True-False Quiz

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. $\displaystyle\lim_{x\to 4}\left(\frac{2x}{x-4}-\frac{8}{x-4}\right)=\lim_{x\to 4}\frac{2x}{x-4}-\lim_{x\to 4}\frac{8}{x-4}$

2. $\displaystyle\lim_{x\to 1}\frac{x^2+6x-7}{x^2+5x-6}=\frac{\lim_{x\to 1}(x^2+6x-7)}{\lim_{x\to 1}(x^2+5x-6)}$

3. $\displaystyle\lim_{x\to 1}\frac{x-3}{x^2+2x-4}=\frac{\lim_{x\to 1}(x-3)}{\lim_{x\to 1}(x^2+2x-4)}$

4. If $\lim_{x\to 5} f(x)=2$ and $\lim_{x\to 5} g(x)=0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

5. If $\lim_{x\to 5} f(x)=0$ and $\lim_{x\to 5} g(x)=0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

6. If $\lim_{x\to 6}[f(x)g(x)]$ exists, then the limit must be $f(6)g(6)$.

7. If p is a polynomial, then $\lim_{x\to b} p(x)=p(b)$.

8. If $\lim_{x\to 0} f(x)=\infty$ and $\lim_{x\to 0} g(x)=\infty$, then $\lim_{x\to 0}[f(x)-g(x)]=0$.

9. A function can have two different horizontal asymptotes.

10. If f has domain $[0,\infty)$ and has no horizontal asymptote, then $\lim_{x\to\infty} f(x)=\infty$ or $\lim_{x\to\infty} f(x)=-\infty$.

11. If the line $x=1$ is a vertical asymptote of $y=f(x)$, then f is not defined at 1.

12. If $f(1)>0$ and $f(3)<0$, then there exists a number c between 1 and 3 such that $f(c)=0$.

13. If f is continuous at 5 and $f(5)=2$ and $f(4)=3$, then $\lim_{x\to 2} f(4x^2-11)=2$.

14. If f is continuous on $[-1,1]$ and $f(-1)=4$ and $f(1)=3$, then there exists a number r such that $|r|<1$ and $f(r)=\pi$.

15. If f is continuous at a, then f is differentiable at a.

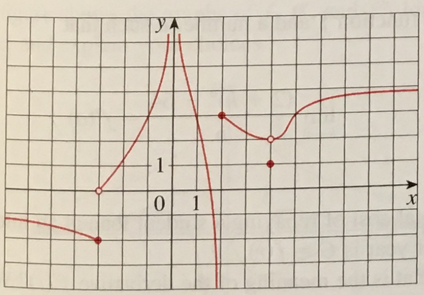
16. If $f'(r)$ exists, then $\lim_{x\to r} f(x)=f(r)$.

17. $\displaystyle\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

18. If $f(x)>1$ for all x and $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x)>1$.

Exercises

1. The graph of f is given.
   (a) Find each limit, or explain why it does not exist.
   (i) $\lim_{x\to 2^+} f(x)$ (ii) $\lim_{x\to -3^+} f(x)$
   (iii) $\lim_{x\to -3} f(x)$ (iv) $\lim_{x\to 4} f(x)$
   (v) $\lim_{x\to 0} f(x)$ (vi) $\lim_{x\to 2^-} f(x)$
   (vii) $\lim_{x\to\infty} f(x)$ (viii) $\lim_{x\to -\infty} f(x)$
   (b) State the equations of the horizontal asymptotes.
   (c) State the equations of the vertical asymptotes.
   (d) At what numbers is f discontinuous? Explain.



2. Sketch the graph of an example of a function f that satisfies all of the following conditions:
$$\lim_{x\to -\infty} f(x)=-2,\quad \lim_{x\to\infty} f(x)=0,\quad \lim_{x\to -3} f(x)=\infty,$$
$$\lim_{x\to 3^-} f(x)=-\infty,\quad \lim_{x\to 3^+} f(x)=2,$$
f is continuous from the right at 3

3–18 Find the limit.

3. $\displaystyle\lim_{x\to 1} e^{x^3-x}$

4. $\displaystyle\lim_{x\to 3}\frac{x^2-9}{x^2+2x-3}$

5. $\displaystyle\lim_{x\to -3}\frac{x^2-9}{x^2+2x-3}$

6. $\displaystyle\lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3}$

7. $\displaystyle\lim_{h\to 0}\frac{(h-1)^3+1}{h}$

8. $\displaystyle\lim_{t\to 2}\frac{t^2-4}{t^3-8}$

9. $\displaystyle\lim_{r\to 9}\frac{\sqrt{r}}{(r-9)^4}$

10. $\displaystyle\lim_{v\to 4^+}\frac{4-v}{|4-v|}$

11. $\displaystyle\lim_{u\to 1}\frac{u^4-1}{u^3+5u^2-6u}$

12. $\displaystyle\lim_{x\to 3}\frac{\sqrt{x+6}-x}{x^3-3x^2}$

13. $\displaystyle\lim_{x\to\pi^-}\ln(\sin x)$

14. $\displaystyle\lim_{x\to -\infty}\frac{1-2x^2-x^4}{5+x-3x^4}$

15. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2-9}}{2x-6}$

16. $\displaystyle\lim_{x\to\infty} e^{x-x^2}$

17. $\displaystyle\lim_{x\to\infty} \left(\sqrt{x^2+4x+1}-x\right)$

18. $\displaystyle\lim_{x\to 1} \left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right)$

19–20 Use graphs to discover the asymptotes of the curve. Then prove what you have discovered.

19. $y = \dfrac{\cos^2 x}{x^2}$

20. $y = \sqrt{x^2+x+1} - \sqrt{x^2-x}$

21. If $2x - 1 \le f(x) \le x^2$ for $0 < x < 3$, find $\lim_{x\to 1} f(x)$.

22. Prove that $\lim_{x\to 0} x^2 \cos(1/x^2) = 0$.

23. Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.

(i) $\displaystyle\lim_{x\to 0^+} f(x)$ (ii) $\displaystyle\lim_{x\to 0^-} f(x)$ (iii) $\displaystyle\lim_{x\to 0} f(x)$

(iv) $\displaystyle\lim_{x\to 3^-} f(x)$ (v) $\displaystyle\lim_{x\to 3^+} f(x)$ (vi) $\displaystyle\lim_{x\to 3} f(x)$

(b) Where is f discontinuous?

(c) Sketch the graph of f.

24. Show that each function is continuous on its domain. State the domain.

(a) $g(x) = \dfrac{\sqrt{x^2-9}}{x^2-2}$ (b) $h(x) = xe^{\sin x}$

25–26 Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

25. $2x^3 + x^2 + 2 = 0, \quad (-2, -1)$

26. $e^{-x^2} = x, \quad (0, 1)$

27. The displacement (in meters) of an object moving in a straight line is given by $s = 1 + 2t + \frac{1}{4}t^2$, where t is measured in seconds.

(a) Find the average velocity over each time period.

(i) $[1, 3]$ (ii) $[1, 2]$

(iii) $[1, 1.5]$ (iv) $[1, 1.1]$

(b) Find the instantaneous velocity when $t = 1$.

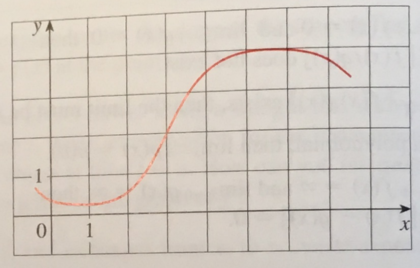
28. According to Boyle's Law, if the temperature of a confined gas is held fixed, then the product of the pressure P and the volume V is a constant. Suppose that, for a certain gas, $PV = 800$, where P is measured in pounds per square inch and V is measured in cubic inches.

(a) Find the average rate of change of P as V increases from 200 in^3 to 250 in^3.

(b) Express V as a function of P and show that the instantaneous rate of change of V with respect to P is inversely proportional to the square of P.

29. For the function f whose graph is shown, arrange the following numbers in increasing order:

$0 \qquad 1 \qquad f'(2) \qquad f'(3) \qquad f'(5) \qquad f''(5)$



30. (a) Use the definition of a derivative to find $f'(2)$, where $f(x) = x^3 - 2x$.

(b) Find an equation of the tangent line to the curve $y = x^3 - 2x$ at the point $(2, 4)$.

(c) Illustrate part (b) by graphing the curve and the tangent line on the same screen.

31. (a) If $f(x) = e^{-x^2}$, estimate the value of $f'(1)$ graphically and numerically.

(b) Find an approximate equation of the tangent line to the curve $y = e^{-x^2}$ at the point where $x = 1$.

(c) Illustrate part (b) by graphing the curve and the tangent line on the same screen.

32. Find a function f and a number a such that

$$\lim_{h\to 0} \frac{(2+h)^6 - 64}{h} = f'(a)$$

33. The total cost of repaying a student loan at an interest rate of r% per year is $C = f(r)$.

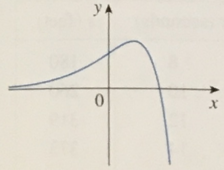
(a) What is the meaning of the derivative $f'(r)$? What are its units?

(b) What does the statement $f'(10) = 1200$ mean?

(c) Is $f'(r)$ always positive or does it change sign?

34–36 Trace or copy the graph of the function. Then sketch a graph of its derivative directly beneath.

34.



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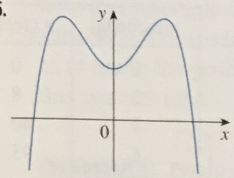

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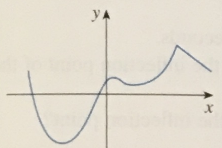
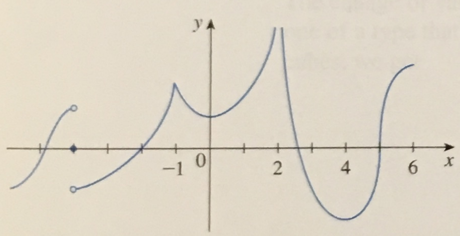


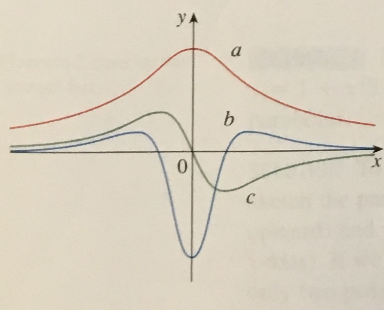
37. (a) If $f(x) = \sqrt{3 - 5x}$, use the definition of a derivative to find $f'(x)$.
(b) Find the domains of f and f'.
(c) Graph f and f' on a common screen. Compare the graphs to see whether your answer to part (a) is reasonable.

38. (a) Find the asymptotes of the graph of $f(x) = \dfrac{4 - x}{3 + x}$ and use them to sketch the graph.
(b) Use your graph from part (a) to sketch the graph of f'.
(c) Use the definition of a derivative to find $f'(x)$.
(d) Use a graphing device to graph f' and compare with your sketch in part (b).

39. The graph of f is shown. State, with reasons, the numbers at which f is not differentiable.



40. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.

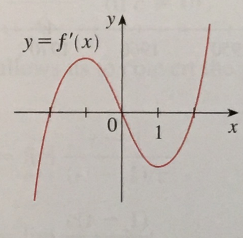


41. Let $C(t)$ be the total value of US currency (coins and banknotes) in circulation at time t. The table gives values of this function from 1980 to 2000, as of September 30, in billions of dollars. Interpret and estimate the value of $C'(1990)$.

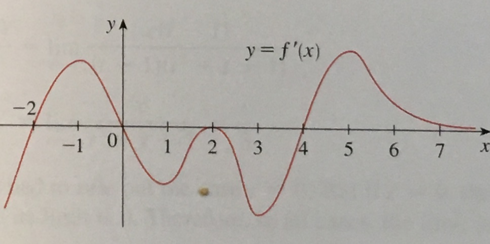
t	1980	1985	1990	1995	2000
$C(t)$	129.9	187.3	271.9	409.3	568.6

42. The cost of living continues to rise, but at a slower rate. In terms of a function and its derivatives, what does this statement mean?

43. The graph of the *derivative* f' of a function f is given.
(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f have a local maximum or minimum?
(c) Where is f concave upward or downward?
(d) If $f(0) = 0$, sketch a possible graph of f.



44. The figure shows the graph of the *derivative* f' of a function f.
(a) Sketch the graph of f''.
(b) Sketch a possible graph of f.



45. Sketch the graph of a function that satisfies the given conditions:

$f(0) = 0, \quad f'(-2) = f'(1) = f'(9) = 0,$

$\lim_{x \to \infty} f(x) = 0, \quad \lim_{x \to 6} f(x) = -\infty,$

$f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$, and $(9, \infty)$,

$f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$,

$f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$,

$f''(x) < 0$ on $(0, 6)$ and $(6, 12)$