Exam 2 - Loss Models 1

Question 1. (9 pts) Losses follow inverse Pareto distribution with $\theta = 50$ but unknown τ. You are told that 3 recorded loss amounts are 40, 50, 70, and one is less than or equal to 100. Find the MLE for τ.

Question 2. (9 pts) You are given:

(i) Losses follow a Pareto distribution where $\theta = 2000$.

(ii) A random sample of 20 losses is distributed as follows:

Loss Range	Frequency
$(0,\ 4000]$	10
$(4000,\ \infty)$	10

Calculate the maximum likelihood estimate of α.

Question 3. (9 pts)

Suppose that N follows a geometric distribution and you observe the following data:

# of Claims	0	1	2	3	4	5+
# of Observations	5	3	2	1	1	0

Find the MLE of p_1.

Question 4. (9 pts) A random sample of n claims $x_1, \ldots, x_n$ is taken from a distribution with the following survival function:

$$S(x) = x^{-\alpha}, \qquad x > 1.$$

Determine the maximum likelihood estimator of α.

Question 5. (9 pts) You have observed that the following three loss amounts:

$$186 \quad 91 \quad 66$$

Seven other amounts are known to be less than or equal to 60. Losses follow an inverse exponential with distribution function

$$F(x) = e^{-\theta/x}, \quad x > 0.$$

Calculate the maximum likelihood estimate of the population mode.

Question 6. (9 pts) A random sample of size n is drawn from an inverse gamma distribution where $\alpha = 2$. Calculate the asymptotic variance of the maximum likelihood estimator of θ.

(For Questions 7–9) You are given:

    (i) Claims have been observed from a lognormal distribution with unknown parameters μ and σ.

    (ii) The maximum likelihood estimates are $\mu = 5$ and $\sigma = 2$.

    (iii) The covariance matrix of μ and σ is:

$$\begin{pmatrix} 0.004 & 0 \\ 0 & 0.002 \end{pmatrix}$$

    (v) The expected claim is E.

    (vi) An approximate 95% confidence interval for the expected claim is $[L, U]$.

Question 7. (9 pts) Find the MLE of E.

Question 8. (9 pts) Find the asymptotic variance of the MLE of E.

Question 9. (9 pts) Calculate U.