QUADRUPLE TANK APPARATUS

EEE 588 Design Of Multivariable Control Systems

SHUBHAM. S. GHARAT.

ARIZONA STATE UNIVERSITY (FALL 2020)

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1 Overview

The plant analyzed in this report is the Quadruple Tank Apparatus, also called as the four tank process. This system has two inputs which are the voltages to the two pumps. The outputs are the liquid levels in the bottom two tanks. This process has four states which are the levels in all the four tanks. I reviewed the literature related to this apparatus and shortlisted some characteristics which were essential for our project. This plant has two operating points(Minimum and Non-minimum phase) and I have analysed both the cases in this report. The control designs analysed are LQR, Kalman Filter, LQG, H-Infinity. Through this project opportunity I was able to work on designing controllers and especially designing them for real world systems. I wish to mention that LQR was effective for this plant whereas LQG was tedious to design according to the required specifications. There was a tradeoff between the peak values of Sensitivity and Complementary Sensitivity, aswell as between the overshoot and the settling time for different control designs.

2 Relevant Literature

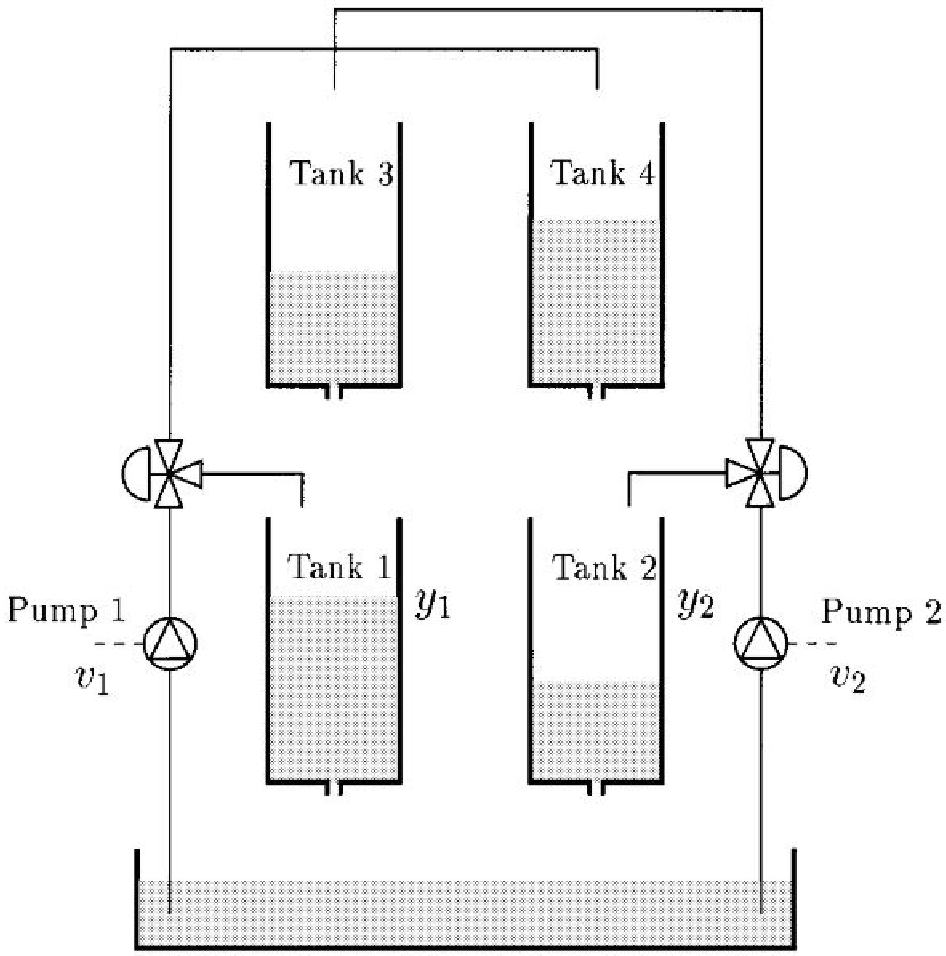
•Dr. Armando Rodriguez. Analysis and Design of Multivariable Feedback Control Systems. CONTROL3D LLC, 2004.
•Johansson, K. H., Nunes, J. L. R. (1998). A multivariable laboratory process with an adjustable zero. Proceedings of the American Control Conference, 4(3), 2045–2049. `https://doi.org/10.1109/ACC.1998.702986`
•Grebeck, M. (1998). A Comparison of Controllers for the Quadruple Tank System. `https://portal.research.lu.se/portal/en/publications/a-comparison-of-controllers-for-the-quadrupl` `.html`
•Victor, A. O. (2010). Developing Advanced Control strategies for a 4-Tank Laboratory process. `https://www.halvorsen.blog/documents/projects/student_projects/master/2010/4-Tank/master_thesis_report.pdf`

3 Description of MIMO system to be controlled

3.1 Introduction

The quadruple tank process is a **two-input, two-output**(TITO) system with a movable zero composed of the interconnection of two double tank processes. The two inputs are the voltages of the two water pumps, and the two outputs are the water levels of the two lower tanks[7] p.4. The quadruple-tank process's linearized model has a multivariable zero, located in either the left or the right-half-plane, by simply changing a valve [11] p.1. The output of each pump is split into two using a three-way valve. Pump 1 is shared by tank 1 and tank 3, while pump 2 is shared by tank 2 and tank 4. Thus, each pump output goes to two tanks, one lower other one into the upper diagonal tank and the flow to these tanks are controlled by the position of the valve represented as γ. The position of the two valves determines whether the system is in the minimum phase or in the non- minimum phase [8] p.2.

3.2 Figure of Quadruple Tank Apparatus



[11]p.2

Figure 1: Quadruple Tank Apparatus

3.3 Relevant Model Information

Variables	Parameters	units	Definition
Inputs	(v_1, v_2)	Volts(V)	Voltages applied to pump 1 and pump 2
Outputs	(h_1, h_2)	Centimeters(Cm)	Level of the liquid in tank 1 and tank 2
States	$(\mathrm{h}_1, h_2, h_3, h_4)$		Levels in all four tanks

Table 1: Input, Output and State Variable

Inputs:
•The two Inputs of this system are the voltages applied to the two respective pumps[Pump 1 and Pump 2](v_1, v_2).The voltage applied to the pumps is measured in Volts(V).
Outputs:
• The two Outputs of this system are the levels of liquid in the lower two tanks [Tank 1 and Tank 2] (y_1, y_2). The level in the tanks are measured in Centimeters(cm).
States:
•This system has four states which are the levels of the liquid in the four tanks[Tank 1, Tank 2, Tank 3, Tank 4].
Variables:
•Manipulated Variables: The manipulated variables are the valve positioning and the pump flow rates [14]p.12.
•Controlled Variables: The level in this process that we want to control, which can be made to follow any pattern or kept constant [14]p.12.
•Load disturbances: This is the flow rate that is entering the Four-tank process. They are usually set by the upstream or downstream parts of the process which is at the valve position, and our control system should be designed in such a way that it keeps the Four-tank process under control not been influenced by the disturbances [14]p.12.

3.4 Key Issues in Controlling this system / Fundamental Performance limitations

Sources of uncertainty in the four-tank system include [13]p.5 :-
(1) unmodeled and neglected dynamics;
(2) valve position uncertainty, which affects the ratios of flows across the top and bottom tanks;
(3) uncertainty in the actuators; and
(4) non-linearities in the system.
• Right half-plane zeros impose restrictions on the sensitivity function because if the sensitivity is forced to be small in one of the frequency bands, it will be large in the other leading to a bad performance.[9]
•The achievable bandwidth for closed-loop performance is limited by input saturation and the location of the Right half plane zero. [13]p.5.
•High uncertainty at steady state causes limitations on the steady-state performance of the system [13]p.5.
•Back-flow problem is observed with the valves after the pumps have been OFF resulting in all the water flowing into only one of the tanks.Pump voltage is turned on high for a couple of seconds and then turned back down to the equilibrium level to overcome this problem [7]p.6.
•Pump Saturation:- The voltage range sets a limit on the control systems ability to control the level of liquid in the lower tanks. In cases when a rapid increase or decrease in the level is required it can command an input outside of the voltage specifications of this system[5]p25.
•Tank Level Saturation:-

• The upper and lower tanks have identical volume with orifices located at the bottom of each tank, these orifices are adjustable in diameters allowing for increased and decreased draining of each tank.

•When a drastic increase in fluid height is commanded in the bottom tank a rapid inflow of fluid is introduced to the upper tank, because the flow into the tank can be much greater than the flow out, due to orifice size, a scenario can occur where the tracking system commands a higher height in the upper tank than is physically possible[5]p26.

3.5 Reasonable Control System Design Specifications

•The input and output voltages are considered between 0-10 [V] [8]p.2.
•The settling time is ten times longer for the non-minimum. The offset of the tanks ranges in the value from -0.6cm to 2cm [11]pp.7-8.
•The dynamics of this system is accurate but a delay is added due to the length of the tubes. The gain model is correct for water levels at 10cm, but the gain for the real system is higher between 12-14cm [4]p.33.
•The height of each tank is 20 cm and the diameter is about 6 cm. The pumps are gear pumps with a capacity of 2.5 l/min. The tanks and the pumps are connected by flexible tubings, each with a diameter of 6 mm [11]p.3.

4 Non-Linear Model of the System

4.1 Variable Definitions and Nominal Values

Variables	Parameters	units	Definition
Inputs	(v_1, v_2)	Volts(V)	Voltages applied to pump 1 and pump 2
Outputs	(h_1, h_2)	Centimeters(Cm)	Level of the liquid in tank 1 and tank 2
States	$(\mathrm{h}_1, h_2, h_3, h_4)$		levels in all four tanks

Table 2: Input, Output and State Variable

Parameters	*units*	*Nominal Values*	*State/Parameters*
A_1, A_3	[cm^2]	28	*Areas of the tanks*
A_2, A_4	[cm^2]	32	*Areas of the tanks*
a_1, a_3	[cm^2]	0.071	*Area of the drain in tank i*
a_2, a_4	[cm^2]	0.057	*Area of the drain in tank i*
k_c	[V/cm]	0.50	
g	[cm/s^2]	981	*Gravitational Constant*

Table 3: Nominal Parameter Values

Symbols	*units*	P_-	P_+	*States/Parameters*
(h_1^0, h_2^0)	[cm]	$(12.4, 12.7)$	$(12.6, 13.0)$	*Nominal levels*
(h_3^0, h_4^0)	[cm]	$(1.8, 1.4)$	$(4.8, 4.9)$	*Nominal levels*
(v_1^0, v_2^0)	[V]	$(3.00, 3.00)$	$(3.15, 3.15)$	*Nominal pump setting*
(k_1, k_2)	[cm^3/Vs]	$(3.33, 3.35)$	$(3.14, 3.29)$	*pump proportionality constant*
(γ_1, γ_2)		$(0.70, 0.60)$	$(0.43, 0.34)$	*ratio of flows in the valves*
(T_1, T_2)		$(62, 90)$	$(63, 91)$	*Time constants*
(T_3, T_4)		$(23, 30)$	$(39, 56)$	*Time constants*

Table 4: Nominal Parameter Values

4.2 Model Development

•Modelling of a process is required to investigate how the behaviour of a process changes with time under influence of changes in the external disturbances and manipulated variables, to consequently design an appropriate controller.The representation is given in terms of a set of mathematical equations which gives the dynamic behaviour of the process [8]p.3.
•For each tank i=1. . . 4, the mathematical modelling is done by consideration of mass balance equation and Bernoulli's law yields:
[Rate of Accumulation of Mass in System] = [Mass flow rate into the system] − [Mass flow out of the system]. Some considerations before deriving the mathematical equation of the systems [8]p.3:
•The input to the pump 1 be v_1 and for pump 2 be v_2.
•The valve priority set for the flow is γ_1 γ_2 $\sum$ [0, 1].
• The flow through the pump 1 when v_1 voltage is applied is $k_1 v_1$ and for pump 2 when v_2 voltage is applied is $k_2 v_2$.
• The flow through the pump is directly proportional to the input voltage applied for the pump.
• The flow in the tank 1 after crossing the valve 1 is $\gamma_1 k_1 v_1$ and for tank 2 after crossing the valve 2 is $\gamma_2 k_2 v_2$.
• The flow in the tank 4 after crossing the valve 1 is (1- $\gamma_1) k_1 v_1$ and for tank 3 after crossing the valve 2 is (1-$\gamma_2) k_2 v_2$.

The non-linear model of the Quadruple tank process is given[8]p.3:
Mass balance equation states that [Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]
Using the law of conservation of mass,

$$\frac{dm_T}{dt} = m_{in} - m_{out} \tag{1}$$

where, m_T= mass accumulated in the Tank
m_{in}=input mass flow rate
m_{out}= output mass flow rate
Mass accumulated, m_T= volume of tank (v) * density of liquid in the tank(ρ)
Input mass flow rate (m_{in}) = volumetric flow rate(q_{in})* density of liquid in the inlet stream($\rho 1$)
Output mass flow rate ($mout$) = volumetric flow rate (q_{out})* density of liquid in the outlet stream($\rho 2$)

$$\frac{d\rho v}{dt} = \rho 1 q_{in} - \rho 2 q_{out} \tag{2}$$

Modelling of non-linear Quadruple tank process is[8]pp.3-4:

$$A_i \frac{dh_i}{dt} = q_{in_i} - q_{out_i} \tag{3}$$

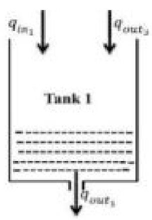
Where A_i denotes the cross sectional area of the tank, h_i is the water level, q_{out_i} is the in-flow of the tank and q_{out_i} is out-flow of the tank i=1...4.
The q_{in_i} only depends on the input voltage supplied to the pump and the q_{out_i} , depends on the acceleration due to gravity and the head of the water in the tank. The q_{out_i} can be determined by the using Bernoulli's equation and flow rate of the liquid.
Therefore,

$$q_{in_1} = k_1 V_{11}$$
$$\mathrm{q}_{in_2} = k_2 V_{22}$$
$$q_{in_3} = k_2 V_2\,(1 -_2)$$
$$q_{in_4} = k_1 V_1\,(1 -_1)$$

Where k_1,k_2 are the pumps constant, V_1, V_2 are the velocities of the flow of water through pump 1 and pump 2, $\gamma 1$ $\gamma 2$ are the valve ratios.

$$\mathrm{q}_{\mathrm{out}_i} = a_i\sqrt{2gh_i} \tag{4}$$

Where, a_i = cross sectional area of the outlet pipes, g = acceleration due to gravity, h_i = represents level of the water in each tanks i=1...4.
Tank 1



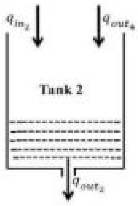
[8]

Figure 2: Mass-balance equation for Tank 1

Using the law of conservation of mass, [Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_1 \frac{dh_1}{dt} = q_{in_1} + q_{\mathrm{out}_3} - q_{\mathrm{out}_1} = \gamma_1 \mathrm{k}_1 \mathrm{V}_1 + \mathrm{a}_3\sqrt{2gh_3} - \mathrm{a}_1\sqrt{2gh_1} \tag{5}$$

Tank 2
Using the law of conservation of mass, [Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

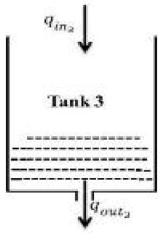


[8]

Figure 3: Mass-balance equation for Tank 2

$$A_2\frac{dh_2}{dt} = q_{in_2} + q_{\mathrm{out}_4} - q_{\mathrm{out}_2} = \gamma_2 \mathrm{k}_2 \mathrm{V}_2 + \mathrm{a}_4\sqrt{2gh_4} - \mathrm{a}_2\sqrt{2gh_2} \tag{6}$$

Tank 3



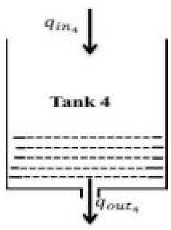
[8]

Figure 4: Mass-balance equation for Tank 3

Using the law of conservation of mass, [Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_3\tfrac{dh_3}{dt} = q_{in_3} - q_{out_3} = (1-\gamma_2)\,\mathrm{k}_2\mathrm{V}_2 - \mathrm{a}_3\sqrt{2gh_3} \tag{7}$$

Tank 4



[8]

Figure 5: Mass-balance equation for Tank 4

Using the law of conservation of mass, [Rate of accumulation] = [Rate of in-flow] – [Rate of out-flow]

$$A_4\tfrac{dh_4}{dt} = q_{in_4} - q_{out_4} = (1-\gamma_1)\,\mathrm{k}_1\mathrm{V}_1 - \mathrm{a}_4\sqrt{2gh_4} \tag{8}$$

The Final equations are given by,

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1\\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2\\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)\,k_2}{A_3}v_2\\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)\,k_1}{A_4}v_1
\end{aligned} \tag{9}$$

Where the parameters used above are;
A_i:cross-sectional area of Tank i
a_i:cross-sectional area of the outlet hole
h_i:the water level in Tank i
v_i:voltage applied to pump i
$k_i v_i$:flow from pump i
g:acceleration due to gravity
Johansson[9]p.13 showed that the inverse response (non minimum phase) will occur when $0 < \gamma 1 + \gamma 2 < 1$ and minimum phase for $0 < \gamma 1 + \gamma 2 < 2$.
The valve settings will provide the overall system an entirely different behaviour from a multivariable control point of view.
•Unmeasured disturbances can be applied by pumping water out of the top tanks and into the lower reservoir, this exposes to disturbances rejection as well as reference tracking[13]

5 Linear Model

5.1 Linearizing the Non-Linear Model

There is a need to linearize the model in order to enhance the stability analysis and controller design. The non-linear model in equation (9) can be linearized around the working point by the levels in the tanks h_1^0, h_2^0, h_3^0 and h_4^0[14]p.19.

$Parameters$	$units$	$Values$
A_1, A_3	[cm^2]	28
A_2, A_4	[cm^2]	32
a_1, a_3	[cm^2]	0.071
a_2, a_4	[cm^2]	0.057
k_c	[V/cm]	0.50
g	[cm/s^2]	981

Table 5: Parameter Values for Linear Model

The model and control of the quadruple-tank process are studied at two operating points: P_- at which the system will have minimum-phase characteristics and P_+ at which it will have non minimum-phase characteristics. The chosen operating points correspond to the following parameter values[11]p.2.

$Symbols$	$units$	P_-	P_+
(h_1^0, h_2^0)	[cm]	$(12.4, 12.7)$	$(12.6, 13.0)$
(h_3^0, h_4^0)	[cm]	$(1.8, 1.4)$	$(4.8, 4.9)$
(v_1^0, v_2^0)	[V]	$(3.00, 3.00)$	$(3.15, 3.15)$
(k_1, k_2)	[cm^3/Vs]	$(3.33, 3.35)$	$(3.14, 3.29)$
(γ_1, γ_2)		$(0.70, 0.60)$	$(0.43, 0.34)$

Table 6: Parameter Values for Linear Model

Using Taylor Series applications, a linearized state-space model form $x_i = h_i - h_i^0$ and control variables $u_i = v_i - v_i^0$ for equation (9) can be represented as as given below:

$$\dot{X}_p = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{a_3}{a_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{a_4}{a_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} X_p + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u_p \tag{10}$$

$$Y_p = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} X_p \tag{11}$$

where x=$\begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix}^T$, $u = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$ and $y = \begin{bmatrix} h_1 & h_2 \end{bmatrix}^T$
the time constants are

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, \quad i = 1, \cdots, 4[11]p.3 \tag{12}$$

The corresponding transfer matrix is

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} [11]p.2 \tag{13}$$

where $c_1 = T_1 k_1 k_c / A_1$ and $c_2 = T_2 k_2 k_c / A_2$ *The two operating points* P_- and P_+ we have the following time constants [11]p.3:

Time constants	P_-	P_+
(T_1, T_2)	$(62, 90)$	$(63, 91)$
(T_3, T_4)	$(23, 30)$	$(39, 56)$

Table 7: Time Constant Values

5.2 State space representation of Plant at Minimum Phase

$$\mathrm{A}_P \; min\ phase = \begin{bmatrix} -0.0161 & 0 & 1.2174 & 0 \\ 0 & -0.0111 & 0 & 0.0333 \\ 0 & 0 & -0.0435 & 0 \\ 0 & 0 & 0 & -0.0333 \end{bmatrix} \quad B_P \; min\ phase = \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0628 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix}$$

$$C_P \; min\ phase = \begin{bmatrix} 0.5000 & 0 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 \end{bmatrix} \qquad D_P \; min\ phase = [0]$$

5.3 State space representation of Plant at Non-Minimum Phase

$$\mathrm{A}_P \; non{-}min\, phase = \begin{bmatrix} -0.0159 & 0 & 0.7179 & 0 \\ 0 & -0.0110 & 0 & 0.0179 \\ 0 & 0 & -0.0256 & 0 \\ 0 & 0 & 0 & -0.0179 \end{bmatrix} B_P \; non{-}min\, phase = \begin{bmatrix} 0.0482 & 0 \\ 0 & 0.0350 \\ 0 & 0.0775 \\ 0.0559 & 0 \end{bmatrix}$$

$$C_P \; non - minphase = \begin{bmatrix} 0.5000 & 0 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 \end{bmatrix} \qquad D_P \; non - min\ phase = [0]$$

6 Analysis of Linear Model: Dynamic Properties

6.1 Plant Transfer Function Matrix(TFM)

The transfer function Matrices for the two operating points are given by

$$G_{-}(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.8}{1+90s} \end{bmatrix} \tag{14}$$

and

$$G_{+}(s) = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{bmatrix} \tag{15}$$

6.2 Bode plots

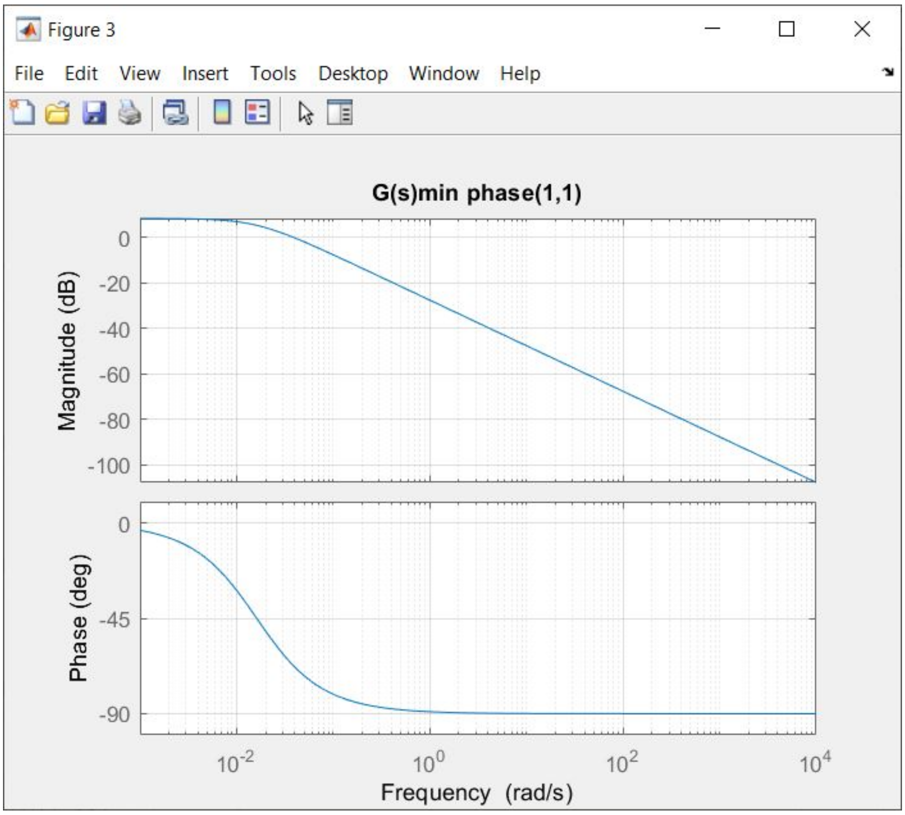


Figure 6: Bode plot of G(s) min phase (1,1)

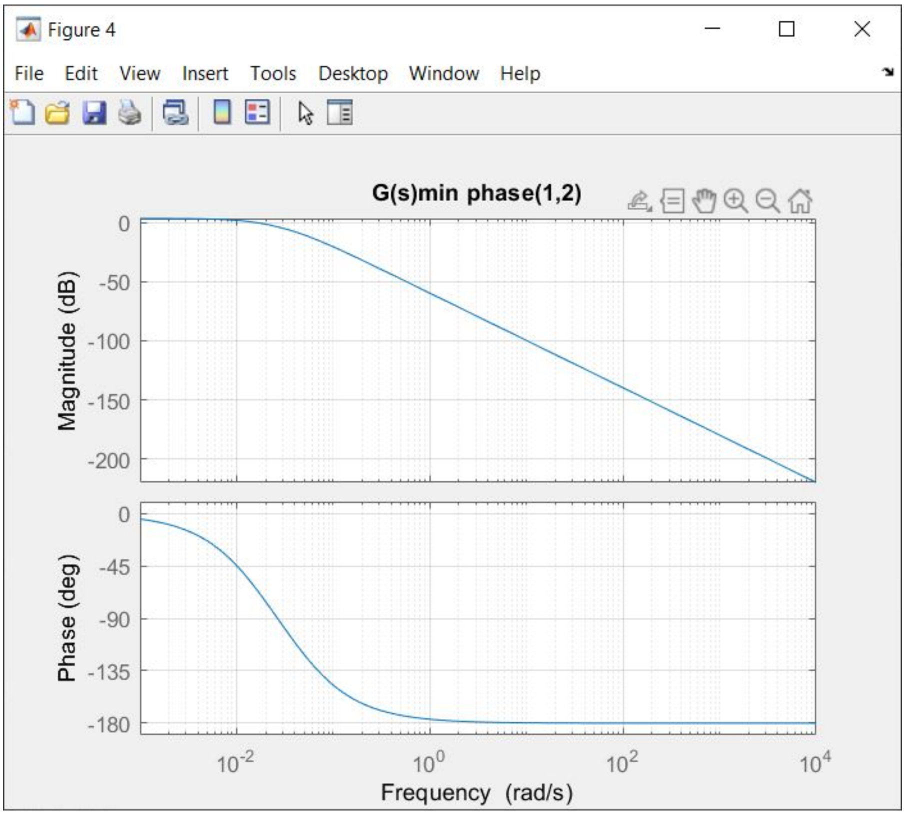


Figure 7: Bode plot of G(s) min phase (1,2)

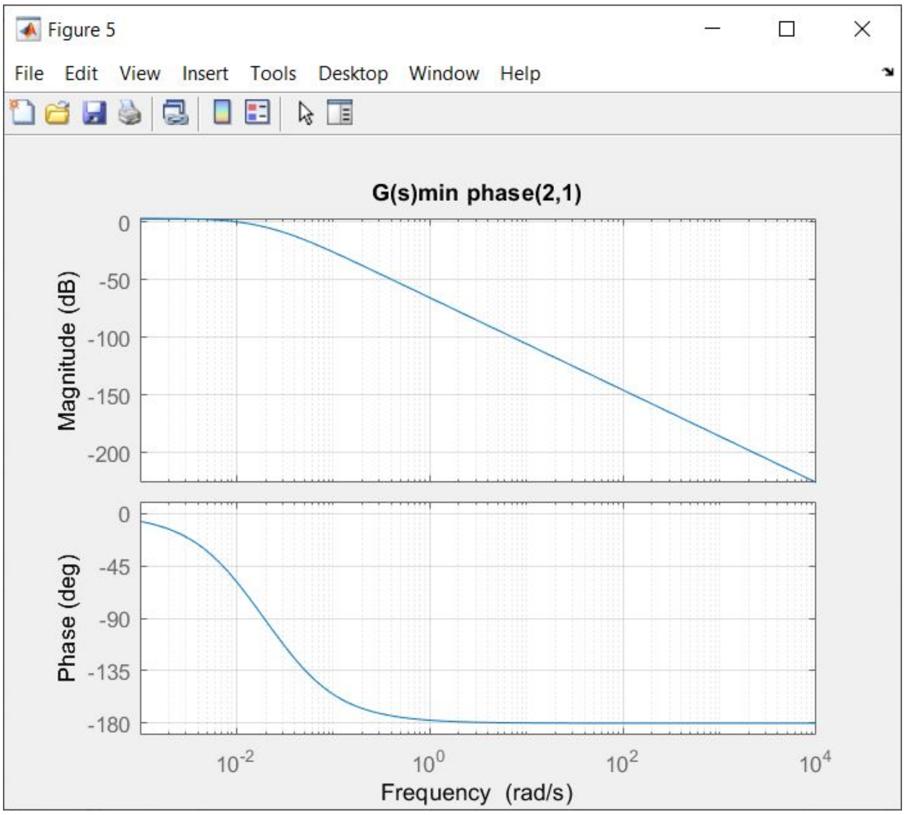


Figure 8: Bode plot of G(s) min phase (2,1)

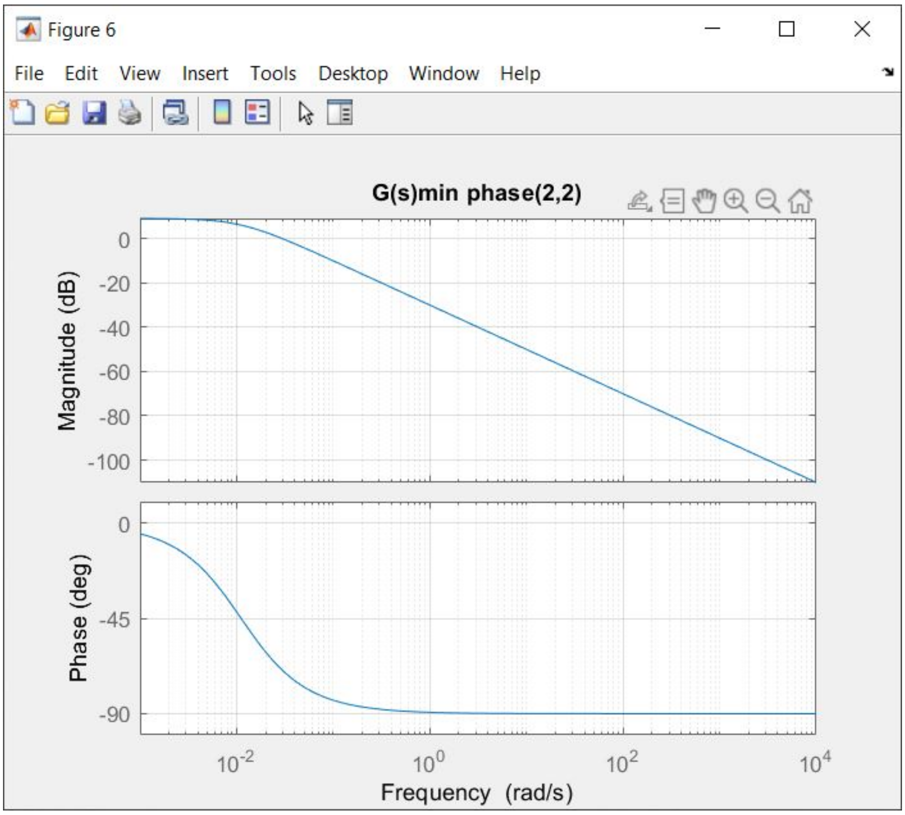


Figure 9: Bode plot of G(s) min phase (2,2)

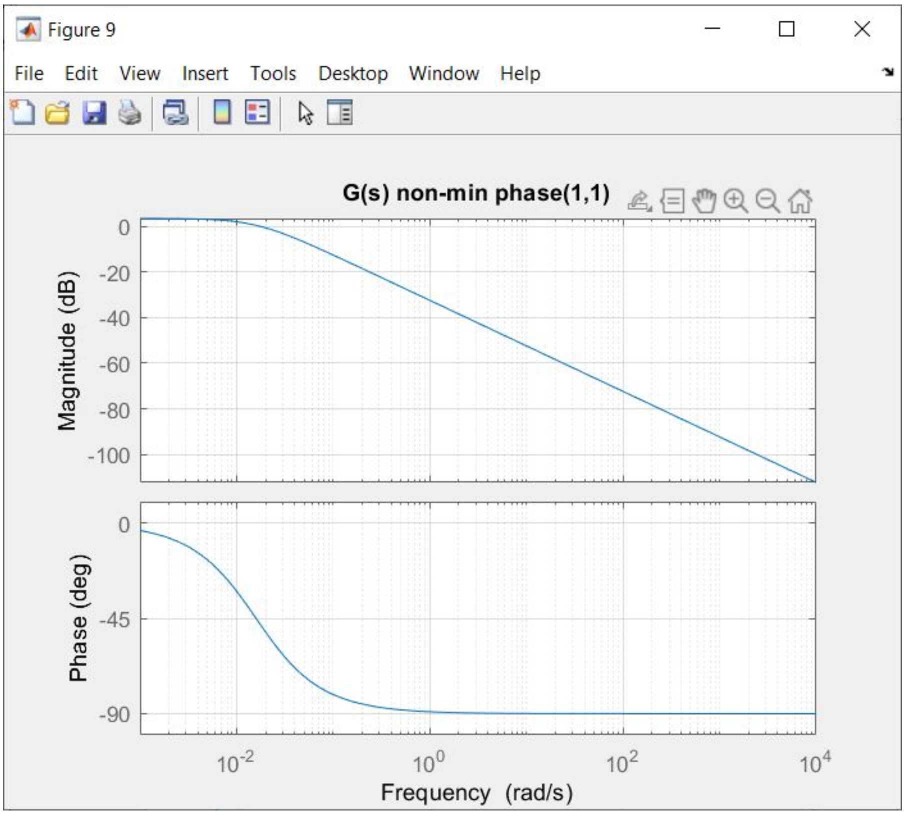


Figure 10: Bode plot of G(s) non min phase (1,1)

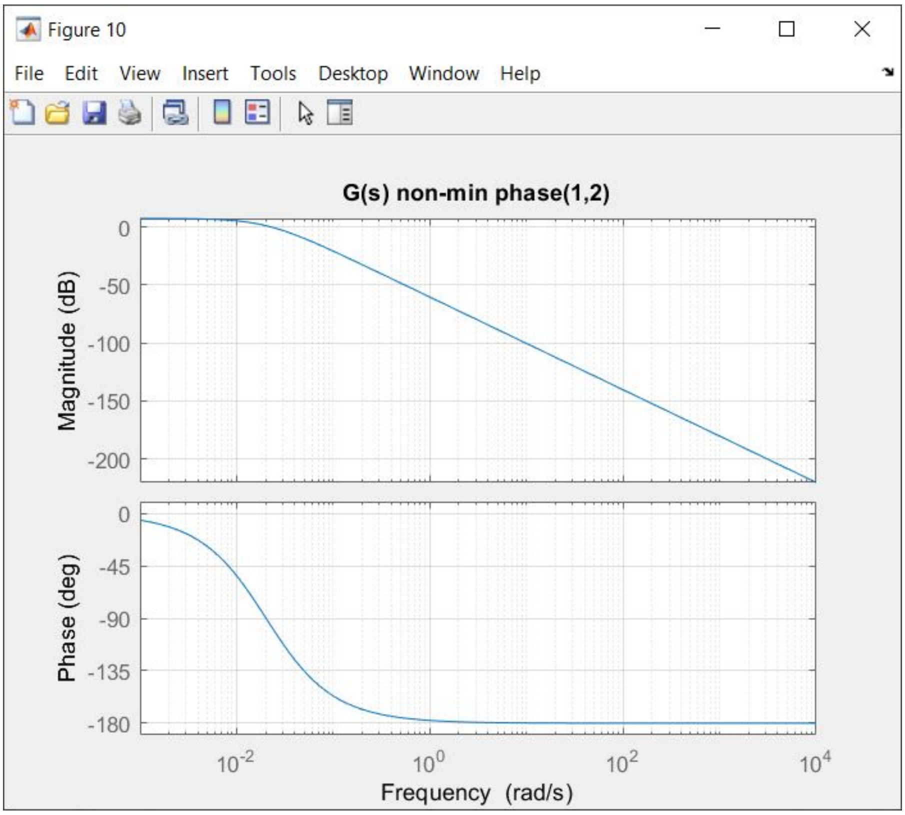


Figure 11: Bode plot of G(s) non min phase (1,2)

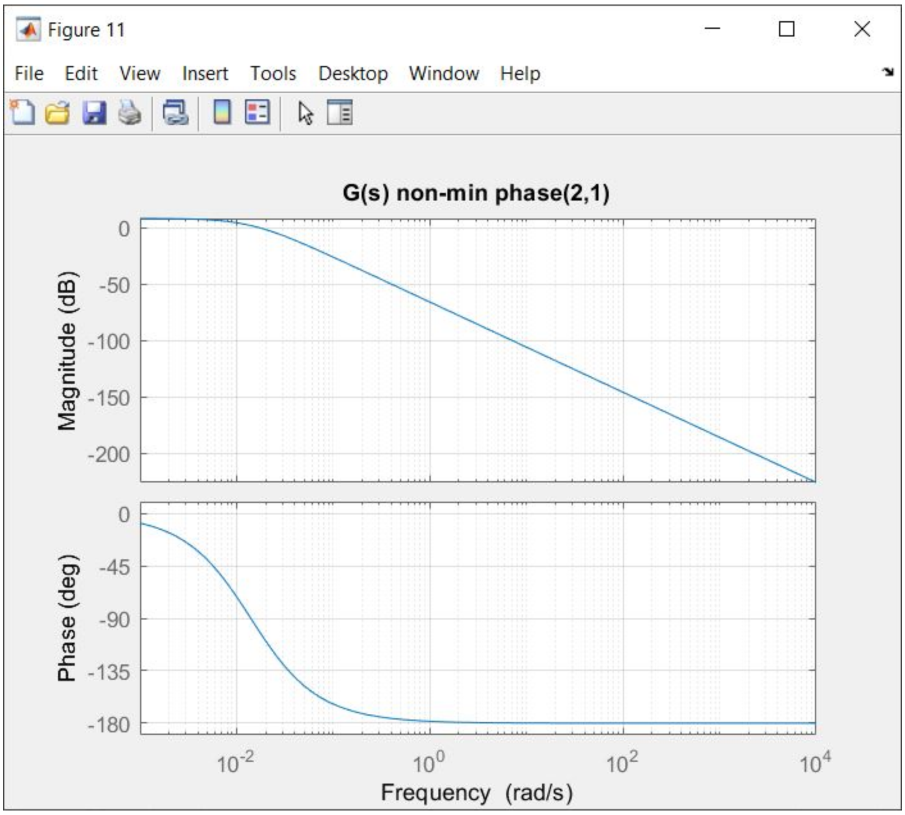


Figure 12: Bode plot of G(s) non min phase (2,1)

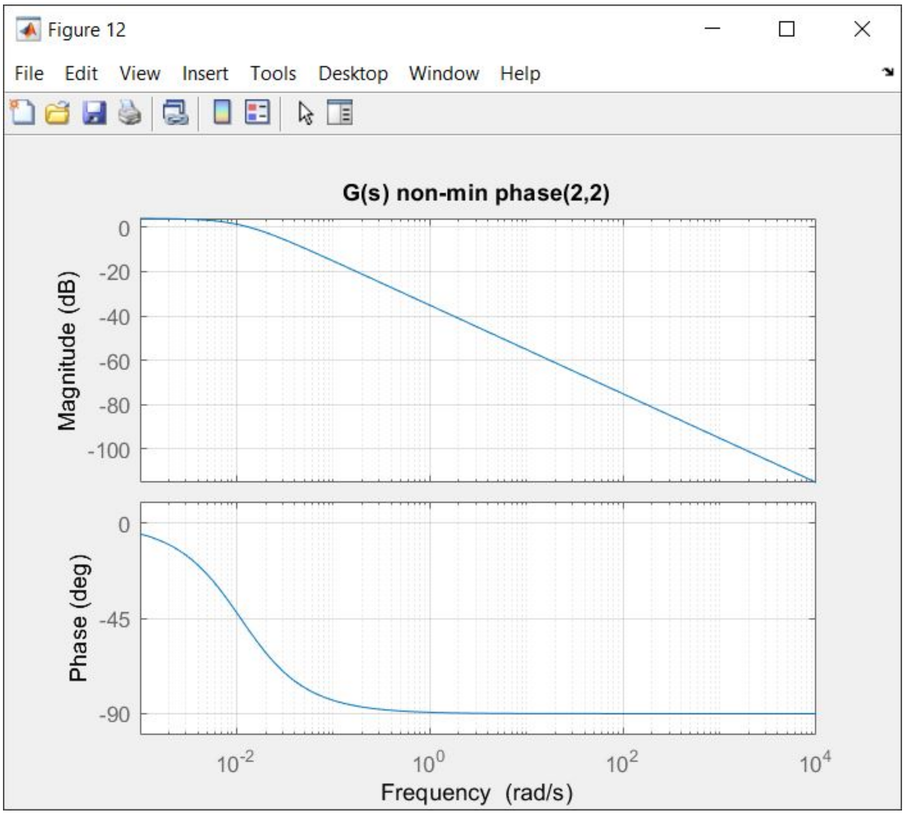


Figure 13: Bode plot of G(s) non min phase (2,2)

6.3 Poles

•Poles and Zeros of a transfer function are the frequencies for which the value of the denominator and numerator of transfer function becomes zero respectively. The values of the poles and the zeros of a system determine whether the system is stable, and how well the system performs.
•Control systems, in the most simple sense, can be designed simply by assigning specific values to the poles and zeros of the system.Physically realizable control systems must have a number of poles greater than the number of zeros.Systems that satisfy this relationship are called Proper.[3]
•Poles can be obtained by calculating the eigenvalues of the matrix A_p.
The minimum phase has the following poles:$P_{min} = -0.0161, -0.0435, -0.0333, -0.0111$
The non-minimum phase has the following poles: $P_{non-min} = -0.0159, -0.0256, -0.0179, -0.0110$

6.4 Stability

•Stability for a MIMO system is defined by the position of the eigenvalues ofA_p, if the eigenvalues are within the unit circle then the system is said to be stable.
•As all the eigenvalues/poles are in the left half plane the system and inside the unit circle the system is stable.
Eigenvalues of A Minimum Phase =[-0.0161, -0.0111, -0.0435, -0.0333]
Eigenvalues of A Non-minimum Phase=[-0.0159, -0.0110, -0.0256, -0.0179]

6.5 Transmission Zeros

•The four-tank dynamics has an adjustable multivariable transmission zero such that its position can be in the LHP or the RHP, and it depends on the ratios of the flow rates between the tanks as determined by γ_1 and γ_2.
•The position of the multivariable zero is a source of motivation for investigating the performance limitations arising from the right-half plane transmission zeroes1. It is also referred to as transmission

zeros.[14]
The zeros of the transfer matrix in (13) are the zeros of the numerator polynomial of the rational function[11]p.4.

$$\det G(s) = \frac{c_1 c_2}{\gamma_1 \gamma_2 \prod_{i=1}^{4} (1 + sT_i)} \times \left[(1 + sT_3)(1 + sT_4) - \frac{(1 - \gamma_1)(1 - \gamma_2)}{\gamma_1 \gamma_2} \right]. \tag{16}$$

•The transfer matrix G has two finite zeros for $\gamma_1 and \gamma_2$ in **{0,1}**.
•One of them is always in the LHP , but the other can be located either in the left or the right half plane.
•The location of the other zero depends on the sign of $\eta = a_3 a_4 A_1 A_2 (\gamma_1 + \gamma_2 - 1 - \gamma_1 \gamma_2) + a_1 a_2 A_3 A_4 \gamma_1 \gamma_2$.
•The multivariable zero is in the right half plane if $\eta < 0$ and will be in the left half plane if η >0.

6.6 Zero Directions

•In Multivariable systems the direction of the zero also plays an important role [11]p.4.
•The output direction of a zero z as a vector $\psi \in R^2$ of unit length such as $\psi^T G(z) = 0$. If ψ is parallel to a unit vector, then the zero is only associated with one output.[11]
•If this is not the case, then the effect of a right half-plane zero may be distributed between both outputs [11].
• For the Transfer function in [13] the zero direction for z $>$ 0is given by:

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}^T \begin{bmatrix} \frac{\gamma_1 c_1}{1+zT_1} & \frac{(1-\gamma_2)c_1}{(1+zT_3)(1+zT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+zT_4)(1+zT_2)} & \frac{\gamma_2 c_2}{1+zT_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \tag{17}$$

•It follows from this equation that $\psi_1, \psi_2 \neq 0$, so that the zero is never associated with only one output.
•If we solve [13] for and simplify, it is easy to show that :

$$\frac{\psi_1}{\psi_2} = -\frac{1 - \gamma_1}{\gamma_1} \cdot \frac{c_2 (1 + zT_1)}{c_1 (1 + zT_4)(1 + zT_2)} \tag{18}$$

•From the above equation it is possible to conclude that if ψ_1 is small,then z is mostly associated with the first output and if ψ_1 is close to one, then z is mostly associated with the second output.[11]
•Hence, for a given zero location, the relative size of $\psi_1 and \psi_2$ determines which output the right half-plane zero is related to.[11]

6.7 Pole-Zero plot

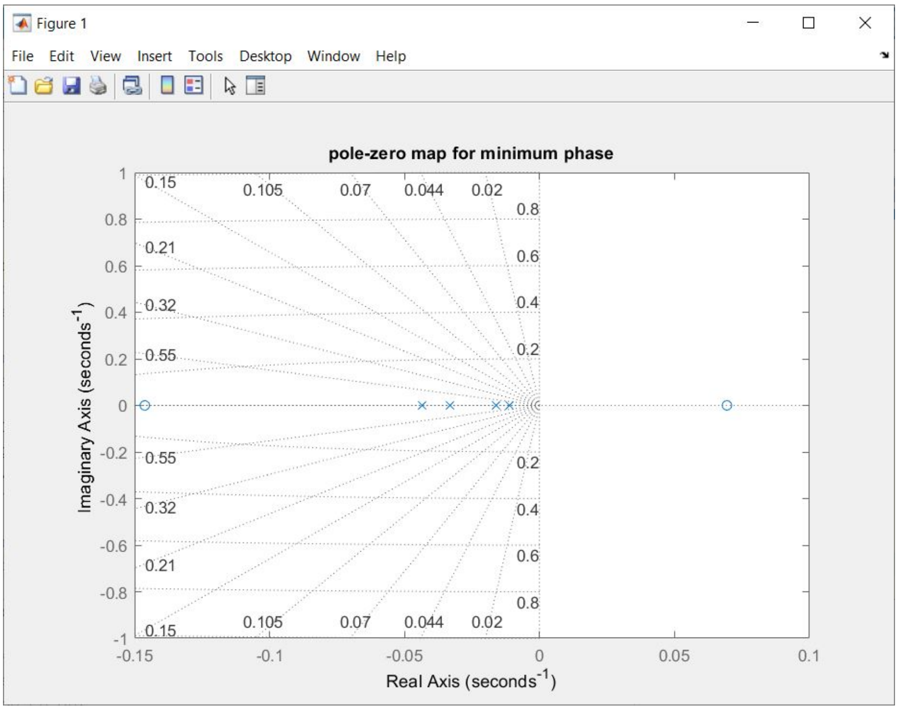


Figure 14: pole-zero plot minimum phase

•The above figure shows the pole zero plot for the first operating point. It shows characteristics similar to a system having Minimum Phase.

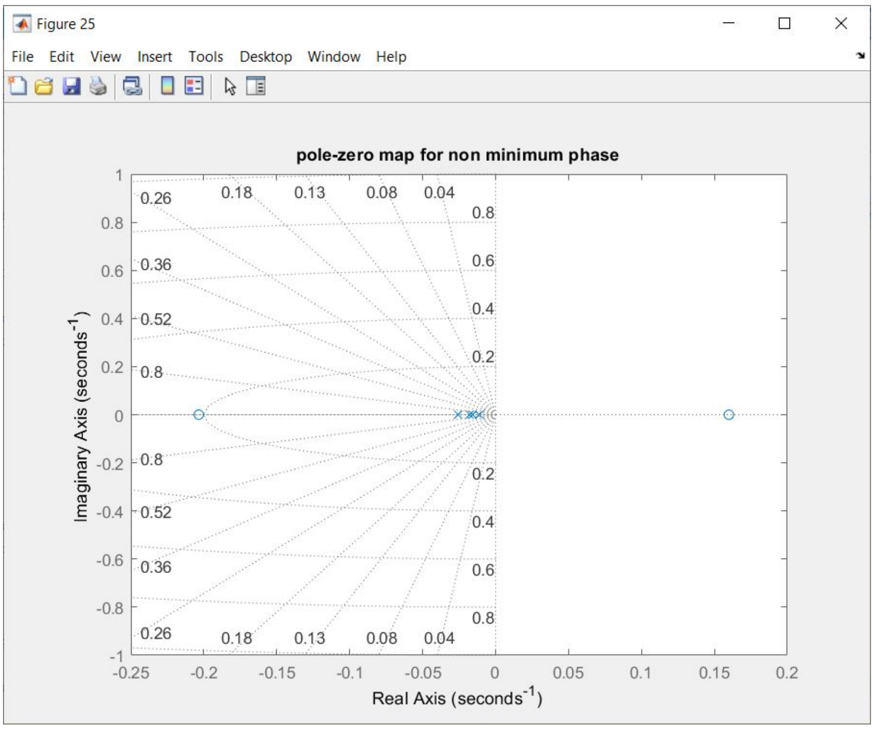


Figure 15: pole-zero plot non-minimum phase

•The above figure shows the pole zero plot for the Second operating point. It shows characteristics similar to a system having Non-Minimum Phase.

6.8 Singular Plots

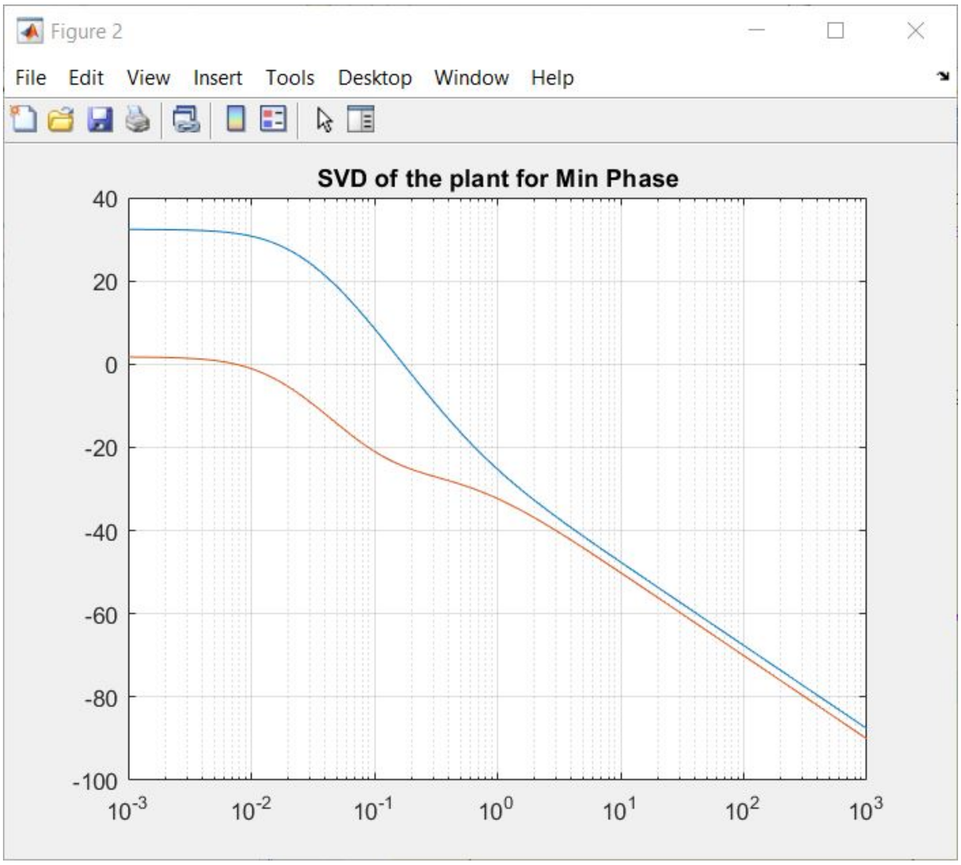


Figure 16: Singular Plot of Plant in Minimum Phase

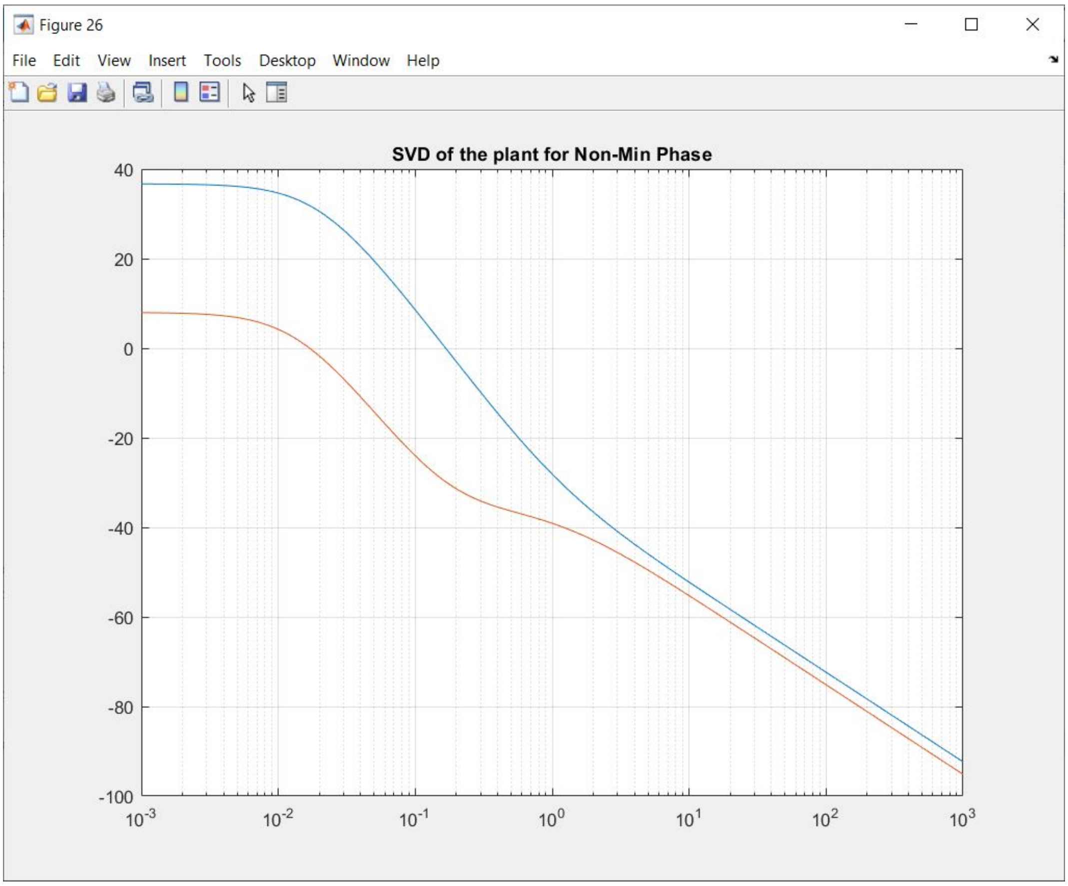


Figure 17: Singular Plot of Plant in Non-Minimum Phase

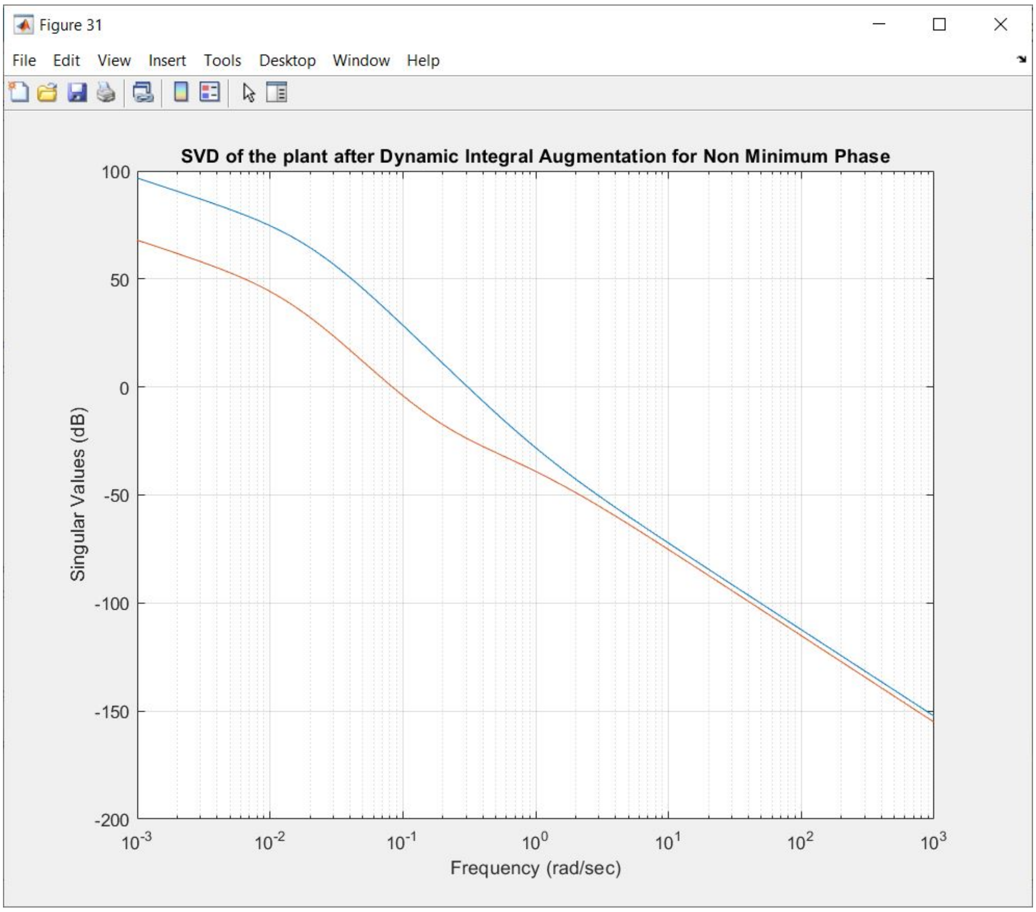


Figure 18: Singular Plot of Plant with Dynamic Augmentation of Integrator

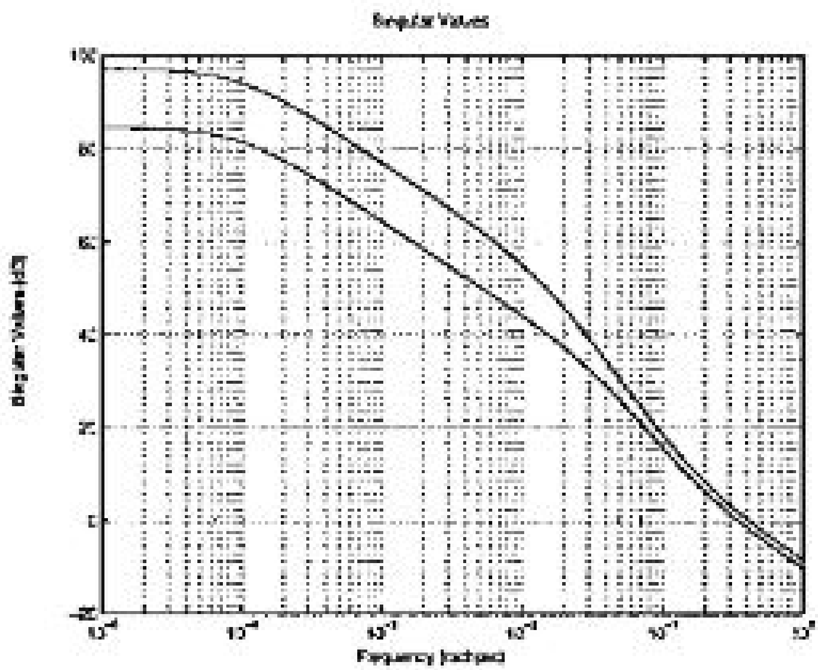


Figure 19: Principal gains of return ratio using the augmented plant [7]

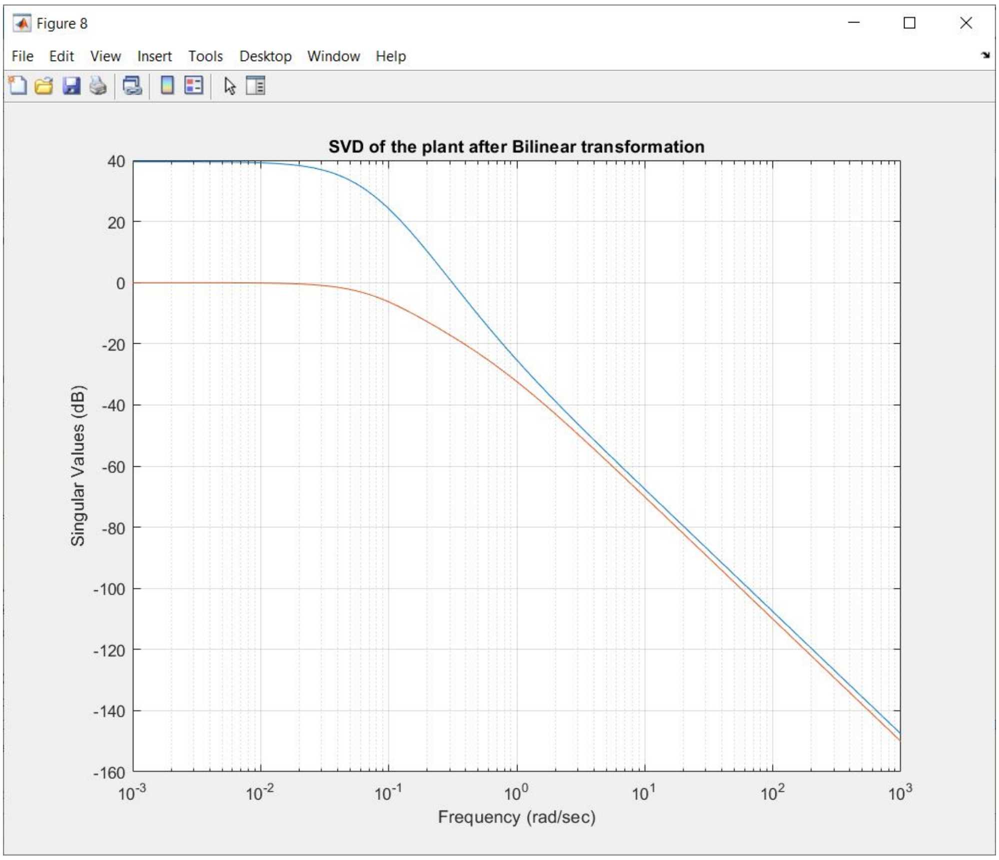


Figure 20: Singular Plot of Plant with Bilinear Transformation for Min Phase

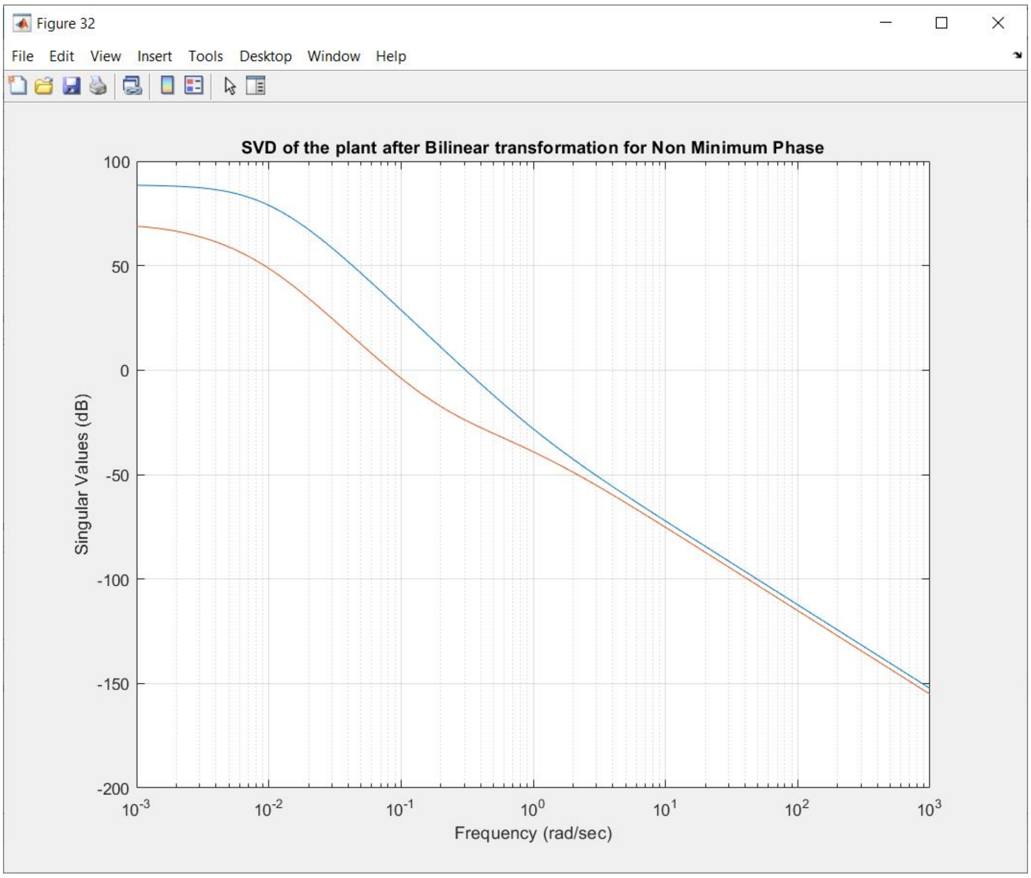


Figure 21: Singular Plot of Plant with Bilinear Transformation for Non-Minimum Phase

6.9 Dc Gain Matrix

•The DC Gain matrix for the Quadruple Tank Apparatus for the two operating points are given by:

G(s) minimum phase $= \begin{bmatrix} 2.5808 & 1.4747 \\ 1.4133 & 2.8266 \end{bmatrix}$ $\quad G(s)\ non-minimum\ phase = \begin{bmatrix} 1.1590 & 2.3314 \\ 2.6664 & 1.5905 \end{bmatrix}$

• The above matrices shows that the system has its inputs coupled to each other and is not similar to an Single Input Single Output(SISO) system.

6.10 Controllability

•Complete state controllability (or simply controllability) describes the ability of an external input to move the internal state of a system from any initial state to any other final state in a finite time interval.[1]

•The n $\times np$ *controllability matrix is given as* $C = \begin{bmatrix} B & AB & A^2B \cdots A^{n-1}B \end{bmatrix}$
The system is said to be controllable when the rank of the system matrix A_p is equal to the rank of the controllablity matrix C.

$$\text{Controllabiltiy minimum phase} = \begin{bmatrix} 0.0833 & 0 & -0.0013 & 0.0583 & 0.0000 & -0.0035 & -0.0000 & 0.0002 \\ 0 & 0.0628 & 0.0010 & -0.0007 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0 & 0.0479 & 0 & -0.0021 & 0 & 0.0001 & 0 & -0.0000 \\ 0.0312 & 0 & -0.0010 & 0 & 0.0000 & 0 & -0.0000 & 0 \end{bmatrix}$$

rank of controllability matrix minimum phase=4

Controllabiltiy non minimum phase=

$$\begin{bmatrix} 0.0482 & 0 & -0.0008 & 0.0557 & 0.0000 & -0.0023 & -0.0000 & 0.0001 \\ 0 & 0.0350 & 0.0010 & -0.0004 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0 & 0.0775 & 0 & -0.0020 & 0 & 0.0001 & 0 & -0.0000 \\ 0.0559 & 0 & -0.0010 & 0 & 0.0000 & 0 & -0.0000 & 0 \end{bmatrix}$$

rank of controllability matrix non-minimum phase = 4

•From above we can say that this system is controllable at both the operating points.

6.11 Observability

•A system is said to be observable if, for any possible evolution of state and control vectors, the current state can be estimated using only the information from outputs (physically, this generally corresponds to information obtained by sensors).[2]

•The Observability matrix is given as O= $\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$

• The system is said to be observable when the rank of the system matrix A_p is equal to the rank of the Observability matrix O.

$$
\text{Observability minimum phase} = \begin{bmatrix} 0.5000 & 0 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 \\ -0.0081 & 0 & 0.6087 & 0 \\ 0 & -0.0056 & 0 & 0.0167 \\ 0.0001 & 0 & -0.0363 & 0 \\ 0 & 0.0001 & 0 & -0.0007 \\ -0.0000 & 0 & 0.0017 & 0 \\ 0 & -0.0000 & 0 & 0.0000 \end{bmatrix}
$$

rank of observability matrix minimum phase = 4

$$
\text{Observability non minimum phase} = \begin{bmatrix} 0.5000 & 0 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 \\ -0.0079 & 0 & 0.3590 & 0\ 0 \\ -0.0055 & 0 & 0.0089 & \\ 0.0001 & 0 & -0.0149 & 0 \\ 0 & 0.0001 & 0 & -0.0003 \\ -0.0000 & 0 & 0.0005 & 0 \\ 0 & -0.0000 & 0 & 0.0000 \end{bmatrix}
$$

rank of observability matrix non minimum phase = 4

•From above we can say that this system is observable at both the operating points.

7 Control Design

7.1 LQR(Linear Quadratic Regulator) Controller

LQR Problem Statement([12]pp.116-117):

$$\min_u J(u) \stackrel{\text{def}}{=} \frac{1}{2}\int_0^{\infty} \left(x^T Q x + u^T R u\right) d\tau \tag{19}$$

which is subject to the general state space representation with inital condition $x(0) = x_0$, and the assumption that (A,B,M) is stabilizable and detectable .
Solution: The solution to the LQR problem is a linear full-state feedback control law:

$$u = -Gx \tag{20}$$

where the control gain matrix G in R^{mxn} is given by:

$$G = R^{-1} B^T K \tag{21}$$

and K is the unique symmetric (at least) positive semidefinite solution of the following Control Algebraic Riccati Equation (CARE):

$$0 = KA + A^T K + M^T M - KBR^{-1}B^T K \tag{22}$$

.

7.1.1 Minimum Phase Characteristics

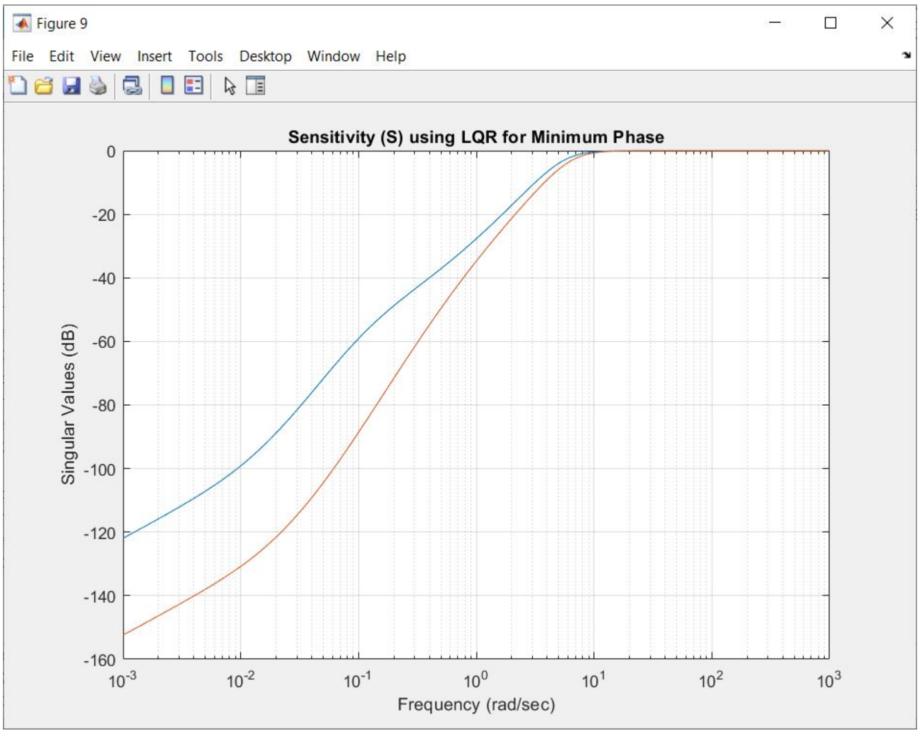


Figure 22: Closed Loop Sensitivity using LQR for Minimum phase

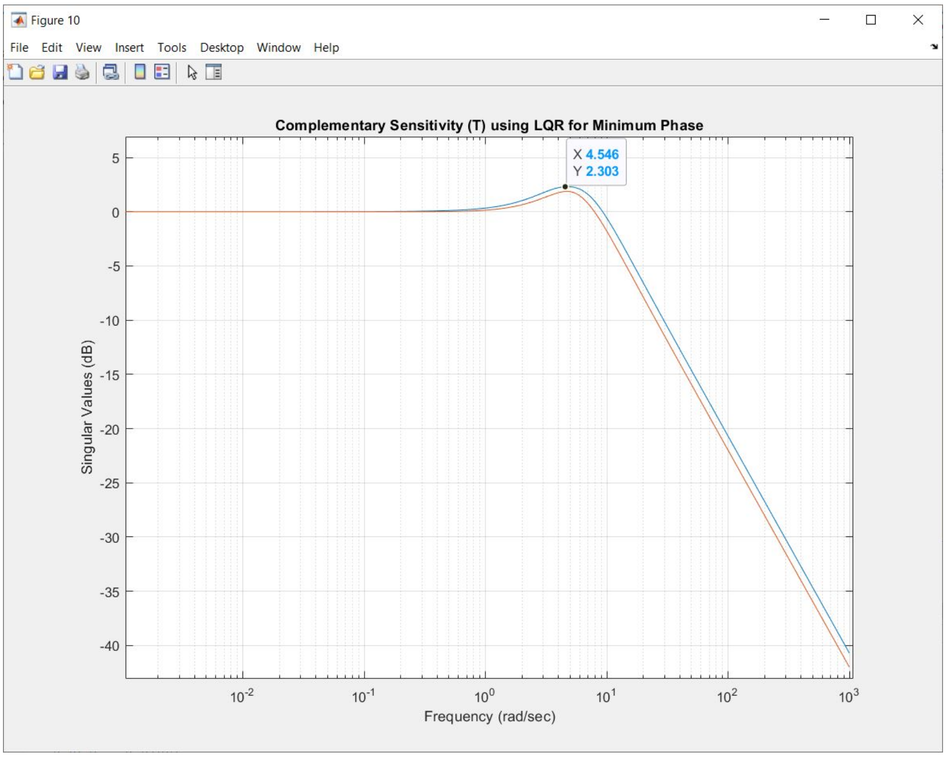


Figure 23: Closed Loop Complementary Sensitivity using LQR for Minimum phase

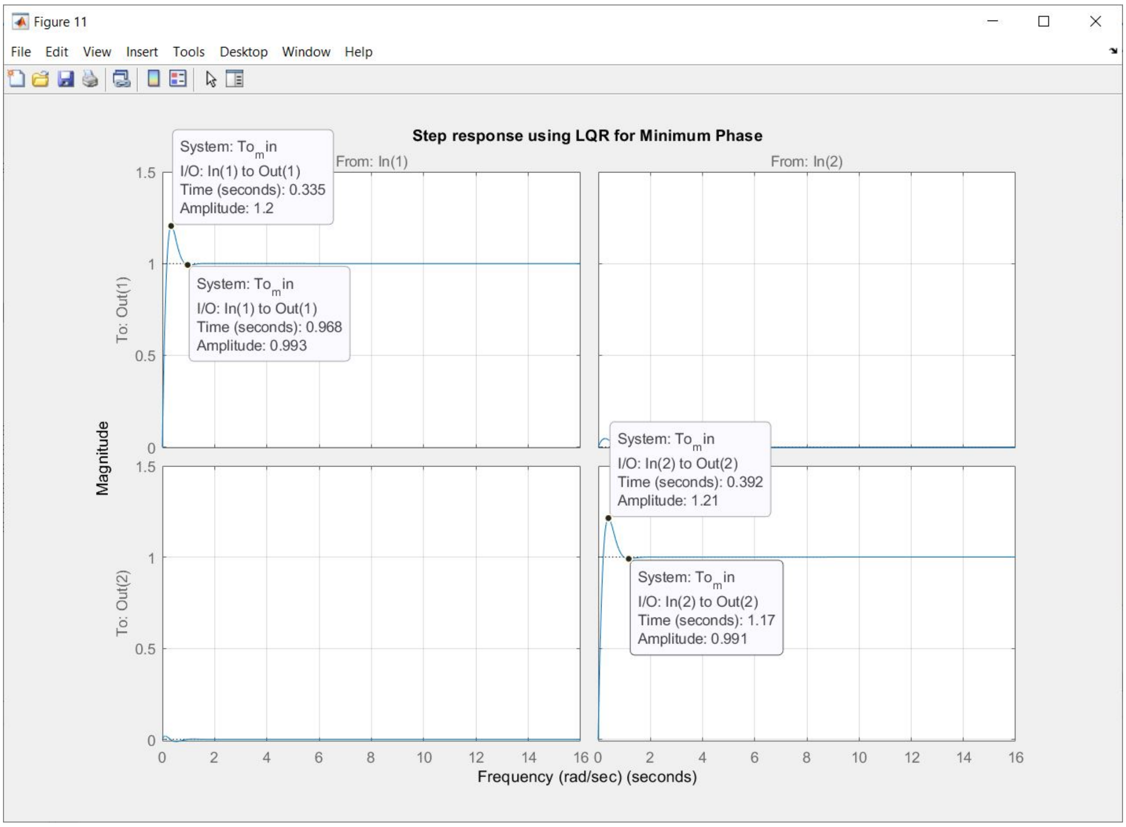


Figure 24: Step Response of LQR for Minimum Phase

7.1.2 Non-Minimum Phase Characteristics

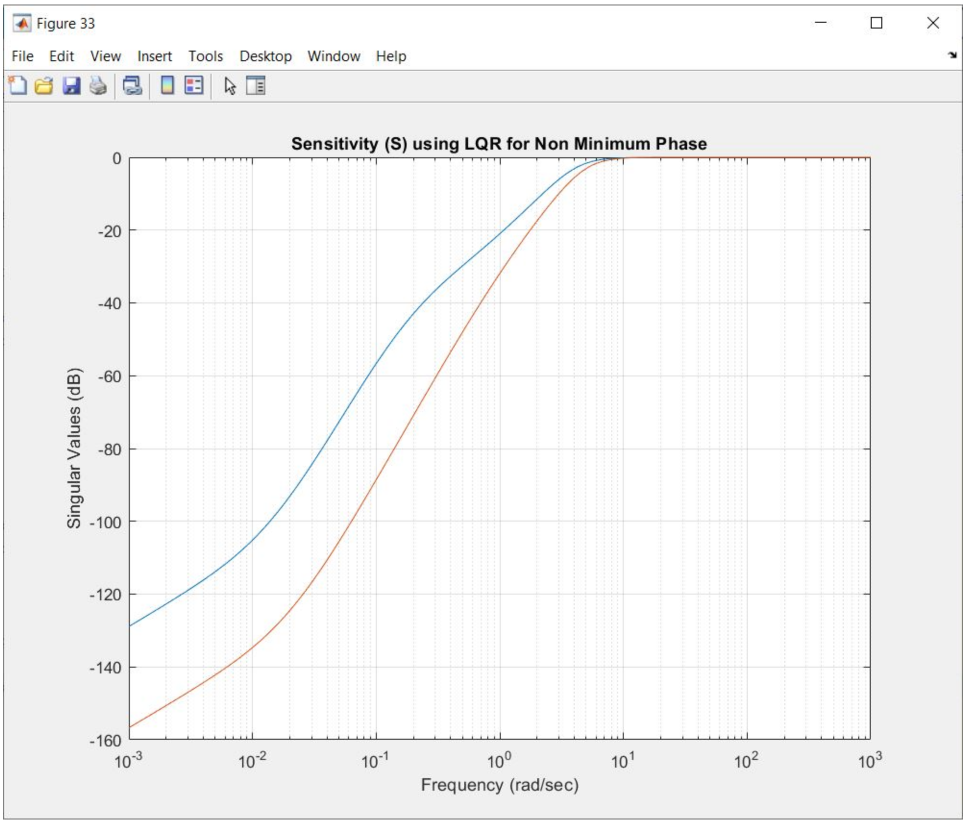


Figure 25: Closed Loop Sensitivity using LQR for Non-Minimum phase

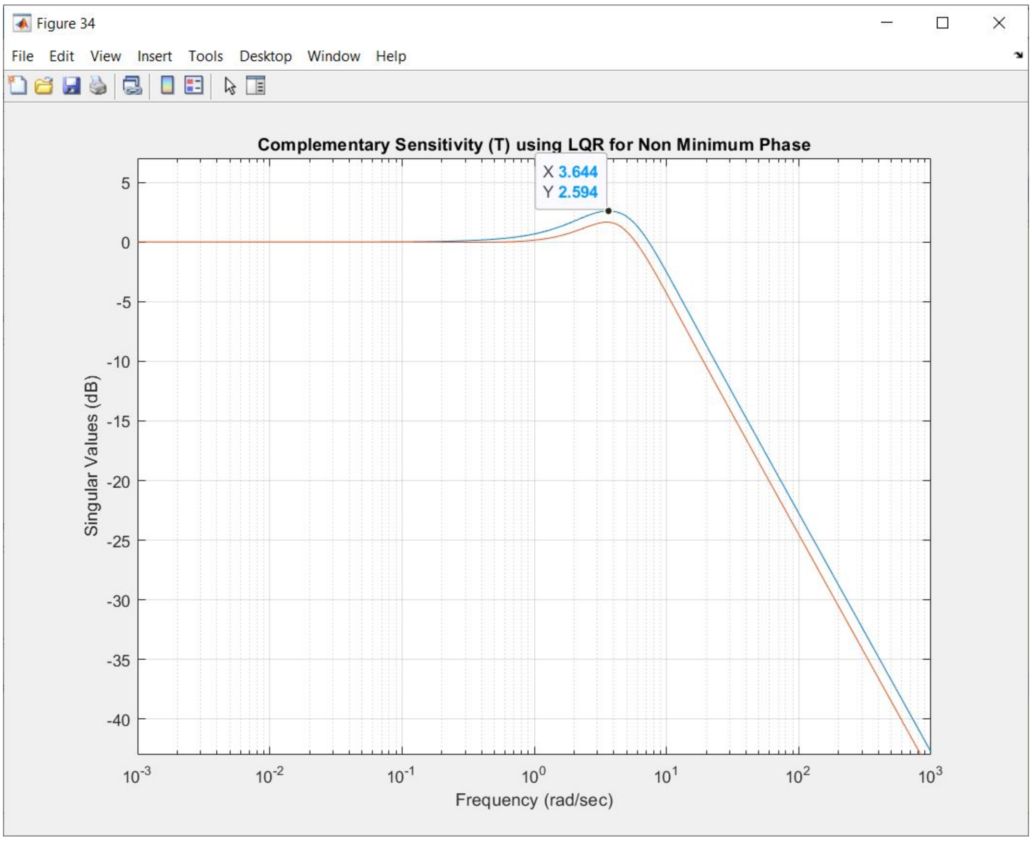


Figure 26: Closed Loop Complementary Sensitivity using LQR for Non-Minimum phase

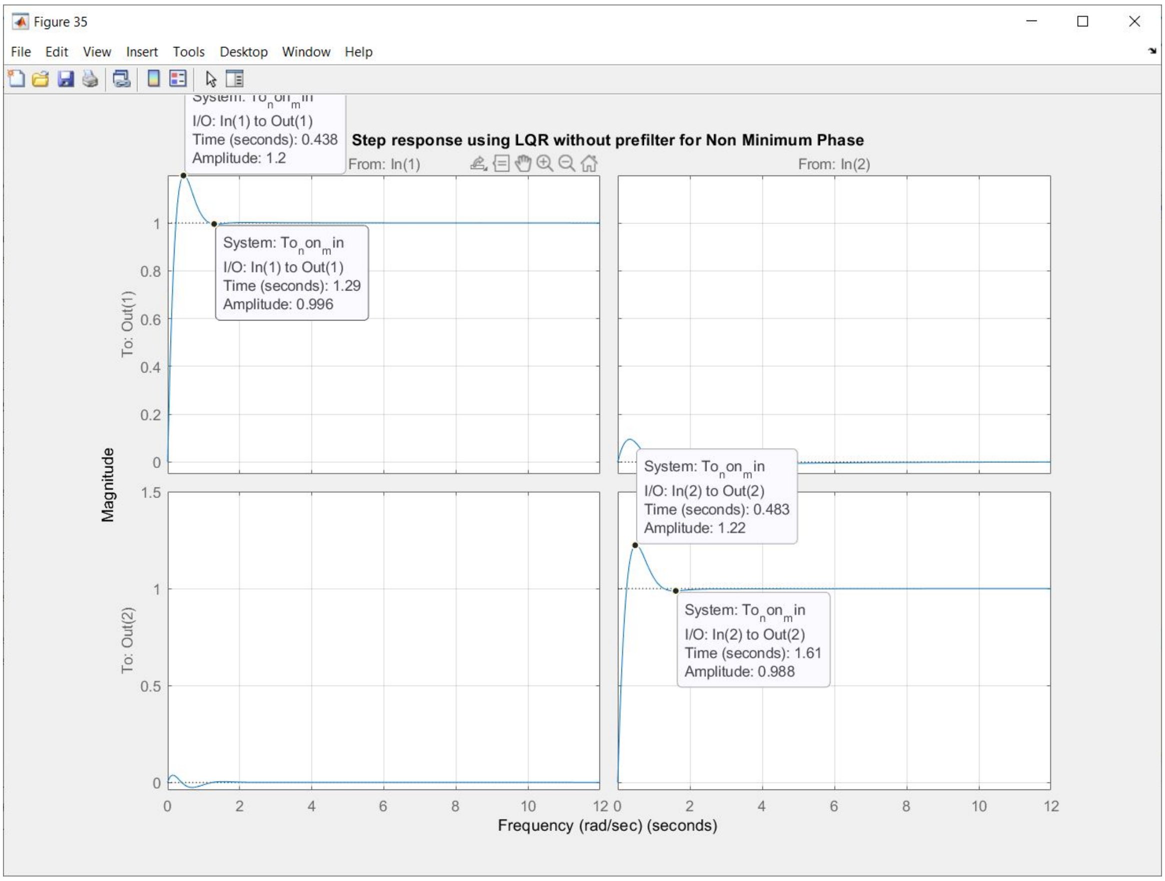


Figure 27: Step Response of LQR for Non- Minimum Phase

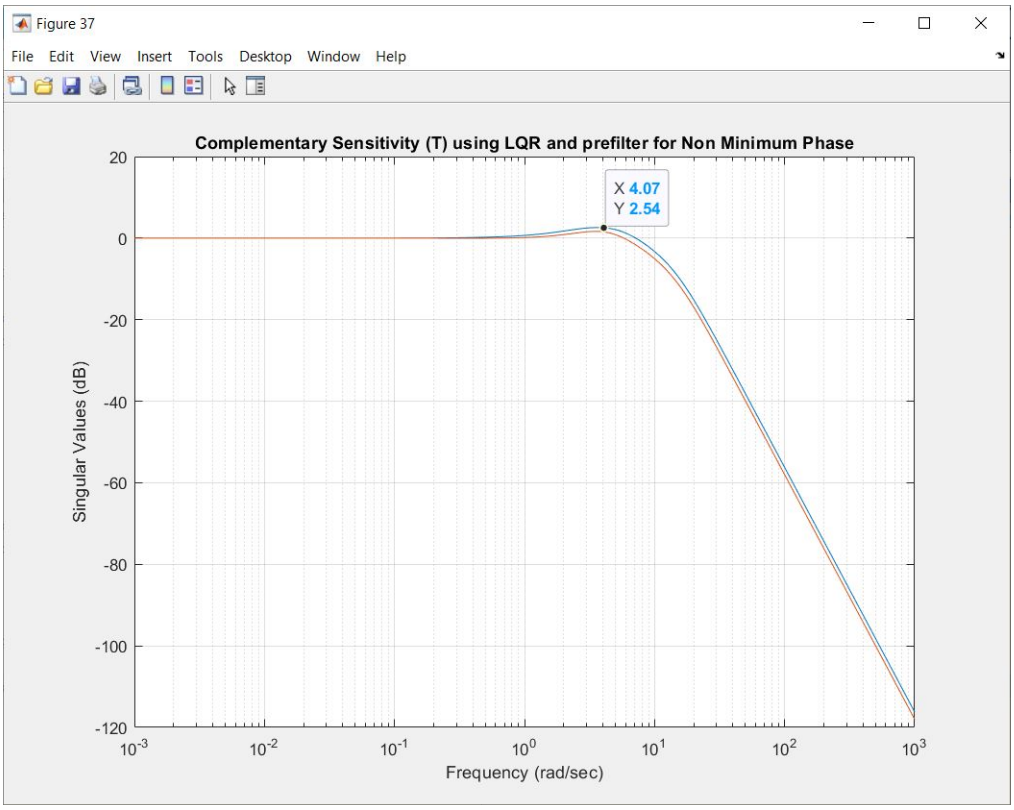


Figure 28: Closed Loop Complementary Sensitivity using LQR with Prefilter for Non-Minimum phase

7.2 Kalman-Bucy Filter

The Kalman-Bucy filter is analogous or dual to the LQR(Linear Quadratic Regulator).
KBF Problem Statement([12]pp.161-163):

$$\min_{\hat{x}(u,y)} J(\hat{x}) \stackrel{\text{def}}{=} \lim_{t_\alpha \to -\infty} \sqrt{E(\tilde{x}\bar{x})} \tag{23}$$

which is subject to the general state space representation with inital condition $x(0) = x_0$, and the assumption that (A,L) is stabilizable and (A,M) detectable . Solution: The solution to this problem is provided by the following model based state estimation/observer structure:

$$\dot{x} = Ax + Bu + Lm_\xi + H(y - \hat{y}) \quad \hat{x}\,(t_o) = m_o \tag{24}$$

$$\dot{y} = C\hat{x} + Du + m_\theta \tag{25}$$

where x̂ is the optimal estimate of the state x;
•$\hat{y}$ is an estimate of the (known) output y;
•m_o is the mean of the initial condition x_o; it represents our best a priori estimate for x_o
•m_ϵ is the mean of the process noise ξ; it represents our best a priori estimate for ξ
•m_0 is the mean of the measurement noise θ; it represents our best a priori estimate for θ;
•the term $H(y - \hat{y})$ provides a feedback mechanism which will keep $\dot{x}$ close to x.
•The matrix H in $R^{n\times p}$ is called the filter gain matrix. It is analogous to the control gain matrix G in the LQR problem where the filter gain matrix H in $R^{n\times p}$ is given by

$$H = \Sigma C^T \Theta^{-1} \tag{26}$$

and Σ is the unique symmetric (at least) positive semidefinite solution of the following Filter Algebraic Riccati Equation (FARE):

$$0 = A\Sigma + \Sigma A^T + LL^T - \Sigma C^T \Theta^{-1} C\Sigma \tag{27}$$

7.2.1 Minimum Phase Characteristics

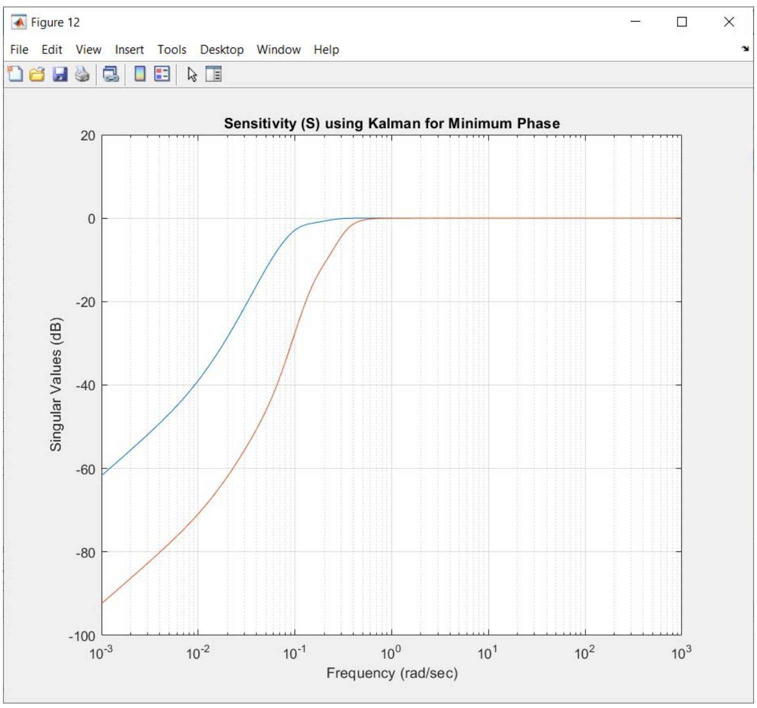


Figure 29: Closed Loop Sensitivity using Kalman for Minimum phase

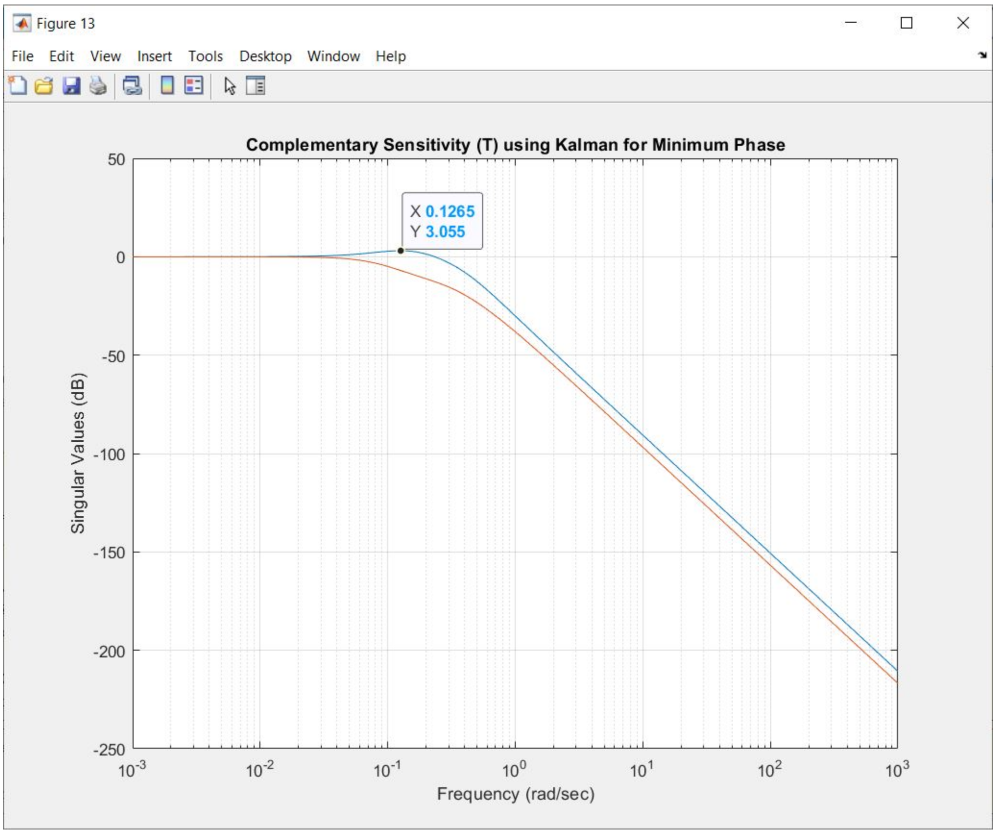


Figure 30: Closed Loop Complementary Sensitivity using Kalman for Minimum phase

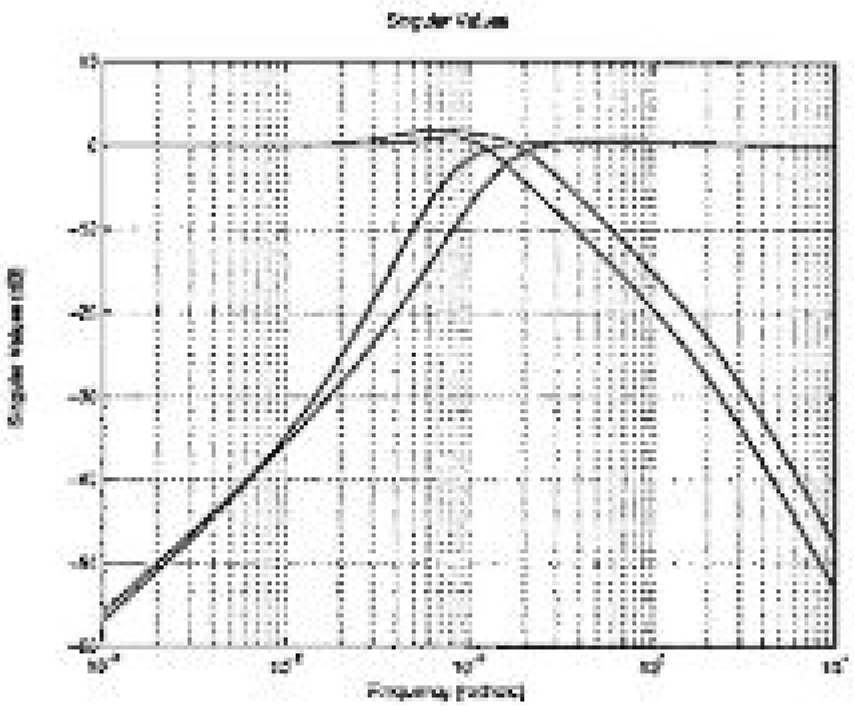


Figure 31: Kalman Filter Sensitivity and Complementary Sensitivity for Minimum Phase from Reference [7]

7.2.2 Non-Minimum Phase Characteristics

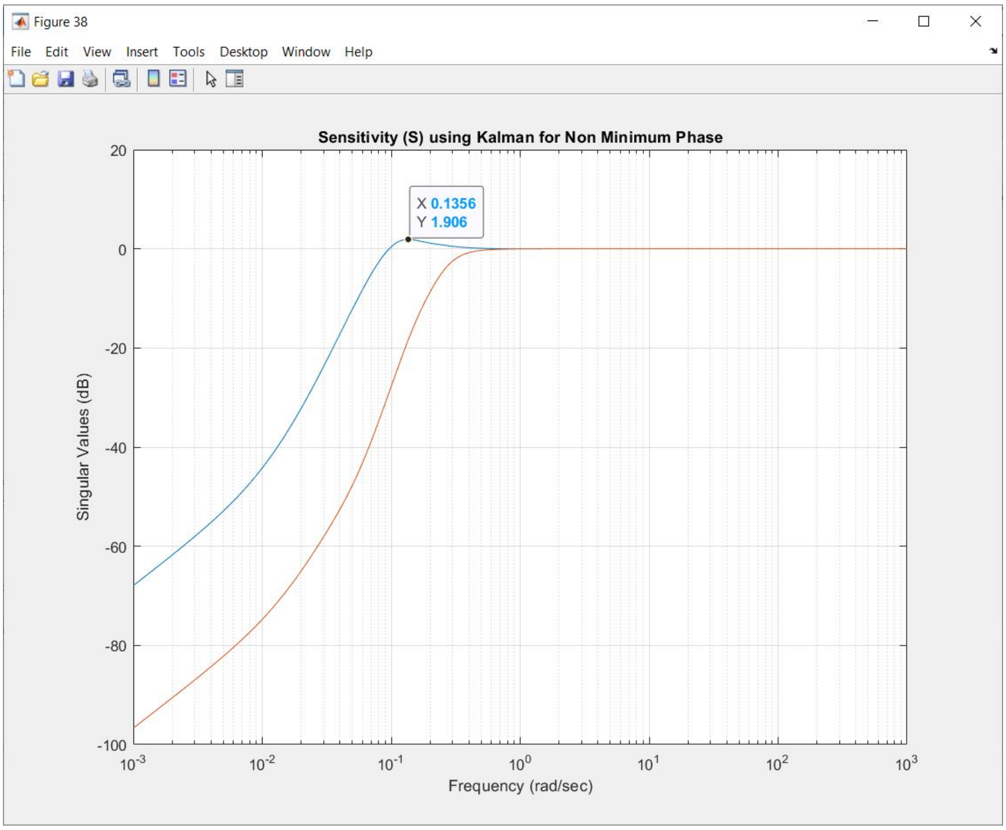


Figure 32: Closed Loop Sensitivity using Kalman for Non-Minimum phase

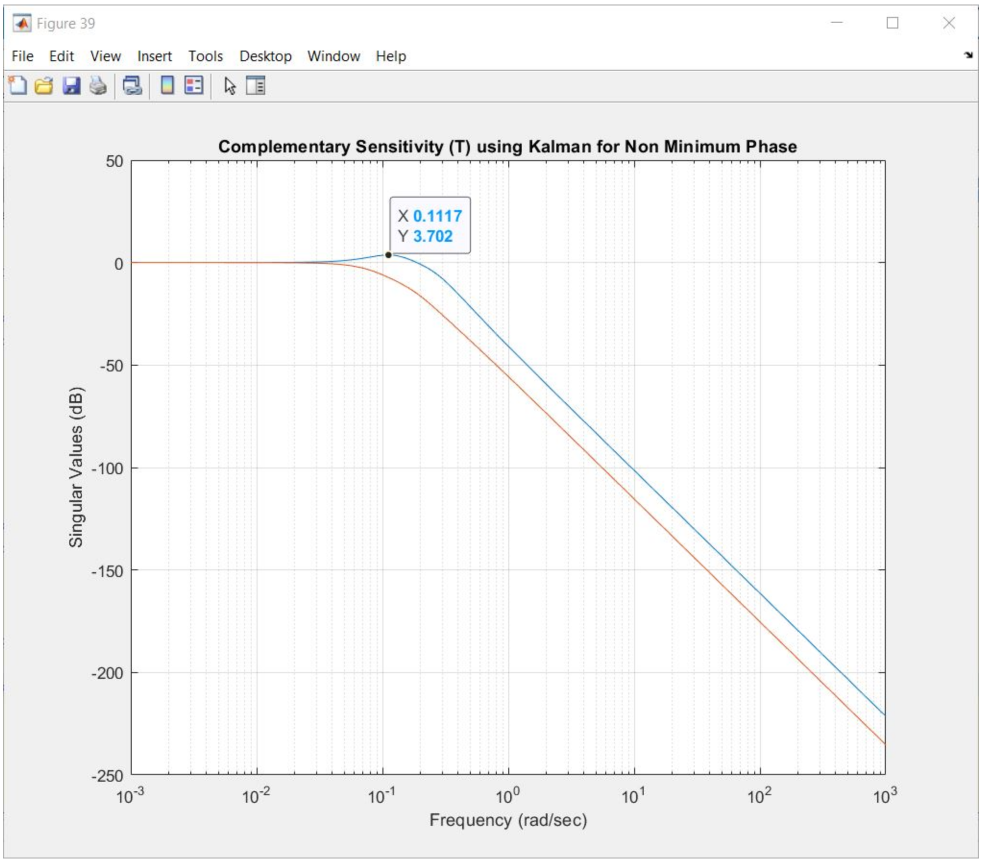


Figure 33: Closed Loop Complementary Sensitivity using Kalman for Non-Minimum phase

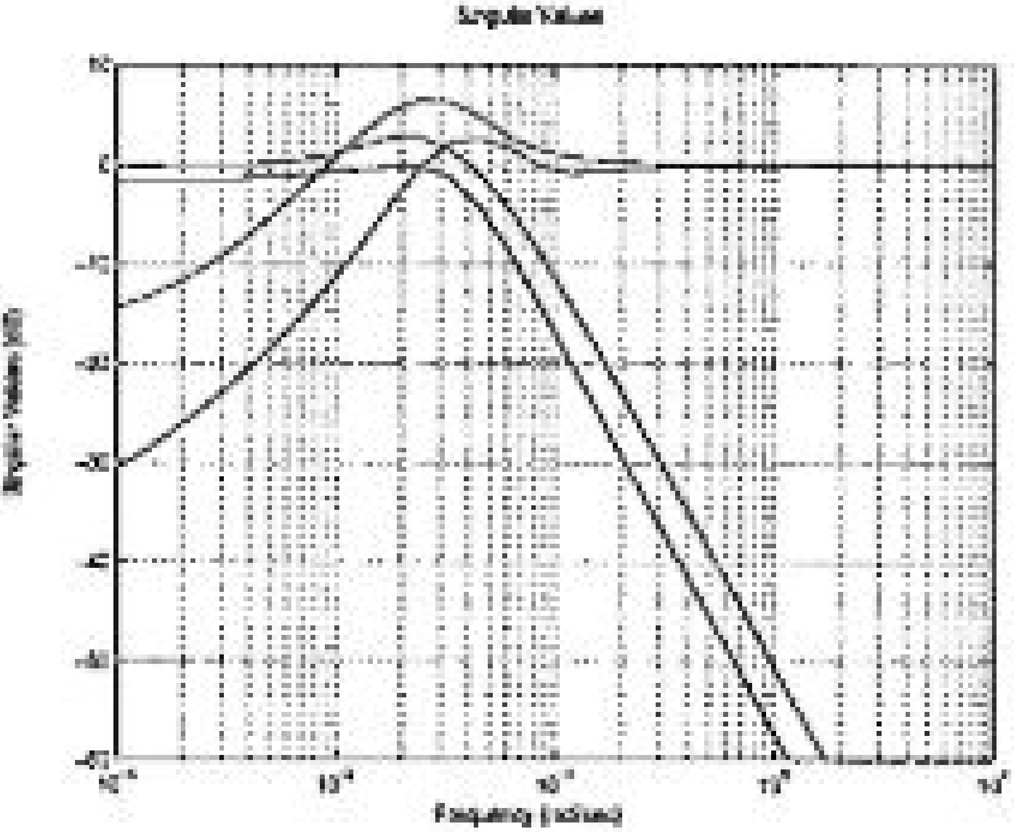


Figure 34: Kalman Filter Sensitivity and Complementary Sensitivity for Non-Minimum Phase from Reference [7]

7.3 LQG(Linear Quadratic Gaussian) Controller

The LQG(Linear Quadratic Gaussian) design methodology is a combination of the LQR(Linear Quadratic Regulator) and the Kalman-Bucy Filter[12].
Problem Statement([12]pp.183): The goal of the LQG problem is to minimize the following stochastic quadratic cost functional

$$J(u,y) = E\left[\int_0^{\infty} \left(x^T M^T M x + u^T R u\right) dt\right] \tag{28}$$

subject to the following dynamic constraint:

$$\begin{aligned} \dot{x} = &\quad Ax + Bu + L\xi \quad x(0) = x_o \\ y &\qquad = Cx + \theta \end{aligned} \tag{29}$$

over all linear functionals of u and y. The assumptions includes (A,B,M) is stabilizable and detectable. Solution:
Optimal Compensator Structure: The optimal LQG controller (compensator) is a dynamic system and may be written as follows:

$$\begin{aligned} \dot{x} &= (A - BG - HC)\hat{x} + Lm_{\xi} + H\left(y - m_0\right) \\ \hat{u} &= -G\hat{x} \end{aligned} \tag{30}$$

Closed Loop Properties: The resulting closed loop dynamical system enjoys a separation principle with all closed loop poles - eigenvalues of $A - BG$ and $A - HC$ - in the open left half plane. When x_o, $\xi,\ and\ \theta$ are Gaussian, the above control law is the optimal control law (without further qualification)([12]pp.183-184).

7.3.1 Minimum Phase Characteristics

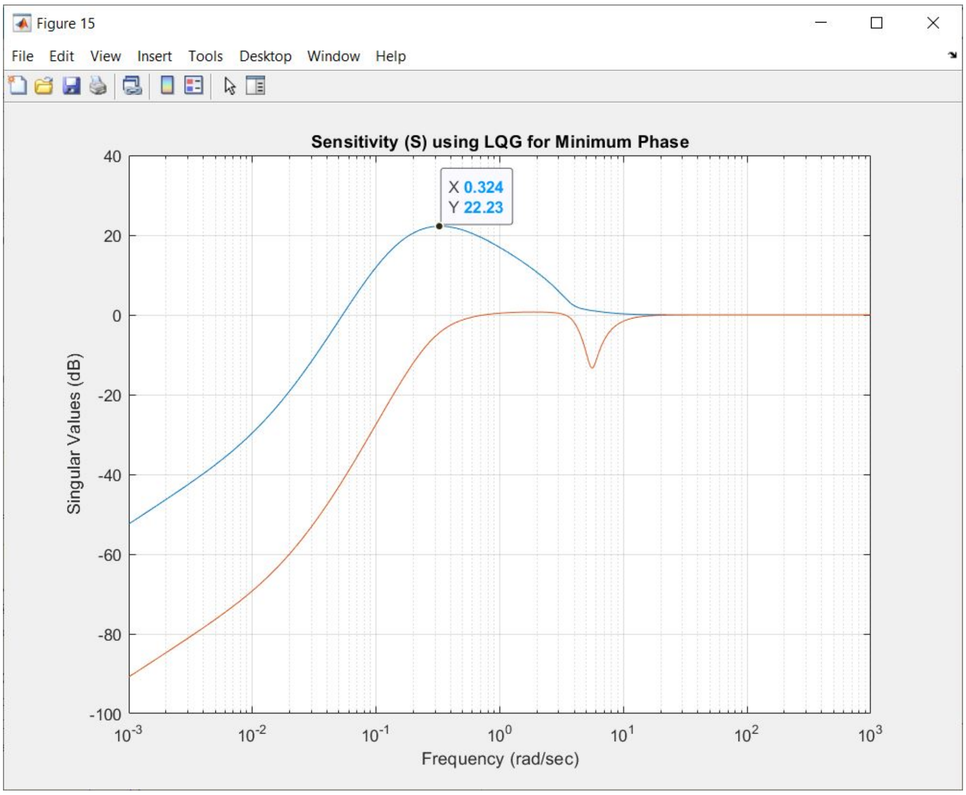


Figure 35: Closed Loop Sensitivity using LQG for Minimum phase

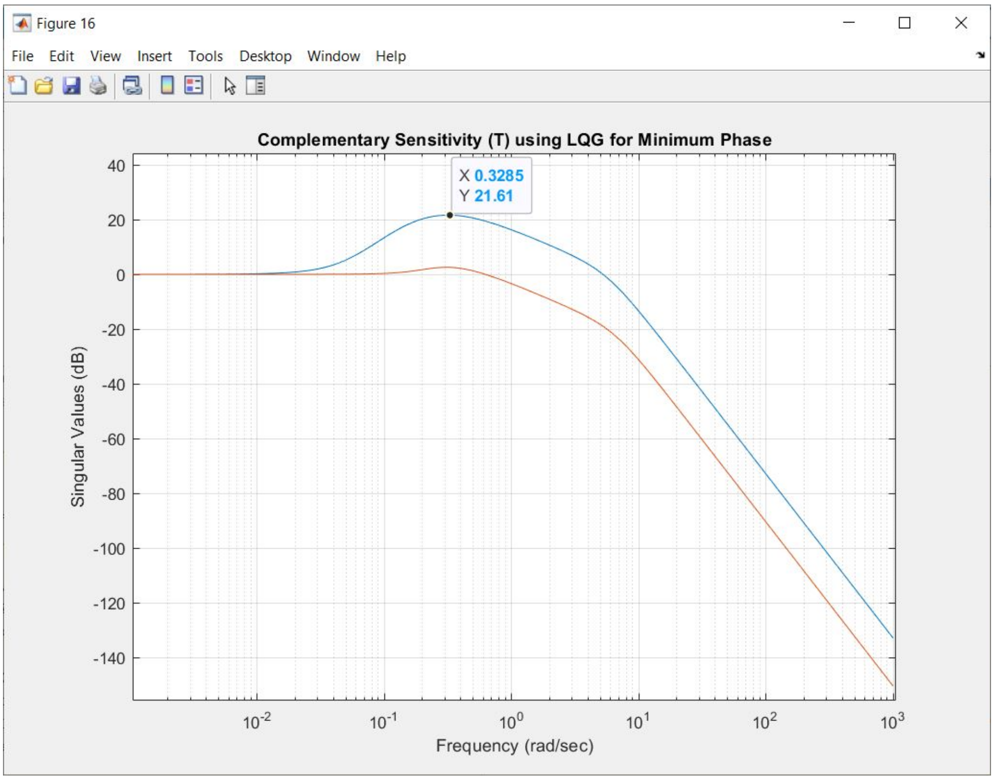


Figure 36: Closed Loop Complementary Sensitivity using LQG for Minimum phase

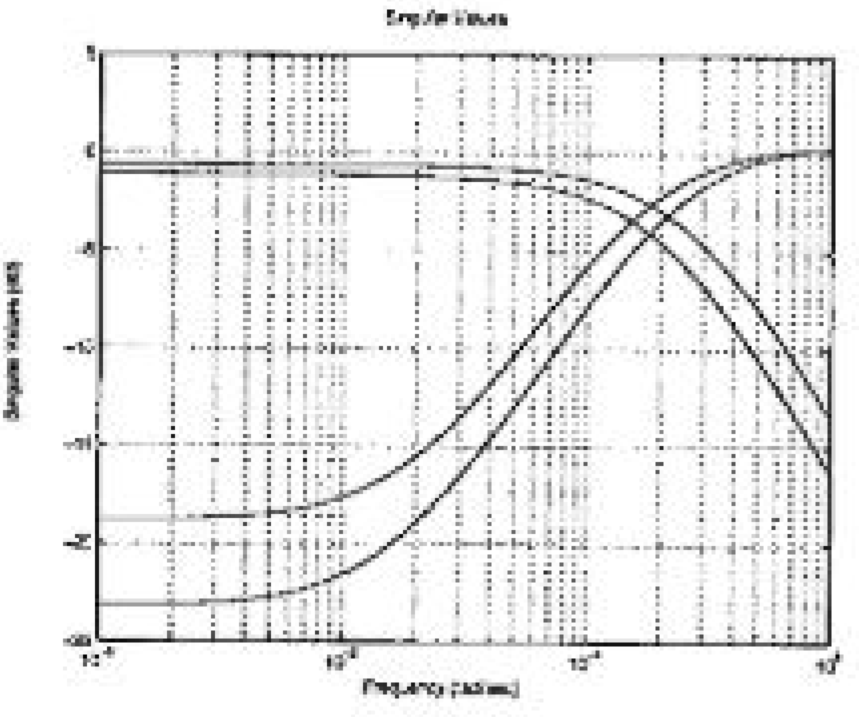


Figure 37: Sensitivity and Complementary Sensitivity using LQG for Minimum Phase from Reference

[7]

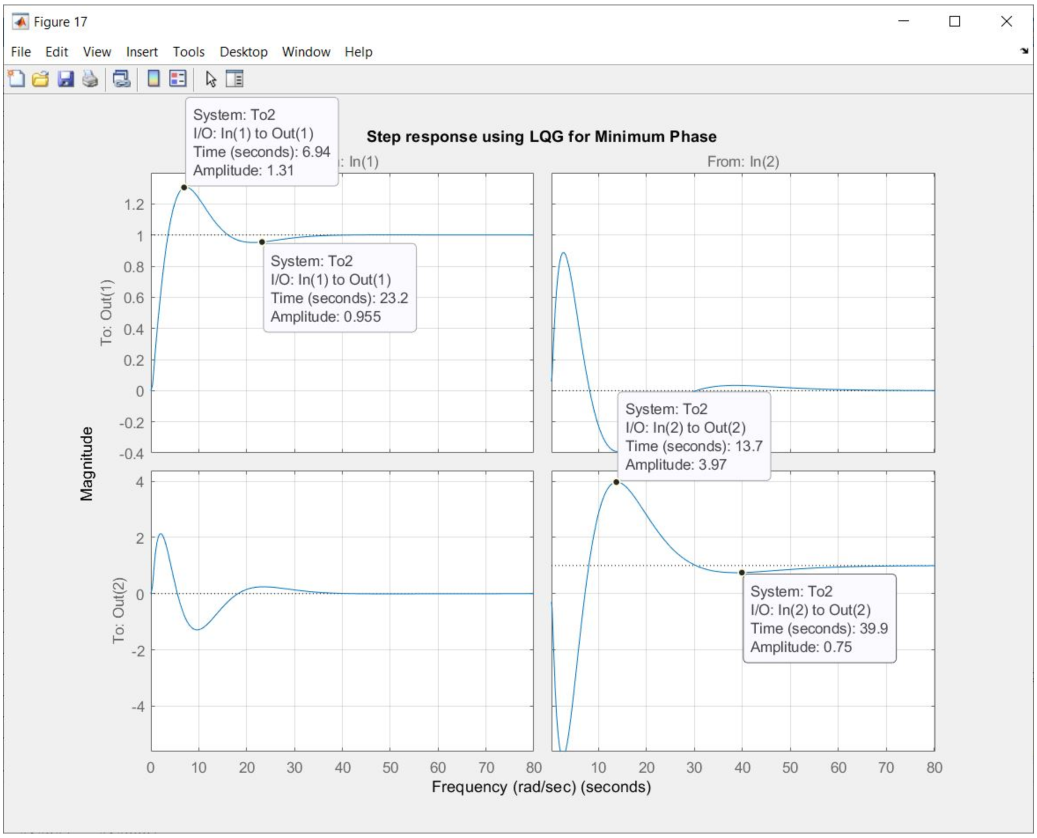


Figure 38: Step Response of LQG for Minimum Phase

7.3.2 Non-Minimum Phase Characteristics

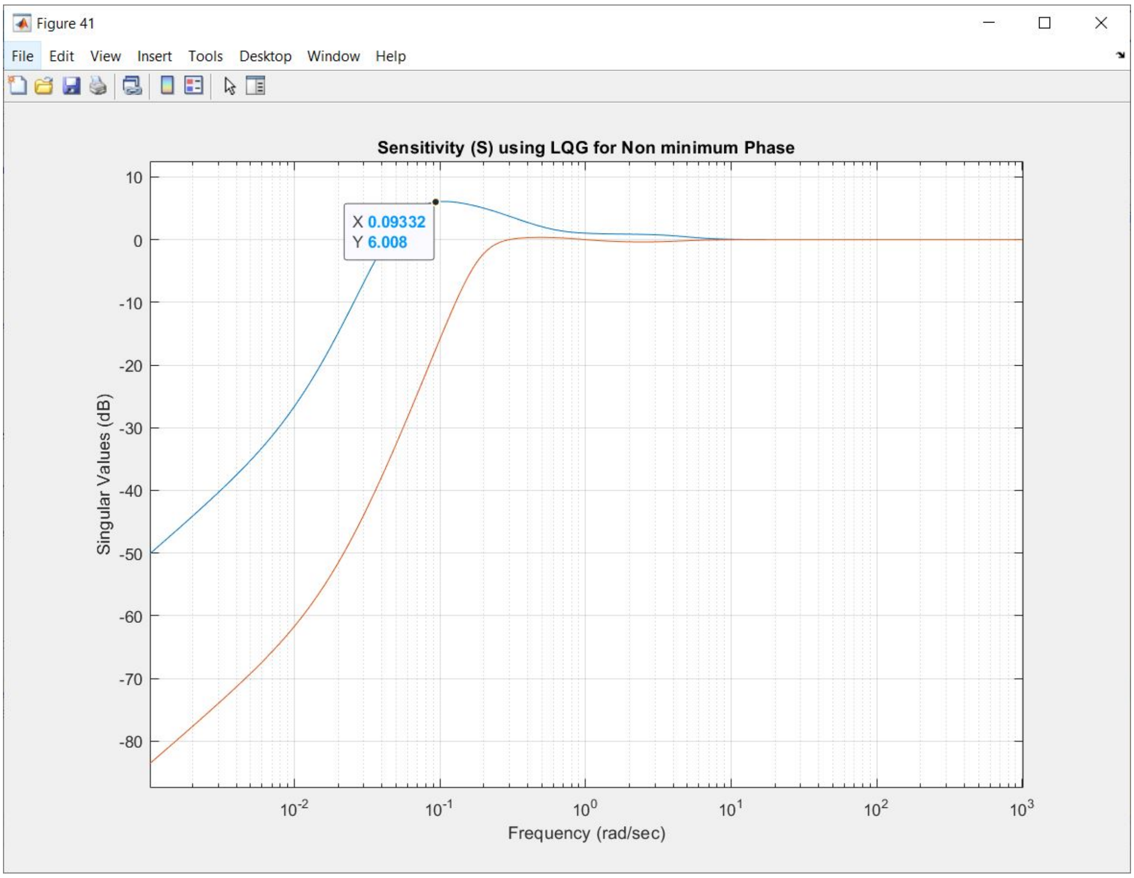


Figure 39: Closed Loop Sensitivity using LQG for Non-Minimum phase

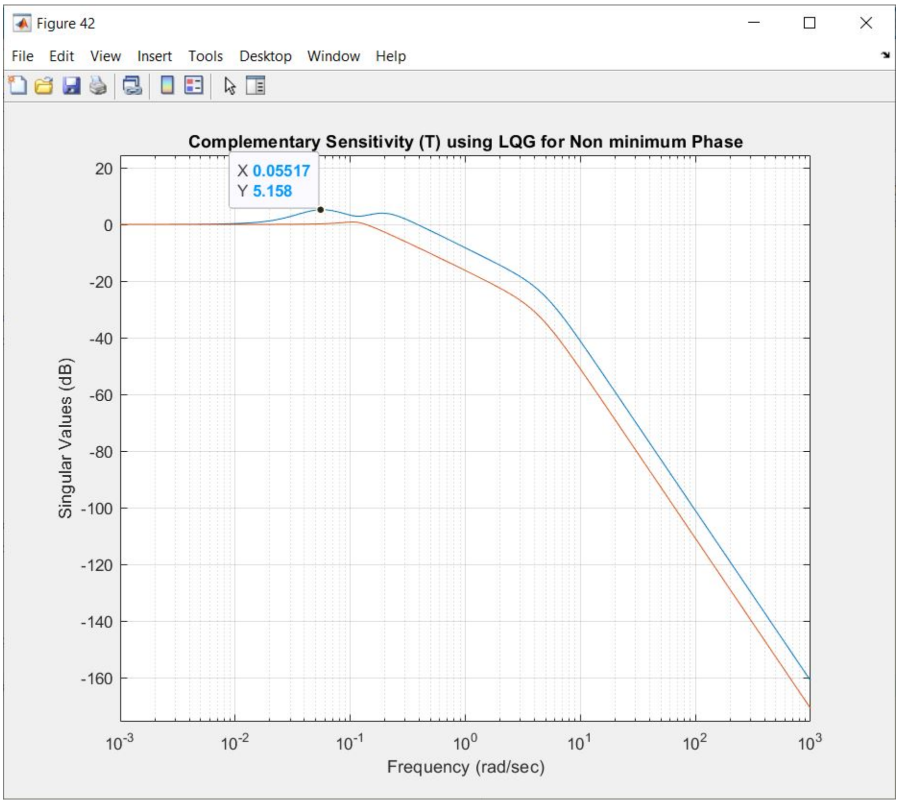


Figure 40: Closed Loop Complementary Sensitivity using LQG for Non-Minimum phase

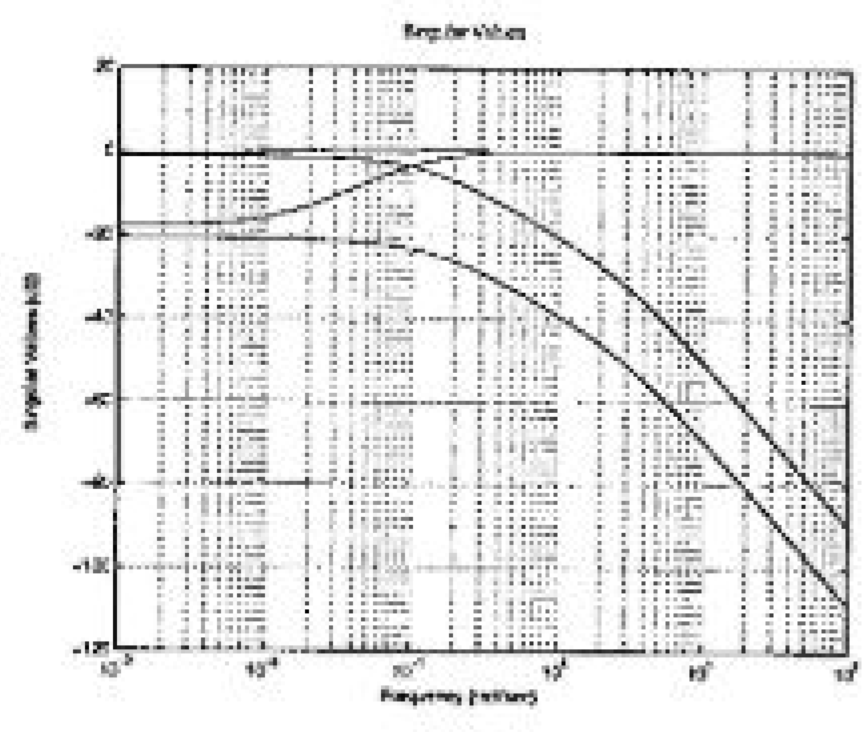


Figure 41: Sensitivity and Complementary Sensitivity using LQG for Non-Minimum Phase from Reference

[7]

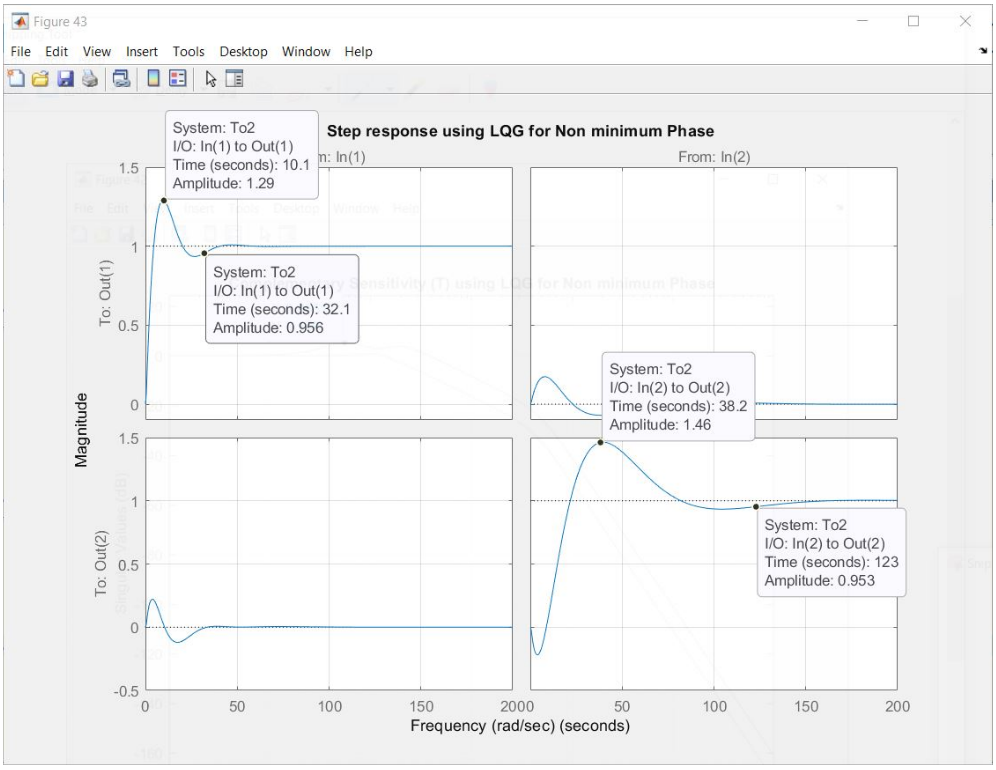


Figure 42: Step Response of LQG for Non- Minimum Phase

7.4 H-Infinity Controller

7.4.1 Minimum Phase Characteristics

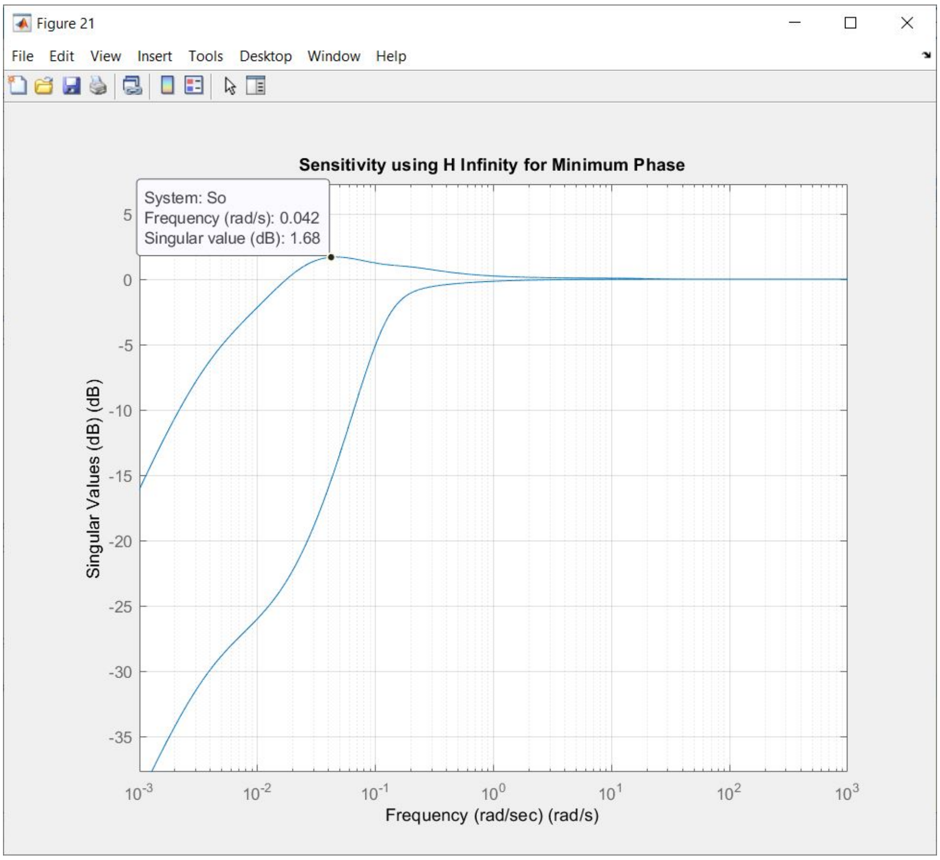


Figure 43: Closed Loop Sensitivity using H-Infinity for Minimum phase

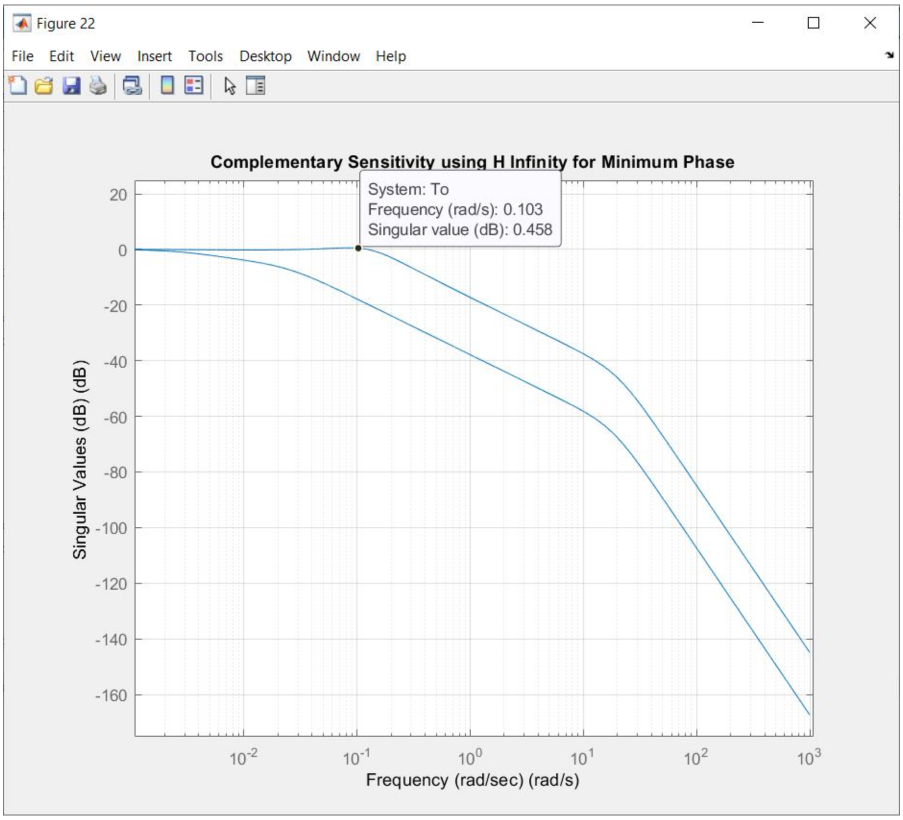


Figure 44: Closed Loop Complementary Sensitivity using H-Infinity for Minimum phase

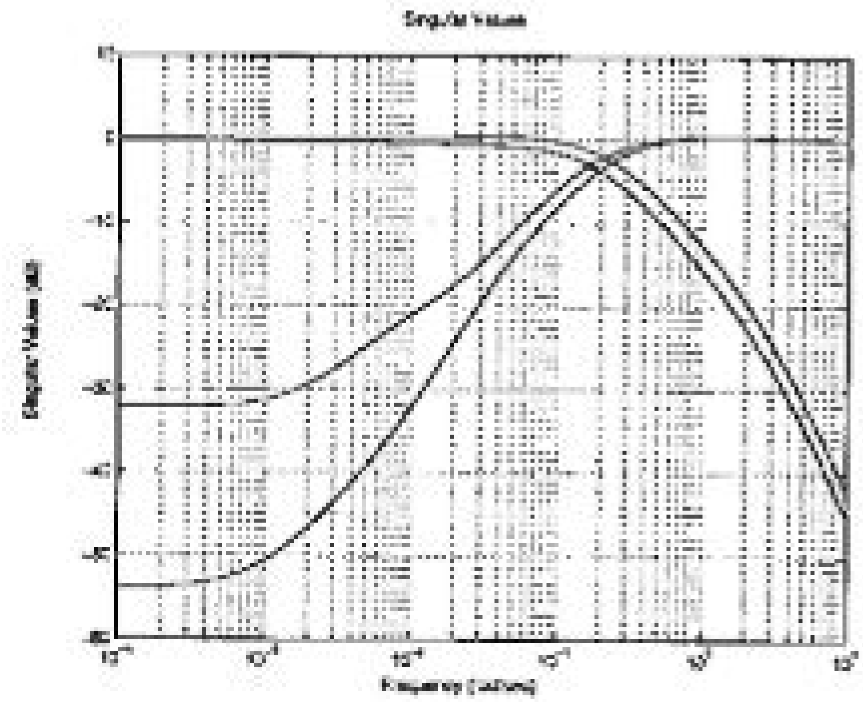


Figure 45: Sensitivity and Complementary Sensitivity using H-Infinity for Minimum Phase from Reference

[7]

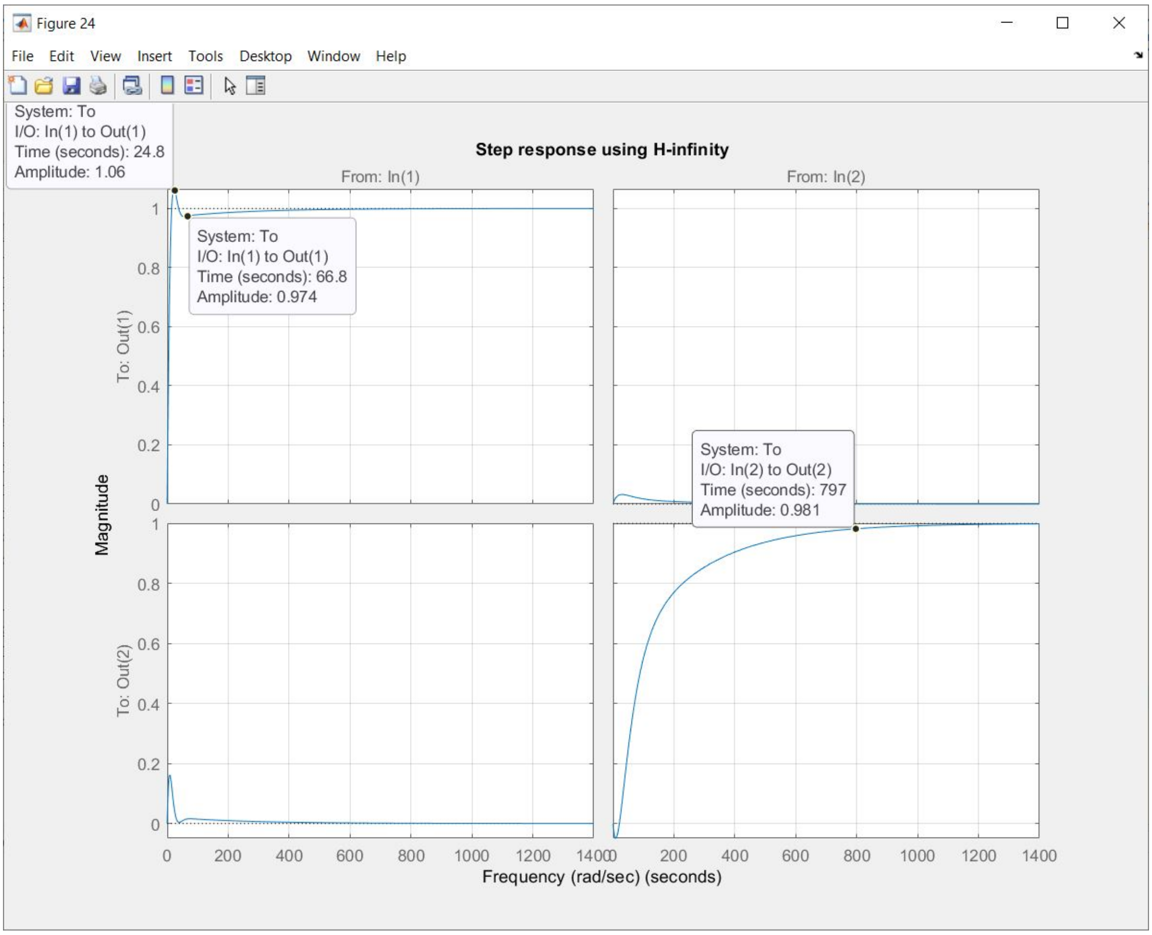


Figure 46: Step Response of H-Inf for Minimum Phase

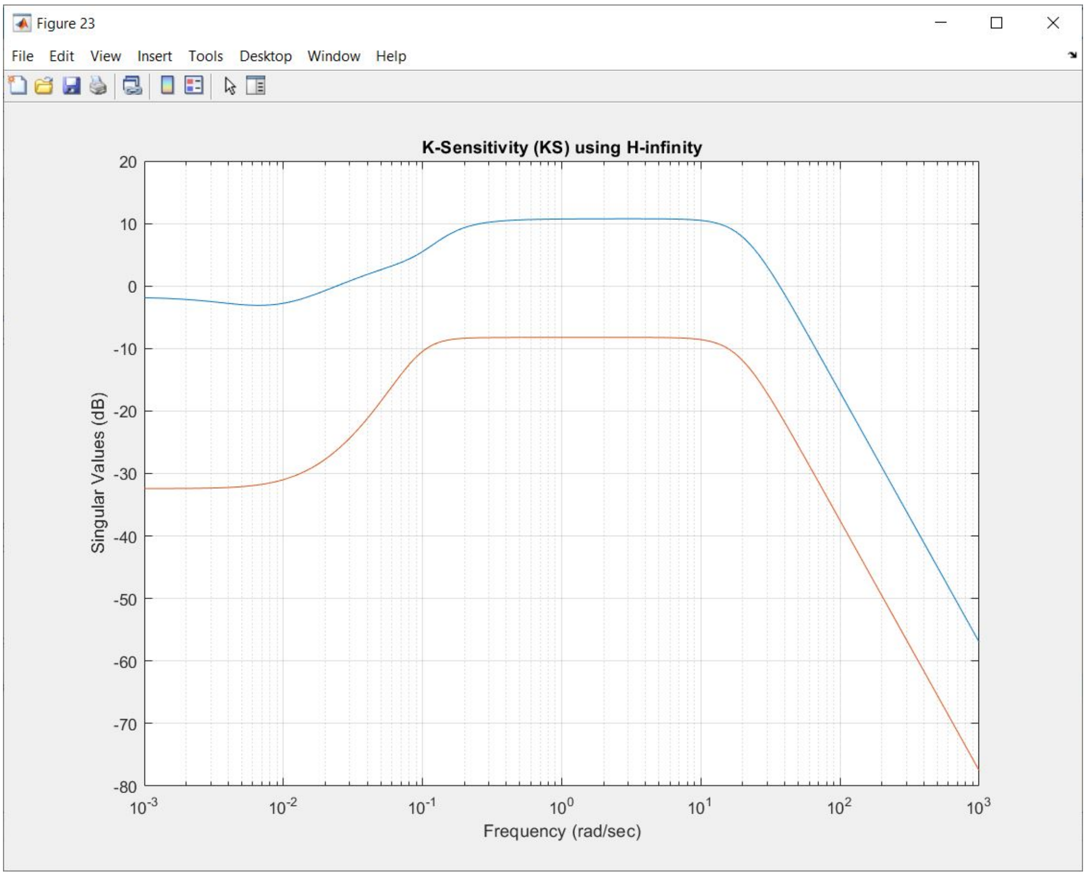


Figure 47: K-Sensitivity frequency response using H-Infinity for Minimum Phase

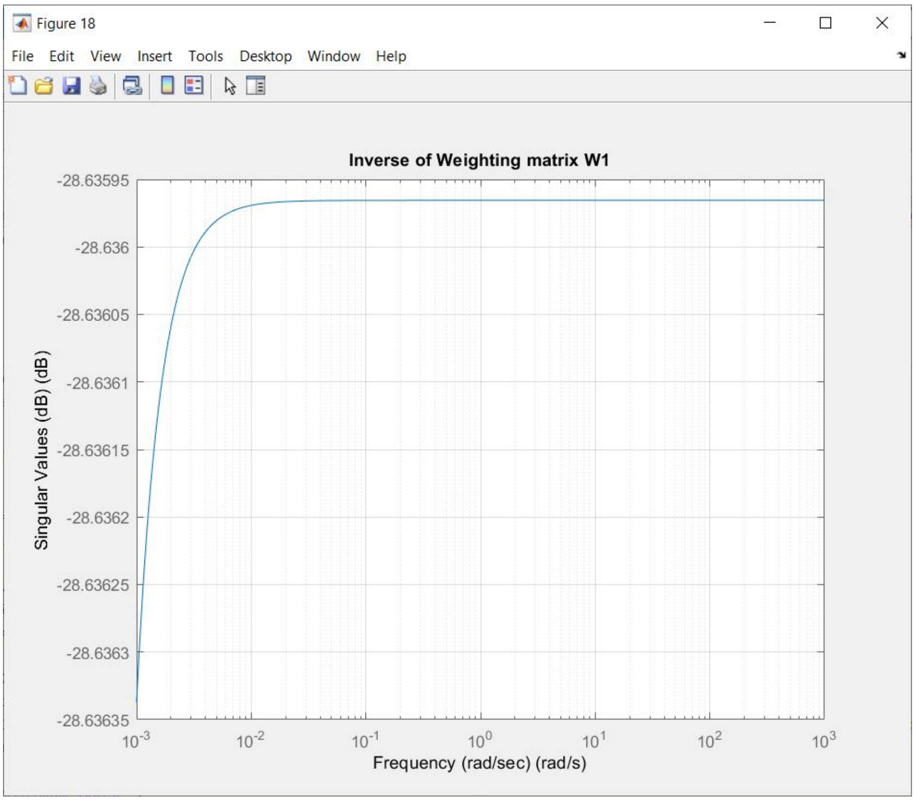


Figure 48: Uncertainity Weighting Function W1 for Minimum phase

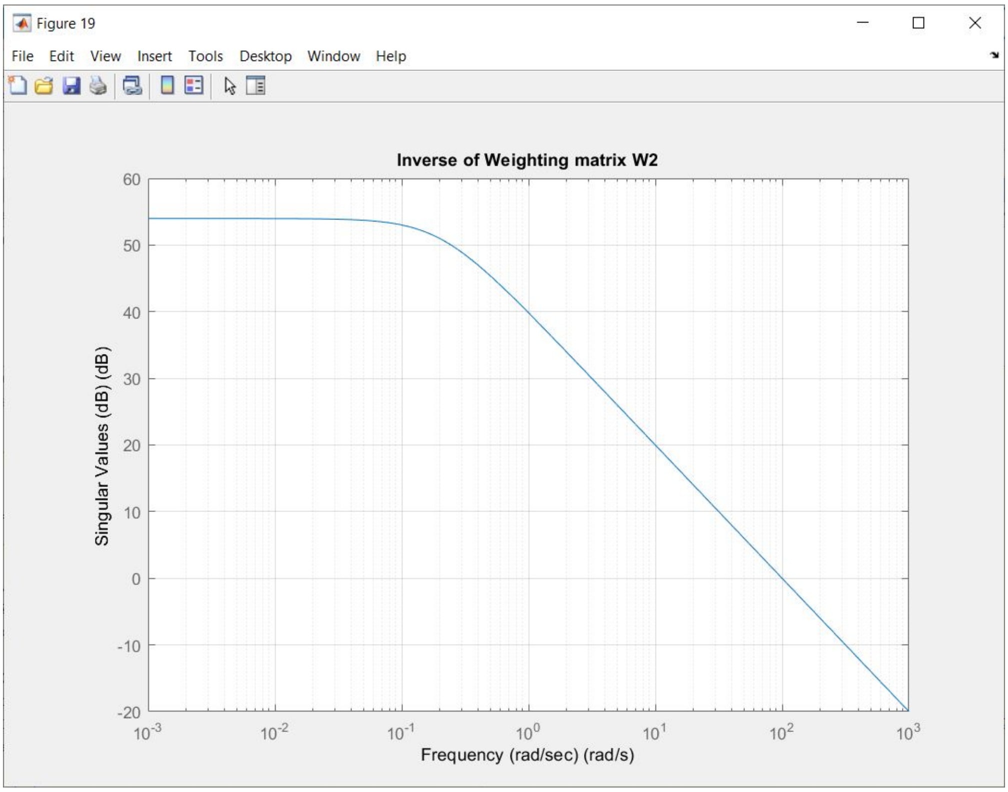


Figure 49: Uncertainity Weighting Function W2 for Minimum phase

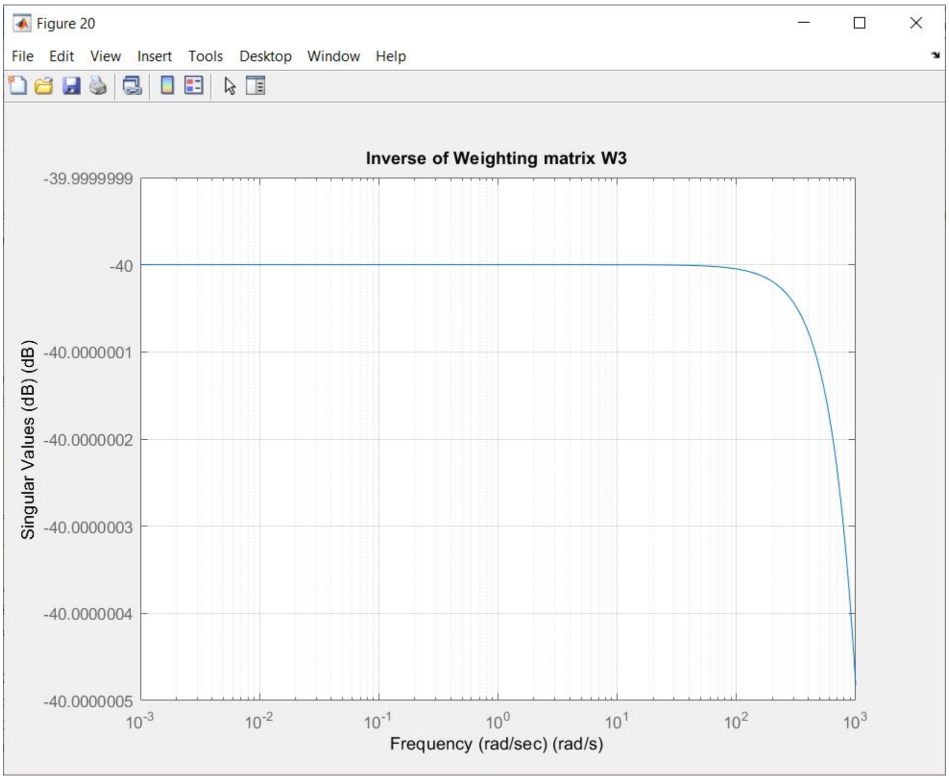


Figure 50: Uncertainity Weighting Function W3 for Minimum phase

7.4.2 Non-Minimum Phase Characteristics

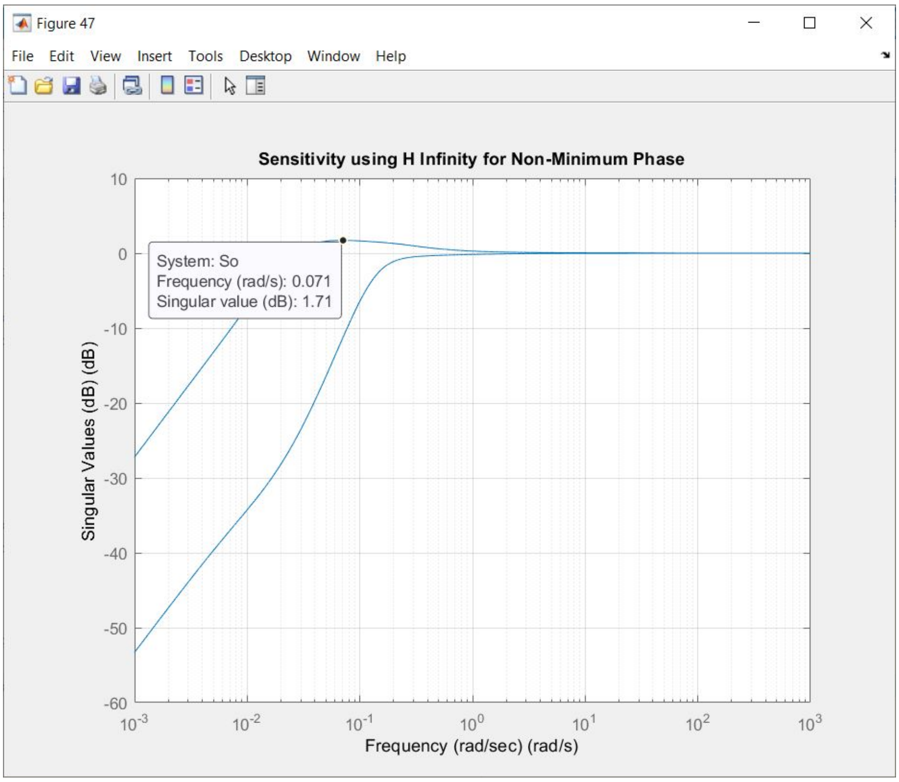


Figure 51: Closed Loop Sensitivity using H-Infinity for Non-Minimum phase

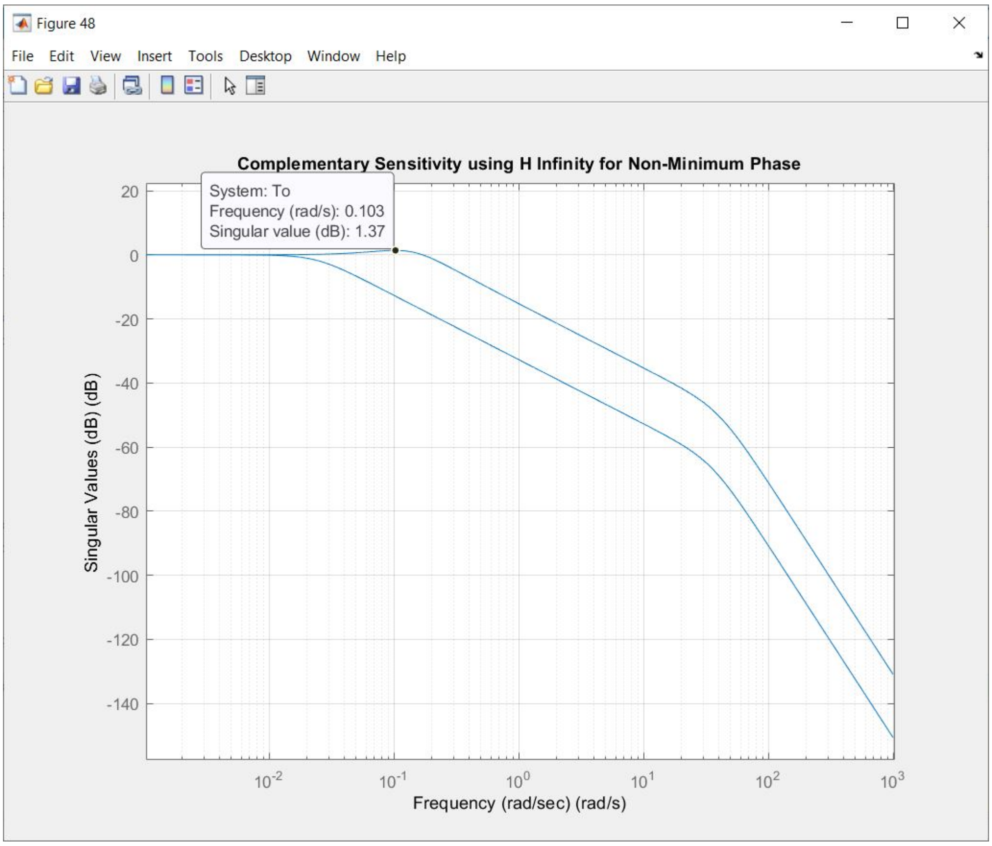


Figure 52: Closed Loop Complementary Sensitivity using H-Infinity for Non-Minimum phase

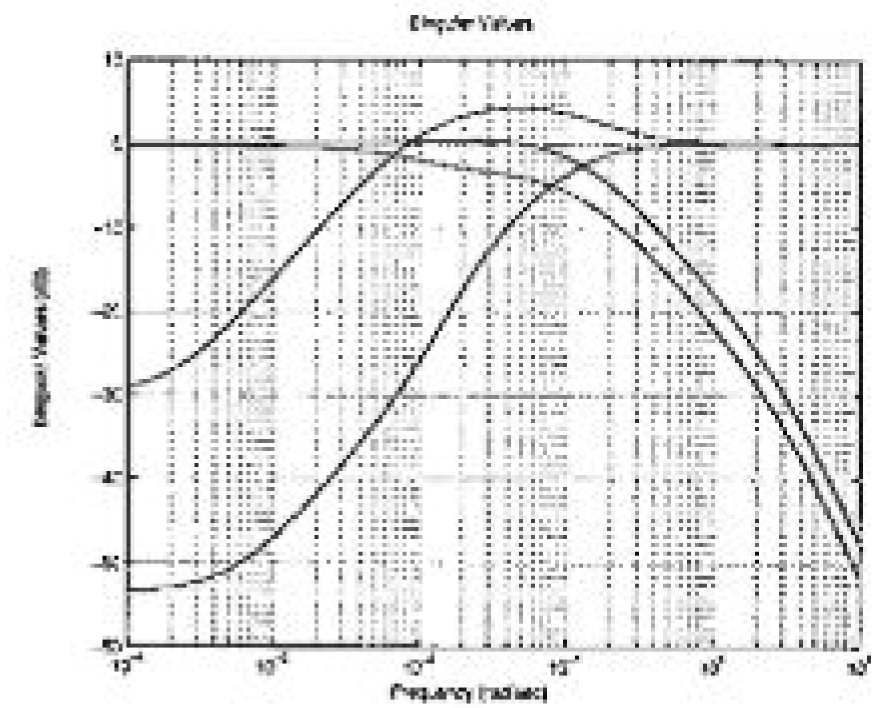


Figure 53: Sensitivity and Complementary Sensitivity using H-Infinity for Non-Minimum Phase from Reference

[7]

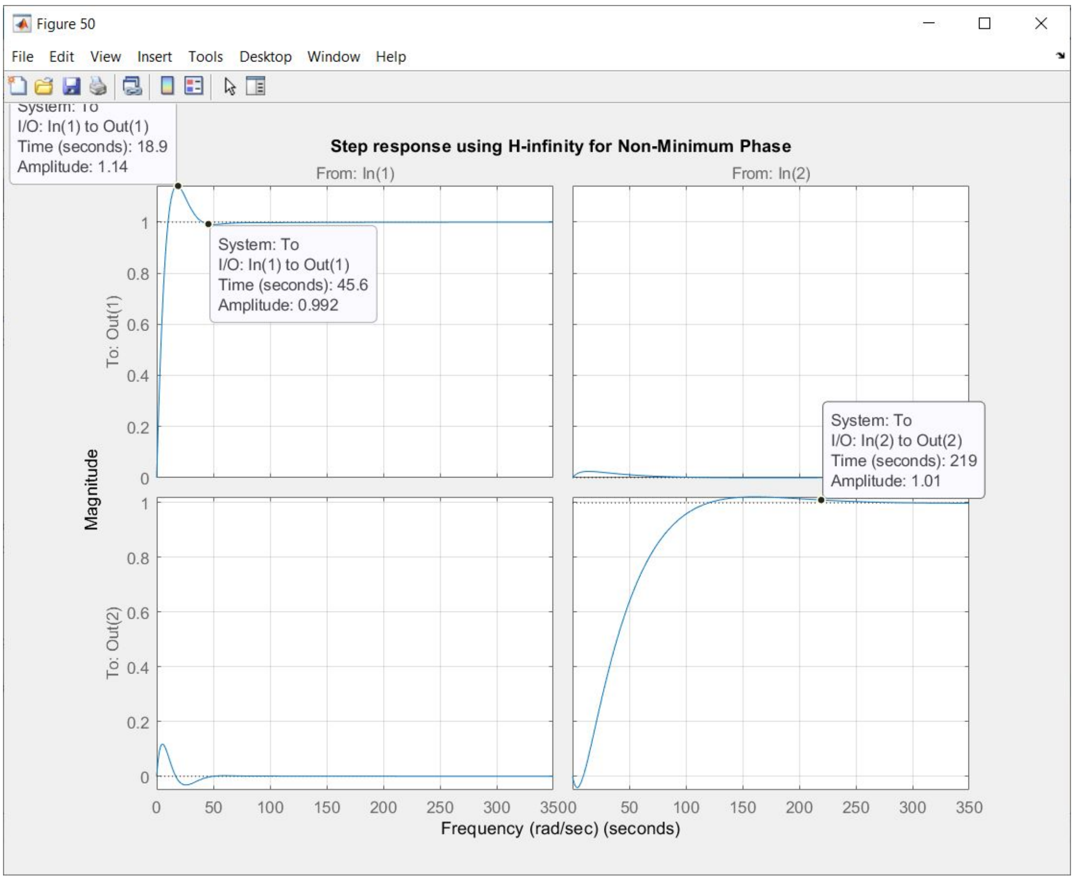


Figure 54: Step Response of H-Infinity for Non- Minimum Phase

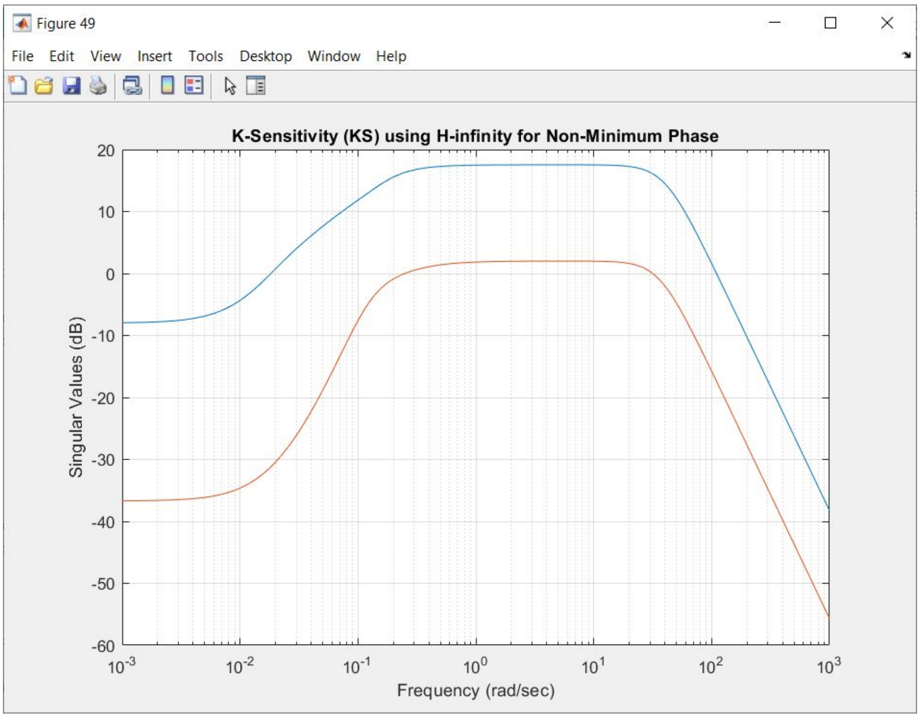


Figure 55: K-Sensitivity frequency response using H-Infinity for Non-Minimum Phase

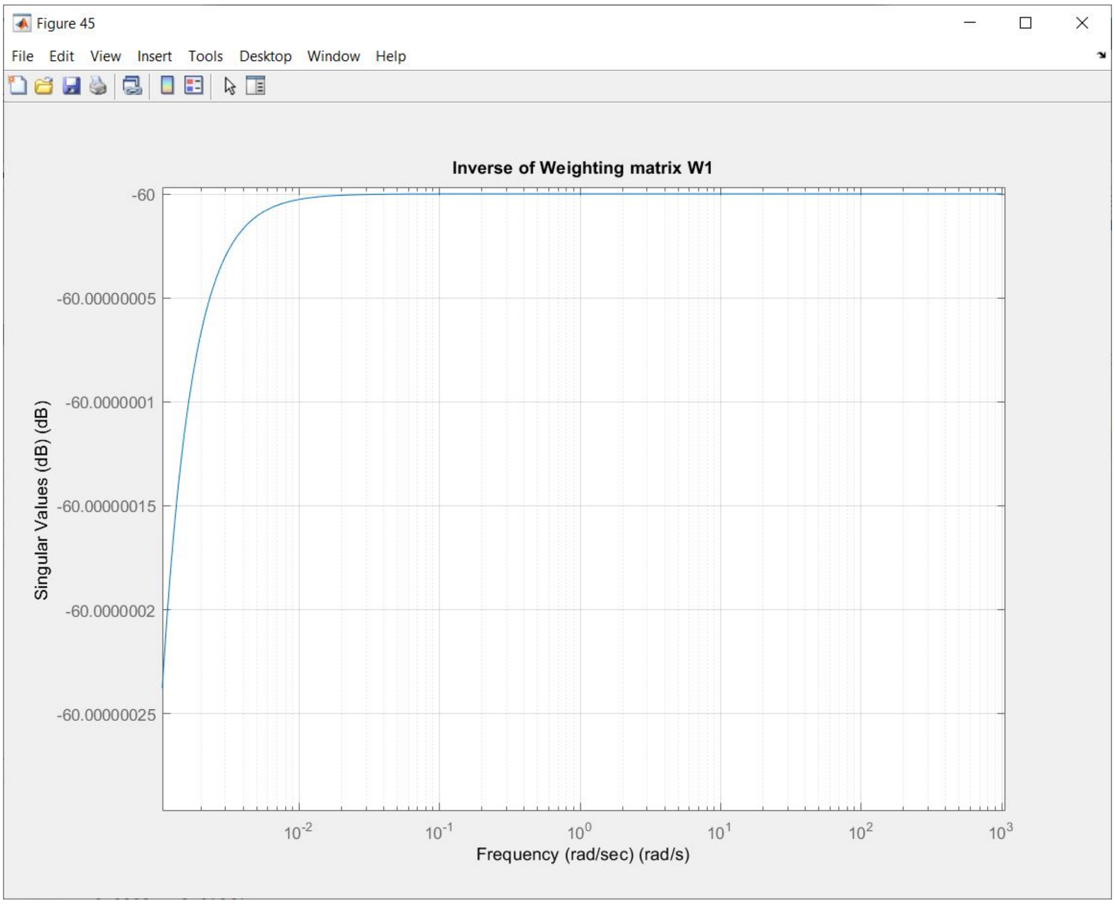


Figure 56: Uncertainity Weighting Function W1 for Non-Minimum phase

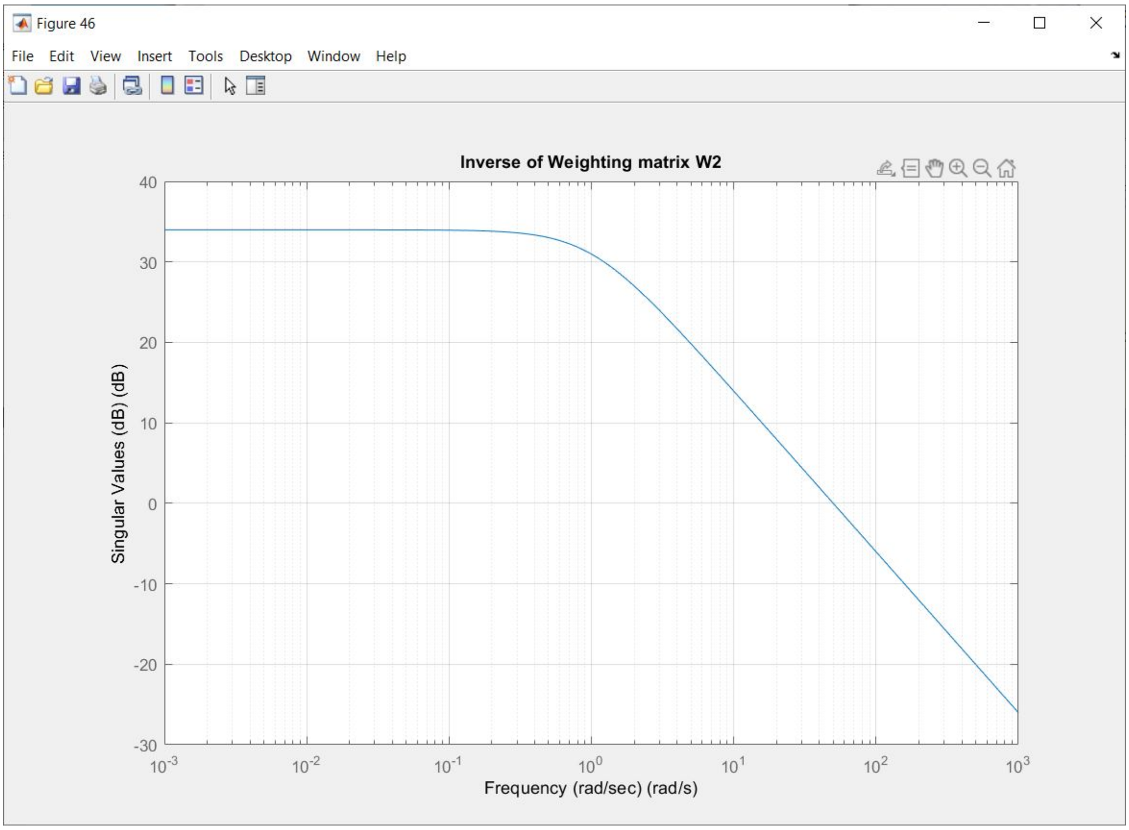


Figure 57: Uncertainity Weighting Function W2 for Non-Minimum phase

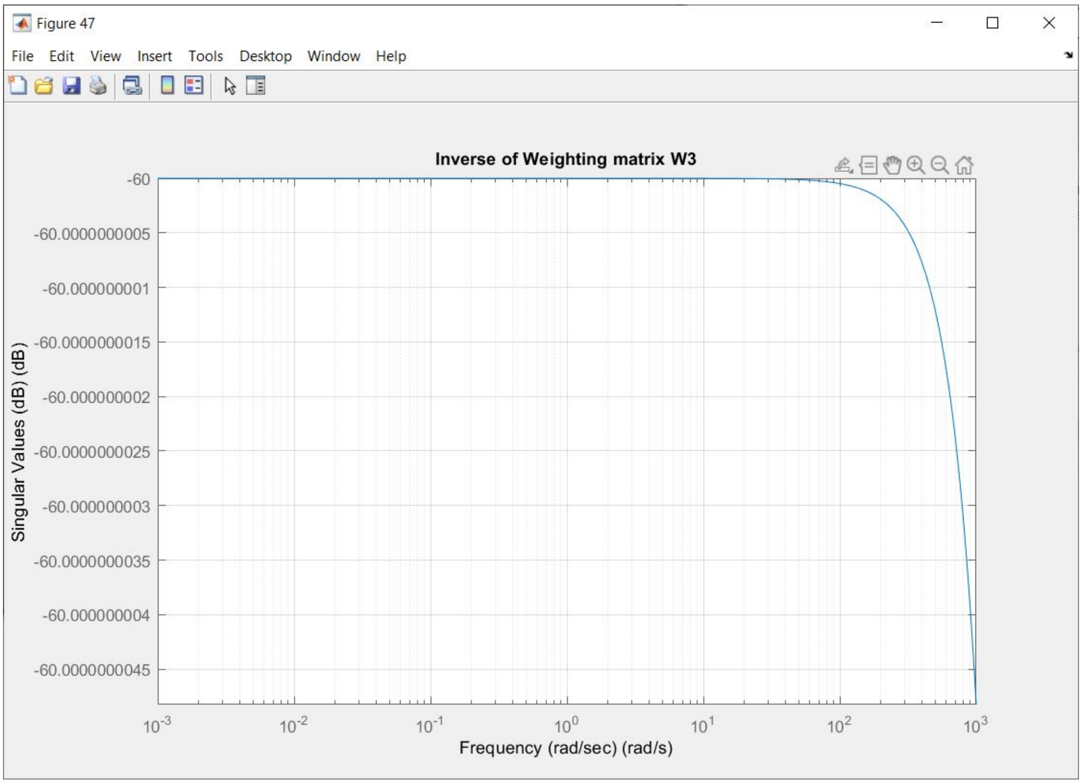


Figure 58: Uncertainity Weighting Function W3 for Non-Minimum phase

7.5 Other Controllers from the Literature

7.5.1 PI Controller [10][11]

•Dashed line represents PI control with static decoupling and the solid line represents Glover-McFarlane designs.
•The bandwidth for the non minimum-phase case is almost a magnitude lower than for the minimum-phase case. This is due to the performance limitation that the multivariable right half-plane zero imposes[10].

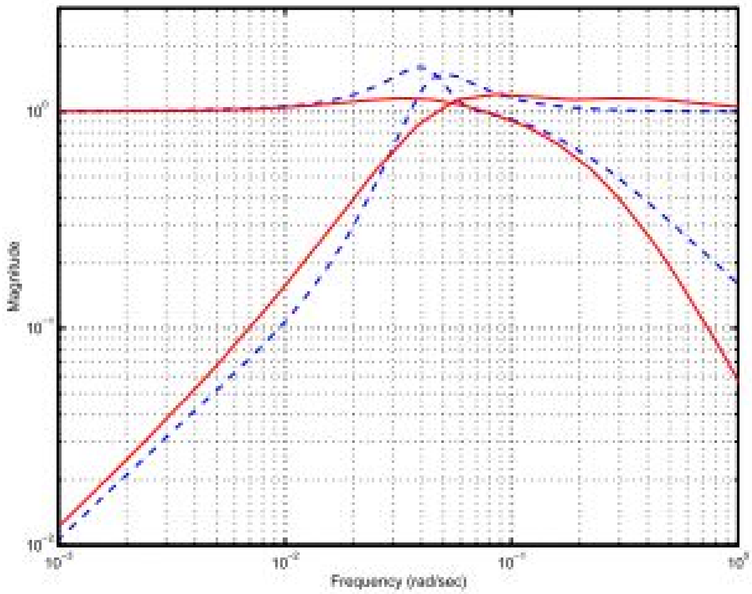


Figure 59: Sensitivity functions and complementary sensitivity functions for the minimum-phase setting

[10]

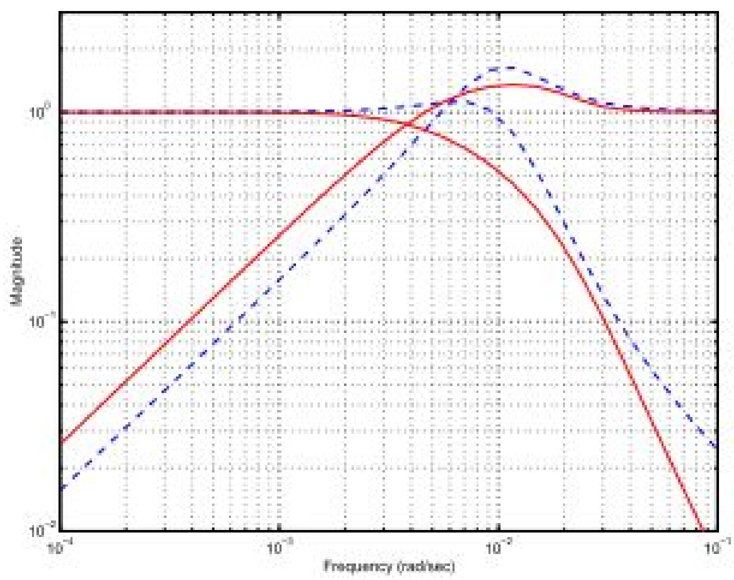


Figure 60: Sensitivity functions and complementary sensitivity functions for the Non-minimum-phase setting

[10]

•The dashed line represents the non-Linear model simulation whereas the solid black line represents the linear model

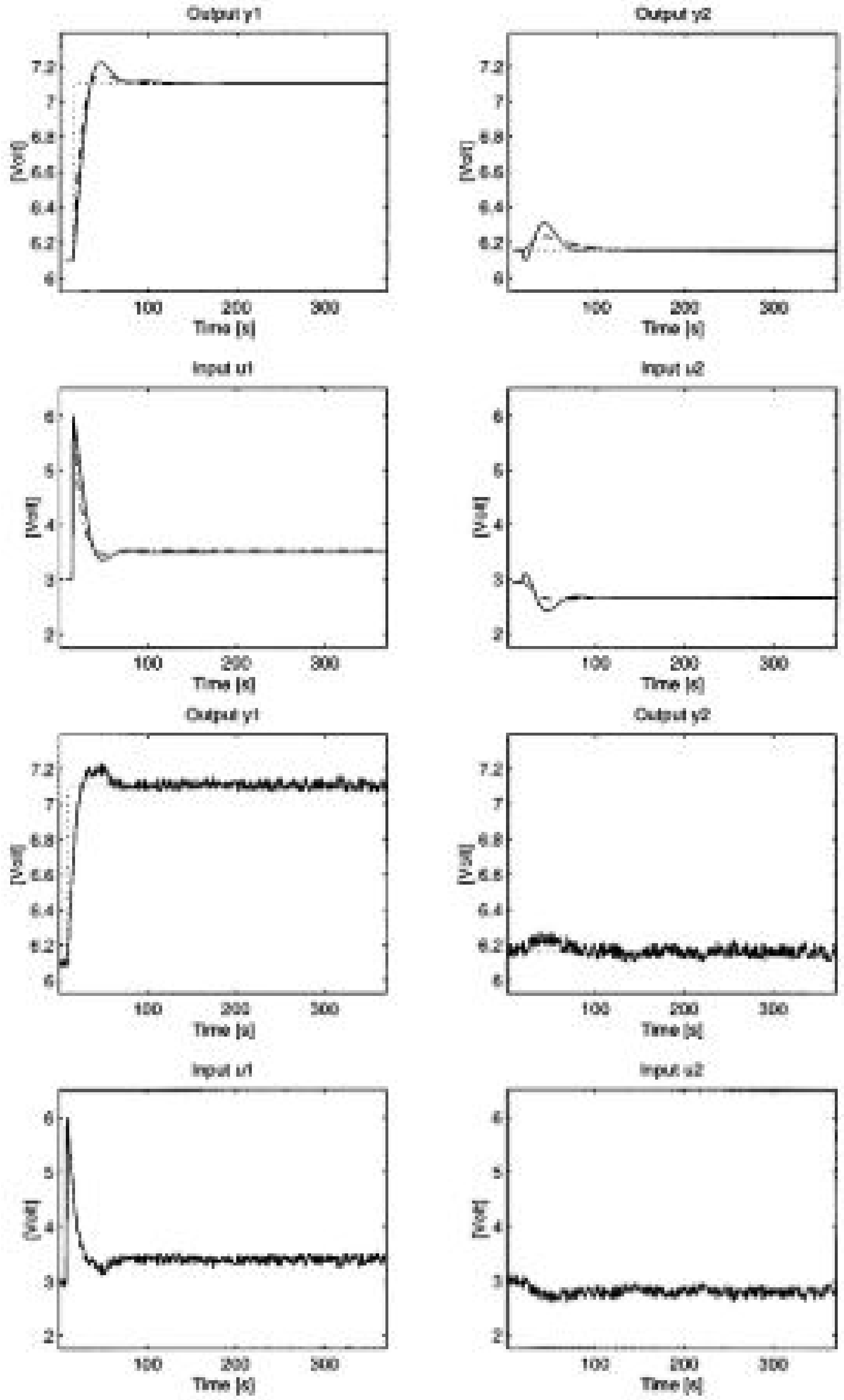


Figure 61: Minimum phase PI Controller
[11]

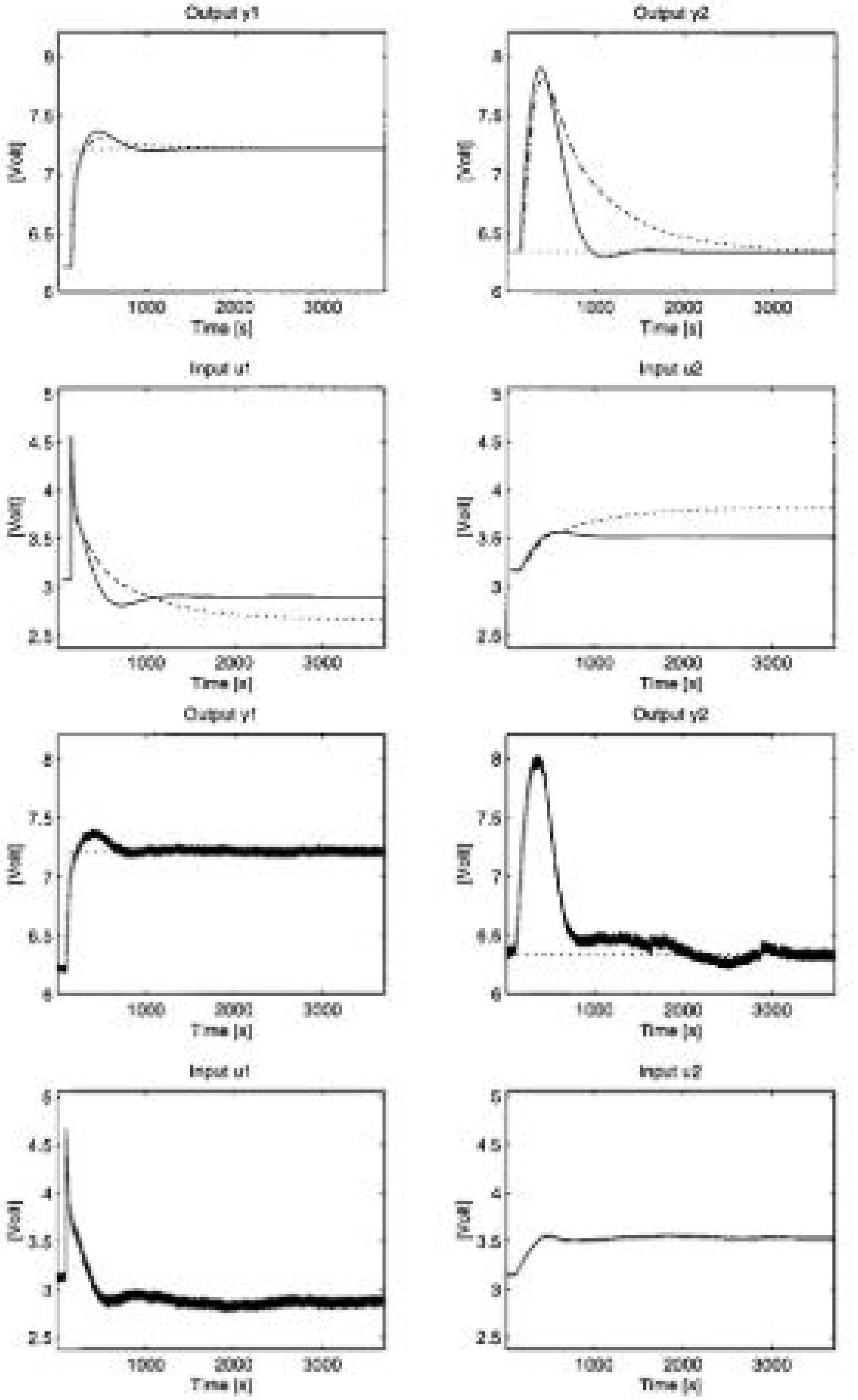


Figure 62: Non-Minimum Phase PI Controller
[11]

7.5.2 Loop-Shaping [7]

•The general idea behind loop shaping is to construct an open loop transfer function L which meets desired performance requirements.
•This simple method works for plants which do not contain a RHP zero, if a plant contains a RHP zero then more advance loop shaping techniques can be used.
•Loop shaping creates a 4^{th} order controller.

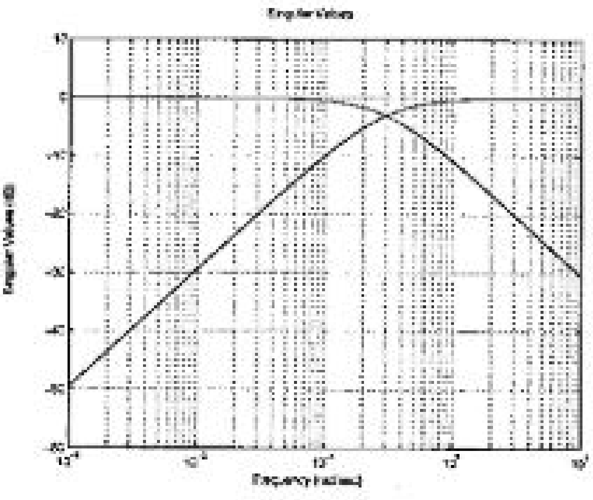


Figure 63: Loop Shaping Sensitivity and Complementary sensitivity [7]

8 Hardware Results

8.1 Simulation Results

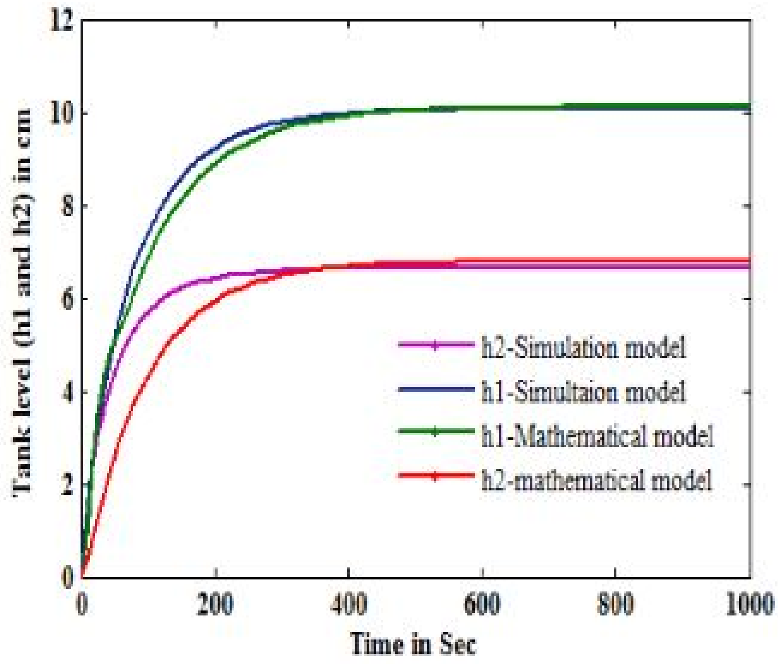


Figure 64: Open Loop Responses in Non-Minimum Phase
[6]

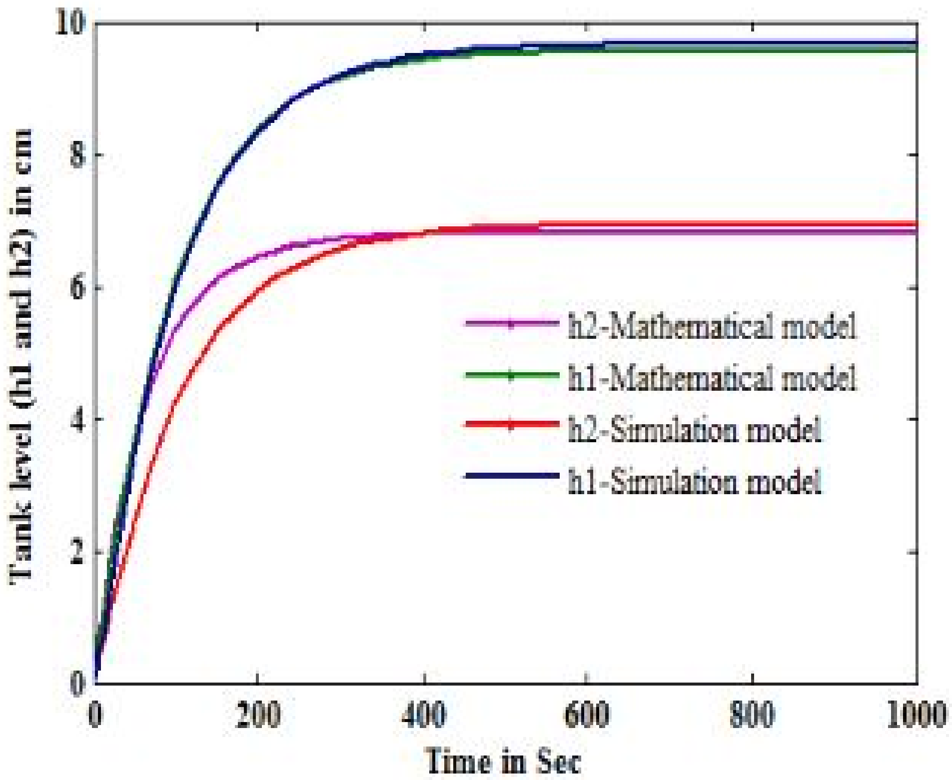


Figure 65: Open Loop Responses in Minimum Phase
[6]

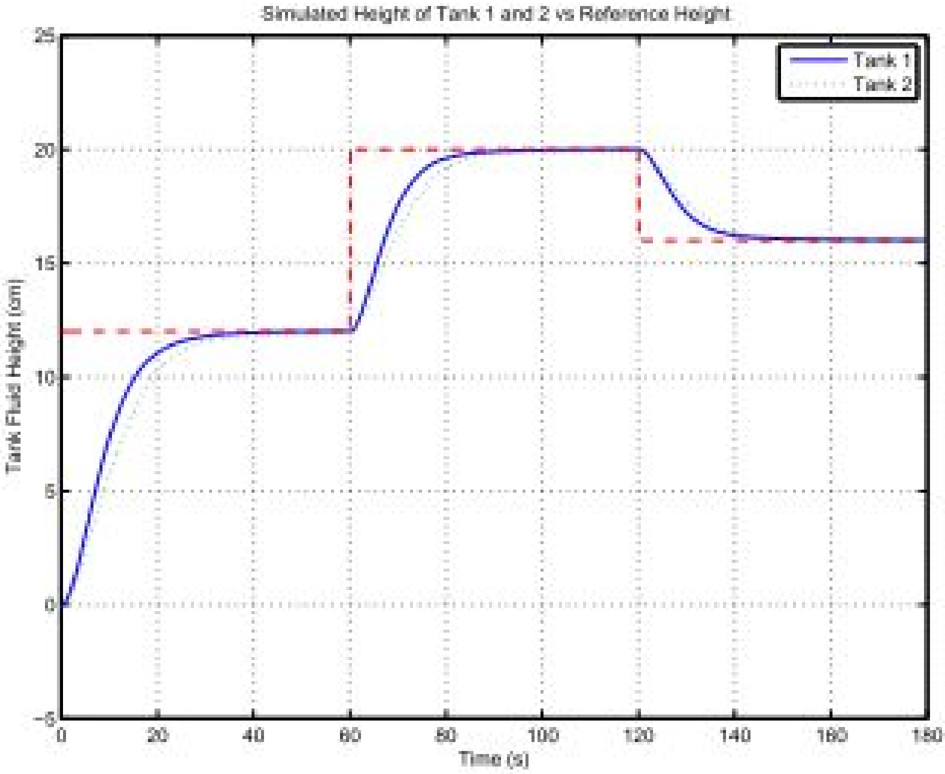


Figure 66: A simulated comparison of commanded reference input to tank fluid height QTP. [5]

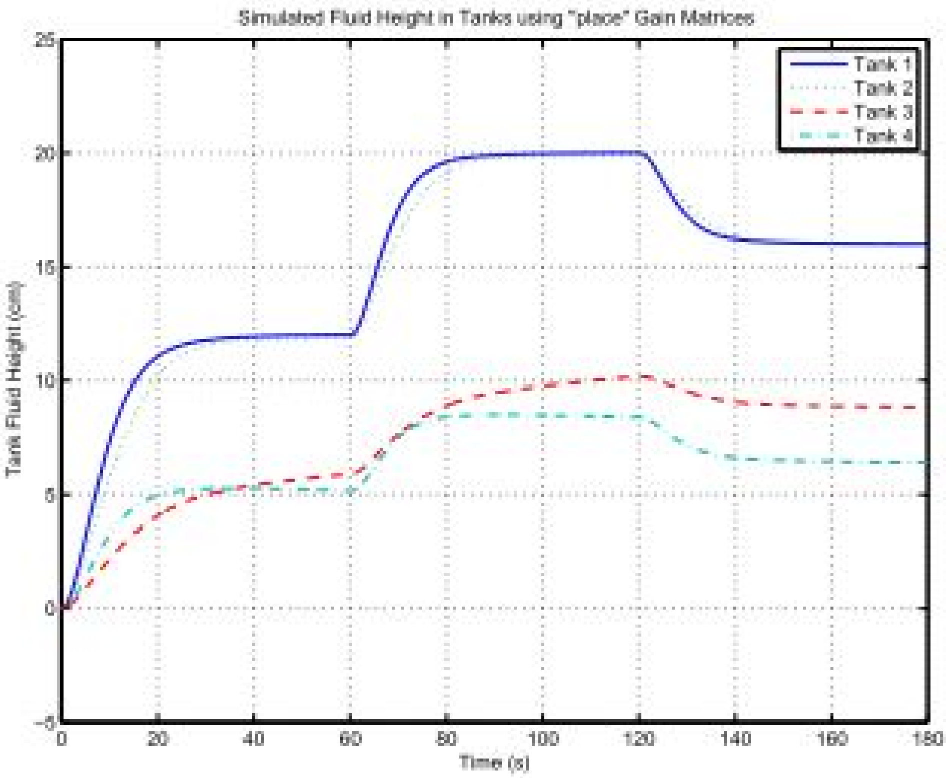


Figure 67: Simulated tank fluid height responses [5]

8.2 Experimental Results

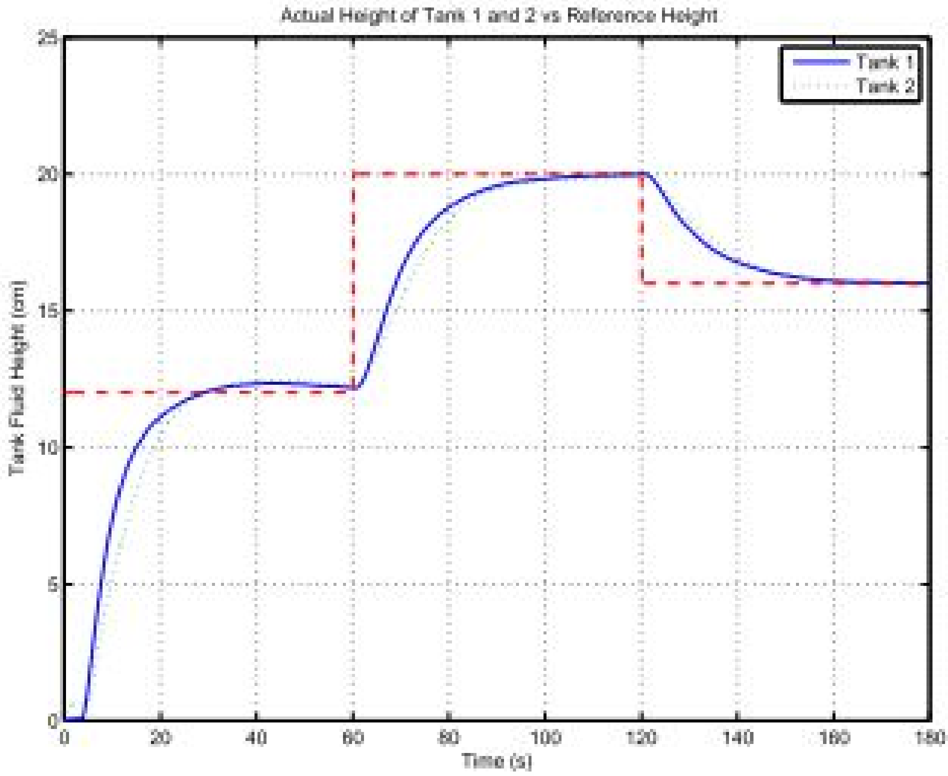


Figure 68: Actual comparison of commanded reference input to tank fluid height. [5]

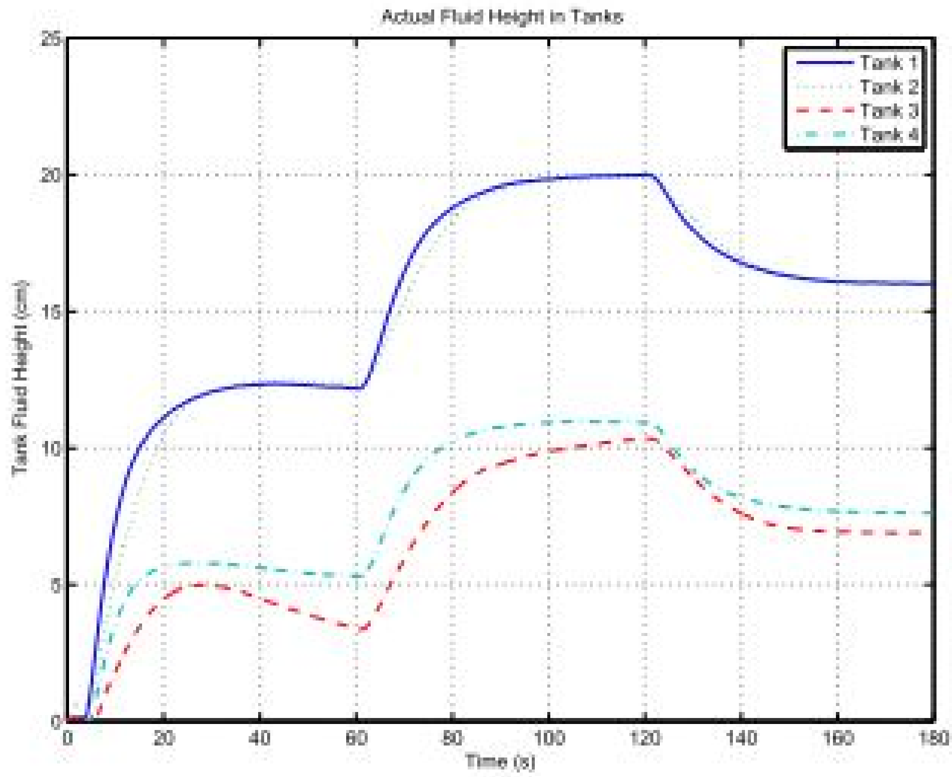


Figure 69: Actual tank fluid height responses [5]

We have compared the simulation and the experimental results above.
•From the comparison, it can be seen that both the responses are almost identical.
Hence it can be used for developing of control law for the control of bottom two tanks[5].

8.2.1 Tracking

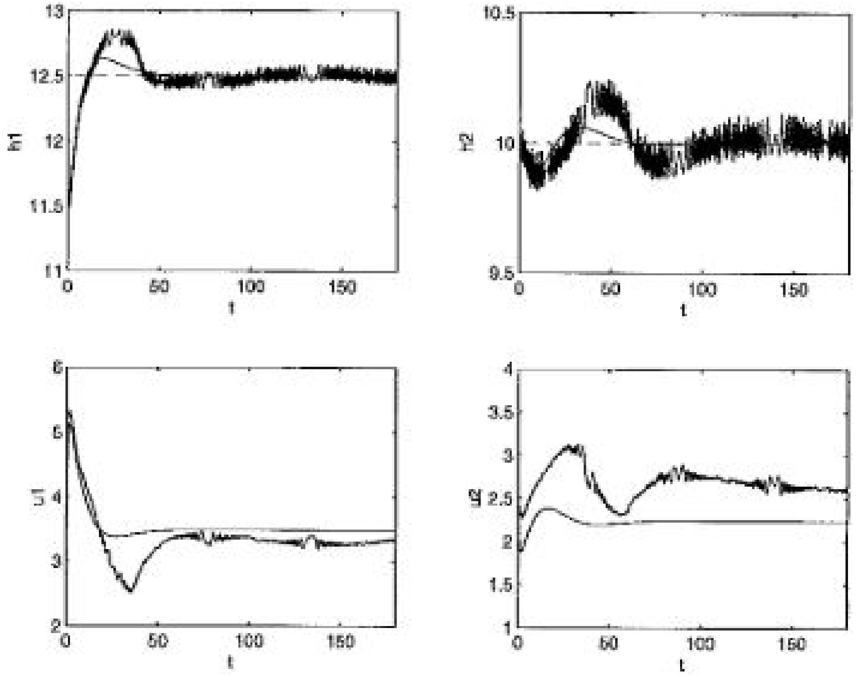


Figure 70: Minimum phase Kalman Step Response.
[7]

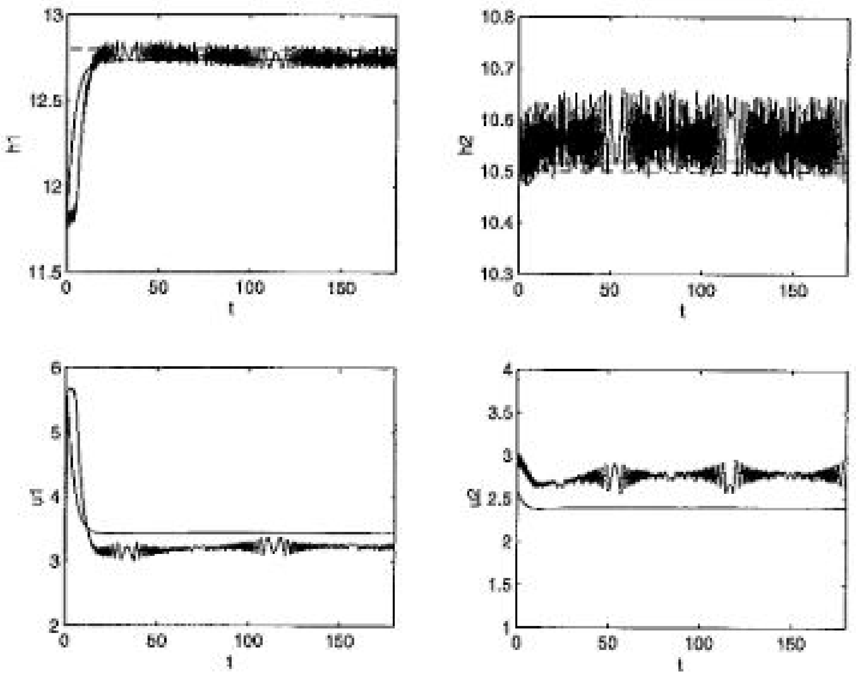


Figure 71: Minimum phase LQG Step Response
[7]

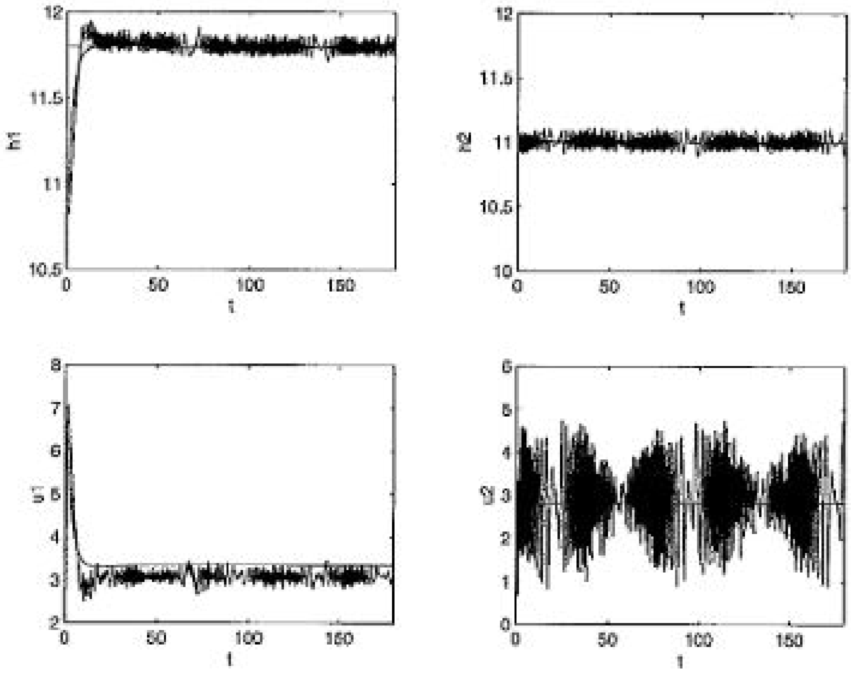


Figure 72: Minimum phase H-Infinity Step Response
[7]

8.2.2 Disturbance Rejection

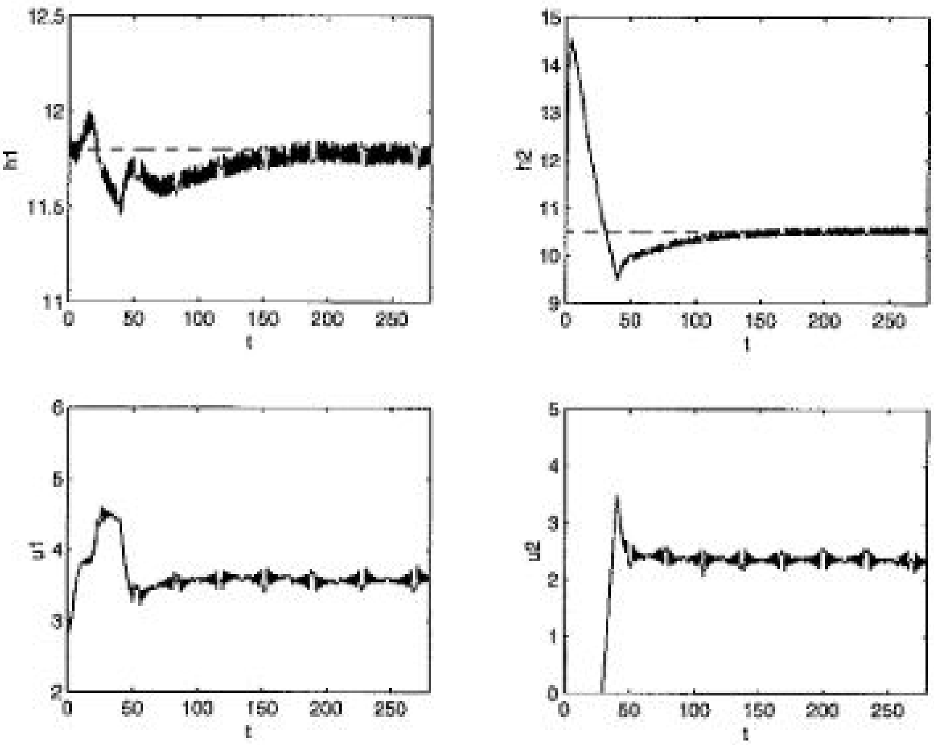


Figure 73: Minimum phase LQG Disturbance Rejection.
[7]

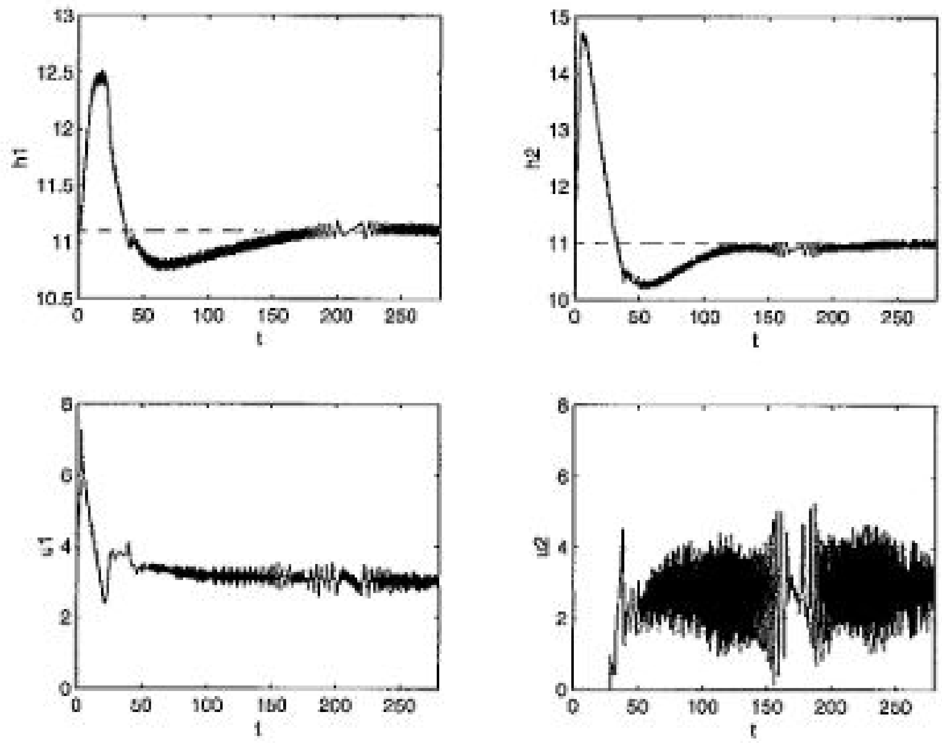


Figure 74: Minimum phase H-Infinity Disturbance Rejection
[7]

9 Summary and Conclusions

From the above analysis it is observed that the settling time for LQR controller is the least compared to all the other designs. Most of the control designs satisfied the conditions of

$$| S | \leq 2dB \ and \ \ | T | \leq 6dB.$$

•LQG does not guarantee nice properties. In addition, if RHP zeros or RHP poles are present then it may not give good results. The H-Infinity controllers have less overshoot as compared to others.

Closed Loop Controllers	**95% Settling Time (seconds)**	**Sensitivity Peak**	**Complimentary Sensitivity Peak**	**Overshoot (Percentage)**
LQR Minimum Phase	0.968, 1.17	0	2.303	20,17
LQR Non-Minimum Phase	1.29, 1.61	0	2.594	20,22
Kalman Filter Minimum Phase	-	0	3.055	-
Kalman Filter Non-Minimum Phase	-	1.906	3.702	-
LQG Minimum Phase	23.2, 39.9	22.23	21.61	31, 290
LQG Non-Minimum Phase	32.1, 123	6.008	5.158	29, 46
H-Infinity Minimum Phase	66.8, 797	1.68	0.458	6,0
H-Infinity Non-Minimum Phase	45.6,219	1.71	1.37	14,1

Table 8: Performance Parameter Analysis

• The smallest settling time of the control methods is LQR, and the highest one is H-Infinity.
•In LQG an external noise is added to the model which affects the results, but the controller attains steady state.

10 Cited References

References

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[2] Observability(wikipedia).

[3] Poles and zeros(wikibooks).

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[6] E. Govinda Kumar, B. Shiva Ram, U. B. Deepak, and G. Sabarinathan. The quadruple tank process with an interaction: A mathematical model. *International Journal of Engineering and Technology(UAE)*, 7(2):172–176, 2018.

[7] Michael Grebeck. A Comparison of Controllers for the Quadruple Tank System. Technical report, 1998.

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[9] Karl Henrik Johansson. The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on Control Systems Technology*, 8(3):456–465, 2000.

[10] Karl Henrik Johansson, Alexander Horch, Olle Wijk, and Anders Hansson. Teaching Multivariable Control Using the Quadruple-Tank Process. *Electrical Engineering*, pages 1–6, 1999.

[11] Karl Henrik Johansson and José Luís Rocha Nunes. A multivariable laboratory process with an adjustable zero. *Proceedings of the American Control Conference*, 4(3):2045–2049, 1998.

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[13] R. Vadigepalli, E. P. Gatzke, and F. J. Doyle. Robust control of a multivariable experimental four-tank system. *Industrial and Engineering Chemistry Research*, 40(8):1916–1927, 2001.

[14] Ademu Okpanachi Victor. Developing Advanced Control strategies for a 4-Tank Laboratory process(Master Thesis), 2010.

11 Relevant Matlab Code

```
%% Quadruple Tank Apparatus/ Four Tank Process for Minimum and
↪ Non-Minimum Phase
% SHUBHAM S GHARAT ASU ID 1217348912
clear all;close all;clc;
s=tf("s");



%%  Minimum Phase Charecteristics
%minimum phase Time constants
A1=28;A3=28;A2=32;A4=32;
alpha1=0.071;alpha3=0.071;alpha2=0.057;alpha4=0.057;
Kc=0.50;g=981;
T1=62;T2=90;T3=23;T4=30;
h1=12.4;h2=12.7;h3=1.8;h4=1.4;
v1=3;v2=3;k1=3.33;k2=3.35;
gamma1=0.70;gamma2=0.60;
A_min_phase=[-(1/T1) 0 A3/(1*T3) 0; 0 -(1/T2) 0 A4/(A2*T4);0 0
↪ -(1/T3) 0;0 0 0 -(1/T4)];
B_min_phase=[(gamma1*k1)/A1 0;0 (gamma2*k2)/A2;0
↪ ((1-gamma2)*k2)/A3; ((1-gamma1)*k1)/A4 0];
```

```
C_min_phase=[Kc 0 0 0;0 Kc 0 0 ];
D_min_phase=0;
c1=(T1*k1*Kc)/A1;c2=(T2*k2*Kc)/A2;
P1_min=ss(A_min_phase,B_min_phase,C_min_phase,D_min_phase);
G_s_min_phase=[ (gamma1*c1)/(1+s*T1)
 ↪  ((1-gamma2)*c1)/((1+s*T3)*(1+s*T1));
 ↪  ((1-gamma1)*c2)/((1+s*T4)*(1+s*T2)) ((gamma2*c2)/(1+s*T2))]
eigenvalues_A_min_phase= eig(A_min_phase)
pzmap(P1_min);
title('pole-zero map for minimum phase')
grid on;figure;

w=logspace(-3,3,1000);
sv=sigma(P1_min,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('SVD of the plant for Min Phase');
grid on;figure;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');



disp("Poles of Minimum Phase operating point")
p1_min=pole(G_s_min_phase(1,1))
p2_min=pole(G_s_min_phase(1,2))
p3_min=pole(G_s_min_phase(2,1))
p4_min=pole(G_s_min_phase(2,2))
tx_zeroes = tzero(P1_min)

bode((G_s_min_phase(1,1)),{0.001,10000});
title('G(s)min phase(1,1)')
ylim([-100 10]);
grid on;figure;
bode(G_s_min_phase(1,2),{0.001,10000});
title('G(s)min phase(1,2)')
ylim([-190 10]);
grid on;figure;
bode(G_s_min_phase(2,1),{0.001,10000});
title('G(s)min phase(2,1)')
ylim([-190 10]);
```

```
grid on;
figure;grid on;
bode(G_s_min_phase(2,2),{0.001,10000});
title('G(s)min phase(2,2)')
ylim([-100 10]);
grid on;

% TFM minimum phase
A1=28;A3=28;A2=32;A4=32;
alpha1=0.071;alpha3=0.071;alpha2=0.057;alpha4=0.057;
Kc=0.50;g=981;s=0;
T1=62;T2=90;T3=23;T4=30;
h1=12.4;h2=12.7;h3=1.8;h4=1.4;
v1=3;v2=3;
k1=3.33;k2=3.35;
gamma1=0.70;gamma2=0.60;

A_min_phase=[-(1/T1) 0 A3/(1*T3) 0; 0 -(1/T2) 0 A4/(A2*T4);0 0 -(1/T3) 0;0 0 0 -(1/T4)];
B_min_phase=[(gamma1*k1)/A1 0;0 (gamma2*k2)/A2;0 ((1-gamma2)*k2)/A3; ((1-gamma1)*k1)/A4 0];
C_min_phase=[Kc 0 0 0;0 Kc 0 0 ] ;
disp("D_min_phase = 0")

c1=(T1*k1*Kc)/A1;c2=(T2*k2*Kc)/A2;
disp('Dc Gain Matrix min phase')
G_s_min_phase=[ (gamma1*c1)/(1+s*T1) ((1-gamma2)*c1)/((1+s*T3)*(1+s*T1)); ((1-gamma1)*c2)/((1+s*T4)*(1+s*T2)) ((gamma2*c2)/(1+s*T2))]
[udc_min,sdc_min,vdc_min]=svd(G_s_min_phase)
Controllabiltiy_min_phase=ctrb (A_min_phase,B_min_phase);
rank_controllability_matrix_min_phase=rank(Controllabiltiy_min_phase)
Observability_min_phase=obsv(A_min_phase,C_min_phase);
rank_observability_matrix_min_phase=rank(Observability_min_phase)

[ns nc]=size(B_min_phase);
no=nc; %Number of outputs

%Augmenting plant with integrator for zero steady state error to step
%inputs in each control channel
```

```
A_Min=[A_min_phase B_min_phase
    0*ones(nc,ns) 0*ones(nc,nc)];
B_Min=[0*ones(ns,nc)
    eye(nc)];
C_Min=[C_min_phase 0*ones(nc,nc)];
D_Min=0*ones(nc,nc);
P=ss(A_Min,B_Min,C_Min,D_Min);
pp_=eig(P);
p_tz=tzero(P);
figure;
sv=sigma(P,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('SVD of the plant after Dynamic Integral Augmentation');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

%Augmented plant has slope of -20dB/dec at low frequencies
%Plant has complex poles on LHP near imag axis so Bilinear
↪   transformation
%has to be performed
p1=-0.09; p2=-1e20;
[A,B,C,D]=bilin(A_Min,B_Min,C_Min,D_Min,1,'Sft_jw',[p2 p1]);
sys=ss(A,B,C,D);
poles=eig(sys);
zeros=tzero(sys);
%Using BLT, complex LHP poles are moved to RHP
figure;
sv=sigma(sys,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('SVD of the plant after Bilinear transformation');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

%LQR Design with augmented plant
q=C'*C;
rho =1e-6;
```

```
% rho_1=1e-15;
r = rho*eye(nc);
% r_1 = rho1*eye(nc);
[G, poles, K] = lqr(A,B,q,r);
% [G_1, poles_1, K_1] = lqr(A,B,q,r_1);
Glqr1=ss(A,B,G,0);
disp('EigenValues with LQR');
lqrp1=eig(A-B*G)
disp('Transmission Zeros with LQR');
lqrtz1=tzero(Glqr1)
[ak1,bk1,ck1,dk1]=bilin(Glqr1.A,Glqr1.B,Glqr1.C,Glqr1.D,-1,'Sft_jw',[p2
 ↪  p1]);
K1=ss(ak1,bk1,ck1,dk1);%K1=L=PK
So_min=inv(eye(size(K1))+K1);
To_min=eye(size(K1))-So_min;%Te=PK*inv[I+PK]
figure;
sv=sigma(So_min,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Sensitivity (S) using LQR for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
sv=sigma(To_min,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using LQR for Minimum
 ↪  Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

%check tzero of K1
tzero(K1);
figure;
step(To_min);
title('Step response using LQR for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude');
```

```
%Kalman filter Design
pinteg=eye(nc);
mu=5;
minteg=mu*eye(nc);
[Kkf, H, sig]=kalman(sys,pinteg,minteg);
Gkf=ss(A-H*C,H,-C,0);
kfp=eig(A-H*C)
kftz=tzero(Gkf);
[akal,bkal,ckal,dkal]=bilin(Gkf.A,Gkf.B,Gkf.C,Gkf.D,-1,'Sft_jw',[p2 p1]);
K_l=ss(akal,bkal,ckal,dkal);
[Lo_l,Li_l,So_l,Si_l,To_l,Ti_l,KS_l,SP_l]=f_CLTFM(P,K_l);
figure;
sv=sigma(So_l,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Sensitivity (S) using Kalman for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
sv=sigma(To_l,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using Kalman for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

figure;
step(To_l);
title('Step response using Kalman for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude')


%LQG Design------------------------------
Am=A-B*G-H*C;
```

```
Bm=H;
Cm=G;
Dm=0;
Glqg=ss(Am,Bm,Cm,Dm);
lqgp=eig(A-B*G-H*C);
lqgtz=tzero(Glqg);
[ak,bk,ck,dk]=bilin(Glqg.A,Glqg.B,Glqg.C,Glqg.D,-1,'Sft_jw',[p2
 ↪ p1]);
K2=ss(ak,bk,ck,dk);
[Lo2,Li2,So2,Si2,To2,Ti2,KS2,SP2]=f_CLTFM(P,K2);
figure;
sv=sigma(So2,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Sensitivity (S) using LQG for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
sv=sigma(To2,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using LQG for Minimum
 ↪ Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
% figure;
% sv=sigma(KS2,w);
% tsv=20*log10(sv);
% semilogx(w,tsv);
% title('K-Sensitivity (KS) using LQG');
% grid on;
% xlabel('Frequency (rad/sec)');
% ylabel('Singular Values (dB)');
tzero(K2)
figure;
step(To2);
title('Step response using LQG for Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
```

```
ylabel('Magnitude');


%H-infinity Control design

% *******************************************************
% Form the state-space model of the scaled plant
%P=ss(ap,bp,cp,dp);
P=ss(A_min_phase,B_min_phase,C_min_phase,D_min_phase);

% *******************************************************
% Weights
% Standard first order weights are chosen
% For any non-standard/higher-order weights, form the transfer functions
% appropiately
% See AAR's book for details on higher-order weights
%  M11=0.22; w11=0.12; Eps11=0.40; M12=0.22; w12=0.12; Eps12=0.40;
M11=0.037;w11=0.00025;Eps11=0.0001; M12=0.037;w12=0.00025;Eps12=0.0001;
W1 = [tf([1/M11 w11], [1 w11*Eps11]) 0; 0 tf([1/M12 w12], [1 w12*Eps12])];
figure;
w=logspace(-3,3,1000);
sigma(inv(W1),w)
title('Inverse of Weighting matrix W1');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
% %
 M21=500; w21=100; Eps21=0.0001; M22=500; w22=100; Eps22=0.0001;
% % M21=100; w21=10000; Eps21=0.1; M22=100; w22=10000; Eps22=0.1;
W2 = [tf([1 w21/M21], [Eps21 w21]) 0; 0 tf([1 w22/M22], [Eps22 w22])] ;
figure;
w=logspace(-3,3,1000);
sigma(inv(W2),w)
title('Inverse of Weighting matrix W2');
grid on;
```

```
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
% % Mu=0.25;
% % W2=Mu;

%
M31=0.01; w31=30000; Eps31=0.00001; M32=0.01; w32=30000; Eps32=0.00001;
% M31=2; w31=3.1; Eps31=0.01; M32=2; w32=3.1; Eps32=0.01;
W3 = [tf([1 w31/M31], [Eps31 w31]) 0; 0 tf([1 w32/M32], [Eps32 w32])] ;
figure;
w=logspace(-3,3,1000);
sigma(inv(W3),w)
title('Inverse of Weighting matrix W3');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');


% **********************************************************
% Generalized plant
% Standard weight augmentation using Matlab's augw command
% For any non-standard augmentation of weights, form the Generalized plant
% manually using state-space methods
GenP=augw(P,W1,W2,W3);

% ***********************************************************
% Obtain controller using Mablab's hinfsyn command
% See help on hinfsyn for more options
K=hinfsyn(GenP);
%  ncont = 2;
%  nmeas = 2;
% [K,CL,gamma] = hinfsyn(GenP,nmeas,ncont);
% *************************************************************
% Form closed loop maps
% f_CLTFM.m is a matlab function for computing OL and CL maps in a standard
% output feedback structure. This file must be in the current Matlab folder
```

```
[Lo,Li,So,Si,To,Ti,KS,PS] = f_CLTFM(P,K);

% Can add prefilter if needed

% PLOTS
wvec=logspace(-3,3,10000);

figure;
sigma(So,wvec);
title('Sensitivity using H Infinity for Minimum Phase')
grid
xlabel('Frequency (rad/sec)')
ylabel('Singular Values (dB)')



figure;
sigma(To,wvec);
title('Complementary Sensitivity using H Infinity for Minimum
 ↪  Phase')
grid
xlabel('Frequency (rad/sec)')
ylabel('Singular Values (dB)')

figure;
sv=sigma(KS,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('K-Sensitivity (KS) using H-infinity');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

figure;
step(To);
title('Step response using H-infinity');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude');
figure;

```

```
%% Non-Minimum Phase Charecteristics
%TFM non minimum phase
A1=28;A3=28;A2=32;A4=32;
alpha1=0.071;alpha3=0.071;alpha2=0.057;alpha4=0.057;
Kc=0.50;g=981;s=0;
T1=63;T2=91;T3=39;T4=56;
h1=12.6;h2=13;h3=4.8;h4=4.9;
v1=3.15;v2=3.15;k1=3.14;k2=3.29;
gamma1=0.43;gamma2=0.34;

A_non_min_phase=[-(1/T1) 0 A3/(1*T3) 0; 0 -(1/T2) 0 A4/(A2*T4);0 0 -(1/T3) 0;0 0 0 -(1/T4)];
B_non_min_phase=[(gamma1*k1)/A1 0;0 (gamma2*k2)/A2;0 ((1-gamma2)*k2)/A3; ((1-gamma1)*k1)/A4 0];
C_non_min_phase=[Kc 0 0 0;0 Kc 0 0 ] ;
disp("D_non_min_phase = 0");
c1=(T1*k1*Kc)/A1;
c2=(T2*k2*Kc)/A2;
disp('Dc Gain Matrix non min phase');

G_s_non_min_phase=[ (gamma1*c1)/(1+s*T1) ((1-gamma2)*c1)/((1+s*T3)*(1+s*T1)); ((1-gamma1)*c2)/((1+s*T4)*(1+s*T2)) ((gamma2*c2)/(1+s*T2))]
[udc_non_min,sdc_non_min,vdc_non_min]=svd(G_s_non_min_phase)
Controllabiltiy_non_min_phase=ctrb (A_non_min_phase,B_non_min_phase);
rank_controllability_matrix_non_min_phase=rank(Controllabiltiy_non_min_pha
Observability_non_min_phase=obsv(A_non_min_phase,C_non_min_phase);
rank_observability_matrix_non_min_phase=rank(Observability_non_min_phase)

clear all
s=tf("s");
A1=28;A3=28;A2=32;A4=32;
alpha1=0.071;alpha3=0.071;
alpha2=0.057;alpha4=0.057;
Kc=0.50;g=981;

% Non-minimum phase Time constants
T1=63;T2=91;T3=39;T4=56;
h1=12.6;h2=13;h3=4.8;h4=4.9;
v1=3.15;v2=3.15;
```

```
k1=3.14;k2=3.29;
gamma1=0.43;gamma2=0.34;

A_non_min_phase=[-(1/T1) 0 A3/(1*T3) 0; 0 -(1/T2) 0 A4/(A2*T4);0 0 -(1/T3) 0;0 0 0 -(1/T4)];
B_non_min_phase=[(gamma1*k1)/A1 0;0 (gamma2*k2)/A2;0 ((1-gamma2)*k2)/A3; ((1-gamma1)*k1)/A4 0];
C_non_min_phase=[Kc 0 0 0;0 Kc 0 0 ] ;
D_non_min_phase=0;

c1=(T1*k1*Kc)/A1;c2=(T2*k2*Kc)/A2;

G_s_non_min_phase=[ (gamma1*c1)/(1+s*T1) ((1-gamma2)*c1)/((1+s*T3)*(1+s*T1)); ((1-gamma1)*c2)/((1+s*T4)*(1+s*T2)) ((gamma2*c2)/(1+s*T2))]
eigenvalues_A_non_min_phase= eig(A_non_min_phase)
P1_non_min=ss(A_non_min_phase,B_non_min_phase,C_non_min_phase,D_non_min_ph
pzmap(P1_non_min);
title('pole-zero map for non minimum phase')
grid on;figure;


w=logspace(-3,3,1000);
sv=sigma(P1_non_min,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('SVD of the plant for Non-Min Phase');
grid on;
figure;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

disp("Poles of Non Minimum Phase operating point")
p1_non_min=pole(G_s_non_min_phase(1,1))
p2_non_min=pole(G_s_non_min_phase(1,2))
p3_non_min=pole(G_s_non_min_phase(2,1))
p4_non_min=pole(G_s_non_min_phase(2,2))

bode(G_s_non_min_phase(1,1),{0.001,10000});
title('G(s) non-min phase(1,1)')
ylim([-100 10]);
```

```
grid on;
figure;

bode(G_s_non_min_phase(1,2),{0.001,10000});
title('G(s) non-min phase(1,2)')
ylim([-190 10]);
grid on;
figure;

bode(G_s_non_min_phase(2,1),{0.001,10000});
title('G(s) non-min phase(2,1)')
ylim([-190 10]);
grid on;
figure;

bode(G_s_non_min_phase(2,2),{0.001,10000});
title('G(s) non-min phase(2,2)')
ylim([-100 10]);
grid on;

[ns nc]=size(B_non_min_phase);
no=nc; %Number of outputs
%Augmenting plant with integrator for zero steady state error to step
%inputs in each control channel

A_non_Min=[A_non_min_phase B_non_min_phase
0*ones(nc,ns) 0*ones(nc,nc)];
B_non_Min=[0*ones(ns,nc)
eye(nc)];
C_non_Min=[C_non_min_phase 0*ones(nc,nc)];
D_non_Min=0*ones(nc,nc);
P_nm=ss(A_non_Min,B_non_Min,C_non_Min,D_non_Min);
pp_nm=eig(P_nm);
p_nm_tz=tzero(P_nm);
figure;
sv=sigma(P_nm,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('SVD of the plant after Dynamic Integral Augmentation for Non Minimum Phase');
```

```
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

%Augmented plant has slope of -20dB/dec at low freq.s
%Plant has complex poles on LHP near imag axis at -0.00236 +/-
 ↪ 0.0975j
%Bilinear transformation has to be done
p1=-0.009; p2=-1e20;
[A_nm,B_nm,C_nm,D_nm]=bilin(A_non_Min,B_non_Min,C_non_Min,D_non_Min,1,'Sft
 ↪ p1]);
sys=ss(A_nm,B_nm,C_nm,D_nm);
poles_nm=eig(sys);
zeros_nm=tzero(sys);
%Using BLT, complex LHP poles are moved to RHP at 0.0708 +/-
 ↪ 0.0975i, along
%with integrator states
figure;
sv=sigma(sys,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('SVD of the plant after Bilinear transformation for Non
 ↪ Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

%LQR Design-----------------------------
q=C_nm'*C_nm;
rho = 1e-6;
r = rho*eye(nc);
[G, poles_nm, K_nm] = lqr(A_nm,B_nm,q,r);
Glqr=ss(A_nm,B_nm,G,0);
lqrp=eig(A_nm-B_nm*G);
lqrtz=tzero(Glqr);
[ak,bk,ck,dk]=bilin(Glqr.A,Glqr.B,Glqr.C,Glqr.D,-1,'Sft_jw',[p2
 ↪ p1]);
K1=ss(ak,bk,ck,dk);%K1=L=PK
So_non_min=inv(eye(size(K1))+K1);
To_non_min=eye(size(K1))-So_non_min;%Te=PK*inv[I+PK]
figure;
```

```
sv=sigma(So_non_min,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Sensitivity (S) using LQR for Non Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
sv=sigma(To_non_min,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using LQR for Non Minimum
    ↪ Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
%check tzero of K1
tzero(K1)
figure;
step(To_non_min);
title('Step response using LQR without prefilter for Non Minimum
    ↪ Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude');
%Prefilter
dom_zero_real=-10.1033;
dom_zero_imag=10.4622;
dom_zero=dom_zero_real+dom_zero_imag*j;
fil_num=dom_zero*conj(dom_zero);
fil_den=(s-dom_zero)*(s-conj(dom_zero));
fil=fil_num/fil_den;
fil=fil*eye(2);
Try_f= fil * To_non_min;
figure;
sv=sigma(Try_f,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using LQR and prefilter for
    ↪ Non Minimum Phase');
grid on;
```

```
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');


%Kalman filter Design-------------------
pinteg=eye(nc);
mu=20;
minteg=mu*eye(nc);
[Kkf, H, sig]=kalman(sys,pinteg,minteg);
Gkf=ss(A_nm-H*C_nm,H,-C_nm,0);
kfp=eig(A_nm-H*C_nm);
kftz=tzero(Gkf);
[akal,bkal,ckal,dkal]=bilin(Gkf.A,Gkf.B,Gkf.C,Gkf.D,-1,'Sft_jw',[p2
 ↪ p1]);
K_l=ss(akal,bkal,ckal,dkal);
[Lo_l,Li_l,So_l,Si_l,To_l,Ti_l,KS_l,SP_l]=f_CLTFM(P_nm,K_l);
figure;
sv=sigma(So_l,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Sensitivity (S) using Kalman for Non Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
sv=sigma(To_l,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using Kalman for Non Minimum
 ↪ Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
step(To_l);
title('Step response using Kalman for Non minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude');

%LQG Design-----------------------------
```

```
Am=A_nm-B_nm*G-H*C_nm;
Bm=H;
Cm=G;
Dm=0;
Glqg=ss(Am,Bm,Cm,Dm);
lqgp=eig(A_nm-B_nm*G-H*C_nm);
lqgtz=tzero(Glqg);
[ak,bk,ck,dk]=bilin(Glqg.A,Glqg.B,Glqg.C,Glqg.D,-1,'Sft_jw',[p2 p1]);
K2=ss(ak,bk,ck,dk);
[Lo2,Li2,So2,Si2,To2,Ti2,KS2,SP2]=f_CLTFM(P_nm,K2);
figure;
w=logspace(-3,3,1000);
sv=sigma(So2,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Sensitivity (S) using LQG for Non minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
figure;
sv=sigma(To2,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('Complementary Sensitivity (T) using LQG for Non minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
tzero(K2)
figure;
step(To2);
title('Step response using LQG for Non minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude');

% H-infinity Control design

% ***********************************************************
% Form the state-space model of the scaled plant
```

```
%P=ss(ap,bp,cp,dp);
P_nm=ss(A_non_min_phase,B_non_min_phase,C_non_min_phase,D_non_min_phase);

% ********************************************************
% Weights
% Standard first order weights are chosen
% For any non-standard/higher-order weights, form the transfer functions
% appropiately
% See AAR's book for details on higher-order weights
%  M11=0.22; w11=0.12; Eps11=0.40; M12=0.22; w12=0.12; Eps12=0.40;
M11_nm=0.001;w11_nm=0.00025;Eps11_nm=0.0001; M12_nm=0.001;w12_nm=0.00025;Eps12_nm=0.0001;
W1_nm = [tf([1/M11_nm w11_nm], [1 w11_nm*Eps11_nm]) 0; 0 tf([1/M12_nm w12_nm], [1 w12_nm*Eps12_nm])];
figure;
w=logspace(-3,3,1000);
sigma(inv(W1_nm),w)
title('Inverse of Weighting matrix W1');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
% %
M21_nm=50; w21_nm=50; Eps21_nm=0.0001; M22_nm=50; w22_nm=50; Eps22_nm=0.0001;
% % M21=100; w21=10000; Eps21=0.1; M22=100; w22=10000; Eps22=0.1;
W2_nm = [tf([1 w21_nm/M21_nm], [Eps21_nm w21_nm]) 0; 0 tf([1 w22_nm/M22_nm], [Eps22_nm w22_nm])] ;
figure;
w=logspace(-3,3,1000);
sigma(inv(W2_nm),w)
title('Inverse of Weighting matrix W2');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');
% % Mu=0.25;
% % W2=Mu;

%
```

```
M31_nm=0.001; w31_nm=30000; Eps31_nm=0.00001; M32_nm=0.001; w32_nm=30000; Eps32_nm=0.00001;
% M31=2; w31=3.1; Eps31=0.01; M32=2; w32=3.1; Eps32=0.01;
W3_nm = [tf([1 w31_nm/M31_nm], [Eps31_nm w31_nm]) 0; 0 tf([1 w32_nm/M32_nm], [Eps32_nm w32_nm])] ;
figure;
w=logspace(-3,3,1000);
sigma(inv(W3_nm),w)
title('Inverse of Weighting matrix W3');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');


% ******************************************************
% Generalized plant
% Standard weight augmentation using Matlab's augw command
% For any non-standard augmentation of weights, form the Generalized plant
% manually using state-space methods
GenP=augw(P_nm,W1_nm,W2_nm,W3_nm);

% ********************************************************
% Obtain controller using Mablab's hinfsyn command
% See help on hinfsyn for more options
K_nm=hinfsyn(GenP);
%  ncont = 2;
%  nmeas = 2;
% [K,CL,gamma] = hinfsyn(GenP,nmeas,ncont);
% ********************************************************
% Form closed loop maps
% f_CLTFM.m is a matlab function for computing OL and CL maps in a standard
% output feedback structure. This file must be in the current Matlab folder
[Lo,Li,So,Si,To,Ti,KS,PS] = f_CLTFM(P_nm,K_nm);

% Can add prefilter if needed

% PLOTS
wvec=logspace(-3,3,10000);
```

```
figure;
sigma(So,wvec);
title('Sensitivity using H Infinity for Non-Minimum Phase')
grid on;
xlabel('Frequency (rad/sec)')
ylabel('Singular Values (dB)')



figure;
sigma(To,wvec);
title('Complementary Sensitivity using H Infinity for Non-Minimum Phase')
grid on;
xlabel('Frequency (rad/sec)')
ylabel('Singular Values (dB)')

figure;
sv=sigma(KS,w);
tsv=20*log10(sv);
semilogx(w,tsv);
title('K-Sensitivity (KS) using H-infinity for Non-Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Singular Values (dB)');

figure;
step(To);
title('Step response using H-infinity for Non-Minimum Phase');
grid on;
xlabel('Frequency (rad/sec)');
ylabel('Magnitude');

%INCLUDE THIS IN SAME FOLDER
function [Lo,Li,So,Si,To,Ti,KS,SP] = f_CLTFM(P,K)

% OL and CL frequency responses (ss based)
% Works for MIMO P and K
% Inputs:
```

```
% P: Plant in state space form
% K: Control in state space form
% Outputs:
% Lo, Li: Open loop tfs in ss
% So,Si,To,Ti,KS,SP: Closed loop tfs in ss

[Ap, Bp, Cp, Dp] = ssdata(P);
n_e = size(P,1);
n_u = size(P,2);
n_p = size(P,'order');
[Ak, Bk, Ck, Dk] = ssdata(K);
n_k = size(K,'order');

%% Lo = PK
A_Lo = [Ap Bp*Ck; zeros(n_k,n_p) Ak];
B_Lo = [Bp*Dk; Bk];
C_Lo = [Cp Dp*Ck];
D_Lo = Dp*Dk;
Lo = ss(A_Lo,B_Lo,C_Lo,D_Lo);

%% Li = KP
A_Li = [Ak Bk*Cp; zeros(n_p,n_k) Ap];
B_Li = [Bk*Dp; Bp];
C_Li = [Ck Dk*Cp];
D_Li = Dk*Dp;
Li = ss(A_Li,B_Li,C_Li,D_Li);

%% Mo
Mo = inv(eye(n_e)+Dp*Dk);
%% Mi
Mi = inv(eye(n_u)+Dk*Dp);

%% So = inv(I+PK)
A_So = [Ap-Bp*Dk*Mo*Cp Bp*Ck-Bp*Dk*Mo*Dp*Ck; -Bk*Mo*Cp
 ↪  Ak-Bk*Mo*Dp*Ck];
B_So = [Bp*Dk*Mo; Bk*Mo];
C_So = [-Mo*Cp -Mo*Dp*Ck];
D_So = Mo;
So = ss(A_So,B_So,C_So,D_So);

%% Si = inv(I+KP)
```

```
A_Si = [Ak-Bk*Dp*Mi*Ck Bk*Dp*Mi*Dk*Cp-Bk*Cp; Bp*Mi*Ck
  ↪  Ap-Bp*Mi*Dk*Cp];
B_Si = [-Bk*Dp*Mi; Bp*Mi];
C_Si = [Mi*Ck -Mi*Dk*Cp];
D_Si = Mi;
Si = ss(A_Si,B_Si,C_Si,D_Si);

%% To = PKinv(I+PK)
A_To = [Ap-Bp*Dk*Mo*Cp Bp*Ck-Bp*Dk*Mo*Dp*Ck; -Bk*Mo*Cp
  ↪  Ak-Bk*Mo*Dp*Ck];
B_To = [Bp*Dk*Mo; Bk*Mo];
C_To = [Mo*Cp Mo*Dp*Ck];
D_To = Mo*Dp*Dk;
To = ss(A_To,B_To,C_To,D_To);

%% Ti = inv(I+KP)KP
A_Ti = [Ak-Bk*Dp*Mi*Ck Bk*Dp*Mi*Dk*Cp-Bk*Cp; Bp*Mi*Ck
  ↪  Ap-Bp*Mi*Dk*Cp];
B_Ti = [-Bk*Dp*Mi; Bp*Mi];
C_Ti = [Mi*Ck -Mi*Dk*Cp];
D_Ti = -Dk*Dp*Mi;
Ti = ss(A_Ti,B_Ti,C_Ti,D_Ti);

%% KS
A_ks = [Ap-Bp*Dk*Mo*Cp Bp*Ck-Bp*Dk*Mo*Dp*Ck; -Bk*Mo*Cp
  ↪  Ak-Bk*Mo*Dp*Ck];
B_ks = [Bp*Dk*Mo; Bk*Mo];
C_ks = [-Dk*Mo*Cp Ck-Dk*Mo*Dp*Ck];
D_ks = Dk*Mo;
KS = ss(A_ks,B_ks,C_ks,D_ks);

%% SP
A_sp = [Ak-Bk*Dp*Mi*Ck Bk*Dp*Mi*Dk*Cp-Bk*Cp; Bp*Mi*Ck
  ↪  Ap-Bp*Mi*Dk*Cp];
B_sp = [-Bk*Dp*Mi; Bp*Mi];
C_sp = [Mo*Dp*Ck Mo*Cp];
D_sp = Mo*Dp;
SP = ss(A_sp,B_sp,C_sp,D_sp);
```