Exam 3 - Loss Models 1

Question 1. (9 pts) You are given the following for a dental insurer:

- Claim counts for individual insureds follow a Poisson distribution.
- Half of the insureds are expected to have 1 claim per year.
- The other half of the insureds are expected to have 2 claims per year.

A randomly selected insured has made 1 claim in each of the first two policy years. Determine the Bayesian estimate of this insured's claim count in the next (third) policy year.

Question 2. (9 pts) The probability distribution function of claims per year for an individual is a Poisson distribution with parameter λ. The prior distribution of λ is a gamma distribution with density

$$\pi(\lambda) = \lambda e^{-\lambda}, \quad 0 < \lambda < \infty.$$

Given an observation of zero claim in a one-year period, what is the density of the posterior distribution of λ?

Question 3. (9 pts) You are given:

i. The annual number of claims for a policyholder has a binomial distribution with probability mass function

$$p(x|q) = \binom{3}{x} q^x (1-q)^{3-x}, \quad x = 0, 1, 2 \text{ or } 3.$$

ii. The prior distribution of q is

$$\pi(q) = 4q^3, \quad 0 < q < 1.$$

The policyholder has no claim in Year 1 and 1 claim in Year 2.

Determine the Bayesian estimate of the expected number of claims in Year 3.

Question 4. (9 pts) The size of a claim for an individual insured follows a gamma distribution, $\mathcal{G}(3, \lambda)$ with the following density function:

$$f(x|\lambda) = \frac{x^2}{2\lambda^3} e^{-x/\lambda}, \quad x > 0.$$

The prior distribution of λ is inverse gamma distribution, $\mathcal{IG}(1, 4)$ with the following density function:

$$\pi(\lambda) = \frac{(4/\lambda)e^{-4/\lambda}}{\lambda\Gamma(1)}.$$

One claim of size 3 has been observed for a particular insured. Which of the following is proportional to the posterior distribution of λ?

(A) $\lambda^{-2}e^{-4/\lambda}$

(B) $\lambda^{-3}e^{-5/\lambda}$

(C) $\lambda^{-4}e^{-6/\lambda}$

(D) $\lambda^{-5}e^{-7/\lambda}$

(E) $\lambda^{-6}e^{-7/\lambda}$

Question 5. (9 pts) You are given:

i. Losses on a company's insurance policies, $X|\theta$, follow a Pareto distribution, $\mathcal{P}a(1, \theta)$.

ii. For half of the company's policies $\theta = 1$; while for the other half $\theta = 3$.

For a randomly selected policy, losses in Year 1 were 4. Determine the posterior probability that losses for this policy in Year 2 will exceed 6.

Question 6. (9 pts) Five models are fitted to a sample of $n = 200$ observations with the following results:

Model	# of Parameters	Loglikelihood
I	1	-200
II	2	-198
III	3	-196
IV	5	-195
V	6	-194

Determine the model favored by the Bayesian information criterion (BIC).

(A) I

(B) II

(C) III

(D) IV

(E) V

Question 7. (9 pts) You are given the following observed claim frequency data collected over a period of 100 days:

Number of Claims per Policy	Observed Number of Policies
0	20
1	60
2	20

We want to test whether the data is from a Binomial distribution with $m = 2$ or not.

Calculate the value of chi-square goodness-of-fit test statistic.

(A) 3.4

(B) 3.6

(C) 3.8

(D) 4.0

(E) 4.2

Question 8. (9 pts) You fit a Pareto distribution to a sample of 200 claim amounts and use the likelihood ratio test to test the hypothesis that $\alpha = 1.8$ and $\theta = 8.0$. You are given:

(i) The MLEs are $\hat{\alpha} = 1.7$ and $\hat{\theta} = 7.8$.

(ii) The natural logarithm of the likelihood function evaluated at the MLEs is -830.22.

(iii) $\sum \ln(x_i + 8.0) = 610.60$.

Determine the result of the test.

(A) Reject at the 0.005 significance level.

(B) Reject at the 0.010 significance level, but not at the 0.005 level.

(C) Reject at the 0.025 significance level, but not at the 0.010 level.

(D) Reject at the 0.050 significance level, but not at the 0.025 level.

(E) Do not reject at the 0.050 significance level.

Question 9. (9 pts) You are given a random sample of observations:

$$0.1 \quad 0.2 \quad 0.5 \quad 0.7 \quad 1.3$$

You test the hypothesis that the probability density function is:

$$f(x) = \frac{3}{(1+x)^4}, \quad x > 0$$

Calculate the Kolmogorov-Smirnov test statistic.