Selection of Multilayer Dielectric Coatings for Fabry-Perot Interferometry

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The parameters appropriate to selection of multilayer dielectric coatings for Fabry-Perot interferometry in the ultraviolet and visible regions of the spectrum are discussed. A representative example is given.

I. Introduction

High-resolution work in optical spectroscopy is becoming increasingly important as more accurate measurements of hyperfine structures and line shapes become necessary. While the marked improvement in diffraction gratings in recent years[1,2] has partially fulfilled the increased demands, the Fabry-Perot interferometer is still unsurpassed for its resolution and luminosity in many regions of the spectrum.[3] Recent work at Berkeley in the field of optical hyperfine structure, using light sources of limited intensities, has compelled a re-examination of the usefulness of the Fabry-Perot interferometer and available reflecting coatings, in order to utilize fully its advantages in the visible and near ultraviolet regions of the spectrum. The usefulness of coatings of silver and aluminum has been thoroughly investigated by others.[4,5] The decision to concentrate on dielectric coatings came about because of their relatively small absorption losses and developments in their technology.

II. Fundamental Considerations

The general theory and practical applications of the Fabry-Perot interferometer have been thoroughly discussed by Meissner.[6] There have been many more recent developments in theory and in specialized applications, but only those references will be cited which have a particular use here. Three properties or parameters are significant for high-resolution work: the free-spectral range, the instrumental width, and the transmission factor.

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The figures in this article have been reproduced larger than is usual for this journal since they are designed to be used in calculations, rather than simply to present information.

A. Basic Equations

The instrumental function of a perfect Fabry-Perot interferometer with a monochromatic source of radiation is the Airy equation

$$\tau_R(\lambda) = \frac{1}{(1 + A/T)^2} \times \frac{1}{1 + (4R\sin^2\pi\Delta/\lambda)/(1 - R)^2},$$

where A is the absorption coefficient, T the transmission coefficient, and R the reflection coefficient; the path difference $\Delta = 2t\cos\theta$ in vacuum. Further, the equation $m\lambda = 2t\cos\theta$ expresses the relationship between the order m, the wavelength λ, the spacer thickness t, and the angle of observation θ of the transmitted radiation.

B. Free-Spectral Range

The free-spectral range, or spectrum interval free from overlapping, is given approximately by $F_\sigma = 1/(2t)$. For example, an interferometer with a 1-cm spacer has a free spectral range of 0.5 cm^{-1} (0.125 Å at 5000 Å).

C. Instrumental Width

The instrumental width is often called the resolving limit, or wavenumber difference between two monochromatic radiation sources, whose wavenumbers differ by the instrumental width at half-maximum transmission. The value of this width is given approximately by

$$\Delta\sigma = \frac{1 - R}{\pi\sqrt{R}} \times \frac{1}{2t}$$

or

$$\Delta\sigma = \frac{1 - R}{\pi\sqrt{R}} \times F_\sigma.$$

The quantity $\pi\sqrt{R}/(1 - R)$ is denoted by N_R, and is called the equivalent number of interfering beams, or *finesse*, by analogy with the often used formula for the

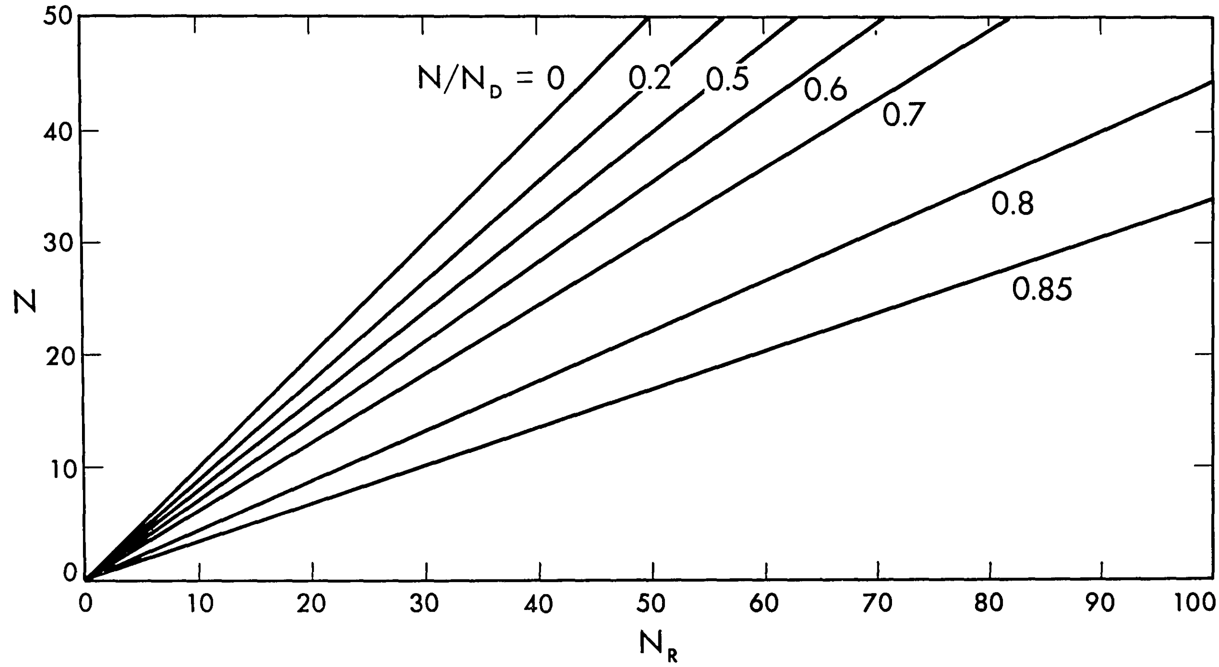

Fig. 1. Finesse vs reflecting finesse, as a function of N/N_D.

resolving power of a spectroscopic instrument $R = mN$. The resolving limit is then $1/N_R$ times the free-spectral range. A typical value of N_R is 20, so that with a 1-cm spacer, the free-spectral range is 0.5 cm^{-1} and the resolving limit is 0.025 cm^{-1} (a resolving power of 800,000 at 5000 Å).

In the case of an actual interferometer with plates not perfectly flat, the above result is modified. The relationships are complex, but have been studied in detail.[7] It is customary to call N the finesse of the interferometer, N_R the finesse owing to the reflection coefficient R, and N_D the limiting finesse owing to the flatness of the plates. If the plates are flat to λ/k, then the limiting finesse $N_D = k/2$. Typical values of N_D run from 10–100.

D. Transmission

The maximum transmission of the interferometer as given by the Airy function is $\tau_R = 1/(1 + A/T)^2$. It differs from unity because the coatings have an absorption coefficient $A = 1 - R - T$. This result must be modified because of the lack of perfect flatness of the interferometer plates.[7] The multiplying factor is called τ_D, making the total transmission $\tau = \tau_R \times \tau_D$.

III. Selection of Parameters

A. General Considerations

Usually the problem at hand is the resolution or separation of some particular lines. The line sharpness and separation fix the upper limit on the allowable instrumental width. It might appear that the only problem is that of the choice of a sufficiently large reflection coefficient R, to give a large value of reflecting finesse, and then to select the spacer thickness large enough to permit resolution of the lines in question. Practically speaking, this solution is unsatisfactory. If the reflectance of the coatings is too high, the light transmission becomes so low as to render the interferometer nearly useless. Also, because of its relatively small free-spectral range, the interferometer must be used in conjunction with a narrow pass filter or auxiliary spectrograph, in order to eliminate blending with other lines emitted by the light source. At the very least, the auxiliary spectrograph must have sufficient resolution and dispersion to separate one free-spectral range of the interferometer. Even this is not sufficient in many cases, since a hyperfine structure pattern is often wider than a free-spectral range, yet has some very closely spaced components. In the past, it has been the practice to allow some overlapping of interferometer orders, and to choose spacer thicknesses of just the proper sizes to result in nonblending of lines. The subsequent unscrambling of patterns often resulted in ambiguous interpretations.

With the recent development of large, well-ruled diffraction gratings, another solution has become practical. The high resolution and dispersion of stigmatic plane grating spectrographs make it possible to resolve the "gross" hyperfine structure with the grating alone, while leaving the Fabry-Perot interferometer the task of resolving the more closely spaced lines.

We can proceed in the following way: first choose the free-spectral range as small as possible consistent with the resolution and dispersion of the auxiliary spectrograph; select the minimum reflectance of the coatings

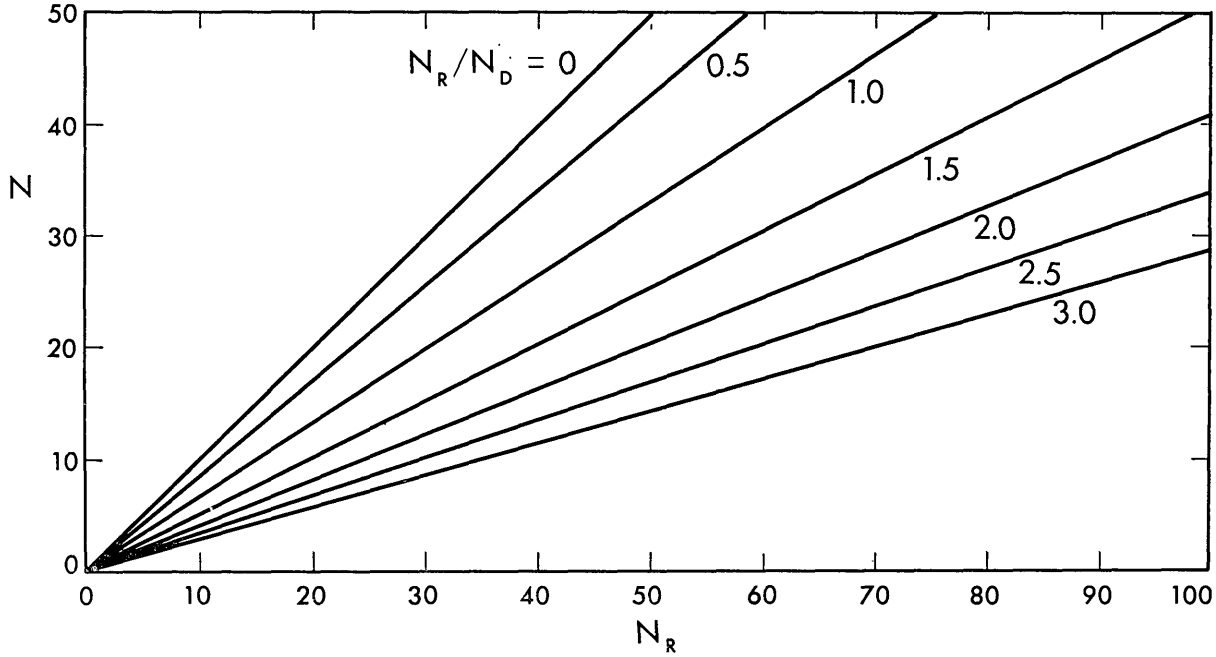

Fig. 2. Finesse vs reflecting finesse, as a function of N_R/N_D.

which will ensure a small enough instrumental width to resolve the closest spaced lines in question; determine the transmission of the interferometer, to ascertain if the experiment is still feasible. Naturally, since these considerations are interrelated, it is often necessary to make some compromises, which are best worked out by experience.

B. Choosing the Free-Spectral Range

Let us consider that the Fabry-Perot rings are focused on the slit of a stigmatic spectrograph. Some minimum length along each ring is necessary for measurement; this length is chosen as the slit-width of the auxiliary spectrograph. Assume that this length is x mm. Also, assume that the plate factor of the spectrograph is y cm^{-1}/mm. Then, the smallest free-spectral range tolerable with no overlapping is the product xy cm^{-1}. A representative example follows: slit-width 0.200 mm, plate factor 1.50 cm^{-1}/mm, minimum free-spectral range 0.300 cm^{-1}, corresponding to a spacer thickness of 16.7 mm. Of course, if the total width of the hyperfine structure pattern is less than this figure, the free-spectral range can be made correspondingly smaller.

C. Choosing the Reflection Coefficient R (the Instrumental Width)

The minimum free-spectral range is now fixed. If we assume that the problem at hand requires that the instrumental width be less than a fixed amount $\Delta\sigma$, the finesse is fixed at $N = F_\sigma/\Delta\sigma$. The appropriate value of reflection coefficient R to produce the required finesse N is found as follows.

First find the ratio N/N_D (see Sec. IIC), and then refer to Fig. 1, which presents graphically[3,7] the relationship between finesse N, and the reflecting finesse N_R. It is possible to have $N = N_R$ only for perfectly flat plates ($N/N_D = 0$). The scale on the ordinate is chosen to be representative of present possibilities. The upper limit is set by the usual operating conditions in the laboratory, since it is very difficult to utilize a finesse greater than 50, simply because the operating conditions are too stringent with respect to distortion of the plates in their holder, temperature, and pressure. (Figure 2 presents the same relationship, but as a function of N_R/N_D. This graph is useful when N_R is known, and N is to be determined. See Sec. IV.)

To find the reflection coefficient R, refer to Fig. 3, which is a plot of the relationship R vs $\pi\sqrt{R}/(1-R)$. Once the value of reflection coefficient has been chosen, the proper dielectric coating must be selected.

D. Choosing Dielectric Coatings

Much has been written about dielectric coatings for Fabry-Perot interferometers.[8-10] The usual practice is to deposit alternate quarter-wave layers of high- and low-index dielectric materials until the desired reflectance is obtained. The high-index material generally is the more durable, and constitutes both the first and last layers. The reflection coefficient can be computed from the following equation:

$$R = (n_H^{2s+2} - n_0 n_L^{2s})^2/(n_H^{2s+2} + n_0 n_L^{2s})^2,$$

where n_H, n_L, and n_0 are the indices of the high index, low index, and substrate materials. The total number of layers in the film is $(2s + 1)$. Conversely,

Table I. Dielectric Coating Materials

Material	Refractive index	Short wavelength limit of transparency (Å)	Loss in a multilayer stack (%)
Cryolite	1.35–1.39	2000	—
ZnS	2.30	4000	0–1
Sb_2O_3	2.1	2900	1–2
PbF_2	1.8	3500	0–1
	1.9	3000	1–2
	2.1	2500	2–5
	2.2	2400	3–10

knowing the value of R desired, the required number of layers can be calculated.

The choice of dielectric materials is somewhat restricted in Fabry-Perot interferometry for practical reasons. The films must be relatively durable under conditions of fairly frequent handling and exposure, and they must be removable without damage to the plates. The materials we have found most useful, together with some of their properties, are listed in Table I. More details are given in the references.[9–11] The losses given in the last column represent those due to absorption, scattering in the films, and surface roughness of the substrate. The figures can only be approximate owing to variations in coating techniques and substrate surfaces. As an example, a 5-layer stack of ZnS and cryolite at 5000 Å has a peak reflectance of about 87%, with losses between 0 and 1 %, and a reflecting finesse of about 23.

E. Determination of Transmission

The final consideration is the transmission of the Fabry-Perot interferometer. The transmission is the product of two factors: one, the maximum of the Airy equation $\tau_R = 1/(1 + A/T)^2$, and the other, a function resulting from the lack of perfect flatness of the plates.[7]

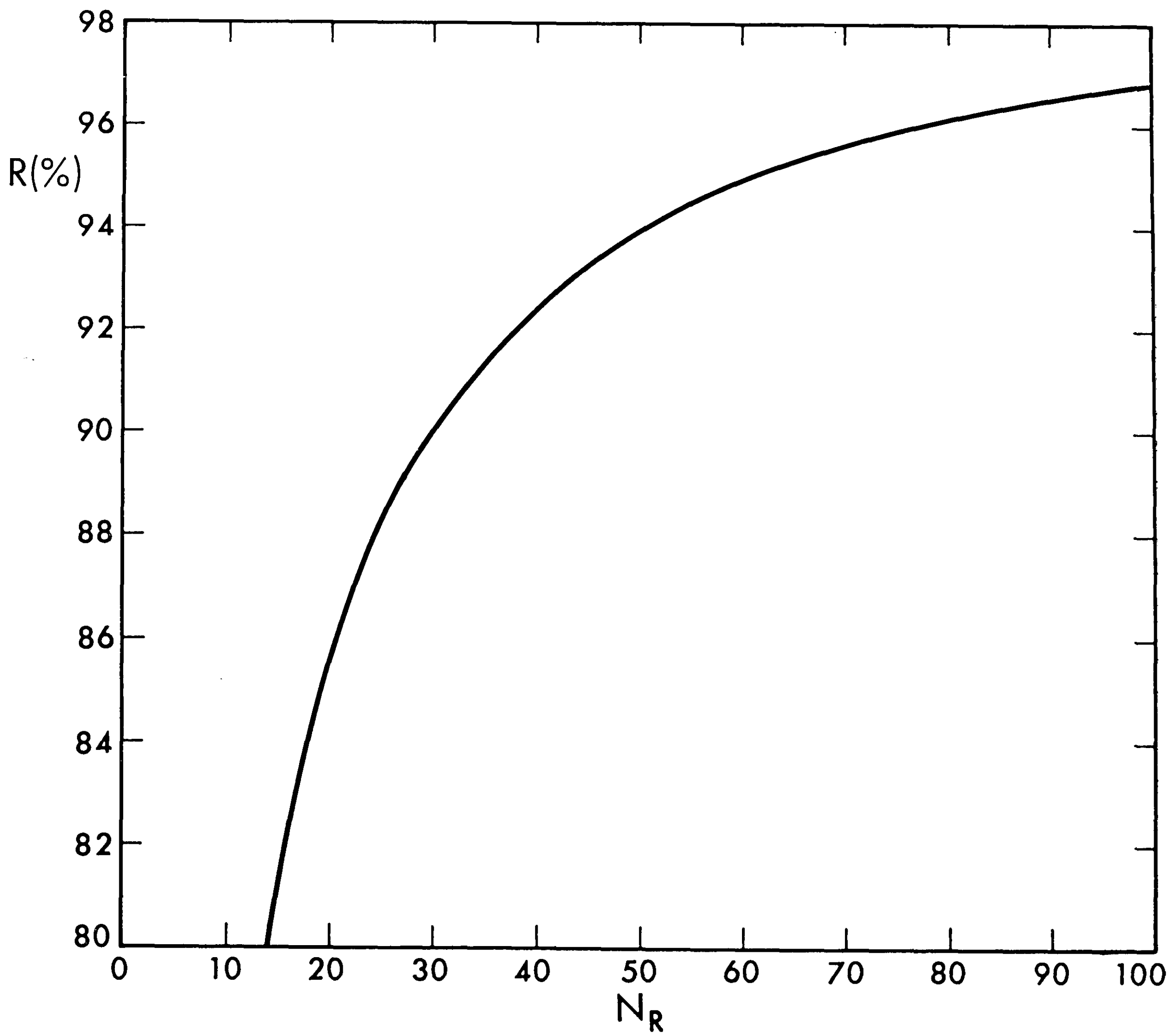

Fig. 3. Reflection coefficient vs reflecting finesse.

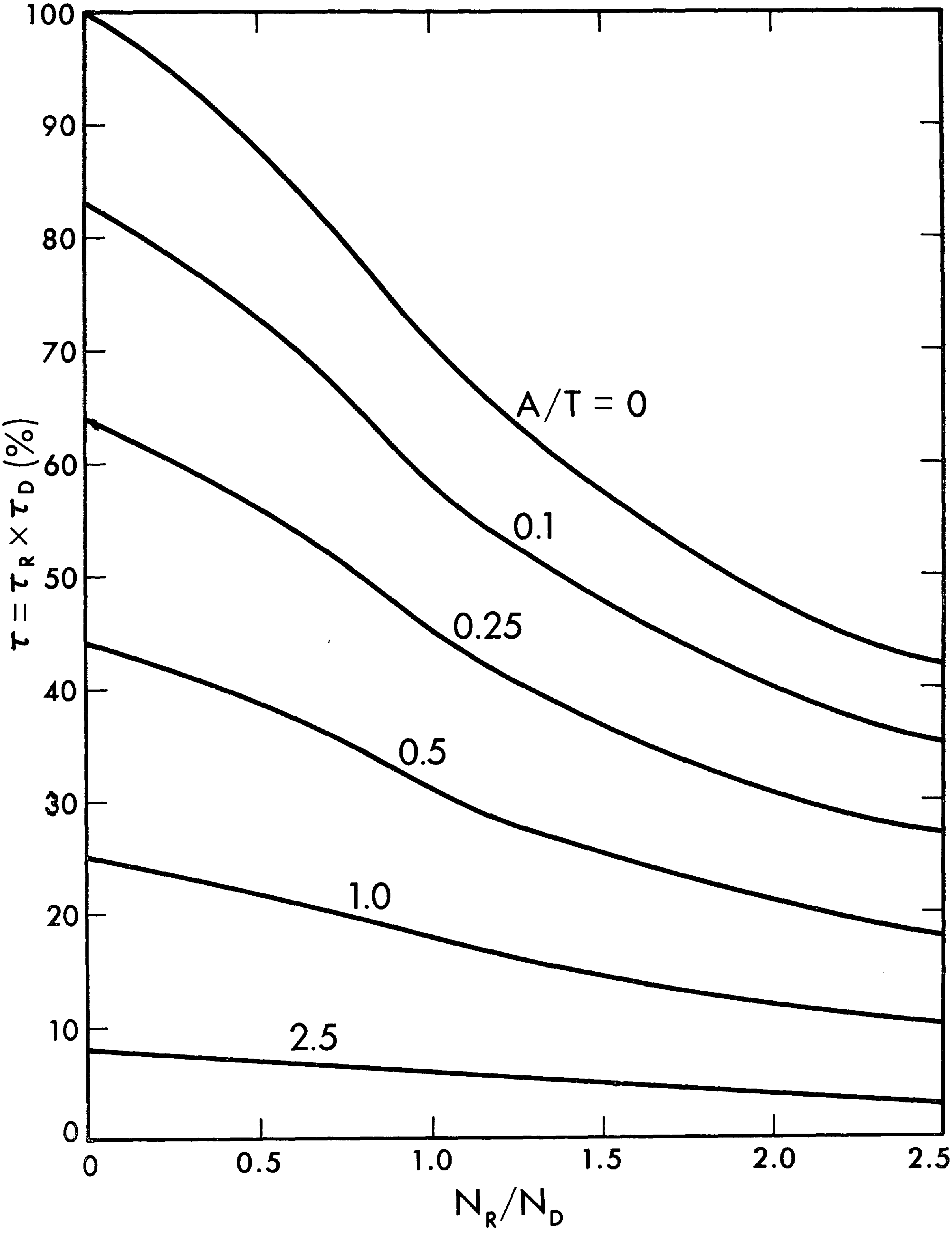

Fig. 4. Transmission of a Fabry-Perot interferometer.

The product of these two factors is graphically presented in Fig. 4, where the transmission is plotted as a function of N_R/N_D, for different values of the ratio A/T. For perfectly flat plates and no absorption losses in the film, the transmission is 100%, signifying that the only function of the interferometer in this case is to cut off light not at the correct angles, without a reduction in the maximum of the transmitted light.

IV. Illustrative Example

Suppose that it is desired to examine the structure of the 2537 Å line of mercury, which has a total width of approximately 0.740 cm^{-1}, and an individual line-width of about 0.035 cm^{-1}. The auxiliary spectrograph to be used has a plate factor of 3.5 cm^{-1}/mm (0.23 Å/mm at 2537 Å). The minimum slit-width (length of Fabry-Perot ring needed for measurement) is 0.2 mm. The minimum allowable free-spectral range is then calculated as F_σ = (slit-width) × (plate factor) = 0.7 cm^{-1}. The spacer thickness is $t = 1/(2F_\sigma) = 7.1$ mm. The required finesse to make the instrumental width equal to the line-width is N = (free-spectral range)/(line-width) = 20. To find the reflecting finesse, it is necessary to know the limiting finesse of the interferometer plates. A representative value is $N_D = 25$ at 2537 Å (plates flat to $\lambda/100$ in the green). Reference to Fig. 1 for $N = 20$ and $N/N_D = 0.8$ yields $N_R = 45$. From Fig. 3, it is found that a reflection coefficient of about 93% is required. A calculation for a 9-layer film of PbF_2 and cryolite (index 1.39 from Table I) on a substrate of fused silica (index 1.52) shows that the theoretical reflection coefficient R is 95%. Let us assume that the losses in the film are about 3%, resulting in a transmission of 2%, and a ratio $A/T = 1.5$. The curves in Fig. 4 show that the transmission of the interferometer is about 8%.

This example is an idealized one, and the exact figures must not be taken too literally. There are wide variations from coating to coating in the fabrication of dielectric films, especially in the ultraviolet region of the spectrum. Often the maximum reflectance does not reach the calculated value, owing to inaccuracies in thicknesses of the individual layers in the stack. Also, the calculation is based on the assumption of non-absorbing films. With absorption and other losses present, it must be regarded as an approximation. The general principles are nevertheless applicable. Specifically, to reach a reflectance of 95% at 2537 Å, it is necessary to use an 11-layer instead of a 9-layer film, as

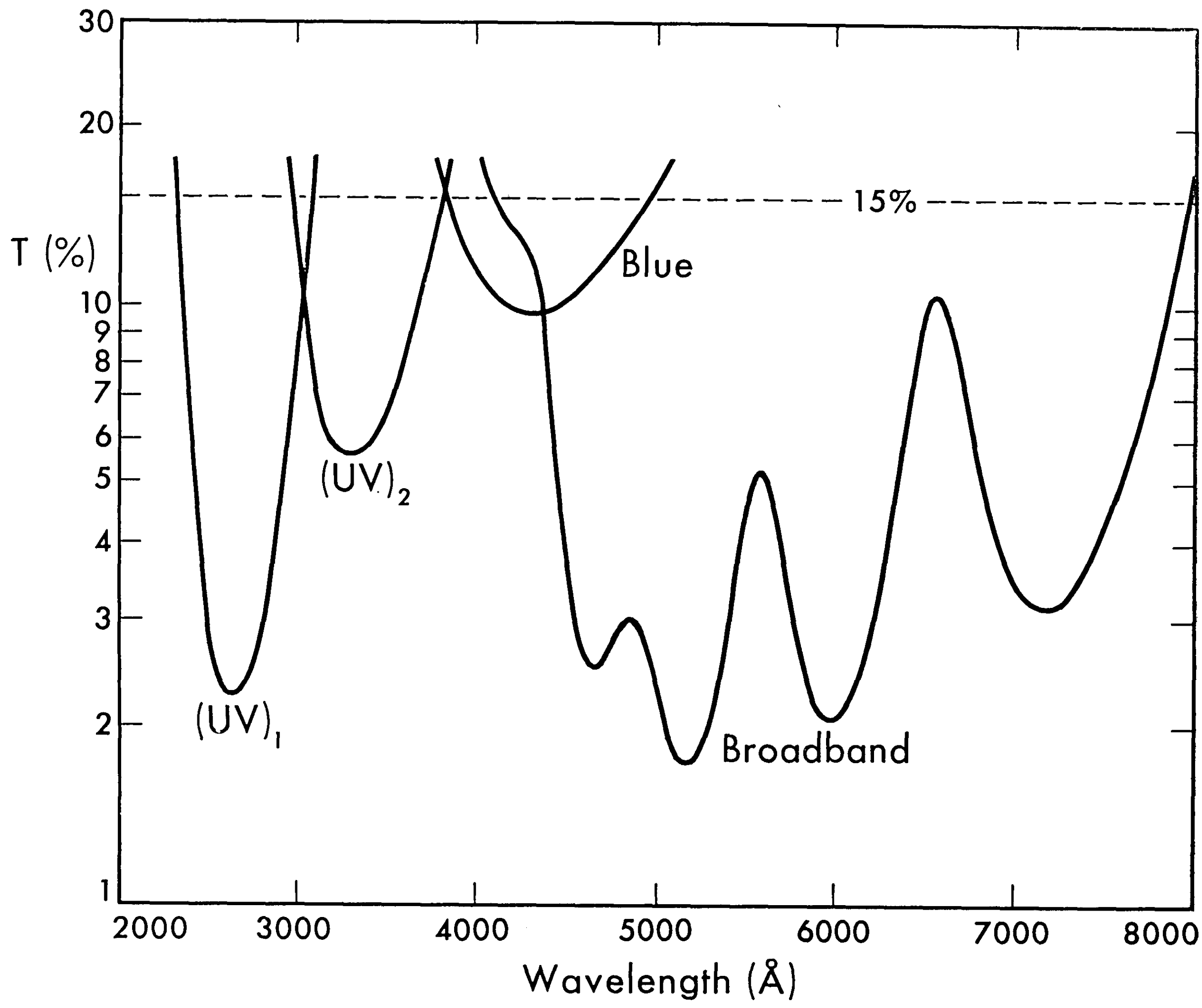

Fig. 5. Representative multilayer dielectric coatings for Fabry-Perot interferometry. $(uv)_1$, 11-layer PbF_2 and cryolite. $(uv)_2$, 7-layer Sb_2O_3 and cryolite. Blue, 5-layer ZnS and cryolite. Broadband, 15-layer ZnS and cryolite.

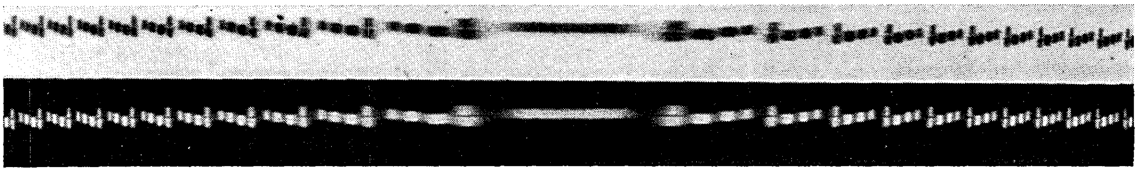

Fig. 6. Interferogram of mercury 2537 Å, taken under conditions outlined in the illustrative example in the text (negative and positive prints).

is shown in Fig. 5. All the curves presented in this figure are transmission curves (the usual laboratory instrument measures transmitted rather than reflected light); absorption losses must be estimated in order to obtain the reflectance. They are representative curves of films practically obtainable. Where the losses are small, the experimental curves closely approach the theoretical ones, as in the case of the "Blue" coating.

The interferogram reproduced in Fig. 6 accurately represents the figures of the example, and was taken with the 11-layer film labeled $(uv)_1$ in Fig. 5.

In practice, a dielectric coating may already be at hand, and the problem is then one of determining the finesse, given the transmission of the coating. The procedure is the reverse of that given in the example with the exception that Fig. 2 is used instead of Fig. 1, since N_R and N_D are known and N is to be determined.

V. General Remarks

The treatment given here is not exhaustive, and presents only a small part of the theoretical knowledge and technical lore of dielectric films and interferometers. Many other conclusions can be drawn about the uses of dielectric films, and many properties have not been mentioned. The aim has been to present those facets worthy of foremost consideration when planning the use of Fabry-Perot interferometers.

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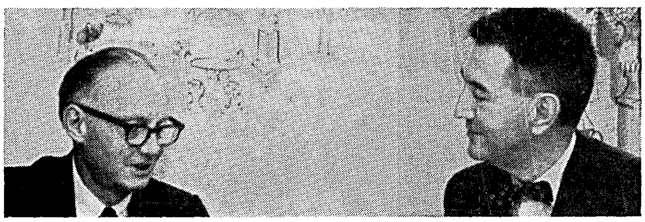

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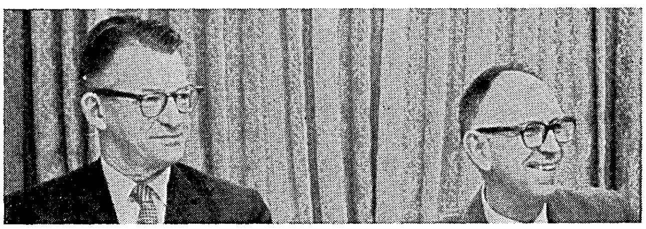

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