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1) $\int_0^1 xe^x\,dx = xe^x\Big|_0^1 - \int_0^1 e^x\,dx$ $\qquad u = x \quad dv = e^x dx$

$\qquad du = dx \quad v = e^x$

$= e - \left(e^x\Big|_0^1\right)$

$= e - e + 1$

$= 1$

2) $\int_0^{\pi} x\sin x\,dx = -x\cos x\Big|_0^{\pi} + \int_0^{\pi} \cos x\,dx$ $\qquad u = x \quad dv = \sin x\,dx$

$\qquad du = dx \quad v = -\cos x$

$= \pi + \left(\sin x\Big|_0^{\pi}\right)$

$= \pi$

3) $\int_1^2 \ln x\,dx = x\ln x\Big|_1^2 - \int_1^2 \frac{x}{x}\,dx$ $\qquad u = \ln x \quad dv = dx$

$\qquad du = \frac{1}{x}dx \quad v = x$

$= 2\ln 2 - \int_1^2 1\,dx$

$= 2\ln 2 - 1$

4) $\int \arctan x\,dx = x\arctan x + C - \int \frac{x}{1+x^2}\,dx$ $\qquad u = \arctan x \quad dv = dx$

$\qquad du = \frac{1}{1+x^2}dx \quad v = x$

$= x\arctan x + C - \frac{1}{2}\int \frac{1}{u}\,du$ $\qquad u = x^2+1 \quad du = 2x\,dx$

$= x\arctan x - \frac{1}{2}\ln|u| + C = x\arctan x - \frac{1}{2}\ln|x^2+1| + C$

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5) $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x + C - \frac{1}{2}\int \frac{x^2}{x} dx$ $u = \ln x$ $dv = x dx$

$du = \frac{1}{x}$ $v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \ln x + C - \frac{1}{2}\int x \, dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

6) $\int x^2 e^x \, dx = x^2 e^x + C - \int 2x e^x dx$ $u = x^2$ $dv = e^x dx$

$du = 2x dx$ $v = e^x$

$= x^2 e^x + C - 2\int x e^x dx$ $u = x$ $dv = e^x dx$

$du = dx$ $v = e^x$

$= x^2 e^x + C - 2(x e^x - \int e^x dx)$

$= x^2 e^x - 2x e^x + C + 2e^x$

$= e^x (x^2 - 2x + 2) + C$

7) $\int x e^{-x^2} dx = -\frac{1}{2}\int e^u du$ $u = -x^2$

$du = -2x \, dx$

$= -\frac{1}{2} e^u + C$

$= -\frac{1}{2} e^{-x^2} + C$

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8) $\int \frac{x}{e^x}\,dx = -xe^{-x} + \int e^{-x}dx + C$ $\quad u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$= -xe^{-x} - e^{-x} + C$

$= -(x+1)e^{-x} + C$

9) $\int e^x \sin x\,dx$ $\quad u = \sin x \quad dv = e^x dx$

$du = \cos x\,dx \quad v = e^x$

$= e^x \sin x - \int e^x \cos x\,dx + C$

$u = \cos x \quad dv = e^x dx$

$= e^x \sin x - (e^x \cos x + \int e^x \sin x\,dx) + C$ $\quad du = -\sin x\,dx \quad v = e^x$

$= e^x(\sin x - \cos x) - \int e^x \sin x\,dx + C$

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) + C$

$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$

10) $\int \sin^2 x\,dx = \int (\sin x)(\sin x)\,dx$ $\quad u = \sin x \quad dv = \sin x\,dx$

$du = \cos x\,dx \quad v = -\cos x$

$= -\sin x \cos x + \int \cos^2 x\,dx + C$

$= -\sin x \cos x + \int \frac{1+\cos 2x}{2}\,dx + C$ $\quad u = 2x \quad du = 2dx$

$= -\sin x \cos x + \frac{1}{4}\int 1 + \cos u\,du + C$

$= -\sin x \cos x + \frac{1}{4}(u + \sin u) + C = -\sin x \cos x + \frac{x}{2} + \frac{1}{4}\sin 2x + C$

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11) $\int \frac{1}{x^2+6x+9}\,dx = \int \frac{1}{(x+3)^2}\,dx$ $\quad u = x+3 \quad du = dx$

$= \int \frac{1}{u^2}\,du$

$= -\frac{1}{u} + C$

$= -\frac{1}{x+3} + C$

12) $\int \frac{2x}{x^2+6x+9}\,dx = \int \frac{2x+6-6}{x^2+6x+9}\,dx$

$= \int \frac{2x+6}{x^2+6x+9} - \frac{6}{x^2+6x+9}\,dx$ $\quad u = x^2+6x+9$

$du = 2x+6$

$= \int \frac{1}{u}\,du - 6\int \frac{1}{(x+3)^2}\,dx$ $\quad v = x+3$

$dv = dx$

$= \ln|u| + C - 6\int \frac{1}{v^2}\,dv$

$= \ln|(x+3)^2| + C - 6\left(-\frac{1}{v}\right)$

$= 2\ln|x+3| + \frac{6}{x+3} + C$

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13) $\int \arcsin x \, dx = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx + C$ $\quad u = \arcsin x \quad dv = dx$

$\quad du = \frac{1}{\sqrt{1-x^2}} \quad v = x$

$= x\arcsin x + C + \frac{1}{2}\int u^{-1/2} du$ $\quad u = 1-x^2$

$\quad du = -2x\,dx$

$= x\arcsin x + \sqrt{u} + C$

$= x\arcsin x + \sqrt{1-x^2} + C$

Ex (a) $\int_1^b \frac{\ln(z)}{z^2} dz$ $\quad u = \ln z$

$\quad z = e^u$

$= \int_0^{\ln b} \frac{u}{e^{2u}} \cdot e^u \, du$ $\quad dz = e^u du$

$= \int_0^{\ln b} \frac{u}{e^u} du$ $\quad u = u \quad dv = e^{-u} du$

$\quad du = du \quad v = -e^{-u}$

$= -ue^{-u}\Big|_0^{\ln b} + \int_0^{\ln b} e^{-u} du$

$= -(\ln b)e^{-\ln b} - e^{-u}\Big|_0^{\ln b}$

$= -(\ln b)e^{\ln \frac{1}{b}} + 1 - e^{-\ln b}$

$= -\frac{1}{b}\ln b + 1 - \frac{1}{b}$

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(b) $\int (\ln(z))^2 \, dz$ $\qquad u = (\ln z)^2 \quad dv = dz$

$\qquad du = 2\frac{\ln z}{z}\,dz \quad v = z$

$= z(\ln z)^2 - 2\int \frac{\ln z}{z} \cdot z\,dz$

$= z(\ln z)^2 - 2\int \ln z\,dz + C$ $\qquad u = \ln z \quad dv = dz$

$\qquad du = \frac{1}{z}dz \quad v = z$

$= z(\ln z)^2 - 2\left(z\ln z - \int \frac{z}{z}\,dz\right) + C$

$= z(\ln z)^2 - 2z\ln z + 2\int 1\,dz + C$

$= z(\ln z)^2 - 2z\ln z + 2z + C$

or

$\int (\ln(z))^2 \, dz$ $\qquad u = \ln z$

$\qquad z = e^u$

$= \int u^2 e^u \, du$ $\qquad dz = e^u du$

$= u^2 e^u - 2\int u e^u \, du + C$ $\qquad u = u^2 \quad dv = e^u du$

$\qquad du = 2u\,du \quad v = e^u$

$= u^2 e^u - 2\left(ue^u - \int e^u du\right) + C$ $\qquad u = u \quad dv = e^u du$

$\qquad du = du \quad v = e^u$

$= u^2 e^u - 2ue^u + 2e^u + C$

$= z(\ln z)^2 - 2z(\ln z) + 2z + C$