Fall 2017-18

Assignment 1

EECS 3401

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Q4. This non-programming question concerns the following situation:

Joe, Sally, Bill, and Ellen are the only members of the Elm Street Bridge Club. Joe is married to Sally. Bill is Ellen's brother. The spouse of every married person in the club is also in the club.

From these facts, most people would be able to determine that Ellen is not married.

   a) **Write some sentences in first-order logic that represent the facts given above. Also provide a glossary where you indicate the intended meaning of your predicate, function, and constant symbols in English.**

Ans. Let *KB* denote the set of facts in our knowledge base. Here are the following facts in our *KB*:

-   Joe, Sally, Bill, and Ellen are the only members of the Elm Street Bridge Club:

$$member(Joe).$$

$$member(Sally).$$

$$member(Bill).$$

$$member(Elen).$$

-   Joe is married to Sally:

$$married(Joe, Sally).$$

-   Bill is Ellen's brother:

$$brother(Bill, Elen).$$

-   The spouse of every married person in the club is also in the club:

$$\forall X, Y \cdot member(X) \wedge married(X, Y) \rightarrow member(Y).$$

Glossary:

-   $member(X)$: is true if X is a member of the Elm Street Bridge Club
-   $married(X, Y)$: is true if X is married to Y
-   $brother(X, Y)$: is true if X is the brother of Y

b) Show that the given facts do not entail that Ellen is not married, i.e., give an interpretation that satisfies the facts but falsifies the conclusion.

Ans. From the definition of entailment, we know, $KB \not\models f \iff KB \cup \neg\{f\}$ is satisfiable.

We know a formula, f, is satisfiable if there is an interpretation (model), I, that makes f true.

Let f denote the predicate that Ellen is not married to someone, i.e $\neg \, married(X, Ellen)$.

We need to show that $KB \not\models f$, or more formerly: $KB \cup \neg\,(\neg\,married(X, Ellen))$ is satisfiable.

Assume $KB \not\models f$. Then $KB = KB \cup \neg\,(\neg\,married(X, Ellen))$. The new set of formulas in our KB now includes the fact $\neg\,(\neg\,married(X, Ellen)) = married(X, Ellen)$.

Let I represent the following interpretation:
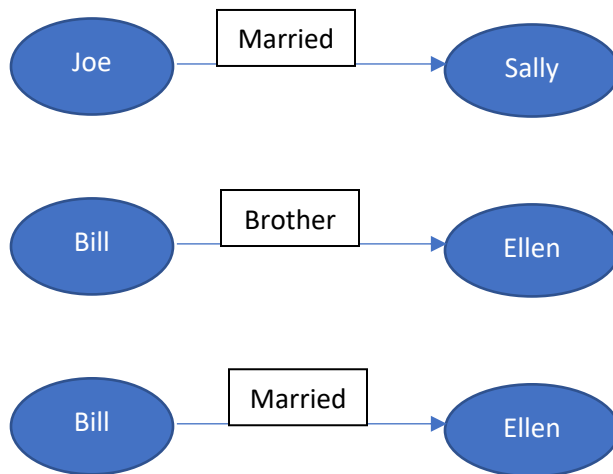
Trivially, the above interpretation, I, satisfies the knowledge base, KB, because for every formula $f \in KB$, f is true under I.

In other words, since in I there was an added scenario which states that Bill is married to Ellen. It thus satisfies the new added fact in KB; someone is married to Ellen i.e $married(X, Ellen)$.

$\therefore$ Q.E.D. Since $KB \cup \neg\,(\neg\,married(X, Ellen))$ is satisfiable we can conclude that the given facts in our KB <u>does not entail</u> that Ellen is not married, $KB \not\models \neg\,married(X, Ellen)$.

c) **Write some sentences in first-order logic that represent the additional knowledge that most people would have and prove that the augmented set of sentences now entails that Ellen is not married. Your proof should use the definition of entailment and refer to interpretations; do not use resolution.**

Ans. We would need to add the following knowledge in our *KB* to make the intuition asserted in part b) more explicit:

$$\neg \, brother(Joe, Sally).$$

$$\neg \, married(Bill, Ellen).$$

$$\forall X, Y \cdot married(X, Y) \rightarrow \neg \, brother(X, Y).$$

From the definition of entailment, we know, $KB \models f \Leftrightarrow KB \cup \neg\{f\}$ is unsatisfiable.

We know a formula, f, is unsatisfiable if there is an interpretation (model), I, such that f contradicts under I.

Let f denote the predicate that Ellen is not married to someone, i.e $\neg \, married(X, Ellen)$.

We need to show that $KB \models f$, or more formerly: $KB \cup \neg (\neg \, married(X, Ellen))$ is unsatisfiable.

Let *I* be the following representation:
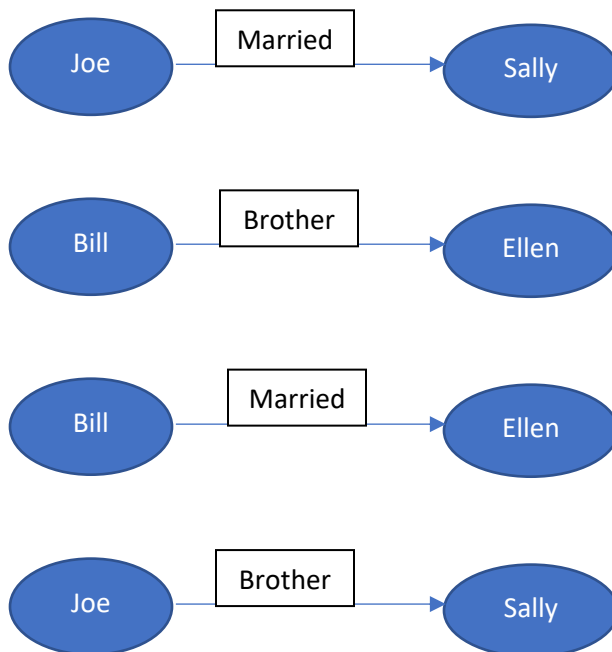
The interpretation, *I*, will not satisfy *KB* because there exists some formula $f \in KB$, such that f contradicts under *I*. For example, $\neg\, married(Bill, Ellen)$ will contradict with $married(X, Ellen)$.

Moreover, now the knowledge base knows that if X and Y are married then they cannot be siblings and conversely, if X and Y are siblings then they cannot be married.

∴ Q.E.D. With the new set of facts in our *KB*, now it <u>entails</u> that Ellen is not married to someone.