Practice Integration Gateway Test

① $\int\left(\sqrt{3}+\frac{1}{t^{1/2}}\right)dt = \boxed{\sqrt{3}t + 2\sqrt{t} + C}$

② $\int x^2 e^{2x}\,dx = \frac{1}{2}x^2e^{2x} - \int x e^{2x}\,dx$

$u = x^2 \quad dv = e^{2x}dx$

$du = 2x\,dx \quad v = \frac{1}{2}e^{2x}$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}x^2e^{2x} - \left[\frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx\right]$

$= \boxed{\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$

③ $\int \frac{x^2+2}{1+6x+x^3}\,dx = \frac{1}{3}\int\frac{du}{u} = \boxed{\frac{1}{3}\ln\left|1+6x+x^3\right| + C}$

$u = 1+6x+x^3 \Longrightarrow$

$du = 6+3x^2dx$

$= 3(x^2+2)dx$

④ $\int \frac{\cos(x)}{1+\sin^2(x)}\,dx = \int\frac{du}{1+u^2} = \boxed{\arctan(\sin(x)) + C}$

$u = \sin(x) \Longrightarrow$

$du = \cos(x)dx$

⑤ $\int_0^{\pi/2}\cos^2(t)\,dt = \int_0^{\pi/2}\frac{1+\cos(2t)}{2}\,dt = \int_0^{\pi/2}\frac{1}{2}dt + \frac{1}{2}\int_0^{\pi/2}\cos(2t)\,dt$

$= \left[\frac{1}{2}t + \frac{1}{4}\sin(2t)\right]_0^{\pi/2}$

$= \left(\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}\cancel{\sin(\pi)}^{0}\right) - \left(0 + \frac{1}{4}\cancel{\sin(0)}^{0}\right) = \boxed{\frac{\pi}{4}}$

⑥ $$\int_e^{e^2} \frac{1}{x(\ln(x))^3}\,dx = \int_{x=e}^{x=e^2} \frac{du}{u^3} = \left[-\frac{1}{2}(\ln(x))^{-2}\right]\Bigg|_e^{e^2}$$

$u=\ln(x) \Rightarrow$

$du=\frac{1}{x}dx$

$$= -\frac{1}{2}\left[\frac{1}{(\ln(e^2))^2} - \frac{1}{(\ln(e))^2}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{4} - 1\right] = \frac{1}{2}\left(\frac{3}{4}\right) = \boxed{\frac{3}{8}}$$

⑦ $$\frac{1}{t^2+t} = \frac{A}{t} + \frac{B}{t+1} \iff 1 = A(t+1) + Bt$$

$t=0$: $1=A$

$t=-1$: $1=-B$

$$\int \frac{dt}{t^2+t} = \int \frac{dt}{t} - \int \frac{dt}{t+1} = \boxed{\ln|t| - \ln|t+1| + C}$$

⑧ $$\int_1^4 \ln(t)\,dt = t\ln(t)\Big|_1^4 - \int_1^4 dt = \Big[t\ln(t) - t\Big]_1^4$$

$u=\ln(t)$ $dv=dt$

$du=\frac{1}{t}dt$ $v=t$

$$= (4\ln(4)-4) - (\cancel{\ln(1)}^{0} - 1)$$

$$= \boxed{4\ln(4) - 3}$$