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$$1) \int_2^3 2x \sin(x^2) dx$$

$$u = x^2$$

$$du = 2x dx$$

$$= \int_4^9 \sin u du$$

$$= -\cos u \Big|_4^9$$

$$= \cos 4 - \cos 9$$

$$2) \int_0^1 x(x^2+1)^4 dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$= \int_1^2 \frac{1}{2} u^4 du$$

$$= \frac{1}{2} \left[ \frac{u^5}{5} \right]_1^2$$

$$= \frac{1}{2} \left[ \frac{32}{5} - \frac{1}{5} \right]$$

$$= \frac{31}{10}$$

$$3) \int \sin^3 x dx = \int \frac{1 - \cos 2x}{2} dx \quad \begin{array}{l} \cos 2x = 1 - 2\sin^2 x \\ \sin^2 x = \frac{1 - \cos 2x}{2} \end{array}$$

$$= \frac{1}{2} \int (1 - \cos 2x) dx$$

$$= \frac{1}{2} (x - \int \cos 2x dx + C)$$

$$u = 2x$$

$$du = 2 dx$$

$$= \frac{1}{2} (x - \frac{1}{2} \int \cos u du + C)$$

$$= \frac{1}{2} x - \frac{1}{4} \sin u + C = \frac{1}{2} x - \frac{1}{4} \sin 2x + C$$

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$$4) \int \frac{x}{x+1} dx = \int \frac{u-1}{u} du$$

$$u = x+1$$

$$x = u-1$$

$$du = dx$$

$$= \int 1 - \frac{1}{u} du$$

$$= u - \ln|u| + C$$

$$= x+1 - \ln|x+1| + C$$

$$= x - \ln|x+1| + C$$

$$1) \int \tan^2 x \sec^2 x dx$$

$$u = \tan x$$

$$du = \sec^2 x dx$$

$$= \int u^2 du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{\tan^3 x}{3} + C$$

$$2) \int_0^1 \frac{x}{\sqrt{x^2+1}} dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$= \frac{1}{2} \int_1^2 \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{2} \int_1^2 u^{-1/2} du$$

$$= \frac{1}{2} \cdot 2 u^{1/2} \Big|_1^2 = \sqrt{2} - 1$$

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3)

$$\int \frac{dx}{3x+1} = \frac{1}{3} \int \frac{1}{u} du$$

$$u = 3x+1$$

$$du = 3dx$$

$$= \frac{1}{3} \ln|u| + C$$

$$= \frac{1}{3} \ln|3x+1| + C$$

$$4) \int_0^1 \frac{dx}{e^{3x}} = \frac{1}{3} \int_0^3 \frac{1}{e^u} du$$

$$u = 3x$$

$$du = 3dx$$

$$= \frac{1}{3} \int_0^3 e^{-u} du$$

$$= -\frac{1}{3} e^{-u} \Big|_0^3$$

$$= \frac{1}{3} - \frac{1}{3} e^{-3}$$

$$5) \int \frac{e^x}{1+e^{2x}} dx = \int \frac{1}{1+u^2} du$$

$$u = e^x$$

$$du = e^x dx$$

$$= \arctan u + C$$

$$= \arctan e^x + C$$

$$6) \int \frac{\ln x}{x} dx = \int u du$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$= \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$$

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$$7) \int_0^{\pi} \cos^2 x \sin x dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$= -\int_1^{-1} u^2 du$$

$$= -\left. \frac{u^3}{3} \right|_1^{-1}$$

$$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$8) \int \cos^2 x \sin^3 x dx$$

$$= \int \cos^2 x \sin^2 x \sin x dx$$

$$= \int \cos^2 x (1 - \cos^2 x) \sin x dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$= -\int u^2 (1 - u^2) du$$

$$= -\int u^2 - u^4 du$$

$$= -\left[ \frac{u^3}{3} - \frac{u^5}{5} + C \right]$$

$$= -\frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C$$

$$\begin{aligned}
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 9) \int_0^3 x e^{-x^2} dx &= -\frac{1}{2} \int_0^{-9} e^u du & u &= -x^2 \\
 & & du &= -2x dx \\
 &= -\frac{1}{2} e^u \Big|_0^{-9} \\
 &= -\frac{1}{2} - \frac{1}{2} e^{-9}
 \end{aligned}$$

$$\begin{aligned}
 10) \int \tan x dx &= \int \frac{\sin x}{\cos x} dx & u &= \cos x \\
 & & du &= -\sin x dx \\
 &= -\int \frac{1}{u} du \\
 &= -\ln|u| + C \\
 &= -\ln|\cos x| + C
 \end{aligned}$$

$$\begin{aligned}
 11) \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx &= 2 \int \cos u du & u &= \sqrt{x} = x^{1/2} \\
 & & du &= \frac{1}{2} x^{-1/2} dx = \frac{1}{2} \frac{1}{\sqrt{x}} dx \\
 &= 2 \sin u + C \\
 &= 2 \sin \sqrt{x} + C
 \end{aligned}$$

$$\begin{aligned}
 12) \int \frac{2x}{\sqrt{1-x^4}} dx &= \int \frac{1}{\sqrt{1-u^2}} du & u &= x^2 \\
 & & du &= 2x \\
 &= \arcsin u + C \\
 &= \arcsin(x^2) + C
 \end{aligned}$$

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$$13) \int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = 2 \int \frac{1}{u} du$$

$$= 2 \ln|u| + C$$

$$= 2 \ln|1+\sqrt{x}| + C$$

$$u = 1 + \sqrt{x} = 1 + x^{1/2}$$

$$du = \frac{1}{2} x^{-1/2} dx$$

$$= \frac{1}{2} \frac{1}{\sqrt{x}} dx$$

$$14) \int \sec x dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$= \int \frac{1}{u} du$$

$$= \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$u = \sec x + \tan x$$

$$du = (\sec x \tan x + \sec^2 x) dx$$