1 Review

Definition Let $f(x)$ be a continuous function on $[a, b]$. Then the **definite integral** of f from a to b is defined as

$$\int_a^b f(x)\,dx =$$

if the limit exists, where $\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x$, and $x_k^* \in [x_{k-1}, x_k]$. The definite integral is the area under the graph of f from a to b.

Example Give three other ways to equivalently define the integral.

Mean Value Theorem: If f is a differentiable function on the interval $[a, b]$, then there exists a number c between a and b such that

$$f'(c) = \_\_\_\_\_$$

2 Fundamental Theorem of Calculus

Fundamental Theorem of Calculus (Evaluation Theorem): If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = \_\_\_\_\_$$

where F is any antiderivative of f, that is, _____.

Proof:

1. Divide the interval $[a, b]$ into n subintervals with endpoints

   $$a = x_0, x_1, x_2, \ldots, x_n = b.$$

   The distance between subsequent x_k's is _____.

2. Let F be an antiderivative of f.

3. We see that

   $$F(b) - F(a) =$$
   $$=$$
   $$= \sum$$

4. Since F is differentiable, it is _____.

5. By the Mean Value Theorem, for each $[x_{k-1}, x_k]$

6. Therefore

   $$F(b) - F(a) =$$

7. Taking the limit as $n \to \infty$ of either side of the above, we see that

The Fundamental Theorem of Calculus:
If $f(x)$ is a continuous function on the interval $[a, b]$ and $f(t) = F'(t)$, then

$$\int_a^b f(t)\ dt = F(b) - F(a)$$

Questions

1. A function has many antiderivatives. If F and G are antiderivatives of $f(x)$, then what is true about $F(x) - G(x)$?

2. The Fundamental Theorem of Calculus tell us that

$$\int_a^b f(x)\,dx = F(b) - F(a) \text{ and } \int_a^b f(x)\,dx = G(b) - G(a).$$

   Does this mean that there is more than one area under the curve? Why or why not?

3. Why does the Fundamental Theorem of Calculus not imply that

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = -\frac{1}{x}\bigg|_{-1}^{1} = -\frac{1}{1} - \left(-\frac{1}{-1}\right) = -2?$$

   Why does this not make sense?

4. Why can't we use FTC to compute $\displaystyle\int_1^3 e^{-x^2}\,dx$?