Speed and Accuracy in Reporting: Model and Results

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1 Model

Players and state I consider model with a single sender and single receiver. There is a binary, persistent state $\theta \in \{0, 1\}$ which is initially unknown to both sender and receiver. Both players are endowed with a symmetric prior on the state: $Pr(\theta = 1) = \frac{1}{2}$.

Sender types The sender may be one of two types, good or bad. Let her type be denoted by $i \in \{G, B\}$. The sender's type is independent of the state. While the sender knows her type, the receiver does not. The receiver is endowed with a prior on the sender's type: $R_0 \equiv Pr(i = G) \in (0, 1)$.

Sender learning The sender has access to an informative signal about the state, and her type indexes her ability. I assume a two-sided Poisson learning process: in each period, $t \in \{1, 2, ..., T\}$, θ is privately revealed to the sender with probability λ_i. More formally, in each period, the sender observes a signal s_i^t, where

$$s_i^t = \begin{cases} \theta & \text{with probability } \lambda_i \\ \emptyset & \text{with probability } 1 - \lambda_i \end{cases}$$

Note that $s_i^t = \emptyset$ indicates that the state was not revealed to the sender in t. We assume that signal is strictly informative, and that good sender is of higher ability than the bad sender:

$$\lambda_G > \lambda_B \geq 0$$

Sender reporting At any point during the game, the sender can choose to send a message $m \in \{0, 1\}$, which will end the game. We will later show that in equilibrium, this message can be interpreted as the sender's *conjecture* about the state. Note further that the sender is not obligated to send a message, i.e., she can stay silent at all t. Therefore T should not be interpreted as a deadline, but rather the final opportunity for the sender to communicate with the receiver. Let τ denote the time at which a sender sends her message. For notational convenience, we let $\tau = T + 1$ denote the lack of a report by the sender, in which case we let $m = \emptyset$.

Receiver signal In period $T + 1$ (i.e., after observing the sender's report), the receiver observes a signal $s \in \{0, 1\}$ about θ. We assume that this signal is symmetric and strictly (but not fully) informative. Formally: $Pr(s = \theta) = \pi \in (\frac{1}{2}, 1)$.

Payoffs and reputation The sender's payoff is equal to her reputation, i.e., the receiver's belief that the sender is of type G, at the end of period $T+1$. Let us denote this belief using **reputation function** R, which denotes the sender's reputation for every realization of report time τ, message m, and private signal s:

$$R : \{1, 2, ..., T\} \times \{0, 1, \emptyset\} \times \{0, 1\} \to [0, 1]$$

2 Defining equilibrium

A **Markov strategy** M_i specifies reporting behavior for a sender of type i at every time and belief:

$$M_i : \{1, 2, ..., T\} \times [0, 1] \to \Delta\{0, 1, \emptyset\}$$

I.e., M_i maps each time period and sender $p \equiv Pr(\theta = 1)$ to a distribution over the possible reports 0 and 1, as well as the option to abstain from making a report, denoted by $\emptyset$. While given the prior and learning process of the sender, only beliefs are ever reached, namely 0, 1, and $\frac{1}{2}$, for convenience, we assume the strategy is defined over all beliefs.

Because the above game is one of cheap talk, we seek to refine the set of equilibria in a sensible way. I do this by assuming that the high-type of sender is a behavioral type. In particular, I assume that the high-type sender reports *truthfully*, i.e., she reports the state if and only if she has learned it, otherwise, she remains silent. I.e., I assume that for all t:

$$M_G(t, p) = \begin{cases} 0 & \text{if } p = 0 \\ \emptyset & \text{if } p = \frac{1}{2} \\ 1 & \text{if } p = 1 \end{cases}$$

Definition 1 (Equilibrium)**.** *An equilibrium is a pair* (R, M_B) *such that*

1. $M_B(t, p)$ *maximizes* $E_s[R(t, m, s)]$ *at all* t *and* $p \in [0, 1]$*.*
2. $R(\tau, m, s)$ *is computed using Bayes Rule, given* (M_G, M_B)

3 Results: Static model

I begin by presenting the results in the static version of the model. Before doing so, I introduce notation and observations relevant to this analysis.

3.1 Notation and observations

- Fixing an R function, we let $V(m, p)$ denote the sender's value from sending message m under belief p. Formally:
$$V(m, p) \equiv \tilde{p}R(m, 1) + (1 - \tilde{p})R(m, 0)$$
where $\tilde{p} = p\pi + (1 - p)(1 - p)$.
- We further let $V(p)$ denote the sender's value under belief p, i.e.,
$$V(p) \equiv \max_m V(m, p)$$

- In the analysis that follows, we work extensively with R. In equilibrium, $R(m, s)$ is computed using Bayes Rule from m_B. Bayes Rule applies in all instances because given the selection criterion and the fact that the receiver's private signal is not fully accurate. Formally Bayes Rule yields:

$$R(m, s) = \frac{1}{1 + L(m, s)(\frac{1-R_0}{R_0})}$$

where $L(m, s)$ is the likelihood ratio of (m, s) realizing across $i = \{B, G\}$:

$$L(m, s) \equiv \frac{Pr(m, s|i = B)}{Pr(m, s|i = G)}$$

where

$$Pr(m, s|i) = \frac{1}{2}Pr(s|\theta = 1)Pr(m|\theta = 1, i) + \frac{1}{2}Pr(s|\theta = 0)Pr(m|\theta = 0, i)$$

3.2 Behavior under arrival

We begin by characterizing the bad type's belief when she is informed about the state. We establish the result that in any equilibrium, the sender must truthfully report the state if she has observed it (Proposition 1). To this end, we establish two lemmas.

Lemma 1 (Truthful reporting upon arrival)**.** *In any equilibrium,*

1. $Pr(m = 1|\theta = 0, B) > 0$
2. $Pr(m = 0|\theta = 1, B) > 0$

Proof. We will now prove 1. 2 will follow symmetrically.

Suppose by contradiction that $Pr(m = 1|\theta = 0, B) = 0$. This then implies that

$$L(1, 1) = L(1, 0) = \frac{\lambda_B m(1, 1)}{\lambda_G} < 1$$

Thus, $R(1, 0) = R(1, 1) > R_0$. It follows that for all p:

$$V(p) \geq V(1, p) > R_0$$

Which implies that the bad sender's reputation will always improve following the first period. This is a violation of Bayes Rule. □

Lemma 2. *Monotonic values In any equilibrium,*

1. *V(1, p) is strictly increasing in p.*
2. *V(0, p) is strictly decreasing in p.*

Proof. We now prove 1. 2 will follow symmetrically. Rearranging the earlier formula for $V(1, p)$, we can write

$$V(1, p) = p(2\pi - 1)[R(1, 1) - R(1, 0)] + [R(1, 1)(1 - \pi) + R(1, 0)\pi]$$

Thus, to show that $V(1,p)$ is strictly increasing in p it suffices to show that $R(1,1) > R(1,0)$. Furthermore, to show $R(1,1) > R(1,0)$ it suffices to show that $L(1,1) < L(1,0)$. Our earlier expression for L yields

$$L(1,1) = \frac{1}{\lambda_G}[\frac{1-\pi}{\pi}Pr(m=1|\theta=0,B) + Pr(m=1|\theta=1,B)]$$

$$L(1,0) = \frac{1}{\lambda_G}[\frac{pi}{1-\pi}Pr(m=1|\theta=0,B) + Pr(m=1|\theta=1,B)]$$

Since by claim 1, $Pr(m = 1|\theta = 0, B) > 0$ and since by assumption $\pi > \frac{1}{2}$, it follows that $L(1,1) < L(1,0)$. ☐

Proposition 1 (Truthful report of arrivals)**.** *In any equilibrium,*

$$m_B(1,1) = m_B(0,0) = 1$$

Proof. I will now show that in any equilibrium, $m_B(1,1) = 1$. That $m_B(0,0) = 1$ follows symmetrically.

First, I show that $m_B(0,1) = 0$. Suppose by contradiction that $m_B(0,1) > 0$. Then, $m = 0$ must be optimal under the belief $p = 1$. Thus, $V(0,1) \geq V(1,1)$. It follows from claim 2 that

$$V(0,p) > V(1,p) \text{ for all } p < 1.$$

Thus, $m_B(1, \frac{1}{2}) = m_B(1,0) = 0$. It follows that

$$Pr(m=1|\theta=0,B) = \lambda_B m_B(1,0) + (1-\lambda_B)m_B(1,\frac{1}{2}) = 0.$$

This is a contradiction of lemma 1.

Next, I will show that $m_B(\emptyset,1) = 0$. Suppose by contradiction that $m_B(\emptyset,1) > 0$. This implies that $\emptyset$ is optimal under $p = 1$. Thus,

$$V(\emptyset,1) \geq V(\emptyset,0). \tag{1}$$

Next, by lemma 1, we know

$$Pr(m=1|\theta=0,B) = \lambda_B m_B(1,0) + (1-\lambda_B)m_B(1,\frac{1}{2}) > 0$$

This implies that if $m_B(1,0) = 0$, then $m_B(1, \frac{1}{2}) > 0$. This in turn implies

$$V(\emptyset,\frac{1}{2}) \leq V(1,\frac{1}{2}). \tag{2}$$

Thus, we have that $V(\emptyset,1) > V(\emptyset,\frac{1}{2})$, which follows from the following chain of inequalities:

$$V(\emptyset,1) \geq V(1,1) > V(1,\frac{1}{2}) \geq V(\emptyset,\frac{1}{2})$$

Where the first inequality follows from (1), the second from lemma 2, and the thrid from (2). By the definition of V, this implies that $R(\emptyset,1) > R(\emptyset,0)$.

Meanwhile, since $m_B(\emptyset, 0) > 0$, it follows that $V(\emptyset, \frac{1}{2}) \leq V(0, \frac{1}{2})$, which because $V(\emptyset, \cdot)$ is strictly increasing in p and $V(0, \cdot)$ is strictly decreasing in p implies $V(\emptyset, 0) < V(0, 0)$. Thus, $m_B(\emptyset, 0) = 0$. This implies that

$$Pr(\emptyset|\theta = 0, B) = (1 - \lambda_B)m_B(\emptyset, \frac{1}{2}) < \lambda_B m_B(\emptyset, 1) + (1 - \lambda_B)m_B(\emptyset, \frac{1}{2}) = Pr(\emptyset|\theta = 1, B)$$

By definition, this implies

$$L(\emptyset, 0) = \frac{(1-\pi)Pr(\emptyset|\theta = 1, B) + \pi Pr(\emptyset|\theta = 0, B)}{(1 - \lambda_G)} < \frac{\pi Pr(\emptyset|\theta = 1, B) + (1-\pi)Pr(\emptyset|\theta = 0, B)}{(1 - \lambda_G)} = L(\emptyset, 1).$$

this in turn implies $R(\emptyset, 0) > R(\emptyset, 1)$, which is a contradiction of the above. □

3.3 Behavior under non-arrival

Here, we analyze the sender's behavior under non-arrival. We first establish that the sender *mixes* when she is uninformed. Specifically, she must send all three messages (0, 1, and $\emptyset$) with positive probability when she is. We then establish that the sender sends 0 and 1 with equal probability.

Proposition 2 (Mixing under non-arrival)**.** *In any equilibrium, the sender sends messages* 0*,* 1*, and* $\emptyset$ *with positive probability under non-arrival.*

Proof. We begin with showing that $\emptyset$ must be sent with positive probability under non-arrival. Suppose by contradiction that the sender sends message $\emptyset$ with zero probability under non-arrival. Since $\emptyset$ is also sent with zero probability under arrival, it follows that the bad type sends $\emptyset$ with zero probability. Because the good type sends $\emptyset$ with strictly positive probability $R(\emptyset, p) = 1$ for all p, while $R(\emptyset, p) < 1$. Thus $\emptyset$ serves as a profitable deviation for the sender.

Next, I show that 1 must be sent with positive probability under non-arrival. Suppose not, by contradiction. By Proposition 1, this implies 1 is only sent under $\theta = 1$. This is a contradiction of Lemma 1. That 0 is sent with positive probability follows analogously. □

Proposition 3 (Equal "faking" of 0 and 1)**.** *In any equilibrium,*

$$m_B(1, \frac{1}{2}) = m_B(0, \frac{1}{2})$$

Proof. Suppose by contradiction that $m_B(1, \frac{1}{2}) \neq m_B(0, \frac{1}{2})$. Without loss of generality, we assume $m_B(1, \frac{1}{2}) > m_B(0, \frac{1}{2})$.

By definition

$$\begin{aligned} L(0, 0) &= \frac{(1-\pi)(1-\lambda)m_B(0, \frac{1}{2}) + \pi(\lambda_B + (1-\lambda_B)m_B(0, \frac{1}{2})}{\pi\lambda_G} \\ &< \frac{(1-\pi)(1-\lambda)m_B(1, \frac{1}{2}) + \pi(\lambda_B + (1-\lambda_B)m_B(1, \frac{1}{2})}{\pi\lambda_G} = L(1, 1) \end{aligned} \tag{3}$$

Furthermore,

$$\begin{aligned} L(0, 1) &= \frac{\pi(1-\lambda_B)m_B(0, \frac{1}{2}) + (1-\pi)(\lambda_B + (1-\lambda_B)m_B(0, \frac{1}{2}))}{(1-\pi)\lambda_G} \\ &< \frac{\pi(1-\lambda_B)m_B(1, \frac{1}{2}) + (1-\pi)(\lambda_B + (1-\lambda_B)m_B(1, \frac{1}{2}))}{(1-\pi)\lambda_G} = L(1, 0) \end{aligned} \tag{4}$$

It follows from these two inequalities, that $R(0,0) > R(1,1)$ and $R(0,1) < R(1,0)$. Thus,

$$V(0, \frac{1}{2} = \frac{1}{2}R(0,0) + \frac{1}{2}R(0,1) > \frac{1}{2}R(1,1) + \frac{1}{2}R(1,0) = V(1, \frac{1}{2})$$

Meanwhile, since $m_B(1, \frac{1}{2}) > 0$, $V(1, \frac{1}{2}) \geq V(0, \frac{1}{2})$. This is a contradiction. □

3.4 Equilibrium characterization

Below, we show that there exists a unique equilibrium. We begin by establishing a class of strategies.

- For $b \in [0,1]$, let m_B^b denote the strategy such that

$$m_B(0,0) = m_B(1,1) = 1 \text{ and } m_B(0, \frac{1}{2}) = m_B(1, \frac{1}{2}) = b/2.$$

- Furthermore, let R^b, V^b, L^b denote the reputation, value and likelihood functions, respectively, consistent with m_B^b.

Note that the above Propositions imply that any equilibrium must belong to this class. Below we establish that there exists a unique such strategy in this class that consistutes an equilibrium.

Theorem 1. *There exists a unique b^* $(m_B^{b^*}, R^{b^*})$ is the unique equilibrium.*

Proof. Note that Propositions 1 and 3 imply that any equilibrium must belong to the above class. For any $b \in [0,1]$, define

$$X(b) \equiv V^b(1, \frac{1}{2}) - V^b(\emptyset, \frac{1}{2}).$$

By Proposition 2, if m_B^b constitutes an equilibrium, $X(b) = 0$. We will now show that there exists a unique $b^* \in (0,1)$ such that $X(b^*) = 0$. To this end, we make two observations about $X(b)$:

(1) $X(b)$ is continuous and strictly decreasing in b. It suffices to show that $V^b(1, \frac{1}{2})$ is continuous and strictly decreasing in b and $V^b(\emptyset, \frac{1}{2})$ is continuous and strictly increasing in b. To show that $V^b(1, \frac{1}{2})$ is continuous and strictly decreasing in b, first note that for all $s \in \{0,1\}$,

$$L^b(1,s) = \frac{(1-\lambda_B)b/2 + Pr(s|\theta=1)\lambda_B}{Pr(s|\theta=1)\lambda_G}$$

which is continuous and strictly increasing in b. This implies that $R^b(1,s)$ is continuous and strictly decreasing in b for $s \in \{0,1\}$, and thus that $V^b(1, \frac{1}{2})$ is continuous and strictly decreasing in b.

To show that $V^b(\emptyset, \frac{1}{2})$ is continuous and strictly increasing in b, first note that

$$L^b(\emptyset, s) = \frac{(1-\lambda_B)(1-b)}{(1-\lambda_G)} \text{ for } s \in \{0,1\}$$

which is continuous and strictly decreasing in b. This in turn implies that $R^b(\emptyset, s)$, and consequently $V^b(\emptyset, \frac{1}{2})$ is continuous and strictly increasing in b for $s \in \{0,1\}$.

(2) $X(0) > 0$ **and** $X(1) < 0$ To show $X(0) > 0$, note that by the above formulai, for $s \in \{0,1\}$,

$$L^0(1,s) = \frac{\lambda_B}{\lambda_G} < \frac{1-\lambda_B}{1-\lambda_G} = L^0(\emptyset, s)$$

Thus, $R^0(1,s) > R^0(\emptyset, s)$ for $s \in \{0,1\}$. Therefore, $X(0) > 0$. To show $X(1) < 0$, note that for $s \in \{0,1\}$, $L^1(1,s) > 0 = L^1(\emptyset, s)$. Thus, $R^1(1,s) < R^1(\emptyset, s)$ for all $s \in \{0,1\}$. Thuse, $X(1) < 0$.

Combining the above two observations, it follows that there exists a unique $b^* \in (0,1)$ such that $X(b^*) = 0$. Thus we have shown that the only candidate equilibrium is $(m_B^{b^*}, R^{b^*})$. It remains to confirm that this is indeed an equilibrium, i.e., that the sender cannot profitably deviate at any possible belief.

Let us begin with the belief $p = \frac{1}{2}$. First, note that under $m_B^{b^*}$, $V^{b^*}(1, \frac{1}{2}) = V^{b^*}(0, \frac{1}{2})$. Thus, because $X(b^*) = 0$

$$V^{b^*}(1/2) = V^{b^*}(m, 1/2) \text{for all } m \tag{5}$$

Next, we will show there does not exist a profitable deviation when $p = 1$. That there does not exist a profitable deviation when $p = 0$ follows symmetrically. To show this, first note by definition of L:

- $L^{b^*}(\emptyset, 0) = L^{b^*}(\emptyset, 1)$
- $L^{b^*}(1, 0) > L^{b^*}(1, 1)$
- $L^{b^*}(0, 1) > L^{b^*}(0, 0)$

It follows from these three inequalities that

- $V^{b^*}(\emptyset, p)$ is constant in p
- $V^{b^*}(1, p)$ is strictly increasing in p
- $V^{b^*}(0, p)$ is strictly decreasing in p

Thus, it follows from (5) that

$$V^{b^*}(1,1) > V^{b^*}(\emptyset, 1) > V^{b^*}(0,1)$$

Thus, $m = 1$ is the unique best response at $p = 1$, and there is no profitable deviation. □

3.5 Comparative statics

Here, we present some comparative statics on the the sender's reporting behavior:

Proposition 4 (Comparative statics)**.** *Holding all other parameters fixed, b^* is*

- *strictly decreasing in π.*
- *strictly increasing in λ_G.*
- *strictly decreasing in λ_B.*
- *strictly decreasing in R_0.*

4 Results: Dynamic model

4.1 Notation and observations

- Let $V(t, m, p)$ denote the sender's value from sending message $m \in \{0, 1\}$ at time t, under belief p. Formally:
$$V(t, m, p) = \tilde{p}R(t, m, 1) + (1 - \tilde{p})R(t, m, 0)$$
where $\tilde{p} = p\pi + (1 - p)(1 - \pi)$.

- Let $V(t, \emptyset, p)$ denote the sender's *continuation value* at t.

- Finally, let $V(t, p)$ denote the sender's value at (t, p):
$$V(t, p) = \max_{m \in \{0,1,\emptyset\}} V(t, m, p)$$

- In equilibrium, $R(t, m, s)$ is computed using Bayes Rule as follows:
$$R(t, m, s) = \frac{1}{1 + L(t, m, s)(\frac{1-R_{t-1}}{R_{t-1}})}$$
where
$$L(t, m, s) = \frac{Pr(t, m, s|B, E_t)}{Pr(t, m, s|G, E_t)}$$
where E_t is the event that no reports are made before t.

- Finally Let R_t denote the sender's reputation conditional on not having reported at or before t, i.e., $R_t = P(G|E_{t+1})$

4.2 Behavior under arrival

Proposition 5 (Arrivals are reported truthfully)**.** *In any equilibrium,* $m_B(t, 1, 1) = m_B(t, 0, 0) = 1$ *for all* t.

4.3 Behavior under non-arrival

Proposition 6 (Indifference under non-arrival)**.** *In any equilibrium,*

$$V(t, \frac{1}{2}) = V(t, 1, \frac{1}{2}) = V(t, 0, \frac{1}{2}) = V(t, \emptyset, \frac{1}{2})$$

Proof. First we show that $V(t, \frac{1}{2}) = V(t, \emptyset, \frac{1}{2})$. To show this, suppose by contradiction that

$$V(t, \frac{1}{2}) > V(t, \emptyset, \frac{1}{2})$$

It folows that the bad sender never abstains at time t. Since the good sender does so with strictly positive probability, the reputation that will be assigned to a sender who abstains is 1. Thus,

$$V(t, \emptyset, \frac{1}{2})$$

Next, we show that $V(t, \frac{1}{2}) = V(t, 1, \frac{1}{2})$. Suppose not, by contradiction. We know from the previous proposition that the sender reports 1 with strictly positive probability under belief $\frac{1}{2}$. Thus, the statement holds.

That $V(t, \frac{1}{2}) = V(t, 0, \frac{1}{2})$ follows analogously. □

Proposition 7 (Equal faking under non-arrival)**.** *At all t, $b_t^0 = b_t^1$.*

Proof. Suppose not, by contradiction. Let t denote the first period such that $b_t^0 \neq b_t^1$. Assume without loss of generality that $b_t^0 > b_t^1$. Note that

$$R(t, 1, 1) = \frac{1}{1 + (\frac{1-R_{t-1}}{R_{t-1}}(\frac{\lambda\pi + \frac{1}{2}b_t^1(1-\lambda_B)}{\lambda_G\pi}))}$$

$$R(t, 0, 0) = \frac{1}{1 + (\frac{1-R_{t-1}}{R_{t-1}}(\frac{\lambda\pi + \frac{1}{2}b_t^0(1-\lambda_B)}{\lambda_G\pi}))}$$

Examining the equalities, we have $R(t, 1, 1) > R(t, 0, 0)$. We can analogously show that $R(t, 1, 0) > R(t, 0, 1)$.

But since

$$V(t, 0, \frac{1}{2}) = \frac{1}{2}R(t, 0, 0) + \frac{1}{2}R(t, 0, 1)$$
$$V(t, 1, \frac{1}{2}) = \frac{1}{2}R(t, 1, 1) + \frac{1}{2}R(t, 1, 0)$$

It follows from these two inequalities that $V(t, 0, \frac{1}{2}) < V(t, 1, \frac{1}{2})$, thus contradicting the previous proposition. □

4.4 Equilibrium characterization

Proposition 8 (Equilibrium existence)**.** *There exists an equilibrium. Furthermore, under any equilibrium, for all t:*

- $m_B(t, 1, 1) = m_B(t, 0, 0) = 1$
- $m_B(t, \emptyset, 1) = m_B(t, \emptyset, 0) \equiv b_t \in (0, 1)$

4.5 Dynamics of reputation

Proposition 9 (R_t is declining)**.** *In any equilibrium, for all $t \in \{1, ..., T\}$, $R_{t-1} > R_t$.*

Proof. First, note that proving this claim is equivalent to show that for all t,

$$b_t > \bar{b} \equiv \frac{\lambda_G - \lambda_B}{1 - \lambda_B}$$

Now, we will prove this claim by backwards induction:

Base case First we show that $b_T < \bar{b}$. Suppose by contradiction that $b_T \geq \bar{b}$. It follows that $V(T, \frac{1}{2}, \emptyset) = R_T > R_{T-1}$.

Meanwhile, $R(T) < R_{T-1}$, but

$$R(T) = qR(T,1,1) + (1-q)R(T,1,0)$$

where $q > 1/2$ and $R(T,1,1) > R(T,1,0)$. It follows that

$$V(T, \frac{1}{2}, 1) = \frac{1}{2}R(T,1,1) + \frac{1}{2}R(T,1,0) < R(T)$$

Thus, $V(T, \frac{1}{2}, \emptyset) > V(T, \frac{1}{2}, 1)$. And thus, the uninformed sender can profitably deviate by never reporting 1.

Inductive step Suppose that $b_{t+1} < \bar{b}$. We wish to show that $b_t < \bar{b}$. Suppose by contradiction that $b_t \geq \bar{b}$. Note that

$$V(t, \frac{1}{2}, \emptyset) = qR(t+1,1,1) + (1-q)R(t+1,1,0)$$

where $q = \pi\lambda_B + (1-\lambda_B)\frac{1}{2} > 0$. Meanwhile

$$V(t, \frac{1}{2}, 1) = \frac{1}{2}R(t(,1,1) + \frac{1}{2}R(t,1,0)$$

Next, I claim that

$$R(t,1,1) < R(t+1,1,1). \tag{6}$$

First note that

$$R(t,1,s) = \frac{1}{1 + (\frac{1-R_{t-1}}{R_{t-1}})(\frac{\lambda_B\pi + (1-\lambda_B)b_t(1/2)}{\lambda_G\pi})}$$

$$R(t,1,s) = \frac{1}{1 + (\frac{1-R_t}{R_t})(\frac{\lambda_B\pi + (1-\lambda_B)b_{t+t}(1/2)}{\lambda_G\pi})}$$

Since, $b_t \geq \bar{b} > b_{t+1}$ and $R_{t-1} \leq R_t$, (6) follows immediately. Furthermore, we can analogously show that

$$R(t,1,0) < R(t+1,1,0) \tag{7}$$

Inequalities (5) and (6), combined with the fact that $R(t,1,1) > R(t,1,0)$ for all t, yield that

$$V(t, \frac{1}{2}, 1) < V(t, \frac{1}{2}, \emptyset)$$

Thus the uninformed sender can profitably deviate by never reporting 1. This is a contradiction. □

Next, we show that the sender's reputation conditional on only the time of her report, $R(t)$, is also strictly declining in t.

Proposition 10 (R(t))**.** *In any equilibrium, for all t, $R(t) > R(t+1)$*

Proof. Suppose by contradiction that $R(t) \leq R(t+1)$, for some t. Now, consider two separate cases:

1. $b(t) < b(t+1)$.
   Note that we can write

$$R(t) = q_t V(t,1,1) + (1-q_t)V(t,1,\frac{1}{2})$$

   where q_t is the proability that, conditional on reporting at time t, the sender is informed. Since $b(t) < b(t+1)$, it follows that $q_t > q_{t+1}$. Since $V(t,1,1) > V(t,1,\frac{1}{2})$ for all t, in order for $R(t) \leq R(t+1)$, it must be that $V(t,1,1) < V(t+1,1,1)$. Thus the *informed* sender can profitably eviate at t by waiting until $t+1$ to report: contradiction.

2. $b(t) \geq b(t+1)$.
   I claim that $R(t,1,1) \leq R(t+1,1,1)$. This becomes obvious by examining the equations for $R(t,1,1)$ and $R(t+1,1,1)$:

$$R(t,1,1) = \frac{1}{1+(\frac{1-R(t)}{R(t)})(\frac{\pi(\lambda_B + b_t(1-\lambda_B))}{\pi\lambda_B + \frac{1}{2}b_t(1-\lambda_B)})}$$

$$R(t+1,1,1) = \frac{1}{1+(\frac{1-R(t+1)}{R(t+1)})(\frac{\pi(\lambda_B + b_{t+1}(1-\lambda_B))}{\pi\lambda_B + \frac{1}{2}b_{t+1}(1-\lambda_B)})}$$

   By analogous reasoning, we can show that $R(t,1,0) \leq R(t+1,1,0)$. Thus

$$V(t,\frac{1}{2},\emptyset) = \lambda_B R(t+1,1) + (1-\lambda_B)[\frac{1}{2}R(t+1,1,1) + \frac{1}{2}R(t+1,1,0)] > \frac{1}{2}[R(t,1,1) + R(t,1,0)] = V(t,\frac{1}{2},1)$$

   Thus, the uninformed sender can profitably deviate by never reporting 1 at time t. Contradiction.

□

4.6 Dynamics in faking

Below, I show that as long as the bad sender is of sufficiently low ability, she will become *more truthful* as time passes. That is, b_t is declining in t. In the analysis that follows I use $R(t,C)$ and $R(t,I)$ to denote the sender's reputation from making a correct and incorrect report, respectively, at time t. Formally, a "correct" report is one that matches the receiver's signal, and an "incorrect" report is one that does not. It is without loss to represent the reputation function in this form due to our above characterization of the sender's equilibrium behavior.

Proposition 11 (Declining $b(t)$)**.** *Fixing all other parameters, there exists a $\bar{\lambda}$ such that when $\lambda_B < \bar{\lambda}$, $b(t)$ is strictly decreasing in t.*

Proof. To prove this statement, first consider the case in which $\lambda_B = 0$. Suppose by contradiction that $b_t \leq b_{t+1}$. It follows that

$$R(t,C) > R(t+1,C) \text{ and } R(t,W) > R(t+1,W)$$

Note that in any equilibrium, the uninformed sender must be indifferent between reporting at time t and waiting until time $t+1$ to report instead. Since in this case $\lambda_B = 0$, this implies

$$V(t,1,\frac{1}{2}) = \frac{1}{1}R(t,C) + \frac{1}{2}R(t,W) > \frac{1}{1}R(t+1,C) + \frac{1}{2}R(t+1,W).$$

Next, note that because the equilibrium b is continuous in λ_B (need to formalize this), it follows that there exists a $\bar{\lambda}$ such that for all $\lambda_B < \bar{\lambda}$, this result holds. □