1 Review

The Chain Rule tells us

From this, we saw that if g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

We called this **u-substitution**.

2 Integration by Parts

From the chain rule for differentiation, we learned a new technique for integrating. Today, we will see that we can similarly get a new technique for integration from looking at the product rule for differentiation.

Product Rule: If f and g are differentiable, then

This tells us that

$$\int [f(x)g'(x) + g(x)f'(x)]\,dx = \underline{\hspace{8cm}}$$

We can then rewrite this as

$$\int f(x)g'(x)\,dx = \underline{\hspace{8cm}}.$$

If we let $u = $ ______________ and $v = $ ______________, then we have what is usually called **integration by parts**.

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Examples Evaluate each of the following:

1. $\int_0^1 xe^x \, dx$

2. $\int_0^\pi x \sin(x) \, dx$

3. $\int_1^2 \ln x \, dx$

4. $\int \arctan x \, dx$

5. $\int x \ln x \, dx$

6. $\int x^2 e^x \, dx$

7. $\int xe^{-x^2}\,dx$

8. $\int \frac{x}{e^x}\,dx$

9. $\int e^x \sin x\,dx$

10. $\int \sin^2 x\,dx = \int \sin x \sin x\,dx$ [Compare this to what we get using u-substitution.]

11. $\displaystyle\int \frac{1}{x^2+6x+9}\,dx$

12. $\displaystyle\int \frac{2x}{x^2+6x+9}\,dx$

13. $\displaystyle\int \arcsin x\,dx$

Example Let $u = \ln z$. Then $z =$ _______, so $dz =$ _______.

(a) Find $\displaystyle\int_1^b \frac{\ln(z)}{z^2}\,dz$.

(b) Find $\displaystyle\int (\ln(z))^2\,dz$.