1 Review

Fundamental Theorem of Calculus I: If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then

Fundamental Theorem of Calculus II: If f is continuous on an interval, then for x and a in that interval

Important: Remember that these theorems are named/numbered differently in the textbook.

2 u-Substitution

So far, to find antiderivatives we have only had limited techniques, such as

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Chain Rule: If f and g are differentiable functions, then

Question Have we found an antiderivative for some function?

Question What does that tell us about the following integral?

$$\int f'(g(x))g'(x)\,dx =$$

What we've done is called **u-substitution**.
The general idea, is that if g' is continuous on $[a, b]$ and f is continuous on the range of g, then to integrate

$$\int f'(g(x))g'(x)\,dx,$$

we define u as follows.

$$\begin{aligned} u &= \\ \frac{du}{dx} &= \\ \int f'(g(x))g'(x)\,dx &= \end{aligned}$$

Note that if we are computing $\int_a^b f'(g(x))g'(x)\,dx$, then we can either see that this equals

$$f(g(x))\Big|_a^b \text{ or } f(u)\Big|_{g(a)}^{g(b)}.$$

Examples Evaluate each of the following.

1. $\displaystyle\int_2^3 2x\sin(x^2)\,dx$

2. $\displaystyle\int_0^1 x(x^2+1)^4\,dx$

3. $\displaystyle\int \sin^2 x\,dx$

4. $\displaystyle\int \frac{x}{x+1}\,dx$

Exercises Evaluate each of the following integrals using u-substitution.

1. $\displaystyle\int \tan^2 x\sec^2 x\,dx$

2. $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+1}}\,dx$

3. $\displaystyle\int \frac{dx}{3x+1}$

4. $\displaystyle\int_0^1 \frac{dx}{e^{3x}}$

5. $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

6. $\displaystyle\int \frac{\ln x}{x}\,dx$

7. $\displaystyle\int_0^{\pi} \cos^2 x \sin x\,dx$

8. $\displaystyle\int \cos^2 x \sin^3 x\,dx$

9. $\int_0^3 xe^{-x^2}\,dx$

12. $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

10. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

13. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$

11. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

14. $\int \sec x\,dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x}\,dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$