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Should the police give priority to violence within criminal organizations? A personnel economics perspective

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Abstract

Even among themselves, criminals are not seen as trustworthy. Consequently, a criminal organization needs to incentivize its members, either by threats of violence or by rewarding good behavior. The cost of using violence depends on the resources police allocate to investigating intraorganizational violence. This means that the police may affect the choice of an incentive scheme by the criminal organization. The design of the optimal strategy for crime control has to take this into account. We develop a model of an infinitely repeated criminal labor market where (i) a criminal organization hires and incentivizes members, and (ii) peripheral crime (crime outside the criminal organization) is a stepping stone to a career in organized crime. We establish that there are two possible optimal strategies for the police. (i) There are situations in which the optimal strategy for the police is to use all of their resources to decrease the efficiency of criminals. (ii) In other situations, the optimal strategy for the police is to spend the minimum amount of resources to ensure that the criminal organization cannot punish disloyal criminals, and spend the rest of their resources to decrease the efficiency of criminals.

KEYWORDS
incentives mechanisms, organized crime, police policy

> They always fudge. They figure they're out doing the job, who wants to give up half of what they get to somebody that's not even there? ... Now the thing is, it's a dangerous game, because if you get caught, you're liable to get whacked—killed. Joseph D. Pistone (better known as Donnie Brasco) on criminals of the American Mafia (Pistone & Woodley, 1987)

1 | INTRODUCTION

Many countries are struggling with increases in organized crime.[1] For many criminal organizations, the threat and use of violence are important tools, both externally and internally. This raises the question of whether public authorities should care about intraorganizational violence. What is the problem if one criminal kills another? In other words, should the police focus on investigating murders committed within criminal organizations?

In this paper, we use standard insights from personnel economics to show that focusing on cases of intraorganizational violence can reduce organized crime in equilibrium. However, this is not without its costs. Surprisingly, suppressing organized crime in this way may attract more people to crime.

Criminal organizations face the problem of maintaining internal discipline. Even among themselves, criminals have a reputation for not being trustworthy (see, e.g., Campana, 2016; Campana & Varèse, 2013; von Lampe, 2016, p. 119; as well as the quote from Pistone at the beginning). Because criminal organizations have good reasons not to use receipts, there are ample opportunities to steal from the organization. Instead, many criminal organizations rely on fear to discipline their members. Threats of physical punishment, death, or harming a member's loved ones are not uncommon.[2] However, maintaining discipline through fear requires that the threat of punishment be credible. In this paper, we explore the advantages and disadvantages for the police of limiting the

[1]A 2011 report by the US National Security Council states: "Transnational organized crime (TOC) poses a significant and growing threat to national and international security" (https://obamawhitehouse.archives.gov/administration/eop/nsc/transnational-crime/threat)

[2]For instance, Hopkins et al. (2013) provide evidence that homicide and violence are used in retaliation/punishment for theft from the group. According to Campana (2016) and Campana and Varèse (2013) there is typically little trust among criminals which is not backed up by some type of threat. Finally, the UNODC (2014, pp. 35–42) reports the use of violence for internal discipline for some criminal organizations.

criminal organization's options for maintaining internal discipline by increasing the costs of intragang punishments. In particular, we consider an infinitely repeated labor market where a crime boss hires and incentivizes members for her organization (*organized criminals*). Nonhired individuals can choose an occupation as *an upstanding citizen* or as a criminal outside the organization (*peripheral criminal*). Then organized criminals choose whether to be disloyal, that is, steal from the boss. If an organized criminal is disloyal, the boss faces the choice of whether to punish him or not. If she does, she faces a cost. Note that the boss may not incur this cost directly. However, the boss does need to compensate the members who do incur that cost. Consequently, modeling this as a cost directly to the boss simplifies the analysis without affecting the results. This cost increases with the resources police allocate to detecting these punishments. If instead the boss does not punish, she loses credibility and is replaced by a new boss. Therefore, the boss needs to induce loyalty. She can do so by punishing disloyalty, by rewarding loyalty,[3] or both.

Our model has six main assumptions. First, there is just one criminal organization. This allows us to focus on the incentives for organized criminals, rather than on the competition between criminal organizations. Second, the organization consists of a hierarchy of a boss and organized criminals, with the boss as the residual claimant.[4] Third, people die. This may be due to punishment or due to natural causes. Consequently, the criminal organization needs to look for new members periodically. Fourth, the criminal organization seeks new members among peripheral criminals before it invites upstanding citizens.[5] Fifth, peripheral criminals compete with each other, but not with the criminal organization. Although this assumption is made primarily to simplify the analysis, there is evidence that the criminal activities of peripheral criminals and organized crime do not overlap much (see, e.g., Paoli, 2002, 2013). In Section 6, we discuss how this affects our results. Finally, no crime boss will hire any worker who has a history of disloyalty.[6]

We find three equilibria in which the organization is formed.[7] Each of these equilibria is associated with a specific mechanism for controlling members of the organization. We find that the incentives to induce loyalty depend on the cost of punishing disloyalty to the boss and the amount organized criminals can gain by being disloyal (the "strength of the temptation"). If both the cost of punishment and the strength of the temptation are sufficiently small, the boss will punish disloyalty. Wages compensate the criminals for the opportunity costs of not working as an upstanding citizen, and for the expected costs of being caught and convicted.[8] The expected current and lifetime payoffs are the same for individuals in all three types of employment.

[3]Chalfin and McCrary (2017), among others, show that criminals do react to monetary incentives. Levitt and Vankatesh (2000) show that gang members are also likely to be incentivized by the prospect of future payoffs.

[4]According to Schloenhardt (1999): "the profit of organized criminal activities go to people who stand back and are not directly involved in committing the crime." In our model, these people are captured by a single agent: the boss.

[5]There is evidence that past criminal acts are used as signals within the criminal community. For instance, Gambetta (2009, p. 16) writes: "villains can certainly ask a potential partner or recruit to give them evidence of having committed crimes." This assumption simplifies the analysis, but is not necessary by itself. Our analysis requires only that past disloyalty reduces the chance of being hired.

[6]This too relates to trust. See Gambetta (2009) and von Lampe and Johansen (2004).

[7]We assume that if neither reward nor punishment is used, there will be no incentive to form the organization because all organized criminals will become disloyal.

[8]Carvalho and Soares (2016) show that members of a criminal organization do earn higher wages, yet also face additional risks.

Punishment ceases to be credible if the costs of punishing become too large. In that case loyalty needs to be bought. We model this as efficiency wages.[9,10] The higher wages make it more costly for criminals to be fired for disloyalty, especially if future bosses are unlikely to rehire disloyal organized criminals.[11] If wages within the criminal organization are high enough, then this reduction in future wages outweighs the benefits of disloyalty now. However, efficiency wages increase the marginal costs of employing an organized criminal. Consequently, it reduces the optimal size of the organization. Efficiency wages also give organized criminals a better payoff than upstanding citizens. Because the criminal organization primarily hires new members among peripheral criminals, these criminals have a better chance of earning efficiency wages in the future. This attracts more individuals to employment as peripheral criminals. Thus, peripheral crime increases, and the current payoff of peripheral criminals decreases below that of upstanding citizens. Organized criminals do strictly better than both peripheral criminals and upstanding citizens.[12]

The third option is that the threat of punishment is credible but insufficient, so when punishment costs are low and the temptation is large. In that case, the boss needs to supplement the threat of punishment with an efficiency wage. Qualitatively, the consequences of the efficiency wages are the same as without a credible threat of punishment.

Finally, when the temptation is so strong that a criminal organization cannot properly incentivize its members and avoid a loss, there is no organized crime in equilibrium.

The strength of the temptation, which depends on the nature of the activities carried out by the members of the organization, determines how strong the incentives need to be. For example, activities such as transporting illicit goods or running gambling houses offer many opportunities for theft, which can be very tempting for the organized criminal responsible for these operations. This means that from the boss's perspective, the risk of loss to the organization is significant. The strength of the relationship between the boss and members of the organization may also be an important element in assessing the risk of embezzlement.

On the basis of some empirical studies, the three equilibria described in this model do exist. The equilibria differ in two aspects. First, does the organization use (the threat of) violence to discipline its members? Second, do members earn above-market wages? The second aspect is often harder to assess, because the market wage should reimburse the criminal for the risks he faces.

The well-documented Lavin enterprise in the USA is an example of a criminal organization which did not threaten its members with violence, but rather rewarded them for their loyalty (see, e.g., von Lampe, 2016, pp. 130–135). These two elements refer to an equilibrium in which

[9]The original idea of efficiency wages was to compensate for imperfect monitoring (see both Akerloff, 1982; Shapiro & Stiglitz, 1984). Here it compensates for a strong disalignment in preferences.

[10]There are many ways to reward loyalty. For example, Levitt and Vankatesh (2000) show an example of an organization in which the prospect of internal promotions to better paid positions provides incentives. Our main qualitative results do not depend on the specific form of the reward, as any reward raises the expected lifetime payoff of an organized criminal. Modeling the loyalty reward as an efficiency wage has the advantage of simplicity. Moreover, it is renegotiation proof in the context of our model.

[11]There is considerable evidence from the field that criminals are unwilling to work with other criminals, unless the other has a reputation of being trustworthy (see von Lampe, 2016, p. 115).

[12]In this paper, we ignore any hold-up problems which may occur between criminals and criminal organizations. Those considerations warrant a separate study. One reason that hold-up problems are complex in a criminal context is that interaction may give all involved parties incriminating information on each other.

control within the organization is based exclusively on reward/efficiency wages (hereafter referred to as a Reward equilibrium).

The Liang Xiao Min Syndicate (group 35 in UNODC, 2002) is an example of a criminal organization which used both the threat of violence as well as efficiency wages to incentivize its members. The report states on Liang (the boss): "His control over the syndicate relied on two factors. First, the members were bound by generous and regularly paid salaries. Second, the 'house rules' were rigorously enforced. The rules obliged the members, among other things, to report everything to Liang, be absolutely loyal to him and never to leave the syndicate. If a member violated a rule, the punishment could include cutting off a finger or breaking a leg, depending on the severity of the transgression." These elements show an equilibrium in which the organization relies on both the threat of violence and the payment of rewards (hereafter referred to as a Punishment and Reward equilibrium). Note that this organization operates in sectors, such as the trafficking of illicit goods, gambling houses, and nightclubs, where there are relatively good opportunities for members to be disloyal.

Finally, some organizations rely almost exclusively on the threat of violence. Examples from UNODC (2002) include[13] a Yugoslav organization of human traffickers based in the Netherlands and the Balkan (group 7), The Fuk Ching in the USA (group 16), and the Savlokhov Group in Ukraine (group 25). In the same vein, the report refers to an organization in Australia, the Outlaw Motorcycle Gangs (group 10), whose principle of internal control consists in the use of violence: "The threat of violence is widespread, and is essential to ensure group cohesion. The sergeant at arms is responsible for internal discipline and punishment." Note that this criminal organization operates in sectors (human trafficking, trafficking of stolen vehicles, etc.) where members are likely to have fewer opportunities to be disloyal than in the Liang Xiao Min Syndicate. An organization that regulates the behavior of its members solely through the use of violence will hereafter be referred to as a Punishment equilibrium.

Next, we study what the police should do. Obviously, the strategy of the police should take into account how it affects the strategy of the criminal organization. Specifically, we consider the goal of the police to be to minimize the impact of criminals on the welfare of society. We assume that the police allocate their resources over two possible tasks. The first task is to detect all crimes other than intragang punishments. This lowers the efficiency, and thus profitability, of crime overall. The second task is to detect intragang punishments. This increases the expected cost of punishment. Indeed, the police can raise the costs of punishment to the organization by focusing on homicides and cases of violence which seem to result from punishments within the criminal organization. If these costs are high enough, the threat of punishment becomes noncredible, which disrupts criminal organizations. We establish that there are two types of strategies that the police can use. (1) The police use all their resources to detect crimes that are not related to punishments by the criminal organization, thus reducing the efficiency of the criminals. (2) The police spend just enough resources to make the threat of punishment noncredible, and allocate the rest of their resources to lowering the efficiency of criminals. The police have to take into account the interaction between their strategy and the criminal job market to design an optimal policy.

Finally, we discuss several extensions. First, we consider the case where criminals and the boss are more likely to die than upstanding citizens. Second, we allow for imperfect

[13]For most criminal organizations, information on the income and participation of its members is hard to find. Therefore, it is difficult to say with certainty whether members are not also rewarded.

information on the loyalty of organized criminals. Third, we consider nonmaximal punishments, as well as the strategic choice of a retirement option for the boss.

Clearly, we are not the first to study the relationship between the actions of the police and criminal activity. The closest recent paper is Bac (2022). This paper studies crime chains, where each step in a chain creates value for the boss, until either the chain is completed or to the step where the police succeed in disrupting it. The police try to disrupt the chain, either at the source, or at a node in the chain unknown ex ante. In particular, the police allocate their budget over the two disruption strategies (the source or random nodes in the chain). In our model the police allocate their budget over a different choice: fight crime in general (decreasing its profitability) or increase the cost of internal punishments.[14] Friehe and Miceli (2018) also address the issue of police crime fighting. Like in our paper, the police can choose to put in more effort to arrest criminals. Unlike our paper, criminals can make efforts not to get caught. Moreover their paper is focused on individual criminals, rather than criminal organizations. Friehe and Mungan (2021) build a model to explain the excessive practice of police stops. In contrast to our paper, the actions of the police are not examined in light of the need for the criminal organization to maintain internal discipline.

Piccolo and Immordino (2017), Gamba et al. (2018), Acconcia et al. (2013), and Spagnolo (2000, 2004) study the effects of leniency programs on criminal organizations. Like us, they are concerned with the stability of the criminal organization. Unlike us, they look at how governments can tempt organized criminals to become disloyal and inform on the organization. Instead we focus on how a crime boss can enforce loyalty when organized criminals have the opportunity to steal from the organization.

Akerloff and Yellen (1994) and Dur and van der Weele (2013) also consider the role of the police. Akerloff and Yellen suggest that the police may lose support from a neighborhood if the police treat criminals born and raised in the neighborhood too harshly. This may shift how much information the police will obtain. Then a "Beckerian" harsher crackdown is not always productive. On the other hand, Dur and van der Weele (2013) show that there may also be a double dividend in harsher punishments for petty crime. They consider a criminal environment in which criminals would like to be seen as tough. If "weak" criminals dare to do petty crimes, "tough" criminals need to commit severe crimes to reveal their toughness. If however petty crimes become risky enough to scare off weak criminals, a petty crime would reveal that a criminal is tough. Therefore, harsher penalties for petty crime may also reduce severe crimes. In contrast, our paper does not consider the harshness of law enforcement per se. Rather it looks at police priorities: should the police focus on investigating cases of violence within criminal organizations?

Our paper also belongs to the literature on criminal labor markets. Important earlier contributions in this field include Chang et al. (2005) and Polo (1995). Chang et al. (2005) also let individuals decide between a normal job, peripheral crime, and organized crime. In that study, the criminal organization can provide some benefits to the members, such as protection from the law. However, members pay a membership fee. Criminal organizations make profits by selling memberships. In our paper, the focus is on how the criminal organization incentivizes its members, rather than the protection it can provide to them.[15] In addition, we

[14]Baccara and Bar-Isaac (2008) analyze the internal organization of criminal groups but do not address optimal police strategies.

[15]Within our model, we can also consider a criminal organization which both employs and protects its members by lowering the risk or consequences of getting caught for organized criminals. This basically lowers the wage the boss needs to offer to the criminals.

look at another type of criminal organization. In our criminal organization the boss is the residual claimant, rather than the members. Like us, Polo (1995) considers how the boss can incentivize the members of her organization through punishment and wages. Unlike us, he focuses on the ability of the boss to punish, in the absence of law enforcement. In his model punishment is noncostly and always credible. Instead, we focus on the willingness to punish if law enforcement is present. This allows us to study how the police strategy affects the incentive structure within the criminal organization.

Finally, Garoupa (2007) deals, like us, with the optimal size of the criminal organization. Unlike us, he focuses on the boundary of the organization: which tasks should be done by the organization and which should be outsourced when this decision affects the organization's vulnerability to law enforcement. In our case, the size of the firm is not determined by the outsourcing decision, but rather by the incentive scheme to induce loyalty.

The paper proceeds as follows. In Section 2 we present the model and in Section 3 the resulting equilibria. In Section 4 we analyze the implications for optimal police policy. In Section 5 we discuss several possible extensions to the model. Finally, in Section 6 we conclude.

2 | ORGANIZED CRIME MODEL

Consider an infinitely repeated game of a society with crime. Let there be one criminal organization and two types of players. First, in each period $t \in \{1, 2, ...\}$ there is a risk-neutral boss. The boss, she, is the head of the criminal organization. She hires members for the criminal organization and decides upon the punishment of any disloyal member. Second, there is an infinite population of risk-neutral individuals, $\mathcal{N} = \{1, 2, 3 ...\}$. They choose their occupation: criminal (peripheral or organized) or noncriminal, and, if a member of the criminal organization, whether to be loyal or disloyal. Individuals may die during the game. For simplicity, we assume that the population of individuals is countably infinite.[16] The infinite population of living individuals at time t is denoted by $\mathcal{N}^t, \mathcal{N}^t \subseteq \mathcal{N}$. Every player maximizes his or her expected lifetime payoffs.

2.1 | Timing of the game

We start with an overview of the game (see Figure 1). Period t is divided into six steps. In step 1 the boss hires workers for the organization. We call such a worker an *organized criminal* $[\mathrm{oc}]$. In step 2, each organized criminal chooses whether to be *loyal* [loy], or *disloyal* [disl]. In step 3, every other individual decides whether to be an *upstanding citizen* $[\mathrm{uc}]$, having a normal job, or to be a criminal outside the organization, a *peripheral criminal* $[\mathrm{pc}]$.[17] In step 4, payoffs of individuals are obtained. These payoffs include the costs that criminals may face from the criminal justice system. In step 5, for each organized criminal, the boss learns whether that

[16]The focus of this paper is on the internal organization of criminal organizations. To keep this focus we want to keep the population dynamics resulting from deaths, for example, of a punished disloyal criminal, as simple as possible. This assumption allows us to do so. An alternative would be to allow the “birth” of new individuals in the population. Then the analysis would also need to involve the derivation of a steady state size of the population.

[17]In practice ex-criminals are disadvantaged in the regular labor market. This will not affect our main results. We will discuss this in Section 6.

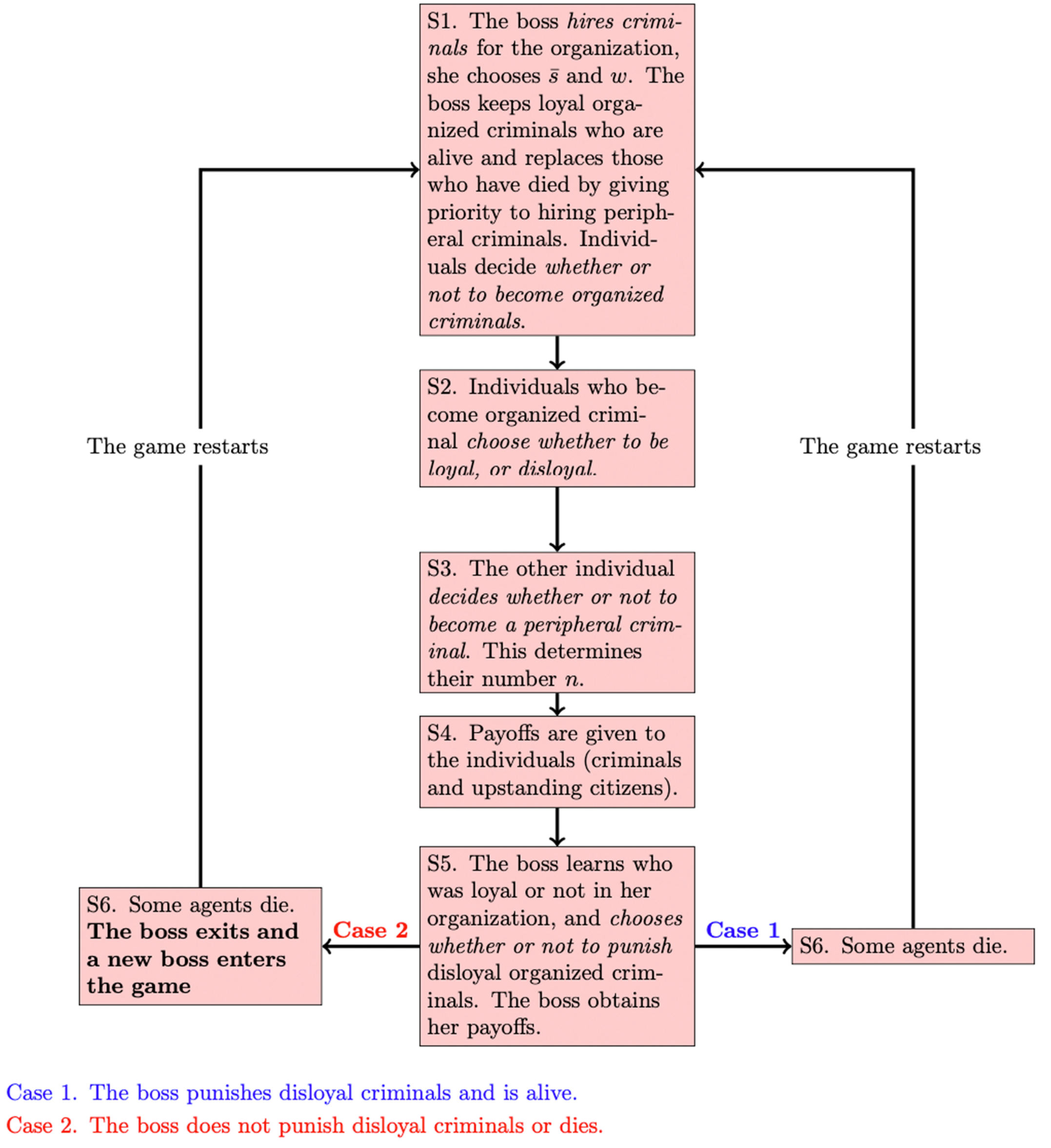

FIGURE 1 Timing of the game.

criminal is loyal or not. If disloyal, the boss decides whether to punish or not, and she obtains her payoffs including the costs imposed by the criminal justice system for intragang violence. In step 6, and finally, some players leave the game. Every player who is not punished dies with probability $(1-\rho)$, where $\rho \in (0, 1)$.[18] Players punished by the boss die and leave the game with probability 1. Also, the boss leaves the game at this moment if she left any disloyalty

[18]Of course, in real life the survival probability of an individual may depend on his occupational choice. Allowing for this does not affect our main results. We discuss this in more detail in Section 5.1.

unpunished. If the boss leaves the game, a new boss enters the game. Police sanctions are applied at step 4 for crimes against upstanding citizens, and at the end of step 5 for intragang violence. Then, period $t + 1$ starts.

2.2 | Criminal organization: Size and wage at *t*

The job of the boss has two aspects: hiring and punishing. Her hiring strategy consists not only of a wage offer, w^t_{oc}, but also the number of positions, that is, the maximum number of individuals that she will hire, $\bar{s}^t$. Given $(\bar{s}^t, w^t_{\text{oc}})$ the boss approaches individuals, one by one, until either $\bar{s}^t$ individuals have accepted, or all eligible individuals have been approached. Let the number of organized criminals hired (the recruitment realized) in period t, s_t, be the size of the organization. In period t, we have $s^t \le \bar{s}^t$.[19] We make the following assumptions on whom the boss approaches and in which order. First, no boss wants to employ an individual who has been disloyal in the past. So if individual i was a disloyal organized criminal at period τ, he is not approached in any period $t > \tau$. Second, the boss approaches individuals who were, at $t - 1$, organized criminals before those who were peripheral criminals, and those who were peripheral criminals before those who were upstanding citizens.[20] Note that the wage w^t_{oc} is not a wage per se. Rather it is the amount of money an organized criminal makes as part of the organization, provided that he does not cheat the boss out of her money.

2.3 | Criminal organization: Profit and punishment

The second task of the boss is to punish disloyal members of her organization. The boss learns from each organized criminal whether he was loyal.[21] If disloyal, she faces a choice: either to kill him or to let him go unpunished. To simplify the analysis, the boss learns and decides for each organized criminal before she learns whether any other organized criminal was disloyal. Each punishment has an expected cost of z. If she punishes each disloyal organized criminal, she stays in control of the organization: she will be the boss in period $t + 1$, unless she dies. If she lets any disloyalty go unpunished, she loses control of the organization and leaves the game at the end of the period.[22]

The boss is the sole residual claimant of the organization. She has the following profits at time t:

$$\pi^t(w^t_{\text{oc}}, s^t, s^t_{\text{loy}}, s^t_{\text{disl}}, s^t_p) = g(s^t_{\text{loy}}) - w^t_{\text{oc}} s^t - z s^t_p - s^t_{\text{disl}} \varepsilon, \tag{1}$$

[19]To avoid technical difficulties, we assume that s^t and $\bar{s}^t$ are continuous variables. The qualitative results do not depend on this assumption.

[20]Before the first period, all individuals are upstanding citizens.

[21]This has the unattractive feature that criminals cannot cheat upon the crime boss without her finding out. In Section 5.2 we consider inconclusive signals.

[22]Consequently, individuals do not consider their own chances of becoming a boss in future. This simplifies the analysis and allows us to focus on the external labor market and wages of criminals. Although the internal labor market for criminal organizations, including internal promotions, is interesting, we believe that it requires a specialized model.

where s^t_{loy} is the number of loyal organized criminals, s^t_{disl} the number of disloyal organized criminals, s^t_{p} the number of organized criminals punished by the boss, and $g(s^t_{\text{loy}})$ the organization's revenue depending on s^t_{loy}. In particular, let $g(\cdot)$ be twice differentiable, $g(0) = 0, g'(\cdot) > 0, g''(\cdot) < 0$, and $\lim_{s\to\infty} g'(s) = 0$.

Furthermore, there are three types of costs. The organization's wage costs are equal to $w^t_{\text{oc}} s^t$. Then, there are punishment costs z per "punished criminal," giving total punishment costs $z s^t_{\text{p}}$. Finally, there is the amount that is stolen from the organization. This is equal to the number of disloyal criminals, s^t_{disl}, times the amount they can steal, ε. As highlighted in the introduction, the value of ε is closely related to the specific nature of criminal activities pursued by the organization. For instance, criminals engaged in the theft of illicit goods may be in a position to steal more from the organization than criminals engaged in human trafficking.

2.4 | Choices and payoffs of the individuals

If $i \in \mathcal{N}^t$, receives an offer by the boss, he decides whether to accept. If i accepts, then he chooses whether to be loyal or disloyal. If i rejects the offer, or if he does not receive any offer, he needs to decide whether to be an upstanding citizen or a peripheral criminal.

Suppose that i receives and accepts the offer of the boss. Then he earns w^t_{oc}. Recall that by being disloyal, for example, stealing from the boss, he increases his payoffs by $\varepsilon > 0$. Finally, criminals can get caught by the authorities. Let p be the probability that a criminal is caught and F the penalty he incurs if he is caught. Then the payoff of an individual i in organized crime given that he chooses either to be loyal, or disloyal, is equal to $U^t_i(\text{oc},\cdot)$, where

$$U^t_i(\text{oc, loy}) = w^t_{\text{oc}} - pF, \tag{2}$$

$$U^t_i(\text{oc, disl}) = w^t_{\text{oc}} - pF + \varepsilon. \tag{3}$$

Although disloyalty benefits the individual in this period, it offers him a bleak future. Either the boss will punish him, in which case he dies and leaves the game, or he lives but no boss will hire him ever again.

If individual $i \in \mathcal{N}^t$ receives no offer or rejects the offer in period t, then he chooses between peripheral crime and being an upstanding citizen. Upstanding citizenship is the outside option to crime. The payoff of upstanding citizenship at t, $U^t_i(\text{uc})$, is equal to $\overline{w}$, regardless of the number of criminals and the period. Suppose instead that he chooses peripheral crime, and let n^t be the number of individuals who choose to become peripheral criminals. Let the income for being a peripheral criminal at period t be given by $w^t_{\text{pc}} = f(n^t)$, where $f(\cdot)$ is continuous, strictly decreasing, $f(0) > \overline{w} + pF$ and $\lim_{x\to\infty} f(x) = 0$. This function captures the idea that due to competition between criminals, the income of a peripheral criminal decreases when the number of peripheral criminals increases. Then his payoff in that period is denoted by $U^t_i(\text{pc},n^t)$, with

$$U^t_i(\text{pc},n^t) = f(n^t) - pF.$$

2.5 | Equilibrium of the game

We consider subgame perfect equilibria of this game which are stationary in the sense that $\bar{s}^t$, s^t, w_{oc}^t, and n^t are constant over time. This has two advantages. First, it simplifies the analysis. Second, it eliminates the incentives for the boss to replace the members of her organization. We also assume that $g(s)$ and $f(n)$ are such that in equilibrium most individuals are upstanding citizens, and most criminals are peripheral criminals. In particular, define $\hat{n}$ and $\hat{s}$ be the solutions to, respectively, $f(\hat{n}) = \overline{w} + pF$ and $g'(\hat{s}) = \overline{w} + pF$. Then $\hat{n} > \hat{s} > 0$. Finally, we are interested in a situation where in equilibrium the boss wants to induce loyalty. Imposing nonnegative wages, $w_{\text{oc}}^t \geq 0$, guarantees that the boss does not want to hire someone she anticipates will be disloyal.

3 | ANALYSIS

In this section we characterize the subgame perfect equilibria of the organized crime game by backward induction. First, we derive the optimal actions. Then we characterize and compare the different equilibria. Equilibrium variable values are denoted by a superscript asterisk (*), and the proofs are in the appendix.

3.1 | Optimal actions

3.1.1 | Credibility of punishment

The last decisions in each period are the punishment decisions by the boss. She faces a simple trade-off. If she punishes disloyalty, she incurs a cost z. If she does not punish, she foregoes the net present value (NPV) of her expected future profits as the boss. If π is the per-period profit in equilibrium, then staying in charge is worth $\frac{\rho}{1-\rho}\pi$ to her. Clearly, incentives to punish increase in the profitability of the organization.

Lemma 1. *Consider an equilibrium of the organized crime game with per-period profits of π. The boss punishes disloyalty if and only if*

$$z \leq \frac{\rho}{1-\rho}\pi. \tag{4}$$

3.1.2 | Choice between peripheral crime and upstanding citizenship

Now consider the choice of individuals between upstanding citizenship and peripheral crime. Individuals can freely choose between these two options.[23] Thus, in equilibrium, individuals are indifferent between the two careers: they have the same NPV of payoffs:

[23] In reality, criminals may find it costly to change back to upstanding citizenship. Criminal records may prevent people from earning the same as individuals without criminal records. This will not affect the equilibrium. It only affects the adjustment time to equilibrium after exogenous shocks which makes upstanding citizenship more attractive.

$$\mathrm{NPV}_{\mathrm{uc}} = \mathrm{NPV}_{\mathrm{pc}} = \frac{1}{1-\rho}\overline{w}. \tag{5}$$

The number n^* of peripheral criminals in equilibrium is determined by this equality in lifetime payoffs. The specific solution to this depends on the wages of organized criminals. The reason is that the two options differ in the probability of giving access to a career as organized criminals. Peripheral crime is a stepping stone towards organized crime, whereas upstanding citizenship is not. If joining a criminal organization is seen as attractive, then this attracts more individuals to peripheral crime. This reduces the current wage until again the expected NPV of both options is the same. We denote the size of the criminal organization in equilibrium by s^* and the wages of loyal organized criminals by w^*_{oc}. Then the probability of being asked in the next period, if still alive, is equal to $\frac{(1-\rho)s^*}{\rho n^*}$, while the expected benefit of joining the criminal organization in the next period is equal to $\rho\frac{w^*_{\mathrm{oc}}-(\overline{w}+pF)}{1-\rho}$. Recalling that $\hat{n} = f^{-1}(\overline{w} + pF)$, we can state the following lemma.

Lemma 2. *Consider the organized crime game. In equilibrium, the size of peripheral crime n^* is uniquely determined by*

$$f(n^*) = \overline{w} + pF - \frac{s^*}{n^*}\Big(w^*_{\mathrm{oc}} - (\overline{w} + pF)\Big). \tag{6}$$

In particular, $n^* \geq \hat{n}$, and $n^* > \hat{n}$ if and only if $w_{\mathrm{oc}} > \overline{w} + pF$.

The lemma implies that in equilibrium peripheral criminals are not better off than upstanding citizens. Equality (6) in discounted lifetime payoffs is specific to peripheral crime and upstanding citizenship. It need not exist between a career in organized crime and the other careers. The reason is that individuals are not free to join criminal organizations: they need to be invited. Therefore it is possible that organized criminals are better off than other individuals. In contrast, in equilibrium it is not possible that organized criminals receive a worse payoff than $\overline{w}$ because, in our model, organized criminals are free to opt out.[24]

3.1.3 | Loyalty choice of organized criminals

Consider an organized criminal and his incentive compatibility constraint. If he is disloyal, he is either punished, losing him all future payoffs, or he is fired. If he is fired, he earns the maximal per-period payoff outside the organization, $\overline{w}$ (see Lemma 2). This gives the following lemma.

[24]In practice, opting out of a criminal organization can be problematic. At first glance this may suggest that payoffs of organized criminals may be worse once they are locked in. At a second glance, this is not obvious for three reasons. First, organized criminals and the boss will tend to have incriminating information on each other. Therefore, it is unclear who is in a stronger position when it comes to renegotiation. Second, the possibility of violence may make renegotiation risky. Finally, in equilibrium individuals are aware of any lock-in effects. Therefore, the boss can hire new organized criminals only if she either compensates prospective organized criminals for the risk of future renegotiations, or if she can commit not to renegotiate.

Lemma 3. *Consider the organized crime game. In any equilibrium with a credible threat of punishment, an organized criminal is loyal if and only if*

$$w_{oc} \geq pF + \frac{1-\rho}{\rho}\varepsilon. \tag{7}$$

If instead there is no credible threat of punishment, an organized criminal is loyal if and only if

$$w_{oc} \geq \overline{w} + pF + \frac{1-\rho}{\rho}\varepsilon. \tag{8}$$

Comparison of inequalities (7) and (8) shows how the threat of punishments reduces the price of loyalty. Punishment takes away a future benefit of $\overline{w}$ per period. Therefore the reward for loyalty needs to increase by $\overline{w}$ per period if punishment is not credible.

3.1.4 | Participation choice of organized criminals

Suppose that an individual receives an offer to join the organization. His outside option is equivalent to a per-period payoff of $\overline{w}$, as shown by Equation (5). From this the participation constraint follows.

Lemma 4. *Consider the organized crime game. In any equilibrium an individual, who plans to be loyal, will accept the offer to join the criminal organization if and only if*

$$w_{oc}^{*} \geq \overline{w} + pF. \tag{9}$$

3.1.5 | The hiring strategy by the boss

Now consider the hiring strategy $(\bar{s}^t, w_{oc}^t)$ of the boss. We first derive the optimal maximum size of the organization $\bar{s}^*(w_{oc}^*)$, as a function of the wage offered w_{oc}^*.

Lemma 5. *Consider a given equilibrium wage w_{oc}^* offered by the boss to join the criminal organization.*

- If $g'(0) > w_{oc}^*$ and w_{oc}^* is such that individuals accept the offer and are loyal, then $\bar{s}^*(w_{oc}^*)$ is the unique solution of

$$g'(\bar{s}^*(w_{oc}^*)) = w_{oc}^*. \tag{10}$$

- If $g'(0) \leq w^*_{\text{oc}}$ or w^*_{oc} does not induce loyalty in the criminal organization, then the boss hires no individual, $\bar{s}^*(w^*_{\text{oc}}) = 0$.

- If no worker accepts the offered w^*_{oc}, then $\bar{s}^*(w^*_{\text{oc}})$ is irrelevant.

Now consider the optimal wage w^*_{oc}. The boss does not benefit from setting higher wages than needed to induce participation and loyalty. This is for two reasons. First, higher wage costs reduce profits as revenues go down, and the costs per member go up. Second, lower profits reduce the incentives to punish, potentially causing the loss of a credible threat to punish. Therefore, in any equilibrium with organized crime, the boss sets the lowest wage for which the participation and incentive compatibility constraints are satisfied. By Conditions (7)–(9), this leads to two candidate optimal wages, namely,

$$w^{\text{c}}_{\text{oc}} = \max\left\{\overline{w} + pF, pF + \frac{1-\rho}{\rho}\varepsilon\right\} \quad \text{or}$$
$$w^{\text{nc}}_{\text{oc}} = \overline{w} + pF + \frac{1-\rho}{\rho}\varepsilon.$$

Lemma 6. *Consider the organized crime game.*

- The boss hires $\bar{s}^*(w^*_{\text{oc}})$ workers, at wage

$$w^*_{\text{oc}} = \begin{cases} w^{\text{c}}_{\text{oc}} & \text{if the threat of punishment is credible at } (w^{\text{c}}_{\text{oc}}, s^*(w^{\text{c}}_{\text{oc}})), \\ w^{\text{nc}}_{\text{oc}} & \text{otherwise,} \end{cases}$$

  and conditional on $g'(0) > w^*_{\text{oc}}$ *(otherwise, she hires* $\bar{s}^* = 0$*).*
- All invited individuals accept the offer and are loyal.

Having analyzed optimal behavior in each step, we are now ready to characterize the possible equilibria.

3.2 | Equilibrium types

The previous analysis shows that the equilibrium is one of four types, depending on which wage is sufficient to induce both participation and loyalty, and whether the organization is profitable at that wage. First, there can be a punishment (Pu) equilibrium, where the threat of punishment is both credible and sufficient to induce loyalty. In this equilibrium the organization's wage, $w^{\text{Pu}}_{\text{oc}}$, is determined by the participation constraint, Condition (9). Second, there can be a Punishment and Reward equilibrium (PuR), in which a credible threat of punishment is supplemented by the promise of a reward (in the form of efficiency wages). Here the organization's wage, $w^{\text{PuR}}_{\text{oc}}$, is given by Condition (7). Third, there can be a Reward (R) equilibrium, in which punishment is not credible, and loyalty is bought. In this equilibrium, the organization's wage, w^{R}_{oc}, is given by the alternative Condition (8). Finally, the price of loyalty may be too high, resulting in a No Organized Criminal (NOC) equilibrium, in which $\bar{s}^* = 0$.

In the following proposition, we characterize the four possible types of equilibria. To do so, we label the size and profitability of crime in each equilibrium type using (1), (6), and (10). For every $k \in \{\text{Pu}, \text{PuR}, \text{R}\}$, we let

$$\bar{s}^k = \bar{s}^*(w_{\text{oc}}^k),$$
$$\pi^k = g(\bar{s}^k) - w_{\text{oc}}^k \bar{s}^k,$$

denote the equilibrium contingent values, and let n^k (uniquely) solve for

$$f(n) = \overline{w} + pF - \frac{s^k}{n}(w_{\text{oc}}^k - (\overline{w} + pF)).$$

Finally, recall that $\hat{n}$ and $\hat{s}$ (uniquely) solve for

$$f(n) = \overline{w} + pF = g'(s).$$

Proposition 1. *Consider the organized crime game. The unique stationary equilibrium is*

- a Pu equilibrium if $\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}, z \leq \frac{\rho}{1-\rho}\pi^{\text{Pu}}$, and $g'(0) \geq w_{\text{oc}}^{\text{Pu}}$. In this equilibrium $w_{\text{oc}}^{\text{Pu}} = \overline{w} + pF, \bar{s}_{\text{oc}}^{\text{Pu}} = \hat{s}$, and $n^{\text{Pu}} = \hat{n}$;
- a PuR equilibrium if $\varepsilon > \frac{\rho}{1-\rho}\overline{w}, z \leq \frac{\rho}{1-\rho}\pi^{\text{PuR}}$, and $g'(0) \geq w_{\text{oc}}^{\text{PuR}}$. In this equilibrium $w_{\text{oc}}^{\text{PuR}} = pF + \frac{1-\rho}{\rho}\varepsilon, \bar{s}^{\text{PuR}} < \hat{s}$, and $n^{\text{PuR}} > \hat{n}$;
- an R equilibrium if ($\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}, z > \frac{\rho}{1-\rho}\pi^{\text{Pu}}$, and $g'(0) \geq w_{\text{oc}}^{\text{R}}$), or ($\varepsilon > \frac{\rho}{1-\rho}\overline{w}$, $z > \frac{\rho}{1-\rho}\pi^{\text{PuR}}$, and $g'(0) \geq w_{\text{oc}}^{\text{R}}$). In this equilibrium, $w_{\text{oc}}^{\text{R}} = \overline{w} + pF + \frac{1-\rho}{\rho}\varepsilon, \bar{s}^{\text{R}} < \hat{s}$, and $n^{\text{R}} > \hat{n}$;
- an NOC equilibrium if ($\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}, z > \frac{\rho}{1-\rho}\pi^{\text{Pu}}$, and $g'(0) < w_{\text{oc}}^{\text{R}}$), or ($\varepsilon > \frac{\rho}{1-\rho}\overline{w}$, $z > \frac{\rho}{1-\rho}\pi^{\text{PuR}}$, and $g'(0) < w_{\text{oc}}^{\text{R}}$). In this equilibrium, $s^{\text{NOC}} = 0$, and $n^{\text{NOC}} = \hat{n}$.

Moreover, in any equilibrium with organized crime, all individuals who receive wage offer w_{oc}^* accept, and all organized criminals are loyal.

Figure 2a,b shows the partitioning of the parameter space into these four ranges, each with a different type of equilibrium. In the Pu equilibrium, the participation wage creates a credible and sufficient threat to punish. It follows that any two Punishment Equilibria are equally profitable. Therefore, the incentives to punish are also the same. This explains the vertical border between the ranges of Pu equilibria and R equilibria.[25] This changes when punishment is insufficient. Then the larger ε is, the more the boss needs to reward loyalty. This lowers her profits and, thus, her willingness to punish. Consequently, the maximum cost of punishment

[25]The border between Pu and PuR equilibria is horizontal. Provided punishment is credible, the incentive to be disloyal does not depend on the cost of punishment.

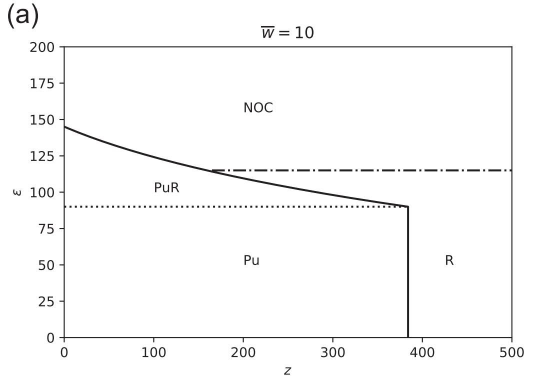

FIGURE 2 $p = 0.10, F = 13, \rho = 0.9, g(s_{\text{loy}}) = 240\log(0.1(s_{\text{loy}}) + 1)$. (a) Transition between equilibria when $\bar{w} = 10$ and (b) transition between equilibria when $\bar{w} = 13$.

for which punishment is credible decreases in ε. This translates into a downward-sloping border between the PuR equilibrium and the R and NOC equilibria.

Comparing Figure 2a,b shows that a better outside option lowers the profitability of the criminal organization. Thus, punishment needs to be cheaper to be credible: the right border shifts inward. Moreover, the better the outside option is, the more costly it is to be punished. Therefore, punishment is sufficient for higher levels of temptation: the upper border shifts upwards if punishment is credible. In contrast, if punishment is not credible, a better outside option increases the wages the boss needs to offer. Then profits decline, and the maximum temptation ε for which the organization is profitable reduces as well.

The different equilibria show different levels of crime and criminal payoffs. In the Pu equilibrium, the participation wage is sufficient to hire loyal organized criminals. This means that all individuals are equally well off: each individual receives a payoff of $\overline{w}$ per period. Any differences in wages just compensate for the risks criminals face. Because organized criminals are cheap to hire, the criminal organization is large. On the flip side, peripheral crime is low. No person becomes a criminal in the hope of being hired by the organization.

This changes when the threat of punishment is either insufficient or noncredible, yet organized crime is still profitable. Then loyalty is bought, resulting in higher payoffs to organized criminals. Because organized criminals are more expensive, the criminal organization will be smaller: $\bar{s}^{\text{R}} < \bar{s}^{\text{PuR}} < \hat{s}$. Due to the rewards organized criminals receive, other individuals want to join the organization. This attracts more people to crime, and peripheral crime is higher: $n^{\text{PuR}}, n^{\text{R}} > \hat{n}$. Consequently, organized criminals earn strictly higher payoffs than others, whereas peripheral criminals earn strictly less: $U^*_{\text{oc}} > \overline{w} = U^*_{\text{uc}} > U^*_{\text{pc}}$. Note that we cannot rank n^{PuR} and n^{R}. On the one hand, we would expect $n^{\text{R}} > n^{\text{PuR}}$ because the benefit of joining the criminal organization is higher in the R equilibrium, as $w^{\text{R}}_{\text{oc}} > w^{\text{PuR}}_{\text{oc}}$. On the other hand, for any given n, the probability of being invited to join the organization is higher in the PuR equilibrium, as $s^{\text{PuR}} > s^{\text{R}}$. The total effect is ambiguous.

Finally, organized crime may be unprofitable. Obviously, this results in the least amount of organized crime, $\bar{s}^{\text{NOC}} = 0$. More surprisingly, it also results in low levels of

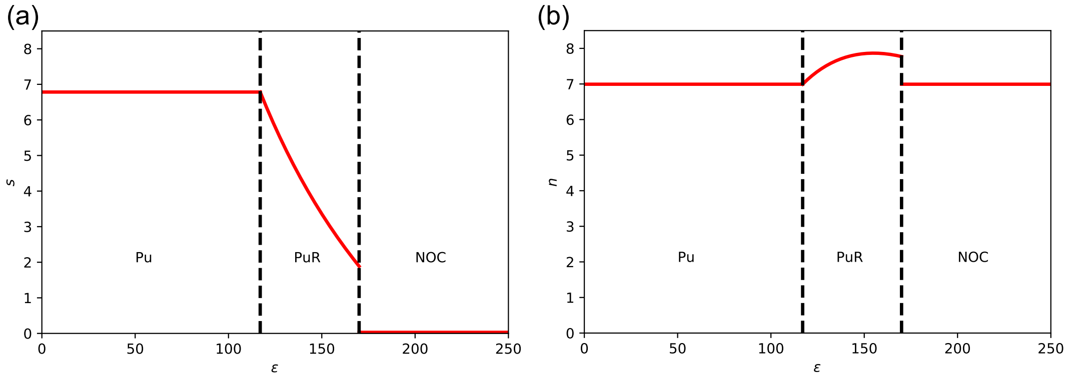

FIGURE 3 Parameters are the same as in Figure 2b except $z = 30$ and $f(n) = \frac{100}{n}$, $n > 0$. (a) Number of organized criminals and (b) number of peripheral criminals.

peripheral crime, as no individuals become criminals in the hope of joining a criminal organization.[26]

Figure 3 provides insight in what happens to the amount of crime as ε increases, starting from a situation in which the threat of punishment is credible. For small ε, when punishment provides ample incentives, nothing happens. This changes when punishment is not enough by itself. Then loyalty is bought, s decreases and n increases.

Figure 3b also shows that, as ε further, at some point n may decrease again. The reason is as follows. Suppose the organization wage w_{oc} is quite high, so that the organization is quite small. Then a small increase in the wage has a relatively large impact on the probability of peripheral criminals of being hired next period, and relatively little effect on the benefit of becoming an organized criminal. Consequently, a small increase in ε reduces the attractiveness of being a peripheral criminal, resulting in a lower n.

When ε becomes too high, the criminal organization becomes unprofitable. Then s drops to 0 and n returns to its original level $\hat{n}$. Note that s is positive at the point where the boss is indifferent between hiring individuals and not. At that point, the cost of punishment z is equal to the NPV of future profits. Because z is positive, profits at this point are positive too, which implies s is positive. If punishment is not credible at all, then there is no discontinuity in s either.

Summarizing, we obtain

$$\overline{w} + pF = w_{\text{oc}}^{\text{Pu}} < w_{\text{oc}}^{\text{PuR}} < w_{\text{oc}}^{\text{R}},$$
$$\bar{s}^{\text{NOC}} = 0 < \bar{s}^{\text{R}} < \bar{s}^{\text{PuR}} < \hat{s} = \bar{s}^{\text{Pu}}, \quad \text{and}$$
$$n^{\text{R}}, n^{\text{PuR}} > \hat{n} = n^{\text{Pu}} = n^{\text{NOC}}.$$

Using these results, we discuss the trade-off for the police regarding the optimal z.

[26]Of course, this depends on our assumption that peripheral crime does not expand into the markets which are now not supplied by organized crime. By continuity, peripheral crime is still lower than in an R or PuR equilibrium if such waterbed effects between organized and peripheral crime are not too strong.

4 | POLICE POLICY

We have established in Section 3 that the parameters of the model have a crucial impact on the labor market for criminals. In this section, we examine how the choices of the police can affect the job market of the criminals and then determine the optimal strategies for the police.[27]

We assume that the amount of crime and the harm it causes to society is increasing in the number of peripheral and organized criminals. For simplicity, we assume the harm to be linear. Specifically, we assume the following welfare function, $W(s, n) = -(\lambda s + (1 - \lambda)n)$. We allow for peripheral and organized criminals to differ in the harm they cause. If $\lambda > 1/2$, then organized criminals are more harmful for society than peripheral ones, while peripheral organized criminals are more harmful for society when $\lambda < 1/2$.

We now focus on the optimal policy for the police. We model this simply by having the police allocate their resources, $\gamma \in (0, 1)$, between two crime-fighting actions, catching criminals (budget b_0) and investigating cases of intragang violence (budget b_z). This requires that the police can distinguish between normal criminal activity and intragang violence. We assume that they can do so perfectly.

Let us explain how the police can allocate their resources. First, it spends $b_0 \in [0, \gamma]$ resources on increasing the probability of detecting criminal activity against upstanding citizens, p. Specifically, we assume that $p = p(b_0)$, with $p(0) \geq 0, p(\gamma) < 1$, and $p'(b_0) > 0$. Clearly, as the probability of being caught increases, criminals need to earn more to offset this increase in costs. Consequently, fewer individuals become peripheral criminals (see Lemma 2) and the organization will hire fewer criminals since it faces higher wage costs per organized criminal (see Lemmas 5 and 6). Therefore, crime of both types decreases. The police spend the remainder of their resources, $b_z = \gamma - b_0$, on increasing the probability of detection of intragang violence. The expected cost incurred by the boss, z, when punishing disloyal criminals is the probability that this punishment is detected, p_z, times the associated cost, c_z. Let $p_z = p_z(b_z)$, with $p_z(0) > 0, p_z(\gamma) < 1$, and $p_z'(b_z) > 0$. Consequently, in equilibrium, we have $p_z(b_z) = p_z(\gamma - b_0)$, and p_z decreases with b_0. To simplify the presentation, we use $z(b_0) = p_z(\gamma - b_0) \cdot c_z, s^{\mathrm{R}}(b_0)$ and $s^{\mathrm{Pu}}(b_0)$ to represent the different equilibrium values when the police allocate b_0 resources to detect regular crimes.

In what follows, we assume ε is low enough such that we will either have a Pu or an R equilibrium, depending on z. We now provide conditions under which the police can affect whether punishments are credible or not. If the police cannot affect this, the optimal policy is trivial: $b_0 = \gamma$, that is, the police invest all their resources in the detection of regular crimes. We let $\Pi^{\mathrm{Pu}}(b_0) = g(s^{\mathrm{Pu}}(b_0)) - (w + p(b_0)F)s^{\mathrm{Pu}}(b_0)$ be the profit function in the Pu equilibrium when the police invest b_0 to improve the probability p to detect regular crimes. As a tie-breaking rule, we assume that the boss does not punish disloyalty, when he is indifferent between punishing and not. This means that for $z(b_0) = \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}(b_0)$ the equilibrium will be of type R.

Perhaps surprisingly, there is a second case where the optimal policy is trivial. Consider Equation (4): the boss will choose to punish if and only if $z < \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}$. Now note that although an increase in b_z increases z, the corresponding decrease in b_0 will increase Π^{Pu} too. By reducing the probability that criminals are caught, crime is more profitable, and the boss has more reason to stay in power. The net effect is unclear. If the latter effect dominates, the

[27]We would like to thank the anonymous referees whose valuable suggestions have greatly contributed to this section.

optimal policy is trivial: $b_0 = \gamma$. Then crime is as costly as possible, and the incentives to punish are as weak as possible too.

Let us now consider a case where allocating resources to cases of intragang violence ($b_0 < \gamma$) can be optimal. In particular assume that (i) $z'(b_0) < \frac{\rho}{1-\rho}[\Pi^{\mathrm{Pu}}]'(b_0)$, for every $b_0 \in (0, \gamma)$, (ii) $z(0) > \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}(0)$, and (iii) $z(\gamma) < \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}(\gamma)$. The first condition guarantees that a decrease in b_0 makes punishment less attractive, whereas the last two conditions guarantee that the police can affect the type of equilibrium. Together, these assumptions imply that there exists a unique $b_0 \in (0, \gamma)$, $b_0^\star$, such that for all $b_0 > b_0^\star$, punishment is noncredible, whereas for all $b_0 < b_0^\star$ punishment is credible.

Lemma 7. *There is a unique $b_0^\star \in (0, \gamma)$ such that $z\left(b_0^\star\right) = \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}\left(b_0^\star\right)$ if*

1. $z'(b_0) < \frac{\rho}{1-\rho}[\Pi^{\mathrm{Pu}}]'(b_0)$, *for every* $b_0 \in (0, \gamma)$,
2. $z(0) > \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}(0)$, *and*
3. $z(\gamma) < \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}(\gamma)$.

Given the equilibrium type, it is optimal to maximize b_0. Therefore, there are only two police strategies that can be optimal: $b_0 = b_0^\star$, resulting in an R equilibrium with $\left(s^{\mathrm{R}}\left(b_0^\star\right), n^{\mathrm{R}}\left(b_0^\star\right)\right)$, and $b_0 = \gamma$, resulting in a Pu equilibrium $(s^{\mathrm{Pu}}(\gamma), n^{\mathrm{Pu}}(\gamma))$. In the following proposition, we establish which of these two pairs is optimal for the police.

Proposition 2. *Let* $\kappa = \frac{1-\rho}{\rho}\frac{\varepsilon}{F}$.

1. *Suppose* $p(\gamma) - p\left(b_0^\star\right) \geq \kappa$. *Then, the police choose the pair* $(b_0, b_z) = (\gamma, 0)$.
2. *Suppose* $p(\gamma) - p\left(b_0^\star\right) < \kappa$.
   (a) *If* $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) < \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$, *then the police choose* $(b_0, b_z) = (\gamma, 0)$.
   (b) *If* $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) > \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$, *then the police choose* $(b_0, b_z) = \left(b_0^\star, \gamma - b_0^\star\right)$.
   (c) *If* $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) = \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$, *then the police are indifferent between the pairs* $(\gamma, 0)$ *and* $\left(b_0^\star, \gamma - b_0^\star\right)$.

Proposition 2 shows that removing the credibility of punishment by focusing on detecting intragang crime is optimal only under two conditions. First, it must lead to a greater reduction in the optimal size of the criminal organization, that is, $s^{\mathrm{R}}\left(b_0^\star\right) < s^{\mathrm{Pu}}(\gamma)$. This is the case if $p(\gamma) - p\left(b_0^\star\right) < \kappa$. This condition is more likely to be met if fewer resources are needed to force the organization into an R equilibrium (so if $b_0^\star$ is close to γ), or if it is more difficult to raise the probability with which individual criminals are caught. Second, by forcing the organization into the reward equilibrium, peripheral crime will increase. The social benefit of the decrease in organized crime needs to outweigh the damage caused by the resulting increase in peripheral crime. This is the case if $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) \geq \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$. In other words, the more harmful organized crime is relative to peripheral crime, the more likely it is that scrutinizing intragang violence is an optimal strategy.

We now provide an example for which there exists an interval for parameters under which $W\left(s^{\mathrm{R}}\left(b_0^\star\right), n^{\mathrm{Pu}}\left(b_0^\star\right)\right) \geq W\left(s^{\mathrm{R}}(\gamma), n^{\mathrm{Pu}}(\gamma)\right)$.

Example 1. Suppose $f(n) = \frac{\beta}{n}$, with $\beta > 0, g(s) = \sqrt{n}, \rho < 1/2, F < \frac{\rho}{4(1-\rho)}$, and $z(b_0) = \frac{1}{a-(\gamma-b_0)}$, with $a \in \left(\min\left(\gamma, \left(\frac{1-\rho}{\rho}\right)(4\overline{w} + \gamma F)\right), \left(\frac{1-\rho}{\rho}\right)4\overline{w} + \gamma\right)$. Note that $z'(b_0) < 0$. Furthermore, we assume $p(b_0) = b_0$ and $p_z(b_z) = b_z$. Because of the value of a, conditions given in Lemma 7 are satisfied. In this example, $z\left(b_0^\star\right) = \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}\left(b_0^\star\right)$ when $\frac{1}{a+b_0^\star-\gamma} = \left(\frac{\rho}{1-\rho}\right)\frac{1}{4\left(\overline{w}+b_0^\star F\right)}$. Consequently,

$$b_0^\star = \frac{\gamma + 4\overline{w}\left(\frac{1-\rho}{\rho}\right) - a}{1 - 4\left(\frac{1-\rho}{\rho}\right)F}.$$

Let $A = \overline{w} + \gamma F, B = \overline{w} + b_0^\star F$, we have $s^{\mathrm{Pu}}(\gamma) = \frac{1}{(2A)^2}$ and $s^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1}{2\left(B + \frac{1-\rho}{\rho}\varepsilon\right)}\right)^2$.

We provide in the appendix the proof of the following result.

1. Suppose $\gamma - b_0^\star \geq \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{F}$. Then the police choose the pair $(\gamma, 0)$.
2. Suppose $\gamma - b_0^\star < \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{F}$. Let $\lambda^\star = \frac{\left(\frac{1-\rho}{\rho}\right)\varepsilon s^{\mathrm{R}}\left(b_0^\star\right) - 4B\beta A s^{\mathrm{Pu}}(\gamma) + \beta}{\left(\left(\frac{1-\rho}{\rho}\right)\varepsilon - B\right)s^{\mathrm{R}}\left(b_0^\star\right) + B(1-4\beta A)s^{\mathrm{Pu}}(\gamma) + \beta}$.
   (a) If $\lambda < \lambda^\star$, then the police choose the pair $(\gamma, 0)$.
   (b) If $\lambda > \lambda^\star$, then the police choose the pair $\left(b_0^\star, b_z^\star\right)$.
   (c) If $\lambda = \lambda^\star$, then the police are indifferent between the pairs $(\gamma, 0)$ and $\left(b_0^\star, \gamma - b_0^\star\right)$.

In other words, if organized criminals are a significant threat to society compared to peripheral criminals, and the detection of intragang violence sufficiently reduces the size of the criminal organization, then the optimal strategy for the police is to invest $b_z^\star = \gamma - b_0^\star$ *in* *the detection of intragang violence. In other cases, the police should allocate all their resources to detecting regular crimes and set $b_z = 0$.*

5 | DISCUSSION

Obviously, the analysis above is based on a model with many simplifying assumptions. In this section we discuss the impact of relaxing some of these simplifications. First, we allow criminal careers to be more dangerous than upstanding citizenship. Second, we allow the boss to get false information about the loyalty decisions. Third, we consider the possibility of other punishment levels as death or firing, as well as retirement options for the organization's bosses.

5.1 | Crime is dangerous

Suppose that life as a criminal is more dangerous than that of an upstanding citizen. We will first look at criminals. After doing so, we look at the boss.

Let ρ_{pc} and ρ_{oc} be survival probability of, respectively, peripheral and organized criminals, where $\rho_{pc}, \rho_{oc} < \rho$. This has several effects. First and foremost, it makes being a criminal less attractive. In equilibrium criminals are compensated for this increased risk, so criminal incomes w_{pc} and w_{oc} will be higher. Consequently, in each type of equilibrium there will be less organized and peripheral crime.

A second effect is that loyalty incentives become less effective. The reason is that both types of punishment, death or losing access to efficiency wages, affect future payoffs. A lower ρ_{oc} reduces the relative importance of future payoffs. In contrast, the benefits of disloyalty, ε, are enjoyed in the current period. Therefore a lower ρ_{oc} makes it more attractive for organized criminals to be disloyal and get the current higher payoff. As a result, threats of punishment may cease to be sufficient, efficiency wages are higher and the criminal organization may become unprofitable.

Now consider the boss. Her incentives to punish decrease whenever she or her criminals are more likely to die. Suppose first that ρ_{oc} decreases. Then both the participation constraint and the individual incentive constraints of criminals increase. Consequently, the optimal wage increases and the profits of the boss decrease. As shown by the individual incentive constraint of the boss, Condition (4), her incentives to punish increase in her per-period profits. Therefore she is less willing to punish if ρ_{oc} decreases. Suppose next that her own survival probability, ρ_b, decreases. Her incentives to punish depend on the value of staying in power: $\frac{\rho_b}{1-\rho_b}(g(\bar{s}) - w_{oc}\bar{s})$. This value decreases when she expects to die sooner. Therefore, the threat of punishment is less likely to be credible if ρ_{oc} or ρ_b is lower. If it ceases to be credible (higher) efficiency wages need to be paid and organized crime declines. This, of course, affects the amount of peripheral crime as well. It is worth noting that an increase of the probability to die of the boss can be seen as an expected loss for her. We should have the same qualitative results if we increase the likelihood of the boss being apprehended by the police, or the magnitude of the penalty she faces.

5.2 | Probabilistic detection of disloyalty and trustworthiness of the boss

In our model, detection of disloyalty is automatic and perfect. In this section, we consider the possibility of false negatives (undetected disloyalty) and false positives (loyalty which is mistaken for disloyalty).

First, consider false negatives. This is straightforward. The possibility to get away with disloyalty makes it more attractive to be disloyal. Therefore, stronger incentives need to be given. Consequently, the threat of punishment is less likely to be sufficient; efficiency wages, if used, are higher, and the organization is less likely to be profitable and exists.

Now consider false positives. There are two effects. First, false positives imply that loyal organized criminals are punished with positive probability. The boss needs to compensate her workers for this risk. Consequently, the income for organized criminals increases and the level of organized crime decreases.

Second, false positives imply that the threat of punishment is not free anymore. Without false positives, a credible threat of punishment gives a free incentive: the boss never needs to follow up on it. This is not true if there are false positives. Then it is possible that the expected cost of punishment is higher than the cost of the efficiency wage that gives an equally strong incentive. Therefore, wages may be lower if the boss uses only efficiency wages than if she (also) threatens with punishment.

Interestingly, whereas typical firms would opt for the cheapest incentive scheme, this is not always true for the criminal organization. Because the boss loses control over the organization if she does not punish, she has strong incentives to punish even if this results in somewhat higher wages. Effectively she is willing to accept lower per-period profits in favor of a longer period in power.

Finally, consider the scenario where the boss may not be trustworthy herself, and may choose to keep the wages of the criminals. Note that the boss has an incentive to commit to trustworthy behavior, if possible. If criminals do not trust her, they will be unwilling to work for her, resulting in zero profits. The question is then whether the boss can commit to pay the wages. This can be due to the threat of violence facing the boss if she steals herself. Another reason why she may be trusted is if it is known that she continues to care enough about future profits. If the boss cheats her criminals now, she may find it more costly, if not impossible, to hire criminals in future, which hurts her future profits. However, what if there are different types of bosses, some of them are known to be trustworthy, whereas other types are known to be untrustworthy? If criminals cannot distinguish between the two types, the trustworthy type is forced to pay a risk premium to his criminals, resulting in higher wages and a smaller optimal organization. Moreover, the organization of untrustworthy bosses is likely to be short-lived. Consequently, doubt on the trustworthiness of the boss can result in less organized crime. At the same time, it gives bosses an incentive to commit to trustworthy behavior if possible.

5.3 | Endogenous punishment

In our model, punishment options are limited. A disloyal criminal is either fired or killed. In practice, more punishments are possible. For instance, a disloyal criminal may be hurt or maimed. It is likely that less severe (physical) punishment will be less costly, so induce a lower z, because lower punishments warrant less police attention and lower penalties in court. This results in a trade-off between the credibility of punishment and the sufficiency of punishment. Punishment is more likely to be credible, the less costly it is. On the other hand, if punishment is softer, it may not be sufficient to deter disloyalty by itself, which is also costly for the boss. In principle the boss will choose a credible level of punishment. Within that range the boss will select a punishment that is just harsh enough to deter disloyalty or, if that is not possible, the harshest punishment that is still credible.

Similarly, the dismissal of the boss after failing to control the organization may be seen as punishment. Such transfers of power can be more or less punishing for the boss. Let w_R be the net present payoff of a boss when he retires, where w_R may be negative. At first glance, bosses may want to allow for a higher w_R. However, standard self-commitment considerations show that a boss can be better off if w_R is reduced. By having w_R as the outside option of not punishing, her incentive compatibility constraint becomes

$$z \leq \frac{\rho}{1-\rho}\pi - w_R.$$

Therefore w_R needs to be low enough to allow for punishment as a loyalty device and the higher profits which that allows.

6 | CONCLUSION

In this paper, we show that the police can reduce organized crime by scrutinizing cases of internal punishments by criminal organizations. This works if focusing on intragang violence removes the credibility of the threat of such punishments. We also show that this is not always beneficial to do. First, it depends on the amount of resources needed to affect the equilibrium type. If this strategy is not sufficiently efficient, those resources are better used in other crime-fighting activities. Second, if this strategy is effective, it is likely to attract additional peripheral crime. Whether this is still desirable depends critically on how harmful organized crime is relative to peripheral crime.

One of the key predictions of our model is that when the cost of punishing disloyal behavior within a criminal organization is very high, or when it is highly advantageous for organized criminals to be disloyal, members of the criminal organization are in a significantly more favorable position than peripheral criminals. In other words, the pay gap between organized and peripheral criminals is large when the criminal organization does not resort solely to violence, but shrinks when violence alone is used to regulate the actions of organized criminals. This is because organized criminals receive a rent associated with the implementation of efficiency wages, while peripheral criminals have wage expectations motivated by the possibility of becoming organized criminals, leading them to accept lower immediate wages. This prediction can be tested empirically. However, the test is not straightforward. We see two main challenges for a proper empirical study. First, the income of criminals involves a compensation for the risk of their career. Not correcting for these risks may lead to finding efficiency wages where there are none, or not finding decreased wages for peripheral criminals when they in fact are lower. In other words, income needs to be correctly translated into payoffs. Second, as Levitt and Vankatesh (2000) show, the prospect of advancement within a criminal organization can also depress wages for lower-level members. This is a second complicating factor that needs to be controlled for when applicable.

In Section 5, we have discussed some of our assumptions. We have considered the possibility that criminals and bosses are more likely to die than others; the possibility that a boss receives false information on the loyalty of the organized criminals; the strategic choice of the severity of punishment as well as the strategic choice of retiring opportunities for bosses.

Of course, other assumptions were made as well. Some of them are innocuous. For instance, the boss may have some probability to stay in power even if he decides not to punish. While this weakens his incentives to punish, it does not affect our results qualitatively. What is important is that he loses power with some probability. Moreover, we assume that criminals can switch to a normal career and receive the same wages as individuals without a criminal past. Criminal records, however, may result in worse noncriminal careers. We would expect that criminal records lead to more constant sorting over the careers, whereas in our current model individuals may switch every period between peripheral crime and a normal job. Moreover, if there is a shock due to which the equilibrium number of criminals would decline, the adjustment process may go slower as some criminals are locked into crime until their deaths. Clearly, in equilibrium, an increase in the number of criminals would go as quickly as before. In contrast to our model, this would also lead to an expansion of peripheral crime when organized crime stops being profitable.

Another important assumption is that peripheral criminals operate in different criminal markets than organized criminals. Obviously, if a criminal organization becomes smaller, or disappears, this may provide new opportunities to peripheral criminals. Partly this leads to the same qualitative results as before, but stronger: peripheral crime increases even more when the boss needs to buy loyalty.

Summarizing, we have shown that scrutinizing intragang violence can be a good police strategy. We have also shown that this is not trivially optimal. Scrutinizing intragang violence is not always able to induce a more costly equilibrium for the organization. Moreover, it depends on the amount of resources that need to be diverted. Finally, the cure may be worse than the disease if the increase in peripheral crime is particularly harmful. It follows that good knowledge and judgment of the local situation by the police is needed before deciding to focus on intragang violence.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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APPENDIX A

Proof of Lemma 1. In every period of a considered equilibrium of the infinitely repeated game, the boss earns a constant per-period profit π. Thus, at the end of given period t, after the boss earned her current profit π, the present value of her future profits is $\rho\pi + \rho^2\pi + \cdots = \frac{\rho}{1-\rho}\pi$.

Subgame perfection requires that any unilateral deviation by an organized criminal be punished. At the end of a given period t, if the boss punishes disloyalty of one member of the organized criminal, she incurs the cost z, and continues to earn $\frac{\rho}{1-\rho}\pi$, given that she also punishes disloyalty at the subsequent periods. If instead, she does not punish disloyalty in period t, she earns 0 and then leaves the game. Thus, the threat of punishment is credible if $\frac{\rho}{1-\rho}\pi - z \geq 0$. $\square$

Proof of Lemma 2. As an individual in an infinite population, an upstanding citizen expects to earn $\overline{w}$ per period. Therefore, $\text{NPV}_{\text{uc}} = \frac{1}{1-\rho}\overline{w}$.

Now consider peripheral criminals. In a given period t, the current profit of a peripheral criminal is $f(n^*) - pF$. In period $t+1$, in expectation there will be $(1-\rho)s^*$ peripheral criminals hired in equilibrium by the boss, among the ρn^* alive peripheral criminals. Thus, a peripheral criminal becomes an organized criminal in the next period with probability $\frac{(1-\rho)s^*}{\rho n^*}$. With probability $1 - \frac{(1-\rho)s^*}{\rho n^*}$, a peripheral criminal remains a peripheral criminal in the next period. Hence, NPV_{pc} solves for

$$\text{NPV}_{\text{pc}} = f(n^*) - pF + \rho\left(\frac{(1-\rho)s^*}{\rho n^*}\text{NPV}_{\text{oc}} + \left(1 - \frac{(1-\rho)s^*}{\rho n^*}\right)\text{NPV}_{\text{pc}}\right).$$

Moreover, in equilibrium organized criminals are loyal at every period, that is,

$$\text{NPV}_{\text{oc}} = \frac{1}{1-\rho}\left(w_{\text{oc}}^* - pF\right).$$

Now from $\text{NPV}_{\text{pc}} = \text{NPV}_{\text{uc}} = \frac{1}{1-\rho}\overline{w}$, we obtain

$$\frac{1}{1-\rho}\overline{w} = f(n^*) - pF + \rho\left(\frac{(1-\rho)s^*}{\rho n^*}\frac{1}{1-\rho}\left(w_{\text{oc}}^* - pF\right) + \left(1 - \frac{(1-\rho)s^*}{\rho n^*}\right)\right.$$
$$\left.\frac{1}{1-\rho}\overline{w}\right),$$

that is, n^* solves for

$$\begin{aligned} f(n) &= \frac{1}{1-\rho}\overline{w} + pF - \rho\left(\frac{(1-\rho)s^*}{\rho n}\frac{1}{1-\rho}\left(w_{\text{oc}}^* - pF\right) + \left(1 - \frac{(1-\rho)s^*}{\rho n^*}\right)\frac{1}{1-\rho}\overline{w}\right) \\ &= \overline{w} + pF - \frac{s^*}{n}\left(w_{\text{oc}}^* - (\overline{w} + pF)\right). \end{aligned}$$

In equilibrium, $w_{\text{oc}}^* - pF \geq \overline{w}$, since otherwise, a peripheral criminal (who earns $\frac{1}{1-\rho}\overline{w}$) would never accept the proposition of the boss to become an organized criminal (who earns $\frac{1}{1-\rho}\left(w_{\text{oc}}^* - pF\right)$). Therefore $\frac{s^*}{n}\left(w_{\text{oc}}^* - (\overline{w} + pF)\right) \geq 0$. Then, for every $s^* \geq 0$ and every p, F, $\overline{w}$, and w_{oc}^* such that $w_{\text{oc}}^* - (\overline{w} + pF) \geq 0$, the above equation (in n) has a unique solution n^*, since as n continuously ranges from 0 (excluded) to infinity, the left-hand side continuously decreases from $f(0) > \overline{w} + pF$ to 0, while the right-hand side continuously increases from $-\infty$ to $\overline{w} + pF$. Moreover, from $w_{\text{oc}}^* - (\overline{w} + pF) \geq 0$, the solution n^* is such that $f(n^*) \leq \overline{w} + pF$. Since f is decreasing, f^{-1} also is decreasing, and we obtain $n^* \geq f^{-1}(\overline{w} + pF) = \hat{n}$, with a strict inequality iff $w_{\text{oc}}^* < \overline{w} + pF$. □

Proof of Lemma 3. If, in a given period t, a member of the criminal organization is disloyal, he earns $w_{\text{oc}} - pF + \varepsilon$ at the end of period t. If he is not punished, he is fired and will never be approached again by the boss. Therefore, he shall never choose to be a peripheral criminal, since $f(n^*) - pF \leq \overline{w}$. Thus, as an alive upstanding citizen, he will earn $\overline{w}$ in every period $t' \geq t + 1$. The period t NPV of such a disloyal criminal is then

$$w_{\text{oc}} - pF + \varepsilon + \frac{\rho}{1-\rho}\overline{w}.$$

If, instead, the member is loyal during period t, he shall be loyal in every subsequent period, and his period t NPV is

$$\frac{1}{1-\rho}(w_{\text{oc}} - pF).$$

Thus, if the threat of punishment is not credible, that is, $z > \frac{\rho}{1-\rho}\pi$, where π is the boss per-period profit, then an organized criminal is loyal in every period *iff*

$$\frac{1}{1-\rho}(w_{oc} - pF) \geq w_{oc} - pF + \varepsilon + \frac{\rho}{1-\rho}\overline{w} \quad \Leftrightarrow \quad \frac{\rho}{1-\rho}(w_{oc} - pF) \geq \varepsilon + \frac{\rho}{1-\rho}\overline{w}$$
$$\Leftrightarrow \quad w_{oc} \geq \overline{w} + pF + \frac{1-\rho}{\rho}\varepsilon.$$

Otherwise, if the threat of punishment is credible (i.e., $z \leq \frac{\rho}{1-\rho}\pi$), a disloyal member in period t leaves the game at the end of the period. Therefore, an organized criminal is loyal in every period *iff* $w_{oc} \geq pF + \frac{1-\rho}{\rho}\varepsilon$. □

Proof of Lemma 4. In equilibrium, the per-period NPV of a loyal organized criminal is $\frac{1}{1-\rho}\left(w^*_{oc} - pF\right)$, whereas the per-period NPV of a peripheral criminal is $\text{NPV}_{pc} = \text{NPV}_{uc} = \frac{1}{1-\rho}\overline{w}$. Hence in an equilibrium with a criminal organization, $w^*_{oc} \geq \overline{w} + pF$. □

Proof of Lemma 5.

- Suppose first that every approached worker accepts the offer, that is, Condition (9) holds, and is loyal at w^*_{oc}, that is, Conditions (4) and (7) (punishment is credible and organized criminals have no incentives to deviate) or Condition (8) (punishment is not credible, but organized criminals have no incentives to deviate) holds. Then, the per-period profit of the boss, as given by (1), is

$$\pi(\overline{s}) = g(\overline{s}) - w^*_{oc}\overline{s}.$$

  Therefore, the optimal size $\overline{s}^*\left(w^*_{oc}\right)$ solves for $g'\left(\overline{s}^*\left(w^*_{oc}\right)\right) = w^*_{oc}$. From $g'' < 0$, and conditional on $g'(0) > w^*_{oc}$, there is a unique strictly positive optimal size of the organization, which is decreasing in w^*_{oc}.

- Suppose now that $g'(0) \leq w^*_{oc}$, or $g'(0) > w^*_{oc}$ and Condition (9) holds, but neither (4) and (7), nor (8) hold. In that case, every hired criminal is disloyal, and from (1), the boss has a negative per-period profit whenever she hires. Thus, she prefers to hire no individuals, and $\overline{s}^*\left(w^*_{oc}\right) = 0$.

- Finally, if Condition (9) does not hold, then no worker accepts the offered wage w^*_{oc}, and then $\overline{s}\left(w^*_{oc}\right)$ does not matter.

□

Proof of Lemma 6. By Conditions (7)–(9), wages w^{c}_{oc} and w^{nc}_{oc} are the minimal wages that induce participation and loyalty of the organized criminals, conditional on the credibility of punishment (and on $g'(0) > w^{c}_{oc}, g'(0) > w^{nc}_{oc}$, respectively). In that case, from (1), the boss' per-period profit is given by $\pi\left(\overline{s}\left(w^*_{oc}\right)\right) = g\left(\overline{s}\left(w^*_{oc}\right)\right) - \overline{s}\left(w^*_{oc}\right)w^*_{oc}$, as a function of the offered wage w_{oc}^* and with, from Lemma 5, $g'\left(\bar{s}\left(w_{oc}^*\right)\right) = w_{oc}^*$. Accordingly,

$$\begin{aligned}\frac{\partial \pi\left(\bar{s}\left(w_{oc}^*\right)\right)}{\partial w_{oc}^*} &= g'\left(\bar{s}\left(w_{oc}^*\right)\right)\bar{s}'\left(w_{oc}^*\right) - \bar{s}'\left(w_{oc}^*\right)w_{oc}^* - \bar{s}\left(w_{oc}^*\right) \\ &= w_{oc}^*\bar{s}'\left(w_{oc}^*\right) - \bar{s}'\left(w_{oc}^*\right)w_{oc}^* - \bar{s}\left(w_{oc}^*\right) = -\bar{s}\left(w_{oc}^*\right) < 0.\end{aligned}$$

Therefore, the boss prefers a lower wage w_{oc}^*. Thus, from $w_{oc}^{c} < w_{oc}^{nc}$, if punishment is credible at $\left(w_{oc}^{c}, \bar{s}^*\left(w_{oc}^{c}\right)\right)$, that is, if $z \leq \frac{\rho}{1-\rho}\pi\left(w_{oc}^{c}, \bar{s}^*\left(w_{oc}^{c}\right)\right)$, and if $w_{oc}^{c} < g'(0)$, then the boss' optimal wage offer is $w_{oc}^* = w_{oc}^{c}$.

If instead punishment is not credible at $\left(w_{oc}^{c}, \bar{s}^*\left(w_{oc}^{c}\right)\right)$, and if $w_{oc}^{nc} < g'(0)$, then the boss optimally offers $w_{oc}^* = w_{oc}^{nc}$.

The remaining situations are $w_{oc}^{c} \geq g'(0)$ (and then $w_{oc}^{nc} \geq g'(0)$) and punishment is credible, or $w_{oc}^{c} < g'(0) \leq w_{oc}^{nc}$ and punishment is not credible. In these situations, from Lemma 5, organized crime is not profitable, and the boss sets $\bar{s}^* = 0$. Then w_{oc}^* is irrelevant. □

Proof of Proposition 1.

- Suppose $\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}$, $z \leq \frac{\rho}{1-\rho}\pi^{Pu}$, and $g'(0) \geq w_{oc}^{Pu}$, with $w_{oc}^{Pu} = \overline{w} + pF$ and $\bar{s}^{Pu} = \hat{s}$. From $z \leq \frac{\rho}{1-\rho}\pi^{Pu}$, punishment is credible. From $\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}$, we have $w_{oc}^{Pu} = \overline{w} + pF \geq pF + \frac{1-\rho}{\rho}\varepsilon$. Then, from Lemma 3 hired organized criminals are loyal. Also from Lemma 4, every offer is accepted. Since $\bar{s}^{Pu}$ solves $g'(\bar{s}^{Pu}) = \overline{w} + pF$ and $g'(0) > w_{oc}^{Pu}$, from Lemma 5, we have $\bar{s}^{Pu} = \bar{s}^*\left(w_{oc}^{Pu}\right)$. From $w_{oc}^{Pu} = \max\left\{\overline{w} + pF, pF + \frac{1-\rho}{\rho}\varepsilon\right\}$ and Lemma 6, we have $w^{Pu} = w_{oc}^*$. Finally, n^{Pu} solves $f(n) = \overline{w} + pF - 0$, and thus $n^{Pu} = \hat{n}$.
- Suppose $\varepsilon > \frac{\rho}{1-\rho}\overline{w}$, $z \leq \frac{\rho}{1-\rho}\pi^{PuR}$, and $g'(0) \geq w_{oc}^{PuR}$, with $w_{oc}^{PuR} = pF + \frac{1-\rho}{\rho}\varepsilon$ and $\bar{s}^{PuR}$ such that $g'(s^{PuR}) = w_{oc}^{PuR}$. From $\varepsilon > \frac{\rho}{1-\rho}\overline{w}$, we have $w_{oc}^{PuR} \geq \overline{w} + pF$, and from Lemma 4, every offer is accepted. Since $\bar{s}^{PuR}$ solves $g'(\bar{s}^{PuR}) = w_{oc}^{Pur}$ and $g'(0) > w_{oc}^{PuR}$, from Lemma 5, we have $\bar{s}^{PuR} = \bar{s}^*\left(w_{oc}^{PuR}\right)$. From $w_{oc}^{PuR} = \max\left\{\overline{w} + pF, pF + \frac{1-\rho}{\rho}\varepsilon\right\}$ and Lemma 6, we have $w^{PuR} = w_{oc}^*$. Finally, since n^{PuR} solves $f(n) = \overline{w} + pF - \frac{\bar{s}^{PuR}}{n}\left(w_{oc}^{PuR} - (\overline{w} + pF)\right) < \overline{w} + pF$, we have $n^{PuR} > f^{-1}(\overline{w} + pF) = \hat{n}$.
- Suppose ($\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}$, $z > \frac{\rho}{1-\rho}\pi^{Pu}$, and $g'(0) \geq w_{oc}^{R}$), or ($\varepsilon > \frac{\rho}{1-\rho}\overline{w}$, $z > \frac{\rho}{1-\rho}\pi^{PuR}$, and $g'(0) \geq w_{oc}^{R}$), with $w_{oc}^{R} = \overline{w} + pF + \frac{1-\rho}{\rho}\varepsilon$ and $\bar{s}^{R}$ such that $g'(\bar{s}^{R}) = w_{oc}^{R}$. From Lemma 3, hired organized criminals are loyal, and from Lemma 4, every offer is accepted. From $w_{oc}^{R} > \overline{w} + pF = w_{oc}^{Pu}$ and $w_{oc}^{R} > pF + \frac{1-\rho}{\rho}\varepsilon = w_{oc}^{PuR}$, we have $\pi^{Pu} > \pi^{R}$

and $\pi^{\mathrm{PuR}} > \pi^{\mathrm{R}}$, respectively. Then $z > \frac{\rho}{1-\rho}\pi^{\mathrm{Pu}}$ or $z > \frac{\rho}{1-\rho}\pi^{\mathrm{PuR}}$ implies that punishment is not credible. Accordingly, from $g'(0) \geq w_{\mathrm{oc}}^{\mathrm{R}}$ and Lemma 5, we have $\bar{s}^{\mathrm{R}} = \bar{s}^{*}\left(w_{\mathrm{oc}}^{\mathrm{R}}\right)$, and from Lemma 6, $w_{\mathrm{oc}}^{\mathrm{R}} = w_{\mathrm{oc}}^{*}$.

- The NOC equilibrium addresses the complementary $g'(0)$ values, not included in the Pu, PuR, and R cases. The respective cases are given by: $(\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}, z > \frac{\rho}{1-\rho}\pi^{\mathrm{Pu}}$, and $g'(0) < w_{\mathrm{oc}}^{\mathrm{R}})$, that complements the first R case, $(\varepsilon > \frac{\rho}{1-\rho}\overline{w}, z > \frac{\rho}{1-\rho}\pi^{\mathrm{PuR}}$, and $g'(0) < w_{\mathrm{oc}}^{\mathrm{R}})$, that complements the second R case, $(\varepsilon \leq \frac{\rho}{1-\rho}\overline{w}, z \leq \frac{\rho}{1-\rho}\pi^{\mathrm{Pu}}$, and $g'(0) < w_{\mathrm{oc}}^{\mathrm{Pu}})$, that complements the Pu case, and $(\varepsilon > \frac{\rho}{1-\rho}\overline{w}, z \leq \frac{\rho}{1-\rho}\pi^{\mathrm{PuR}}$, and $g'(0) < w_{\mathrm{oc}}^{\mathrm{PuR}})$, that complements the PuR case. In all these cases, function g does not allow the boss to offer a sufficiently high wage, given the corresponding credibility of punishment and the corresponding wage to be attained. Hence she optimally chooses $\bar{s}^{\mathrm{NOC}} = 0$.

□

Proof of Lemma 7. Let $\Gamma(x) = z(x) - \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}(x)$, for $x \in (0, \gamma]$. The derivative of Γ with respect to x can be expressed as $\Gamma'(x) = z'(x) - \frac{\rho}{1-\rho}[\Pi^{\mathrm{Pu}}]'(x)$ for $x \in (0, \gamma]$. By Condition (1), $\Gamma'(x) < 0$ for all $x \in (0, \gamma)$. First, we establish the existence of a unique value $b_0^{\star} \in (0, \gamma)$ such that $z\left(b_0^{\star}\right) = \frac{\rho}{1-\rho}\Pi^{\mathrm{Pu}}\left(b_0^{\star}\right)$. The existence of $b_0^{\star} \in (0, \gamma)$ follows from the intermediate value theorem since $\Gamma(x)$ is continuous, $\Gamma(0) > 0$ and $\Gamma(\gamma) < 0$. We now establish the uniqueness of $b_0^{\star}$. To introduce a contradiction, suppose there exist $\alpha, \beta \in (0, \gamma)$, with $\alpha < \beta$, such that $\Gamma(\alpha) = \Gamma(\beta) = 0$. By the Rolle's Theorem, there exists $\alpha' \in (\alpha, \beta)$ such that $\Gamma'(\alpha') = 0$. However, this contradicts the fact $\Gamma'(x) < 0$ for all $x \in (0, \gamma)$. □

Proof of Proposition 2. We divide the proof of Proposition 2 into two lemmas. □

Lemma 8. *Suppose* $p(\gamma) - p\left(b_0^{\star}\right) \geq \frac{1-\rho}{\rho}\frac{\varepsilon}{F}$. *Then, the police choose the pair* $(b_0, b_z) = (\gamma, 0)$.

Proof. The proof is divided into two steps.

1. We show that $n^{\mathrm{R}}\left(b_0^{\star}\right) > n^{\mathrm{Pu}}(\gamma)$. First we establish that $n^{\mathrm{Pu}}\left(b_0^{\star}\right) > n^{\mathrm{Pu}}(\gamma)$. We have $f\left(n^{\mathrm{Pu}}\left(b_0^{\star}\right)\right) = w + p\left(b_0^{\star}\right)F < w + p(\gamma)F = f\left(n^{\mathrm{Pu}}(\gamma)\right)$, since $p\left(b_0^{\star}\right) < p(\gamma)$. It follows that $n^{\mathrm{Pu}}\left(b_0^{\star}\right) > n^{\mathrm{Pu}}(\gamma)$ since f is decreasing. Second, by Proposition 1, we have $n^{\mathrm{R}}\left(b_0^{\star}\right) > n^{\mathrm{Pu}}\left(b_0^{\star}\right)$. The result follows.
2. Since W is decreasing in its two arguments, if $s^{\mathrm{R}}\left(b_0^{\star}\right) \geq s^{\mathrm{Pu}}(\gamma)$, then $W\left(s^{\mathrm{R}}\left(b_0^{\star}\right), n^{\mathrm{R}}\left(b_0^{\star}\right)\right) < W\left(s^{\mathrm{Pu}}(\gamma), n^{\mathrm{Pu}}(\gamma)\right)$. Thus, it is sufficient to show that $s^{\mathrm{R}}\left(b_0^{\star}\right) \geq s^{\mathrm{Pu}}(\gamma)$ if and only if $p(\gamma) - p\left(b_0^{\star}\right) \geq \frac{1-\rho}{\rho}\frac{\varepsilon}{F}$. We have

$$
\begin{aligned}
p(\gamma) - p\left(b_0^\star\right) \geq \frac{1-\rho}{\rho}\frac{\varepsilon}{F} &\Leftrightarrow \frac{1-\rho}{\rho}\varepsilon \leq F\left(p(\gamma) - p\left(b_0^\star\right)\right) \\
&\Leftrightarrow \overline{w} + p\left(b_0^\star\right)F + \frac{1-\rho}{\rho}\varepsilon \leq \overline{w} + p(\gamma)F \\
&\Leftrightarrow [g']^{-1}\left(\overline{w} + p\left(b_0^\star\right)F + \frac{1-\rho}{\rho}\varepsilon\right) \geq [g']^{-1} \\
&\qquad\qquad (\overline{w} + p(\gamma)F),
\end{aligned}
$$

since $[g']^{-1}$ is strictly decreasing. The result follows from the fact that $[g']^{-1}\left(\overline{w} + p\left(b_0^\star\right)F + \frac{1-\rho}{\rho}\varepsilon\right) = s^{\mathrm{R}}\left(b_0^\star\right)$ and $[g']^{-1}(\overline{w} + p'(\gamma)F) = s^{\mathrm{Pu}}(\gamma)$.

□

Lemma 9. *Suppose* $p(\gamma) - p\left(b_0^\star\right) < \frac{1-\rho}{\rho}\frac{\varepsilon}{F}$.

1. *If* $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) < \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$, *then the police choose* $(b_0, b_z) = (\gamma, 0)$.
2. *If* $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) > \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$, *then the police choose* $(b_0, b_z) = \left(b_0^\star, \gamma - b_0^\star\right)$.
3. *If* $s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) = \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right)$, *then the police are indifferent between the pairs* $(\gamma, 0)$ *and* $\left(b_0^\star, \gamma - b_0^\star\right)$.

Proof. From Lemma 8, we know that $s^{\mathrm{Pu}}(\gamma) > s^{\mathrm{R}}\left(b_0^\star\right)$ when $\frac{1-\rho}{\rho}\frac{\varepsilon}{F}$. Hence, the police choose $(b_0, b_z) = (\gamma, 0)$ if and only if $W(s^{\mathrm{Pu}}(\gamma), n^{\mathrm{Pu}}(\gamma)) \geq W\left(s^{\mathrm{R}}\left(b_0^\star\right), n^{\mathrm{R}}\left(b_0^\star\right)\right)$. We have

$$
\begin{aligned}
&\Leftrightarrow s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) \leq \frac{1-\lambda}{\lambda}\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right) \\
&\Leftrightarrow \lambda\left(s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right)\right) \leq (1-\lambda)\left(n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)\right) \\
&\Leftrightarrow -\left(\lambda s^{\mathrm{R}}\left(b_0^\star\right) + (1-\lambda)n^{\mathrm{R}}\left(b_0^\star\right)\right) \leq -(\lambda s^{\mathrm{Pu}}(\gamma) + (1-\lambda)n^{\mathrm{Pu}}(\gamma)) \\
&\Leftrightarrow W\left(s^{\mathrm{R}}\left(b_0^\star\right), n^{\mathrm{R}}\left(b_0^\star\right)\right) \leq W(s^{\mathrm{Pu}}(\gamma), n^{\mathrm{Pu}}(\gamma)).
\end{aligned}
$$

□

To establish the result given in Example 1 we need two lemmas.

Lemma 10. *We have* $s^{\mathrm{Pu}}(\gamma) > s^{\mathrm{R}}\left(b_0^\star\right)$ *if and only if* $\gamma - b_0^\star < \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{F}$.

Proof. By Lemma 8, we know that $s^{\mathrm{Pu}}(\gamma) > s^{\mathrm{R}}\left(b_0^\star\right)$ if and only if $p(\gamma) - p\left(b_0^\star\right) < \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{F}$. The result follows from $p(\gamma) = \gamma$ and $p\left(b_0^\star\right) = b_0^\star$. □

Lemma 11. *Let $A = \overline{w} + \gamma F$ and $B = \overline{w} + b_0^\star F$.*

1. *We have $s^{\mathrm{Pu}}(\gamma) = \frac{1}{(2A)^2}$ and $s^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1}{2\left(B + \frac{1-\rho}{\rho}\varepsilon\right)}\right)^2$.*
2. *Moreover, we have $n^{\mathrm{Pu}}(\gamma) = 4\beta A s^{\mathrm{Pu}}(\gamma)$, and $n^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1-\rho}{\rho}\right)\left(\frac{\varepsilon}{B}\right)s^{\mathrm{R}}\left(b_0^\star\right) + \left(\frac{1-\rho}{\rho}\right)\frac{\rho\beta}{B}$.*

Proof.

1. From the proof of Lemma 8, we have $[g']^{-1}(\overline{w} + \gamma F) = s^{\mathrm{Pu}}(\gamma)$ and $[g']^{-1}\left(\overline{w} + b_0^\star F + \frac{1-\rho}{\rho}\varepsilon\right) = s^{\mathrm{R}}\left(b_0^\star\right)$. Recall that $g = \sqrt{x}$, so $g'(x) = \frac{1}{2\sqrt{x}}$. Therefore, $[g']^{-1}(y) = \frac{1}{4y^2}$, for $y \in \mathbb{R}^+$. It follows that $s^{\mathrm{Pu}}(\gamma) = \left(\frac{1}{2(\overline{w} + \gamma F)}\right)^2$ and $s^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1}{2\left(\overline{w} + b_0^\star F + \frac{1-\rho}{\rho}\varepsilon\right)}\right)^2$.
2. We have $\overline{w} + \gamma F = f(n^{\mathrm{Pu}}(\gamma))$, that is, $\overline{w} + \gamma F = \frac{\beta}{n^{\mathrm{Pu}}(\gamma)}$, and $n^{\mathrm{Pu}}(\gamma) = \frac{\beta}{\overline{w} + \gamma F}$. Therefore,

$$\frac{n^{\mathrm{Pu}}(\gamma)}{s^{\mathrm{Pu}}(\gamma)} = \frac{\frac{\beta}{\overline{w} + \gamma F}}{\left(\frac{1}{2(\overline{w} + \gamma F)}\right)^2} = 4\beta(\overline{w} + \gamma F),$$

and $n^{\mathrm{Pu}}(\gamma) = 4\beta(\overline{w} + \gamma F)s^{\mathrm{Pu}}(\gamma)$. By Lemma 2, we have

$$\begin{aligned}
\overline{w} + b_0^\star F - \frac{1-\rho}{\rho}\left(\frac{s^{\mathrm{R}}\left(b_0^\star\right)}{n^{\mathrm{R}}\left(b_0^\star\right)}\right)\varepsilon = \frac{\beta}{n^{\mathrm{R}}\left(b_0^\star\right)} &\Rightarrow \overline{w} + b_0^\star F - \frac{1-\rho}{\rho}\left(\frac{s^{\mathrm{R}}\left(b_0^\star\right)}{n^{\mathrm{R}}\left(b_0^\star\right)}\right)\varepsilon - \frac{\beta}{n^{\mathrm{R}}\left(b_0^\star\right)} = 0 \\
&\Rightarrow \rho n^{\mathrm{R}}\left(b_0^\star\right)\left(\overline{w} + b_0^\star F\right) - (1-\rho)s^{\mathrm{R}}\left(b_0^\star\right)\varepsilon - \rho\beta = 0 \\
&\Rightarrow n^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1-\rho}{\rho}\right)\frac{s^{\mathrm{R}}\left(b_0^\star\right)\varepsilon}{\overline{w} + b_0^\star F} + \frac{\beta}{\overline{w} + b_0^\star F}.
\end{aligned}$$

It follows that $n^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1-\rho}{\rho}\right)\left(\frac{\varepsilon}{\overline{w} + b_0^\star F}\right)s^{\mathrm{R}}\left(b_0^\star\right) + \frac{\beta}{\overline{w} + b_0^\star F}$. □

Proof of Example 1. By Lemma 10, we know that $s^{\mathrm{Pu}}(\gamma) > s^{\mathrm{R}}\left(b_0^\star\right)$ if and only if $\gamma < \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{F}$. When $\gamma \geq \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{F}$, we have $W\left(s^{\mathrm{Pu}}\left(b_0^\star\right), n^{\mathrm{R}}\left(b_0^\star\right)\right) < W(s^{\mathrm{Pu}}(\gamma), n^{\mathrm{Pu}}(\gamma))$

since $n^{\mathrm{R}}\left(b_0^\star\right) > n^{\mathrm{Pu}}(\gamma)$. Moreover, by Proposition 2, $W\left(s^{\mathrm{Pu}}\left(b_0^\star\right), n^{\mathrm{R}}\left(b_0^\star\right)\right) \geq W(s^{\mathrm{Pu}}(\gamma), n^{\mathrm{Pu}}(\gamma))$ occurs when $\lambda \geq \lambda^\star = \frac{n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)}{s^{\mathrm{Pu}}(\gamma) - s^{\mathrm{R}}\left(b_0^\star\right) + n^{\mathrm{R}}\left(b_0^\star\right) - n^{\mathrm{Pu}}(\gamma)}$. By using Lemma 11, we have $n^{\mathrm{Pu}}(\gamma) = 4\beta A s^{\mathrm{Pu}}(\gamma)$, where $A = \overline{w} + \gamma F$, and $n^{\mathrm{R}}\left(b_0^\star\right) = \left(\frac{1-\rho}{\rho}\right)\frac{\varepsilon}{B} s^{\mathrm{R}}\left(b_0^\star\right) + \frac{\beta}{B}$, where $B = \overline{w} + b_0^\star F$. We have

$$\lambda^\star = \frac{\left(\frac{1-\rho}{\rho}\right)\varepsilon s^{\mathrm{R}}\left(b_0^\star\right) - 4B\beta A s^{\mathrm{Pu}}(\gamma) + \beta}{\left(\left(\frac{1-\rho}{\rho}\right)\varepsilon - B\right)s^{\mathrm{R}}\left(b_0^\star\right) + B(1 - 4\beta A)s^{\mathrm{Pu}}(\gamma) + \beta},$$

and the result follows. □