ENGPHYS 2CM4 Design Project

Optimizing the Input Current of a Railgun

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April 2020

Appendix

Abstract

After witnessing the battle between the Wizard with the Lizard Wizard and the Magnificent Nejat in 2P04 Assignment 2 (See attached Document for background), Dr. Harold Haugen has decided that these puny magic tricks are an insult before the superior discipline that is engineering. To prove this, Dr. Haugen decides to construct a railgun to entirely obliterate the wizard lizard wizard cage, in a display of overwhelming superiority. The railgun is to be mounted at the intersection of University Ave. and Sterling St. and fired at the entry way to BSB (in the area before the stone steps) where the magic show is being held. The railgun has been chosen for this application due to its non-explosive nature. As opposed to using high powered explosives which can be both expensive and dangerous to handle, the railgun will use an electromagnetic force to propel an armature across two parallel rails. The force will be produced by a voltage across the two rails that will create a magnetic field inducing current through them. At a sufficiently high voltage, an armature can be propelled at very high velocities (up to 3,500 m/s). However, this also proposes some challenges in design. The force that propels the armature across the rails, also acts on the entire gun, meaning that the gun will be actively ripping itself apart every time it's fired. This is to say the force produced between the two rails will distort them and decrease the accuracy of the gun overall. Dr. Haugen has been working for some time now on this railgun and as such the basic dimensions of it are constrained (rail radius, rail separation, armature mass, rail material). Thus, what Dr. Haugen now needs to optimize is the input current to the rails. This is to minimize the rail distortion, while maximizing the muzzle velocity. Maximum accuracy must be achieved to ensure destruction of the pathetic Lizard Wizard, and his Wizard compatriots.

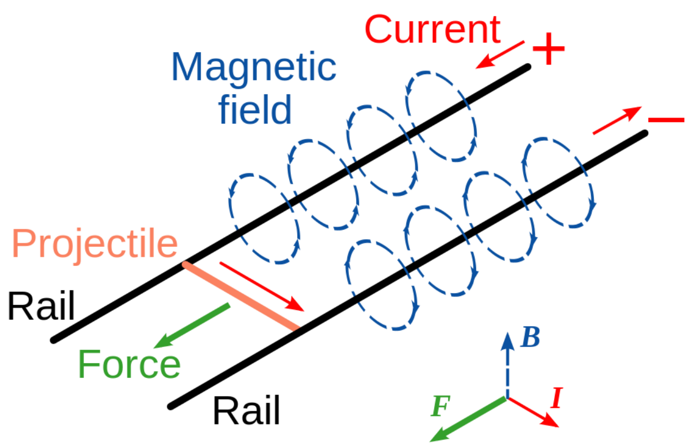
1. Background Physics

A railgun takes advantage of the force produced by a magnetic field and flowing electric current described by the relationship [1]:

$$\boldsymbol{F} = I\boldsymbol{\ell} \times \boldsymbol{B} \quad (1\text{-}1)$$

Where $\boldsymbol{I}$ is the current flowing through some wire of length $\boldsymbol{\ell}$ in the presence of some magnetic field $\boldsymbol{B}$**.**

It shows that a rod with a flowing current can be propelled by applying some magnetic field to it[2].



(Fig. 1-1)

This figure shows a magnetic field produced by a current carrying wire acting on a projectile to propel it forward.

To calculate what this force is, it is first necessary to find the magnetic field that the rails would produce. This is found using the Biot-Savart Law which approximates the magnetic field of any long current carrying wire (a description fit by the rails) [3].

$$\mathbf{B}(s) = \frac{\mu_0 I}{4\pi s}\widehat{\varphi} \quad (1\text{-}2)$$

Here the magnetic field **B** is found at some distance **s** from a wire carrying a current **I** where μ_0 is the permeability constant. And the unit vector **ϕ** is in this case the **z** direction (assuming the rails are in the x-y plane)

This, however, is only true if the wires (or rails) run from x=0 to x=∞. This is not the case, as that would be impossible, we must make an approximation. This is done by adding up the effect of each piece of the wire on each point. Usually we must integrate this but to avoid trivial integration we can take the well-known general expression for the magnetic field created by a semi-infinite current carrying wire as:

$$\mathbf{B}(s) = \frac{\mu_0 I}{4\pi}\left(\frac{1}{s} + \frac{1}{d-s}\right)\hat{z} \qquad (1\text{-}3)$$

Where **d** is the distance between the rails, and s is their radius. Note that this is only true when the length of the wires is much larger than the separation between them **d**.

At this point we are now interested in finding what force this magnetic field will apply on an armature between to rails (the wires in this case). Since the armature will have the same current as the rails (refer to Fig.1-1), the force acting on it can be expressed as:

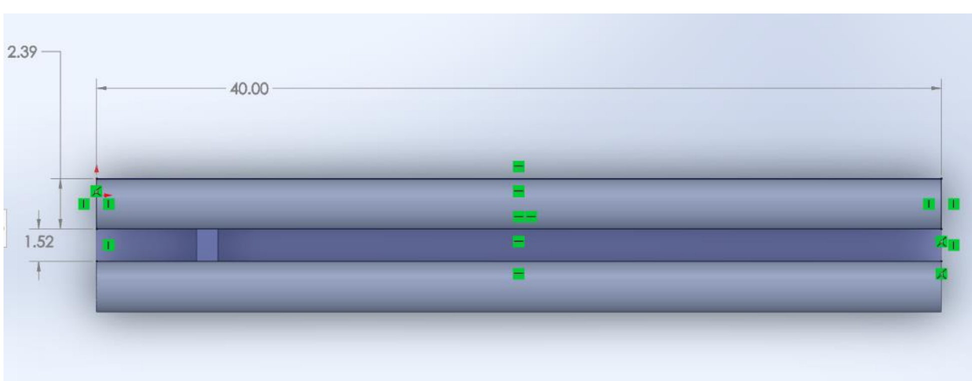
$$\mathbf{F} = I\int_{r}^{d-r} \mathrm{d}\boldsymbol{\ell} \times \frac{\mu_0 I}{4\pi}\left(\frac{1}{s} + \frac{1}{d-s}\right)\hat{z}$$

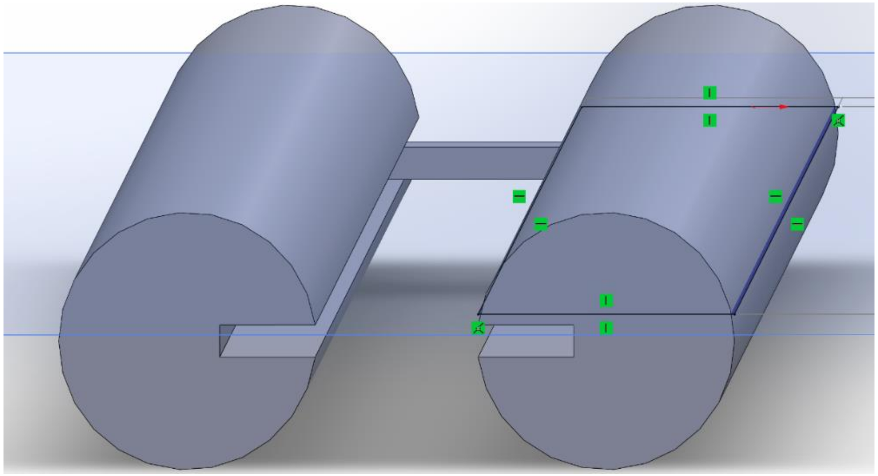
$$= \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{d-r}{r}\right)\widehat{x} \qquad (1\text{-}4)$$

Where **I** is the current supplied to the rails**,** **d** is the separation between them, **r** is their radii, and μ_0 is the permeability constant.

This is essentially the product of summing the force applied by each piece of the rails on the armature over the two rails.

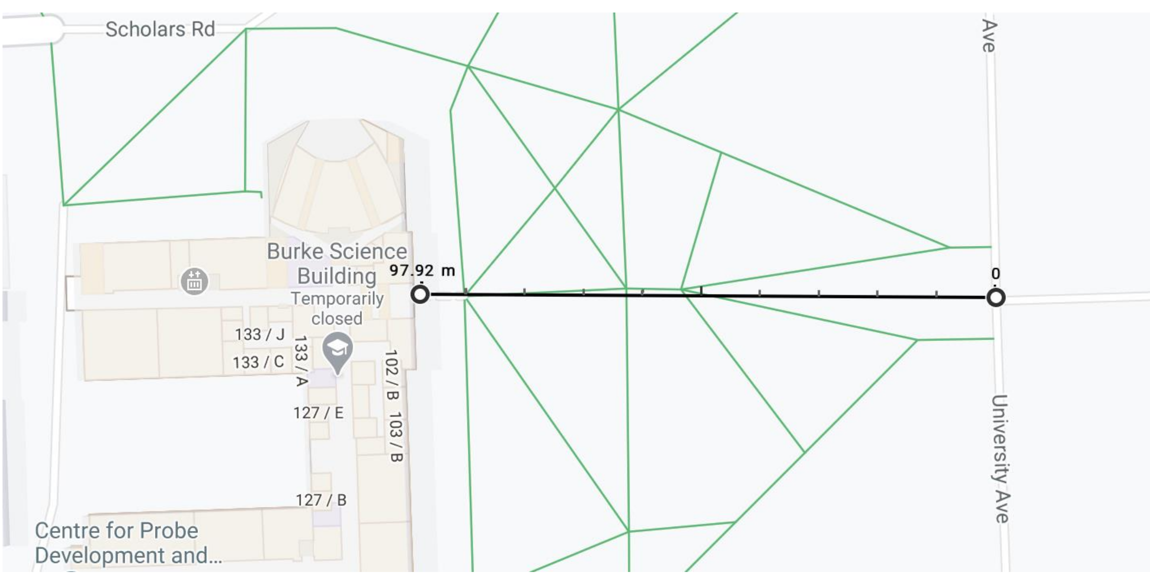
Now that an expression for the force applied on an armature by the rails of the railgun has been obtained, a closer look at the design of the railgun must be taken to better understand the result of this force.





(Fig. 1-2) shows the top and azimuthal views of the two rails with the armature between them. The rails are made of copper (by design of Dr. Haugen) due to their high conductivity.

When the current is applied to one rail, it will shoot the armature forward, propelling it at the Lizard Wizard. As mentioned, the dimensions (shown in Fig. 1-2) are pre constrained and the only variable that Dr. Haugen is now concerned with is the input current, this careful consideration stems from the kinematics of the projectile to be launched. As stated, the magic show is being held just in front of the Burke Science Building, and Dr. Haugen plans to fire the railgun from the intersection of University Ave. and Sterling St.



(Fig. 1-3) Google Maps displays the distance in question

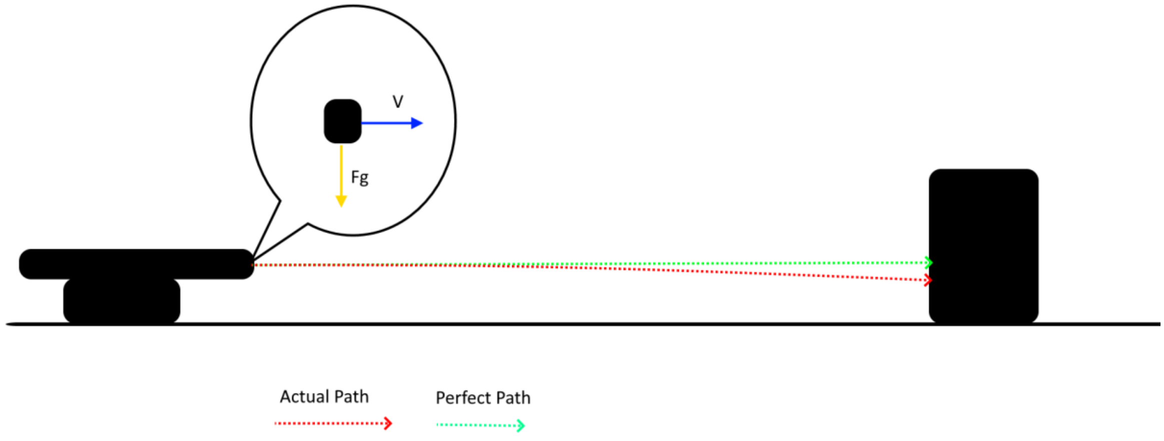
This is a distance of approximately 97.92m as shown by Google Maps. Dr. Haugen intends to fire the railgun straight on with no launch angle and intends to maximize the accuracy. To understand what this has to do with the input current, one must take a look at two effects on the armature: gravity, and rail distortion.

Firstly, the longer the projectile takes to reach the target, the more it will drop due to gravity acting on it. And since the distance is fixed, the only thing that can be changed is the speed of the projectile. As the armature travels on the rails it accelerates from the force applied to it, the exact amount which it accelerates by is shown by Newtons 2^{nd} Law [4]:

$$F = m \cdot a \tag{1-5}$$

Where **F** is the force applied, **m** is the mass of the object, and **a** is the acceleration.

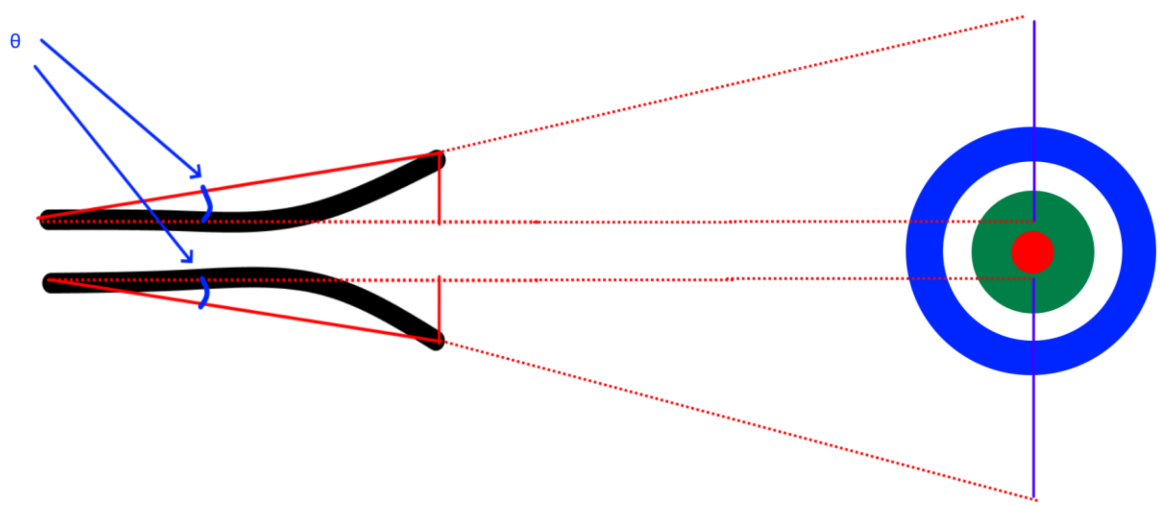
This shows that the acceleration of the armature is directly proportional to the force acting on it. From equation (1-4) it is known that an increase in input current results in an increase in the force acting on the armature (by a factor of n^2), and thus an increase in the input current will increase the resulting muzzle-velocity of the projectile. Following suit, the higher the muzzle-velocity the faster it will reach the target, and the less gravity will cause an error in the accuracy in the y direction.



(Fig. 1-4) A projectiles flight to a target

In Layman's terms, there is a direct relationship between input current and y-axis accuracy.

Secondly, we must consider the effects of the magnetic field on the rails themselves. Since there is a very large current running between them, there will be a very large force acting on them both, large enough that the bending of both rails due to the force cannot be ignored. Ergo, since the same force acting on the armature will also act on the rails, they will distort and bend outwards. The problem with this is that it creates a sort of funnel at the end of the rails.



(Fig. 1-5) The funneled shape of the rails creates an angle that increases the error by a significant amount over a large distance.

Thus, it can be said that the force as described by equation (1-4) will also act on the rails, distorting them. And since the force is directly proportional to the input current, the greater the current, the greater the distortion will be. And since this distortion will give the projectile a larger spread diameter (think blunderbuss vs. sniper rifle), the greater the input current, the greater the error in the x-axis.

In Layman's terms, there is an inverse relationship between the input current and the x-axis accuracy.

The last piece of physics is that which relates the force on the rails, to their degree of distortion.

[5] Every material has an intrinsic Elastic Modulus **E** and poisons ratio **v**. Which dictate the ratio of the applied stress to resulting strain on an object, and the coupling of stress and strain between the different axes of an object respectively. Additionally, every object has a shear modulus which is related to both previous parameters and is defined as:

$$G = \frac{E}{2(1+v)} \qquad (1\text{-}6)$$

The shear modulus **G** shows how much a material shears when a force is applied on the face of a material in a direction parallel with that face.

In actuality, it is a ratio of that stress and the displacement in the direction of the force with respect to the direction of the faces normal.

What can be done with this information is to build a matrix which contains a component for each coupling of axial and shear stress to axial and shear strain for each dimension. 6 rows by 6 columns.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E}{(1+\nu)(1-2\nu)}(1-\nu) & \frac{E}{(1+\nu)(1-2\nu)}\nu & \frac{E}{(1+\nu)(1-2\nu)}\nu & 0 & 0 & 0 \\ \frac{E}{(1+\nu)(1-2\nu)}\nu & \frac{E}{(1+\nu)(1-2\nu)}(1-\nu) & \frac{E}{(1+\nu)(1-2\nu)}\nu & 0 & 0 & 0 \\ \frac{E}{(1+\nu)(1-2\nu)}\nu & \frac{E}{(1+\nu)(1-2\nu)}\nu & \frac{E}{(1+\nu)(1-2\nu)}(1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}$$

(Fig. 1-6)

As seen the matrix is for a material that does not couple shear stresses to axial strains. And is labelled an iso-tropic material. Meaning the resulting strain does not depend on which direction the stress is applied in. This is practical for this purpose since the material being used for the rails is copper, an isotropic material

2. Design and Implementation into Finite Method Numerical Software (FEM)

To calculate the railgun accuracy, it will be simulated in a FEM software, specifically FlexPDE 6 Student Edition. To do this, however, all static parameters must first be defined. This is done below in (Fig. 2-1)

Parameter	Brief Description	Value
E	The Elastic Modulus is a ratio of an objects strain to applied stress	400 GPa (Copper)
v	The Poissons Ratio is a measure of a materials coupling of stresses in certain axes to strain in other axes	0.28
r	Radius of each rail	1.2 m
d	Distance of Separation between the central axes of the rails	3.9 m
μ_0	The permeability of free space	$1.2566 * 10^{-6}[\frac{m * kg}{s^2 A^2}]$
θ	Launce Angle of Railgun	0°
magicShowDist	The Distance from the Railgun to the magic show	97.92 m
G	Shear Modulus shows ratio of shear stress to shear strain	156.25 GPa
armatureMass	Mass of the armature	30 kg

(Fig. 2-1)

Now that all static parameters have been stated, one may describe the implementation of the physics discussed in FlexPDE.

The first step is to understand what variables are being worked with. We are interested in the displacement of the beams in only one direction (as the force applied will only be in one direction). We have arbitrarily assigned that the variable **v** for displacement (we will also include **u** for displacement in the perpendicular direction for readability). Also of interest is the distance in the x-direction of the projectile (for the purposes of calculating at which point it has reached the end of the rails). And the speed of the projectile in both the x and y directions (only x is necessary but y is included for reability). This implementation is shown below.

```
VARIABLES
u(threshold=1e-6)    !Displacement in x
v(threshold=1e-6)!Displacement in y

vx(threshold=1e-6)
vy(threshold=1e-6)
rx(threshold=1e-6)
```

(Fig. 2-2)

Next, we define all the variables shown in (Fig. 2-1). This is trivial and will not be displayed for sake of space.

After this comes the implementation of the Railgun equation described in equation (1-4). This gives the force we expect the projectile to experience as a current is passed through the rails.

```
I = 3e5
Farm= mu0*(I^2)/(2*Pi)*ABS(ln((d-r)/r))!Force produced as a product of the current (I), distance between rails (d), and rail radius (r)
```

(Fig. 2-3) Note the current **I** is variable and the displayed value is not a constant.

Next is described the definitions of the acceleration in the x and y directions, as well as the magnitude of the total velocity.

```
ax = if rx<L then Farm/armatureMass*cos(theta) else 0
ay = 9.81
vel = sqrt((vx*cos(theta))^2+(vy*sin(theta))^2)
```

(Fig 2-4) Noting that the acceleration in x is only affected by the force on the armature while its displacement is not more than the length of the rails. Additionally, the y component of the velocity **vy** is 0 during this time but again is included for readability.

Following this we wish to describe the overall error in the x and y axes, as discussed in section 1., for the y-axis the magnitude of this depends on the effect of gravity on the projectile. As such the flight time is calculated from the distance traveled and muzzle-velocity. Using a simple kinematics formula one calculates the total drop in the y-axis, or the error.

As for the x-axis this depends on the angle created by the displacement of the tip of the rails over the entire distance traveled. Expressed simply by taking the arctangent of the displacement of the rail at the tip to find the angle of displacement (as shown in (Fig. 1-5)) and multiplying that byt the total distance traveled to find the total possible arc length. This is the x-axis error.

Once the two are calculated, the total accuracy is found by adding those components together into a single vector, dividing it over the total target area, and subtracting it from 1.

All together show below.

```
magicShowDist=97.92
flightTime = magicShowDist/vel
dely = 1/2*ay*flightTime^2
delx =ARCTAN (val(v,L,r/2)/L)*180/Pi*magicShowDist
accuracy = 1-(sqrt(dely^2+delx^2)/6)
```

(Fig 2-5)

As a small note the number 6 signifies the target size, this is due to the stage being 4 meters high, the table being 1 meter high, and the cage being 1 meter high. The target is thus taken as a circle of radius 6 centered on the bottom of the stage in the hopes of destroying the entire set up.

As for any concerns about destructive power as long as the projectile makes contact with an element of the show it will cause major damage at speeds of around 200 m/s and a weight of 30 kg.

The last step of definitions is to describe the stress and strain parameters. The matrix of which has been set up as shown in (Fig. 1-6) and all following relationships detailing the dot products of the C matrix with the strain matrix to find the stress. As well as the differential definition of the strain in each direction, which is the differential change in displacement. And the differential definition of the shear strain which is the differential change in displacement of each axis with respect to the perpendicular axis added together.

```
!Stiffness matrix components (isotropic material)
C11 =E*(1-nu)/(1+nu)/(1-2*nu)
C22 = C11
C33 = C11

C12 = E*nu/(1+nu)/(1-2*nu)
C13 = C12
C21 = C12
C23 = C12
C31 = C12
C32 = C12

!! Strain
!Axial Strain
ex=dx(u)
ey=dy(v)

!Engineering Shear Strain
G=E/(2*(1+nu))
gxy=(dx(v)+dy(u))

!mechanical strain
exm=ex
eym=ey

!!Stress via Hooke's law
!Axial Stress
sx = C11*exm+C12*eym
sy = C21*exm+C22*eym

!Shear stress
sxy=G*gxy
```

Lastly it was necessary to describe differential equations which created rules for all the variables we created to follow. These equations dictated the constraints of all the definitions defined before as they cycle through to simulate the scenario. They are the relationships of distance to velocity, velocity to acceleration. And the definition that the differential change in any y displacement with respect to the y-axis (as well as for the x-axis) is zero.

```
EQUATIONSS
u:  dx(sx)+dy(sxy)= 0
v:  dx(sxy)+dy(sy)= 0

rx: dt(rx) = vx
vx: dt(vx) = ax
vy: dt(vy) = ay
```

(Fig. 2-7)

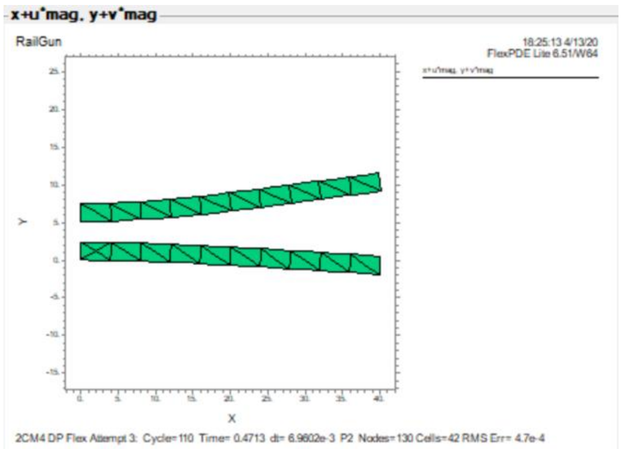
Note that it was also necessary to draw the boundaries in 2D (3D was not done due to this object symmetry in the z axis) as shown in (Fig. 1-2) i.e. birds eye view. But this is trivial and omitted from the explanation for sake of space.

All that changes beyond this implementation is the value of the input current.

For proof of concept a known test case as well as three scaled cases are shown below.

Known Test Case

This case uses the constrained dimensions shown in (Fig. 2-1) and a current of 300 kA.



SUMMARY

RailGun

SUMMARY

val(v,L,r/2)= -2.448608e-3
Farm= 14596.74
I= 300000.0
val(vel,0,0)= 170.0261
val(accuracy,0,0)= 0.722880

This shows bending in the rails as we expect as well as a very high force applied on the armature (and rails). The accuracy however is only about 72% which is not bad, but we can do better.

To shoe scaling with results, below is a case with a current of 100 A; a case with a radius that is doubled; and a case where the distance between the arms is increased by 0.5 meters, the input current is 1.25 kA, and the mass of the projectile is increased to 55 kg. Respectively, from left to right.

SUMMARY

RailGun

SUMMARY

val(v,L,r/2)= -2.699375e-10
Farm= 1.621860e-3
I= 100.0000
val(vel,0,0)= 5.146874e-4
val(accuracy,0,0)= -2.958990e+10

SUMMARY

RailGun

SUMMARY

val(v,L,r/2)= -1.549136e-4
Farm= 8460.065
I= 300000.0
val(vel,0,0)= 115.3231
val(dely,0,0)= 3.536303
val(delx,0,0)= -0.021728
val(accuracy,0,0)= 0.410605

SUMMARY

RailGun

SUMMARY

val(v,L,r/2)= -1.956254e-4
Farm= 1588.943
I= 90000.00
val(vel,0,0)= 41.08583
val(dely,0,0)= 27.86107
val(delx,0,0)= -0.027438
val(accuracy,0,0)= -3.643514

The first case has a current that is far too low, as seen it produces a force of less than 1N, this is barely enough to get the projectile off the rails, as such the accuracy is far less than 0. (Note a value less than 0 or greater than 1 indicates the projectile never makes it to the target).

The second case shows the same current but a much larger rail radius. Equation (1-4) shows that as the rail radius increases the force on the projectile decreases. As such it is moving much slower and the accuracy is lower at only 41%.

The final case displays less a much smaller current which decreases the force on the armature, a larger rail separation which equation (1-4) shows increases the force, and a larger projectile mass, which decreases the acceleration on it as per equation (1-5). All together the decrease in current and increase in mass made it so that the projectile almost did but never reached the target, overpowering the change in rail separation.

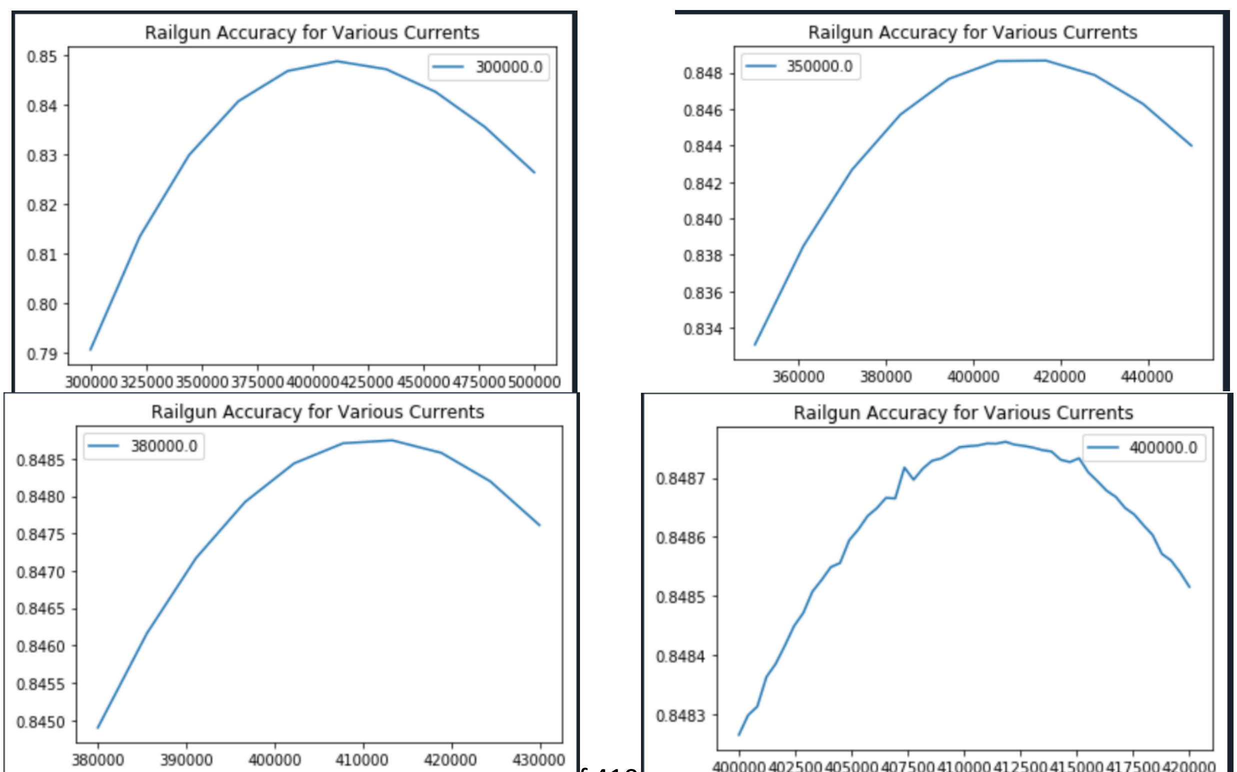
All three cases make sense logically.

3. Optimization with Python

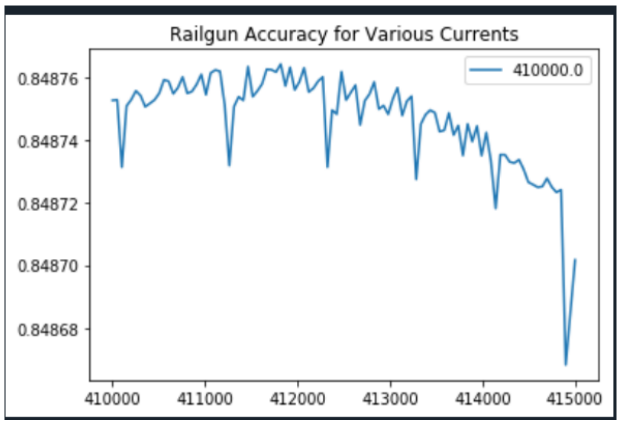
With the FlexPDE model describing the accuracy, it is now of interest to find the optimum input current to achieve the maximum accuracy possible. The stickler for numbers that he is, Dr. Haugen has decided that he will accept an error no greater than 0.0001%. The implementation consists of two steps: scripting and searching.

The first step involves having Python iterate through the FlexPDE model multiple times for an array of currents and place the output accuracy of each case in a list that can be plotted to zero in on an optimum current. This is done by placing the FlexPDE code into Python and taking advantage of the Python write command to create a file with that code and edit the text so that the current is replaced with [%s]. This command allows the string to be edited to change that [%s} value whenever it is called. Next a for loop is run over a very large interval, each time the code exports an answer which Python adds to a list. After the loop is finished the list is plotted so a better interval can be chosen. The results of zeroing in on a small enough interval to optimize are shown below.

(Fig. 3-1)



After deciding on a sufficiently small interval of 410000-415000 A, the script was run over 100 data points. After which all the data points were saved in a list. The next step is searching for the optimum value. To do this a type of binary search method was implemented. The search algorithm began at the start of the list and if the next element was larger than the current one, it moved up by one. It continued this until the next element was smaller than it. At this point it checked that the previous value was smaller than the current. In the case that both criteria were fulfilled, it chose that value as the optimum accuracy. To check the answer a linear search was run on the same interval. The results were are follows




```
414999.99999999953
411818.18181818165
```

(Fig. 3-2)

The first is the binary and the second is the linear search over the smallest interval. These two yield accuracies of 84.8702% and 84.8764%. This is an error of 0.0000073% so the binary search did quite well in optimizing. But it is still better overall to take the linear result.

The reason a binary search was chosen to begin with was due to the limitations of the technology available. Although the linear search yielded a more exact answer, it required some help from the binary search to first shorten the interval by a large amount. At this point the linear search became more feasible.

4. Conclusion

Modelling in FlexPDE and optimizing in Python has allowed the zeroing in on an optimum input current to ensure maximum accuracy of the railgun. With all the variables in place, the optimum current was one which found the perfect balance between increasing the y-axis accuracy by increasing the current and increasing the x-axis accuracy by decreasing the current. Armed with this information, Dr. Haugen is now able to annihilate the wizards and their lizard with nearly 85% certainty. Thus, proving once and for all the superiority of the engineering practice.

The optimization did a good job of not only finding an optimum value, but also displaying the many options available to an engineer to optimize these problems. This specific optimization process required a great deal of manual searching to shrink the interval down to a manageable level. While the question of greater optimization lingers, this process yielded quite satisfactory results and managed to solve our problem.

In reference to the possibilities of expansion with this problem. There are many direction optimizations can go. During research it became apparent just how complicated railgun physics is, and what a monumental engineering challenge their optimization is. This model did not include factors such as melting of components due to massive heat generations, drag forces changing the path of projectiles, aerodynamics of the projectile, bored barrels for greater accuracy, the sources of massive energy inputs. All very complex tasks, especially when combined. But with 2-3 full years of undergrad ahead I believe great strides can be made towards such things.

5. References

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