1 Review

Fundamental Theorem of Calculus I: If $f(x)$ is continuous on the interval $[a,b]$ and $F(x)$ is an antiderivative of $f(x)$, then

Extreme Value Theorem: If $f(t)$ is continuous on the closed interval $[a,b]$, then it has a ______________________________ and a ______________________________ on that interval.

Question If $m \leq f(t) \leq M$ for $a \leq t \leq b$, then

$$\leq \int_a^b f(t)dt \leq$$

2 Fundamental Theorem of Calculus

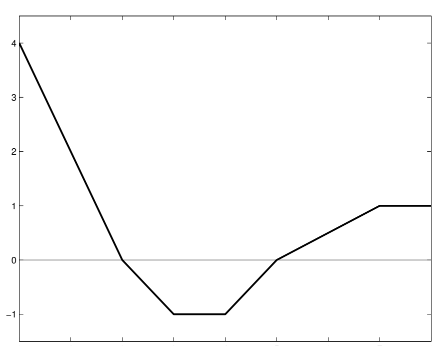
Let $g(x) = \int_0^x f(t)dt$ for a continuous function $f(t)$.

1. What is $g(1)$ geometrically?

2. What is $g(2)$ geometrically?

3. What is $g(x)$ geometrically?

Below is a graph of $h(x)$. Sketch a graph of $k(x) = \int_0^x h(t)dt$. Do you see another relationship between $h(x)$ and $k(x)$?



Suppose $f(t)$ is a continuous function and $g(x) = \displaystyle\int_a^x f(t)dt$. Then

$$\frac{g(x+h)-g(x)}{h} = \frac{\qquad\qquad\qquad\qquad}{h} = \frac{\qquad\qquad\qquad\qquad}{h}$$

Since $f(t)$ is continuous on $[x, x+h]$, ____ tells us it attains a smallest value m and a largest value M. Then,

$$\leq \int_x^{x+h} f(t)dt \leq$$

Let's draw the picture.

Going back, dividing everything by h, we get

$$\leq \frac{\int_x^{x+h} f(t)dt}{h} \leq$$

As $h \to 0$, what happens to the interval $[x, x+h]$?

As $h \to 0$, what happens to m and M?

Therefore, $\frac{d}{dx}\big[g(x)\big] = \lim_{h\to 0} \frac{\int_x^{x+h} f(t)\, dt}{h} =$ ________________.

The Second Fundamental Theorem of Calculus
Let f be continuous on an interval. Then for x and a in that interval

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Exercise

1. Write down an antiderivative for e^{-x^2} and check it.

2. Find the following derivative in two different ways: $\frac{d}{dx}\int_2^x \cos t dt$

3. Let $g(x) = \int_1^x \sqrt{1+t^2}\,dt$.

   What is $g'(x)$?

   What is $g(x^3)$?

   What is $\frac{d}{dx} g(x^3)$?

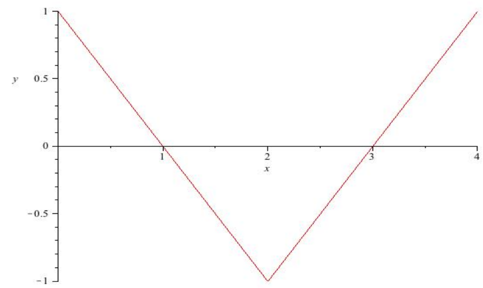
4. Find a function $g(x)$ such that $g'(x) = \sqrt{1+x^2}$ and $g(2) = 0$.

5. Find a function $g(x)$ such that $g'(x) = \sqrt{1+x^2}$ and $g(2) = 10$.

6. Let $g(x) = \int_0^x f(t)\,dt$ where f is continuous.
   If $f(t) > 0$ and $x > 0$, then $g(x)$ is ____________________ and ____________________ .
   If $f(t) > 0$ and $x < 0$, then $g(x)$ is ____________________ and ____________________ .
   If $f(t) < 0$ and $x > 0$, then $g(x)$ is ____________________ and ____________________ .

7. Find $\frac{d}{dx} \int_{x^2}^3 \frac{\sin t}{t}\,dt$.

8. Consider the function $f(x)$, defined by the graph below. Let $F(x) = \int_0^x f(t)\,dt$ for $x \in [0, 4]$.



(a) Find $F(0), F(1), F(2), F(3,),$ and $F(4)$.

x	$F(x)$
0	
1	
2	
3	
4	

(b) Find the average value of $f(x)$ on the interval $[0, 4]$.

(c) Find the critical points of $F(x)$ on the interval $(0, 4)$ making sure to label each as either a maximum or a minimum.