Practice with DEQ

#1. $e^x + c$ satisfy $y' = y$

plug in and check $y' = e^x$

$y = e^x + c$

for $e^x = e^x + c$, we must have $c = 0$.

#2. Let $p(t)$ denote concentration of the drug at time t.

$$\left.\begin{array}{l}\frac{dP}{dt} = -.2P \\ P(0) = 10\end{array}\right\} \Rightarrow \begin{array}{l}P(t) = 10 \cdot e^{-.2t} \\ P(12) = 10\,e^{-2.4t}\end{array}$$

#3. Let $p(t)$ denote concentration of the drug at time t.

$$\left.\begin{array}{l}\frac{dP}{dt} = kP \quad k<0 \\ P(0) = 12 \\ P(2) = 6.5\end{array}\right\} \Rightarrow \begin{array}{l}P(t) = 12e^{kt} \\ 6.5 = 12e^{2k} \Rightarrow k = \ln\left(\frac{6.5}{12}\right)^{\frac{1}{2}}\end{array}$$

$$\Rightarrow P(12) = 12e^{12 \cdot \ln\left(\frac{6.5}{12}\right)^{\frac{1}{2}}} = 12 \cdot \left(\frac{6.5}{12}\right)^6$$

#4. Let $P(t)$ denote # of bacteria.

$$\left.\begin{array}{l}\frac{dP}{dt} = kP = 2.3P \\ P(0) = 20\end{array}\right\} \Rightarrow P(t) = 20e^{2.3t}$$

#5. Let $P(t)$ denote # of bacteria

$$\left.\begin{array}{l}\frac{dP}{dt} = kP \\ P(0) = 30 \\ P(2) = 342\end{array}\right\} \Rightarrow \begin{array}{l}P(t) = 30e^{kt} \\ 342 = 30e^{2k} \Rightarrow k = \ln\left(\frac{342}{30}\right)^{\frac{1}{2}}\end{array}$$

$$\Rightarrow P(10) = 12e^{10 \cdot \ln\left(\frac{342}{30}\right)^{\frac{1}{2}}} = 12\left(\frac{342}{30}\right)^5$$

#6. $y'(t) = 2y(t) - 3 \qquad y(0) = 2$

(a)

Let $v = 2y - 3 = y'$

$v' = 2y' = 2v$.

Then $v = Ce^{2t} \qquad y = \frac{Ce^{2t} + 3}{2}$

plug in $y(0) = 2 \qquad y(0) = \frac{C+3}{2} = 2 \qquad C = 1$

$\therefore \; y = \frac{e^{2t} + 3}{2}$

(b) $y'(t) = 2y(t) - 3 \qquad y(0) = 1$.

From above $y = \frac{Ce^{2t} + 3}{2}$

plug in $y(0) = 1 \qquad y(0) = \frac{C+3}{2} = 1 \qquad C = -1$

$\therefore \; y = \frac{-e^{2t} + 3}{2}$

(C). $y'(t) = -2y(t) + 3 \qquad y(0) = 10$

Let $v = -2y(t) + 3 = y'$

$v' = -2y' = -2v$.

$\therefore \; v = Ce^{-2t} \qquad y = \frac{3 - v}{2} = \frac{3 - Ce^{-2t}}{2}$

plug in $y(0) = 10$.

$y(0) = \frac{3 - C}{2} = 10 \qquad C = -17$

$\therefore \; y = \frac{3 + 17e^{-2t}}{2}$

#7 Let $T(t)$ denote Temperature of the iron at time t.

$$\frac{dT}{dt} = -3(T-60)$$

$$T(0) = 1000$$

Let $u = -3(T-60) = T'$.

then $u' = -3T' = -3u$. $\quad u = ce^{-3t}$.

Then $T = -\frac{c}{3}e^{-3t} + 60$

$$T(0) = -\frac{c}{3} + 60 = 1000 \Rightarrow -\frac{c}{3} = 940$$

$$\Rightarrow T(t) = 940e^{-3x} + 60.$$

#8. $T(t)$ denote Temperature of the dorm

$$\frac{dT}{dt} = k(T-35)$$

$T(0) = 76$

$T(1) = 65$.

$T(7) = ?$

$\Rightarrow$

Let $u = k(T-35) = T'$

$u' = kT' = ku$.

$u = ce^{kt}$.

Then $T = \frac{c}{k}e^{kt} + 35$

$T(0) = 76 \Rightarrow \frac{c}{k} = 41$.

$$\Rightarrow \begin{cases} T = 41e^{kt} + 35 \\ T(1) = 65 \end{cases} \Rightarrow 30 = 41e^{k} \Rightarrow k = \ln\left(\frac{30}{41}\right)$$

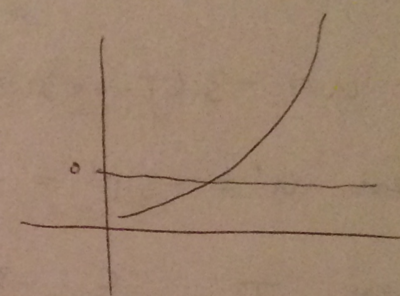
$$\therefore T = 41 \cdot \left(\frac{30}{41}\right)^{t} + 35.$$

#9.

$$\frac{dN}{dt} = 0.03\,N(t) + 35$$

$$N(0) = 800 \qquad \Rightarrow \qquad N = 1966.67\,e^{.03t} - 1166.67.$$

Set $N(t) = 0$ and solve for t.



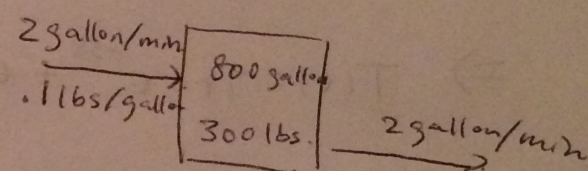
$$1966.67\,e^{.03t} = 1166.67$$

$$e^{.03t} = \frac{1166.67}{1966.69}$$

$$.03t = \ln\left(\frac{1166.67}{1966.69}\right)$$

$$t = \frac{1}{.03}\ln\left(\frac{1166.67}{1966.69}\right)$$

#10.



$$\frac{dS}{dt} = \text{rate in} - \text{rate out}$$

$$= .2 - \frac{S(t)}{800}\,.2$$

$$S(0) = 300$$

$$S(t) = 220\,e^{-0.0025t} + 80.$$

as $t \to \infty$ $\quad S(t) \to 80.$

#11.

$$\frac{dB}{dt} = 1.6 \cdot B - 800.$$

$$\Rightarrow \quad B(t) = Ce^{1.6t} + 500$$

$B(0) = B_0$

plug in $B(0) = B_0 \Rightarrow$

$$B_0 = C + 500.$$

$$C = B_0 - 500.$$

$$\Rightarrow \quad B(t) = (B_0 - 500)e^{1.6t} + 500.$$

Note if $B_0 < 500$, then $B(t) = 0$ for some t.

#12. $P(t)$ denote the fish population

$$\frac{dP}{dt} = -.04P + 2000.$$

$$\Rightarrow \quad P(t) = Ce^{-.04x} + 50000$$

$\therefore$ as $t \to \infty$. $P(t) = 50,000.$