1 Review

u-substitution: If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

integration by parts: If f and g are differentiable, then

Exercise Compute $\int \frac{1}{x+1} + \frac{1}{2x-1}\, dx =$

Exercise Fill in:

$$\frac{1}{x+1} + \frac{1}{2x-1} = \frac{\qquad\qquad\qquad}{(x+1)(2x-1)} = \frac{\qquad}{2x^2+x-1}$$

Question How could I use the previous two excercises to compute the following?

$$\int \frac{3x}{2x^2+x-1}\, dx =$$

2 Partial Fractions

Basically, we can use the opposite method of finding a common denominator to compute

$$\int \frac{1}{x^2+5x+4}\, dx$$

First, we have to find what fractions our integrand is the sum of. Let A and B satisfy

$$\frac{1}{x^2+5x+4} = \frac{A}{\qquad} + \frac{B}{\qquad}.$$

Therefore

$$\int \frac{1}{x^2+5x+4}\, dx =$$

This process is called **partial fractions**.

Exercises Find each of the following.

1. $\displaystyle\int_2^{10} \frac{1}{x^2-1}\,dx$

2. $\displaystyle\int \frac{3}{x^2+x}\,dx$

3. $\displaystyle\int \frac{x}{x^2-5x+6}\,dx$

4. $\displaystyle\int \frac{1}{x^2+9}\,dx$ [Hint: What is $\displaystyle\int \frac{1}{x^2+1}\,dx$? Use your answer and guess and check.]

Example Evaluate $\int \frac{x^3 - 4}{x^2 - 4}\, dx$.

Exercises Evaluate each of the following integrals and check your answer.

1. $\int z\sqrt{z-1}\, dz$

2. $\int_0^1 \tanh(t)\, dt = \int_0^1 \frac{e^t - e^{-t}}{e^t + e^{-t}}\, dt$

3. $\int_{-x}^{x} \cos^2 t\, dt$

4. $\displaystyle\int \tan^3 x \sec^2 x \, dx$

5. $\displaystyle\int \frac{2x}{1+x^4} \, dx$

6. $\displaystyle\int \frac{2}{4-x^2} \, dx$

7. $\displaystyle\int \frac{dx}{1+\sqrt{x}}$