

Formulating the Model

1.1 Defining the elements of the model

$W[C]$ = the total time spent calling a single customer, a function of the number of times a customer is called

t_1 = The time to turn on the phone and dial a number (6 seconds in this case).

$t_{2.1}$ = The time it takes for the call to be determined as no-answer, also the cutoff time for the distribution (25 seconds).

$t_{2.2}$ = The time it takes to identify that the call is busy (3 seconds).

$t_{2.3}$ = The time taken for the customer to pick up, an exponential distribution with mean μ . $0 \leq t_{2.3} \leq t_{2.1}$

μ = The mean time of an answered call (12 seconds).

t_3 = The time taken to end a call (1 second).

C = the number of times a customer is called with data about how the call went (busy, no one will answer, answered)

$p_{2.1}$ = The base probability of taking branch $t_{2.1}$ (0.3)

$P_{2.1}$ = The corrected probability of taking branch $t_{2.1}$ ($p_{2.1} + (1-p_{2.1}-P_{2.2})e^{(-1/\mu)*t_{2.1}}$)

$P_{2.2}$ = The probability of taking branch $t_{2.2}$

$P_{2.3}$ = The probability of taking branch $t_{2.3}$ ($(1-p_{2.1}-P_{2.2})(1-e^{(-1/\mu)*t_{2.1}})$)

1.2 Write the expression for the cumulative distribution function of X , and derive the expression for its inverse

We know the function for the cumulative distribution of an exponential variable:

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} \\ 0 \end{cases}$$

Bounded over the range $x \in [0, \text{INF})$. To invert the expression, we swap x and y :

$$x = 1 - e^{-\lambda y}$$

And solve for y :

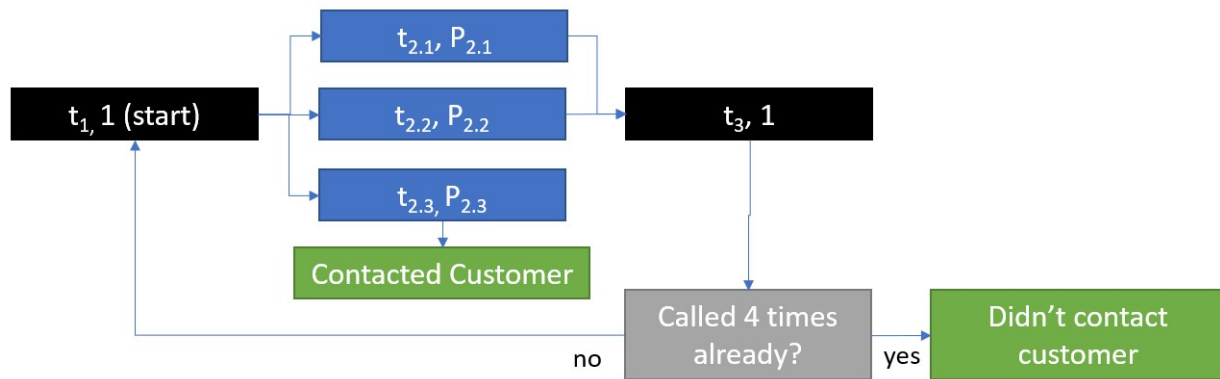
$$\begin{aligned} x &= 1 - e^{-\lambda y} \\ x - 1 &= -e^{-\lambda y} \\ 1 - x &= e^{-\lambda y} \\ \ln|1 - x| &= -\lambda y \\ \frac{\ln|1 - x|}{-\lambda} &= y \end{aligned}$$

For our specific model where $\lambda = 1/12$, the equation would be as such:

$$-12 * \ln|1 - x| = y$$

Assuming we maintain the bounds of the original equation, $x \in [0, 1]$.

1.3 Draw the tree diagram for the calling process



Where black indicates non-decision-making guaranteed nodes, gray indicates decision making nodes, blue indicates probability-attached nodes, and green indicates terminating nodes. Note that time-consuming nodes have a probability associated with them.

Here, variable values were used to accommodate the extension of this diagram into a full Monte Carlo simulation. For a specific diagram with a counter:

