Probability!

Week 5 Lecture 1
Math 122L Section 2

Why study probability?

- what's the weather like tomorrow?
- what are the chances of a drug working?
- is it a boy or girl?
- applications in modern scientific research, compressed sensing, random matrix theory, etc.

Important principle

P79. CP.

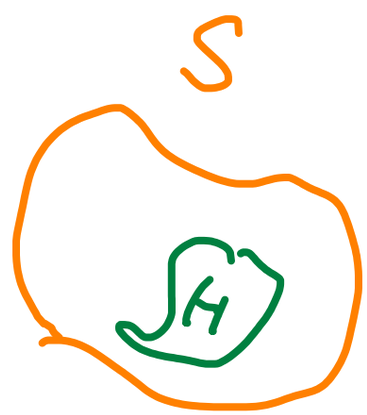
- Probability of any event is in [0,1].
- Sum of probability of all possible outcomes is 1

$$\sum_{\text{all possible}} P(\text{outcome}) = 1$$

- Definition: sample space - the set of all possible outcomes

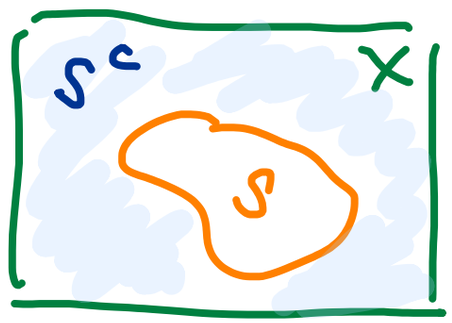
Basic set theory

- Set S: collection of objects
  - example: all the outcomes of a single six-sided die. S = {1, 2, 3, 4, 5, 6} ← sample space.
  - example: empty set ϕ
- Subset H: if every element of H is also in S

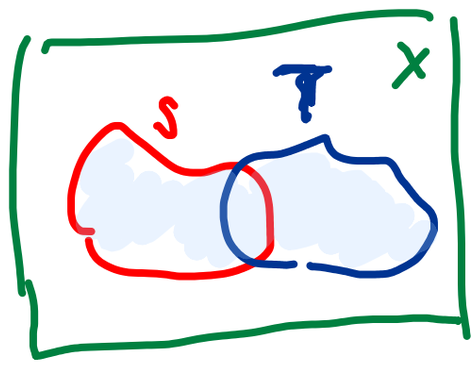


Set operations

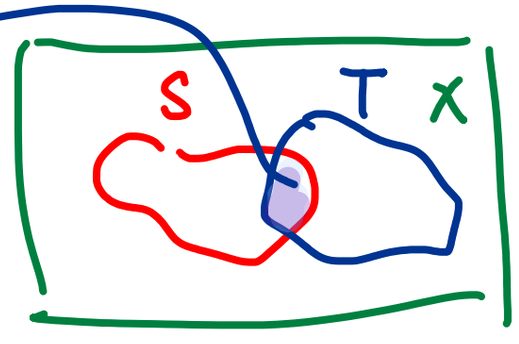
- Complement of S
  - all elements in X not in S



- Union of sets S, T
  - all elements in S or T or both



- Intersection of sets S, T
  - all elements in both S and T



Probability models

- Sample space: set of all the possible outcomes
  - can't overlap
  - must be exhaustive
- Events: subsets of sample space
- Probabilities: how likely events are

Example: roll of two dice

What's the sample space of a roll of two dice?

How many different outcomes?

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

6

$P(\text{rolling a 7}) = \frac{6}{36} = \frac{1}{6}$

5

$\star\ P(A \text{ and } B) = P(A)\,P(B)$.: = Independent

Random Variable

- S is the sample space
- A random variable, X, is a function that assigns a real number to each element of the sample space. X: S —> R

- Example:

  S = possible outcomes of flipping a coin 3 times

  = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

  3 2 2 2 1 1 1 0

  X = number of heads

  range X = {0, 1, 2, 3}.

Example: roll of two dice

Consider the random variable X = sum of two dice and whose domain is S, how many elements are in the domain of X?

What is the range of X?

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

inputs

X

range(X) = {2, 3, ... 12}.

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

outputs.

Mass density

- If Z is a random variable and x a real number, then $p(x) = P(Z = x)$ is called a
  - mass density
  - probability mass density
  - probability density function
  - pdf

Note:

1. sum $\sum_{x \in \mathbb{R}} p(x) = 1$
2. range $[0, 1]$