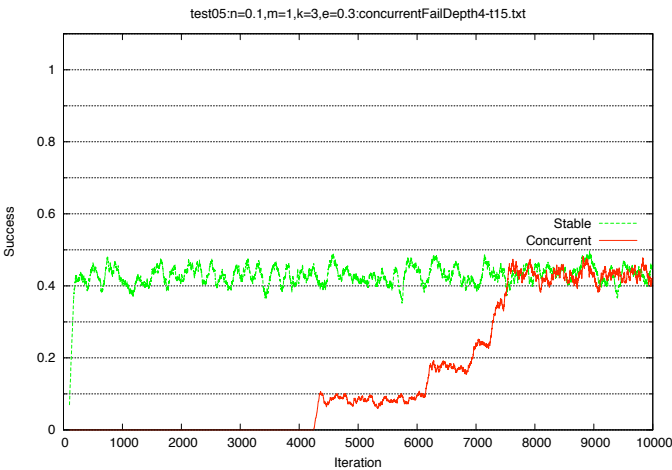
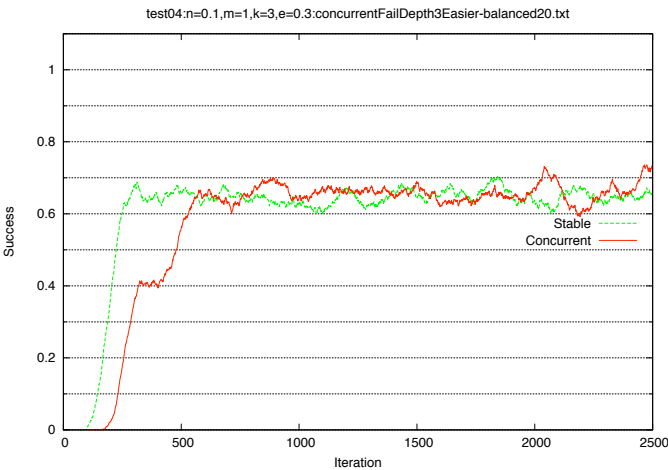
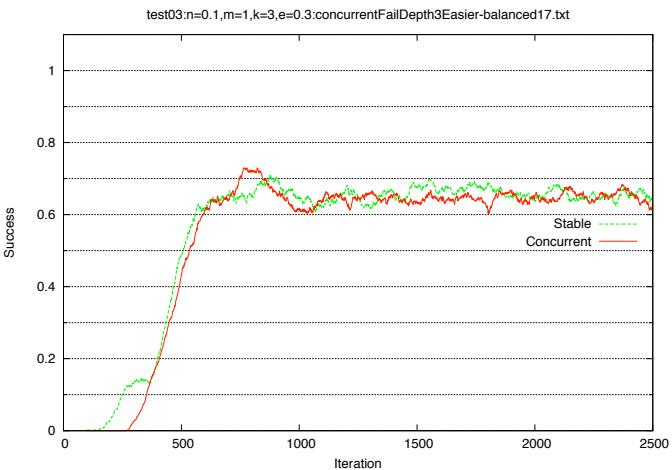
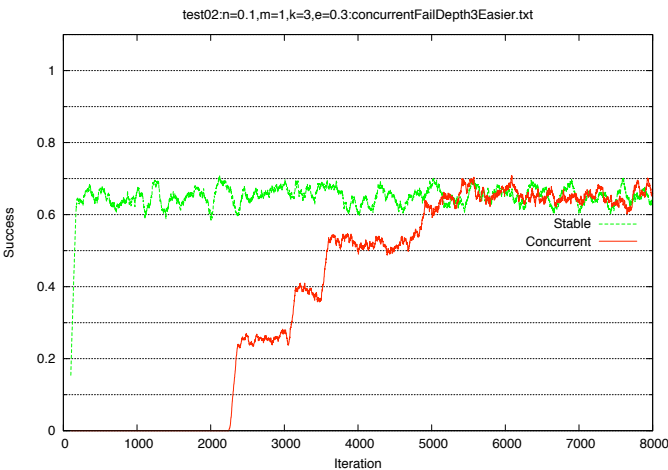
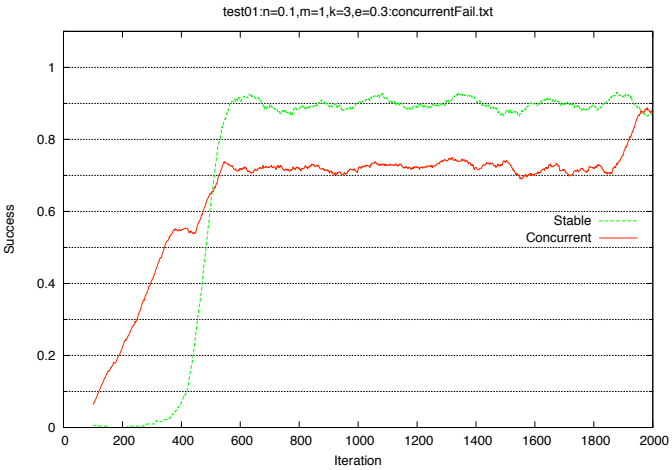
## **5 Acknowledgements**

Revisions of this document are a result of an ongoing discussion and contributions from Stéphane Airiau of the University of Amsterdam, and Sebastian Sardina and Professor Lin Padgham of RMIT University.

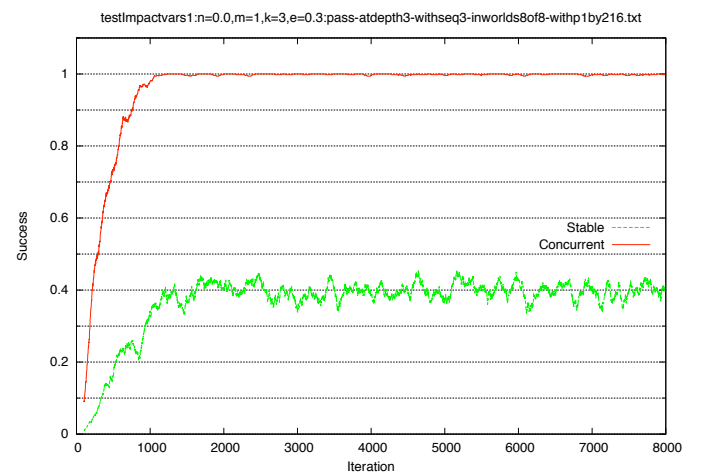
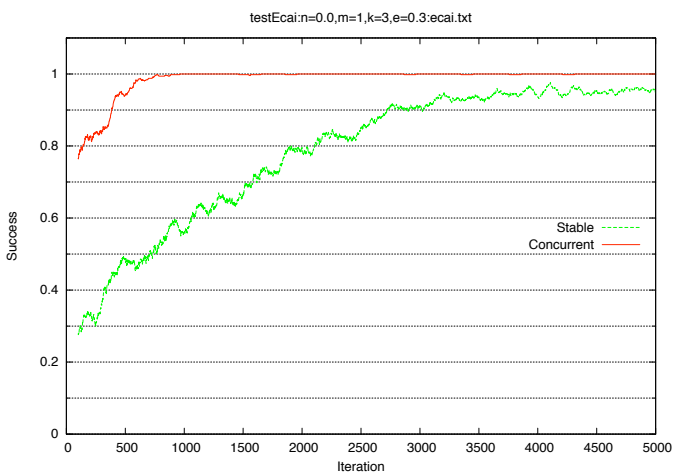
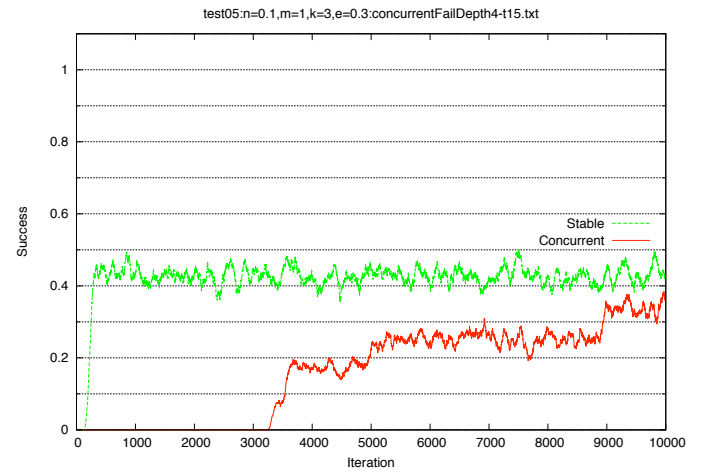
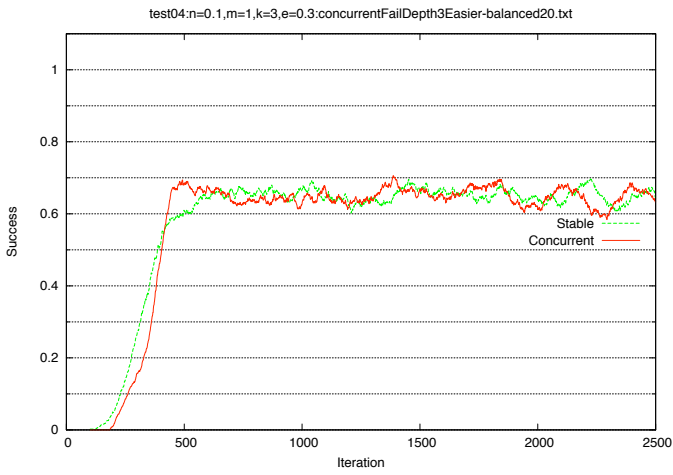
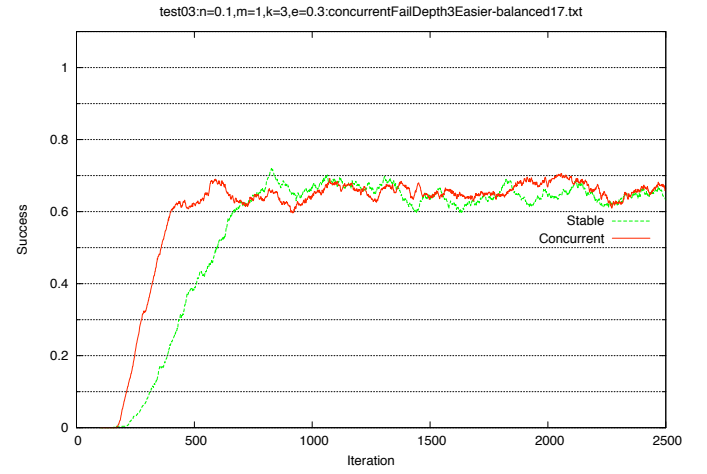
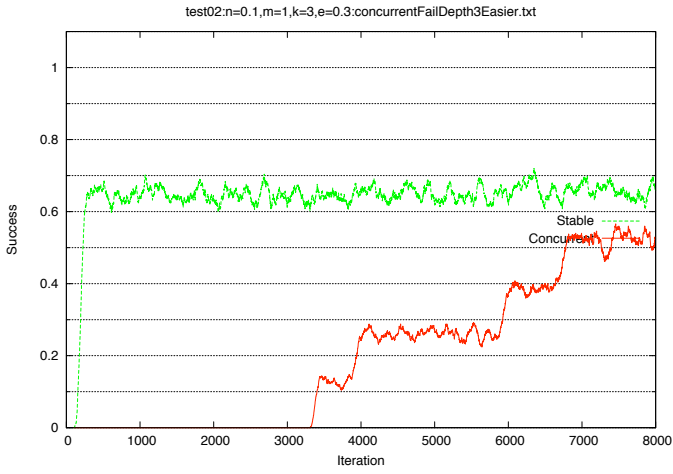
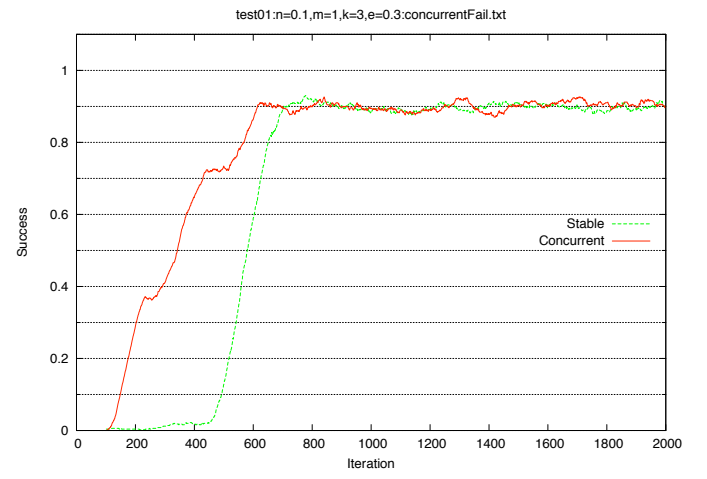
## **A Appendix**

### **A.1 Results for *Stable* Performance I**

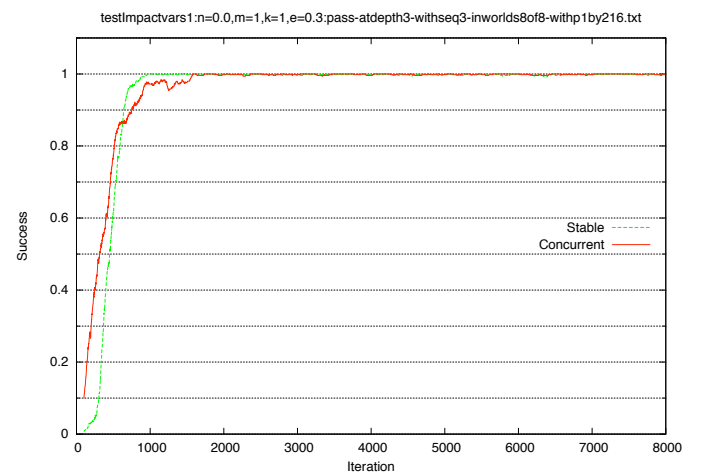
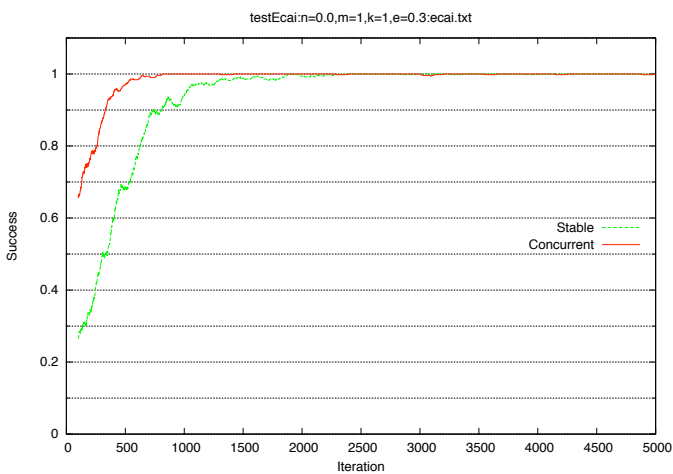
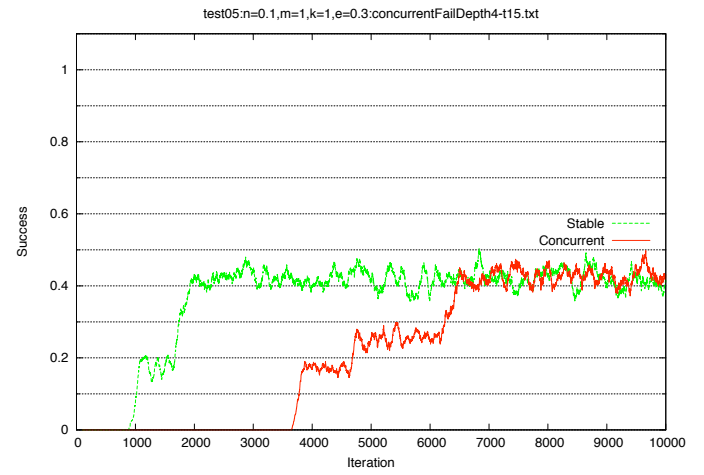
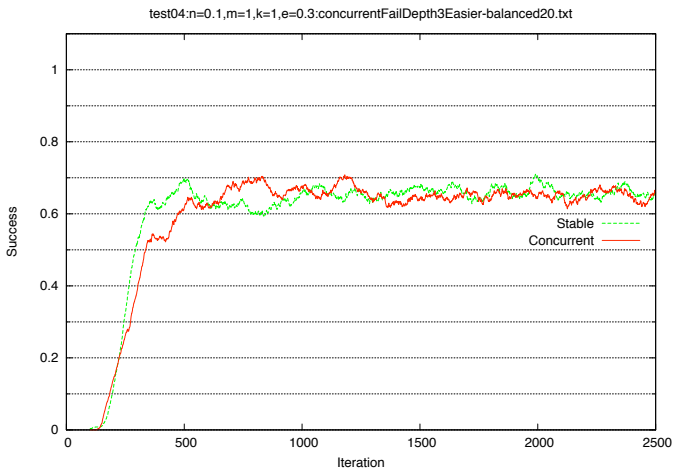
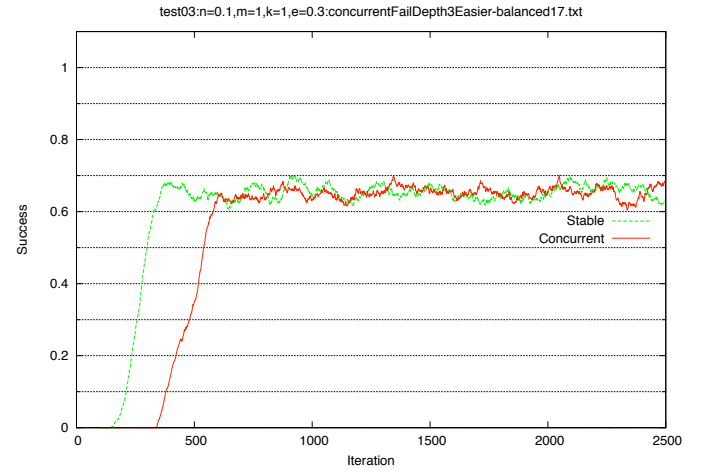
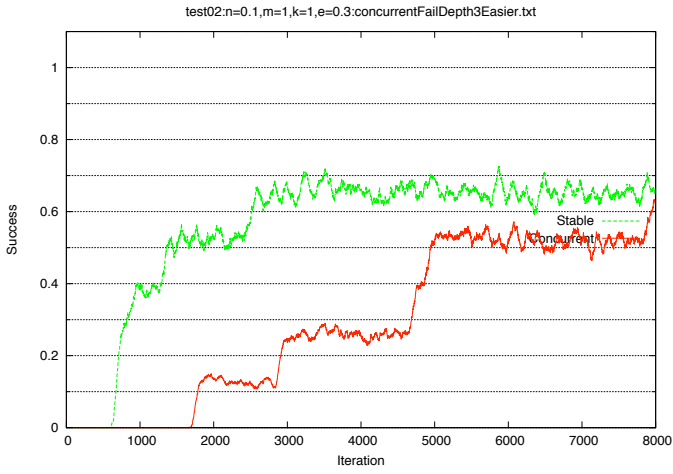
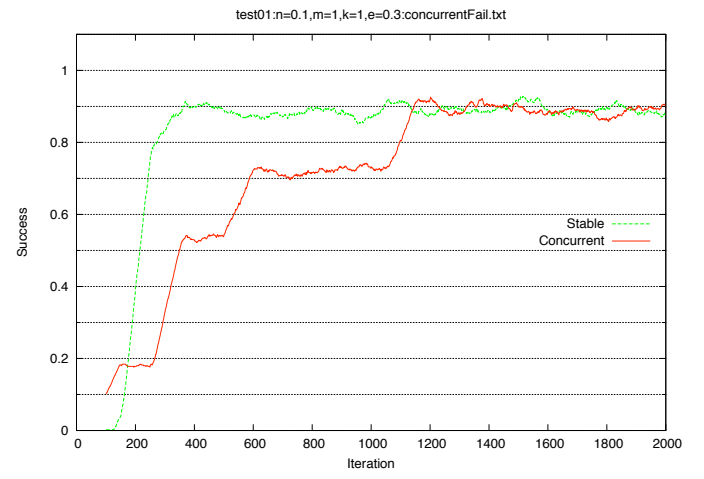
Prior to applying suggested changes  
(Repository Revision 49)



After applying suggested changes using  $k=3$   
(Repository Revision 49+)



After applying suggested changes using  $k=1$   
(Repository Revision 49+)



## A.2 Test coverage

Sebastian and I briefly discussed the *testImpactvars* result and agreed that it exposes some grey area about our understanding of *Concurrent* and *Stable*. Moreover, from our current tests it is difficult to understand the conditions under which one approach performs better than the other.

In considering how we can improve context learning, we need a better understanding of how the two current approaches perform in a combination of factors as listed in Table 2. So far we have experimented with some of these factors but not enough to get a good understanding of how they influence our approaches.

Factor	Tested
Branching factor of tree	Yes
Depth of tree	Yes
Stochastic nature of the environment	Yes
Number of worlds	Yes recently
If the G/P hierarchy is balanced (all sub-trees have the same breath/depth)	Barely (recently)
If the distribution of worlds within the G/P hierarchy is balanced (sub-trees handle an equal share of all possible worlds)	Barely (recently)
If more than one sub-tree holds a solution	No
If failure has a cost	No

Table 2: Factors that impact the performance of *Concurrent* and *Stable*

Our testing strategy is open for discussion. Should we conduct systematic testing to understand the influences of the various factors on the two approaches? If so, what is the best strategy here since the number of combinations is too high.

## A.3 Insights into the workings of *Concurrent* and *Stable*

Finally, here I have collected some insights into the workings of our two approaches in a generalised manner.

- *Stable* performs better when
  1. One solution exists in a deep sub-tree (note that differences between the approaches is amplified when the the probability of hitting that solution is lowered by fine-tuning the breath/depth of the sub-tree); and
  2. At least one other sub-tree is *shallower*; and
  3. the shallower sub-tree *does not* hold a solution.

In this case, *Stable* will realise first that the shallower sub-tree does not hold the solution and that the deeper sub-tree *may*. So it will assign a lower probability to the shallower sub-tree. (*Concurrent* will assign more or less equal probability to all sub-trees since none of them seem to work). In effect the probability of picking the deeper sub-tree increases and therefore *Stable* has a better chance of finding the solution there first.

- *Concurrent* performs better when
  1. One solution exists in a deep sub-tree (same as before); and

2. At least one other sub-tree is also *deep*; and
3. All other *deep* sub-trees *do not* have a solution (the more the number of failing deep sub-trees the more amplified the difference).

In this case, *Concurrent* performs the same as before. *Stable* however takes a long time to be confident that the failing deep sub-trees are in fact fruitless so it does not change their probabilities for a long time. When a solution is finally found, *Concurrent* favours that sub-tree whereas *Stable* still devotes exploration to the fruitless sub-trees until it is confident that no solution exists there.

- At the leaf nodes, the differences between *Concurrent* and *Stable* are minimal, but *Stable* takes longer to be confident that an observation of failure is in fact a true failure and not due to a stochastic environment.