Expected Value!

Week 5 Lecture 2

Math 122L Section 2

Pdf

- If Z is a random variable and x a real number, then p(x) = P(Z = x) is called a pdf

- Consider the experiment: roll a single die until a 6. Let X = number of flips until a 6 appears. What are the possible values of X?

P82.

- p(1) = $\frac{1}{6}$

- p(2) = $\frac{5}{6} \cdot \frac{1}{6}$

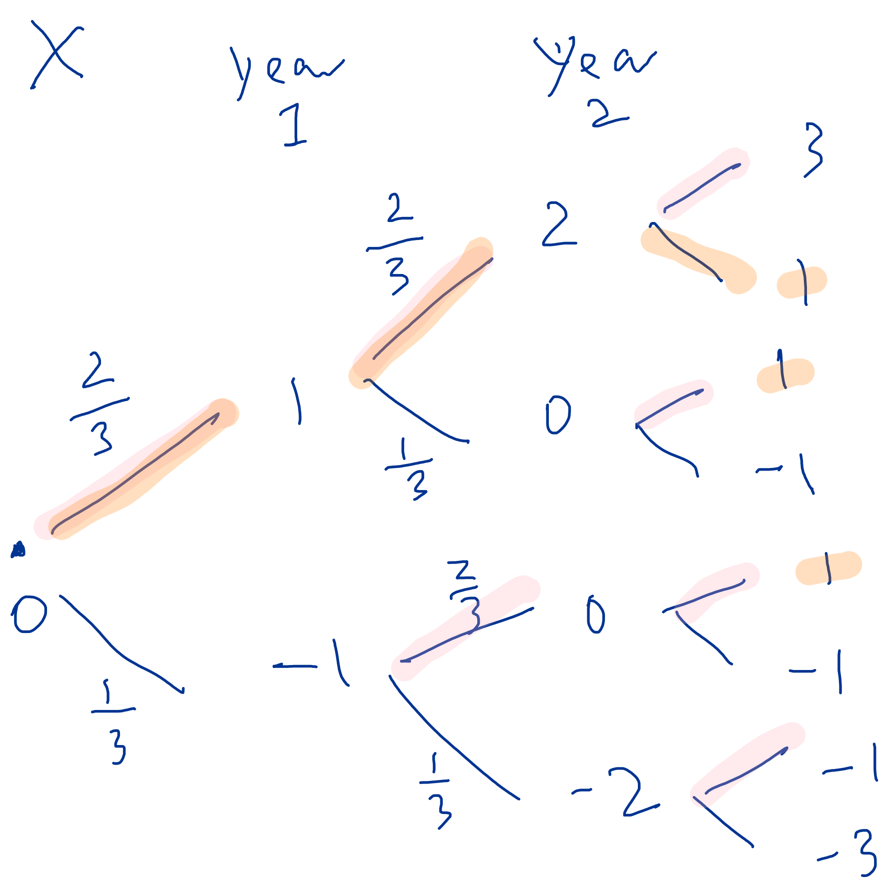
- p(3) = $\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$

- p(n) = $\left(\frac{5}{6}\right)^{n-1} \cdot \frac{1}{6}$.

- Sum $\sum_{k=1}^{\infty} p(k) = \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \cdots$

$= 1.$

CP #6



$$X = \{3, 1, -1, -3\}$$

$$\mathbb{P}(X=3) = \left(\frac{2}{3}\right)^3$$

$$\mathbb{P}(X=-3) = \left(\frac{1}{3}\right)^3$$

$$\mathbb{P}(X=1) = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}$$

$$\mathbb{P}(X=-1) = 1 - \left[\mathbb{P}(X=3) + \mathbb{P}(X=-3) + \mathbb{P}(X=1)\right].$$

CP #7

$$\{\overset{3}{HHH}, \overset{1}{HHT}, \overset{1}{HTH}, \overset{1}{THH}, \overset{-1}{THT}, \overset{-1}{TTH}, \overset{-1}{HTT}, \overset{-3}{TTT}\}$$ sample space

range of $X = \{3, 1, -1, -3\}$.

$X(HHH) = 3$

$X(HHT) = 1$

$p(a) = \mathbb{P}(X = a)$

$\mathbb{P}(X = -1) = \frac{3}{8}$

$\mathbb{P}(X = 1) = \frac{3}{8}$

$\mathbb{P}(X = 3) = \frac{1}{8}$

$\mathbb{P}(X = -3) = \frac{1}{8}$.

Group exercises

P87 #1. #2.

#1 :

① $\dfrac{2+3+3+3+4+4+\cdots+6+6+6+6}{19}$

② $\dfrac{2+3\cdot 3+4\cdot 6+5\cdot 5+6\cdot 4}{19}$

#2. $\dfrac{x_1\cdot p_1+x_2p_2+\cdots x_n\cdot p_n}{n}$

$p_1+p_2+\cdots+p_n$

if $p_1, p_2 \cdots p_n$ are probabilities

then this becomes

$x_1p_1+x_2p_2+\cdots x_np_n$

$= \sum_{k=1}^{n} p_k x_k$.

Expected value

- If p1, p2, … , pn represent the probabilities associated, respectively, with the outcomes x1, …, xn of a random variable, then we call
    - expectation
    - expected value
    - mean
    - average value
- if p(x) is the probability density function for a random variable X, we call $\sum_{\text{all } x} x\,p(x).$ the expected value of x and use the notation $\mathbb{E}(X)$.

CP #4

$P(2 \leq X \leq 5) =$

1	2	3	4	5	6	7
0.52	.27	.11	.05	.02	.02	.01

CP #5

$$\mathbb{P}(Z=0) = \left(\frac{10}{16}\right)^3$$

$$\mathbb{P}(Z=1) = 3 \cdot \left(\frac{6}{16}\right)\left(\frac{10}{16}\right)^2$$

$$\mathbb{P}(Z=2) = 3 \cdot \left(\frac{6}{16}\right)^2\left(\frac{10}{16}\right)$$

$$\mathbb{P}(Z=3) = \left(\frac{6}{16}\right)^3.$$

$$0 \cdot \mathbb{P}(Z=0) + 1 \cdot \mathbb{P}(Z=1) + 2 \cdot \mathbb{P}(Z=2) + 3 \cdot \mathbb{P}(Z=3).$$

CP #6

Let price be P.

X = net profit.

Sample space $\{\phi, \ , \ , \ \}$.

$\mathbb{P}(\phi) = (.8)^3$ $\quad X(\phi) = 3\cdot P - 200$

$\mathbb{P}(\ ) = (.8)^2\cdot(0.2)\cdot \underline{\underline{3}}$ $\quad X(\ ) = 3\cdot P - 4000 - 200$

$\mathbb{P}(\ ) = 3\cdot(0.2)^2(0.8)$ $\quad X(\ ) = 3\cdot P - 8000 - 200$

$\mathbb{P}(\ ) = (.2)^3.$ $\quad X(\ ) = 3\cdot P - 12000 - 200 + 3000$

$P(X = 3\cdot P - 200) = (.8)^3$

$$\mathbb{E}(X) = (3P-200)\cdot(.8)^3 + (3P-4200)\cdot(.8)^2\cdot(.2)\cdot 3 + (3P-8200)\cdot 3\cdot(.2)^2(.8) + (3P-9200)\times(.2)^3 = 500$$

Y = net profit of Rae.

$Y(\phi) = 200$

$Y(\ ) = 200$

$Y(\ ) = 200$

$Y(\ ) = 200 - 3000$