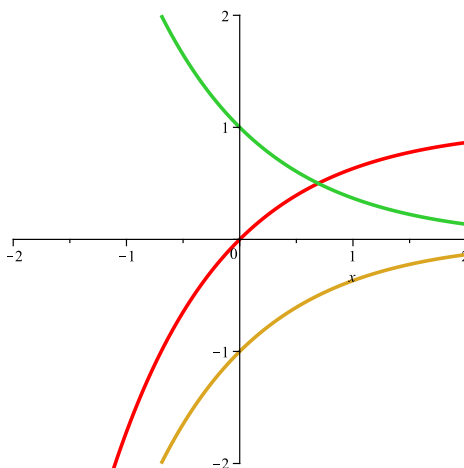


Math 122L - Brief Review of Prerequisites

1. What is the **definition** of the derivative of f at x ?
2. Use the **definition** of the derivative to derive $f'(x)$ for $f(x) = \frac{1}{\sqrt{1+x}}$.
3. On the graph below, identify which graph is f , f' and f'' . Explain how you know.



4. Find the line tangent to the function $f(x) = xe^{kx}$ at $x = 0$. Assuming that $k > 0$, does the linear approximation underestimate or overestimate xe^{kx} near 0? Explain your answer carefully.
5. The table below gives the values of the functions $f(x)$ and $g(x)$ at specified values of x .

x	1	2	3	4	5
f(x)	0	3	6	8	2
g(x)	1	4	5	2	0

- (a) Using the table, estimate the value of the derivative of $f(g(x))$ at $x = 2$.
 - (b) Using the table, estimate the value of the derivative of $g(f(x))$ at $x = 2$.
6. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take t years to pay off. What are the units of $P'(t)$? Is $P'(t)$ positive or negative? Explain.

7. Let

$$f(x) = \begin{cases} c^x + x & \text{if } x < 1 \\ x^c + 2 & \text{if } x \geq 1 \end{cases}$$

Answer the following without using a graphing calculator.

- (a) Define what it means for a function, $g(x)$, to be continuous at the point $x = a$.
 - (b) What value(s) of c make $f(x)$ continuous?
 - (c) Define what it means for a function, $g(x)$, to be differentiable at the point $x = a$.
 - (d) For this value(s) of c that you found in part (b), is $f(x)$ differentiable? Why or why not?
8. If it is possible, draw a graph of a continuous function, f , that satisfies the following conditions:
 f has a horizontal asymptote at 3
 $f(1) = 3$
 $f'(x) > 0$ for $x < 2$, and $f'(x) < 0$ for $x > 2$
If it is not possible, explain why.
9. Find the equation of the line tangent to $y = f(x)$ at $(3, 2)$ if $xy + y^2 = 10$.
10. The position of a particle (in centimeters) at time t (in seconds) is $s(t) = \frac{1}{3}t^3 - 5t^2 + 24t$.
- (a) When is the particle at rest?
 - (b) When is the particle moving to the right?
 - (c) When is the particle speeding up?
 - (d) Find the total distance traveled by the particle over the interval $0 \leq t \leq 10$.
11. Find the following limits, or state that they do not exist, noting that a , b , c , and d are constants greater than 1. Make sure to justify your answers (not with a calculator).
- (a) $\lim_{x \rightarrow \infty} \frac{4a^{-x} + 2b}{3c + d^{-2x}}$
 - (b) $\lim_{x \rightarrow c^-} \frac{|x - c|}{2x - 2c}$
 - (c) $\lim_{x \rightarrow -a} \frac{x^2 - a^2}{(x)(x + a)}$
12. Which point(s) on the graph of $f(x) = \frac{1}{\sqrt{x}}$, for $x > 0$, is closest to $(0, 0)$?