sample Test 1 for Math 2164-002 Fall 2023

Name: ____________________________________

In questions 2-7 you must show your work to receive full credit.

(1) **[12pts]** For each statement, indicate if it is True (T) or False (F) (there won't be this many!)

(a) If the columns of a 4×4 matrix A are linearly independent, then $A\mathbf{x} = \mathbf{0}$ has no trivial solutions.

(b) If the columns of a 3×3 matrix A are linearly independent, then $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is consistent.

(c) If $A\mathbf{x} = \mathbf{b}$ has no solution, then at least one column of A is a non-pivot column.

(d) If A and B have the same reduced echelon form, then $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same set of solutions.

(e) If a 3×5 matrix has 3 pivot columns, then the columns span $\mathbb{R}^3$

(f) The reduced echelon form of the matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ -2 & 1 & 3 & 1 \\ -3 & 2 & 7 & -1 \end{bmatrix}$ is equal to $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(g) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, then one of $A\mathbf{b}$ and $A\mathbf{d}$ is undefined and one of them equals $2\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(h) If a 4×5 matrix A has 4 pivot columns, then $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ in $\mathbb{R}^4$

(i) If the last column of rref(A) is a pivot column, then $A\mathbf{x} = \mathbf{b}$ is not consistent.

(j) If one of the columns of a 4×4 matrix A is not a pivot column, then one of the columns of A is a linear combination of the others.

(k) If A is $m \times n$ and $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$ in $\mathbb{R}^m$, then $m = n$.

(l) If $\mathbf{b}$ is in the span of the columns of A, then the last column of $[A \mid \mathbf{b}]$ is not a pivot column.

(m) If $\mathbf{v}_4$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ then the fourth column of $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4]$ is a pivot column.

(n) If A and B are $m \times n$ and have the same reduced echelon form, then for all $\mathbf{x} \in \mathbb{R}^n$, $A\mathbf{x} = B\mathbf{x}$.

(o) An echelon form of the matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ -2 & 1 & 3 & 1 \\ -3 & 2 & 7 & -1 \end{bmatrix}$ is equal to $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(p) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, then one of $A\mathbf{b}$ and $A\mathbf{d}$ is undefined and one of them equals $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(q) If $A \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 8 \end{bmatrix}$, then $A\mathbf{x} = \begin{bmatrix} 2 \\ 8 \\ 12 \\ 16 \end{bmatrix}$ has infinitely many solutions.

(r) If $A\mathbf{u} = \mathbf{b}_1$ and $A\mathbf{v} = \mathbf{b}_2$, then $A\mathbf{x} = 3\mathbf{b}_1 - 5\mathbf{b}_2$ is consistent.

(2) [**6pts**] Solve the following systems of linear equations (you can use calculator to get rref):

(a)
$$\begin{aligned} x + y - z &= 1 \\ -x - y + 2z &= 3 \\ 2x + y - 3z &= -2 \end{aligned}$$

(b)
$$\begin{bmatrix} 2 & 4 & 3 \\ 8 & 2 & 5 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

(3) [**4pts**] For the system given below (no calculator)
(i) write down the augmented matrix and then (ii) find a value for the constant k so that the system is inconsistent:

$$\begin{aligned} x + y + 5z &= -3 \\ x + 2y - 5z &= 3 \\ 8x + 8y + k\,z &= 37 \end{aligned}$$

(4) [**6pts**] Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

A is the matrix $\begin{bmatrix} 1 & -2 & -1 & -2 & -2 \\ 3 & -6 & -2 & -2 & -2 \\ 4 & -8 & -3 & -4 & -4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

the reduced echelon form of $[A|\mathbf{b}]$ is $\begin{bmatrix} 1 & -2 & 0 & 2 & 2 & -7 \\ 0 & 0 & 1 & 4 & 4 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(a) State which variables are basic variables and which are free variables.

(b) Express the solution set for $A\mathbf{x} = \mathbf{b}$ in parametrized form and then in parametrized vector form.

(c) Express the solution set for $A\mathbf{x} = \mathbf{0}$ as the span of a set of vectors.

(5) **[7pts]** Let $A = \begin{bmatrix} 3 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 5 & 3 & -1 & 4 \end{bmatrix}$, and let $\mathbf{b} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$, and let $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ be the columns of A.

(a) Decide if $\mathbf{b}$ is in the span of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$.

(b) Decide if $\mathbf{a}_4$ can be expressed as a linear combination of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ and explain your reasoning.

(c) Decide if the columns of A span $\mathbb{R}^3$.
You can use your work in parts (a), (b) to justify your answer for part (c).

(6) **[5pts]** Let $A = \begin{bmatrix} 4 & -7 & -3 \\ -8 & -7 & -7 \end{bmatrix}$ and suppose that $A\,\mathbf{u} = \mathbf{b}$

(a) State how many coordinates that each of $\mathbf{u}$ and $\mathbf{b}$ have.

(b) Evaluate $A\,\mathbf{u}$ where $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

(c) Decide if there is a $\mathbf{b}$ such that $A\,\mathbf{x} = \mathbf{b}$ is inconsistent and justify your answer.

(7) [**5pts**] We are thinking of a 2x2 matrix A satisfying

that $A\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ 21 \end{bmatrix}$ and $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$.

(a) Solve the vector equation $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_1\begin{bmatrix} 3 \\ 6 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(b) Determine the value of $A\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Hint: it is the same as $A\left(c_1\begin{bmatrix} 3 \\ 6 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

(c) If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then simplify $A\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $A\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(d) Solve for the matrix A
hint: think about part (c)

Total points: 45