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12) $\int_{-1}^{1} \frac{dx}{x^2}$

$\lim_{b\to 0} \int_{-1}^{b} \frac{1}{x^2}dx = \lim_{b\to 0}\left(-\frac{1}{x}\Big|_{-1}^{b}\right) = \lim_{b\to 0}\left(-\frac{1}{b}-1\right) = \infty$

diverges

so $\int_{-1}^{1} \frac{dx}{x^2}$ diverges

or

$\lim_{b\to 0} \int_{b}^{1} \frac{1}{x^2}dx = \lim_{b\to 0}\left(-\frac{1}{x}\Big|_{b}^{1}\right) = \lim_{b\to 0}\left(\frac{1}{b}-1\right) = \infty$ diverges

So $\int_{-1}^{1} \frac{dx}{x^2}$ diverges

13) $x > \sqrt{x^2-1}$

$\frac{1}{x} < \frac{1}{\sqrt{x^2-1}}$

$\int_0^1 \frac{1}{x}dx < \int_0^1 \frac{1}{\sqrt{x^2-1}}dx$

By 11(b) we know $\int_0^1 \frac{1}{x}dx$ diverges,
so by comparison
$\int_0^1 \frac{1}{\sqrt{x^2-1}}dx$ diverges

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14 (a) $\int_0^\infty \frac{1}{e^2+2^x}\,dx$

$$0 \le \frac{1}{e^2+2^x} \le \frac{1}{2^x}$$

$$0 \le \int_0^\infty \frac{1}{e^2+2^x}\,dx \le \int_0^\infty \frac{1}{2^x}\,dx$$

$$\int_0^\infty \frac{1}{2^x}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{2^x}\,dx = \lim_{b\to\infty}\left(-\frac{1}{\ln 2}\cdot 2^{-x}\Big|_0^b\right)$$

$$= \lim_{b\to\infty}\left(-\frac{1}{\ln 2}\left(2^{-b}-1\right)\right) = \frac{1}{\ln 2}$$

converges so $\int_0^\infty \frac{1}{e^2+2^x}\,dx$ converges

See next page for actual gross computation.

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14) (a) $\int_0^\infty \frac{1}{e^2+2^x}\,dx$

$= \lim_{b\to\infty} \int_0^b \frac{1}{e^2+2^x}\,dx$ $u = e^2+2^x$

$du = (\ln 2)2^x dx$

$= \lim_{b\to\infty} \int_{1+e^2}^{2^b+e^2} \frac{1}{(\ln 2)u(u-e^2)}\,du$ $dx = \frac{1}{(\ln 2)2^x}\,du$

$= \lim_{b\to\infty} \frac{1}{(\ln 2)e^2} \int_{1+e^2}^{2^b+e^2} \frac{1}{u-e^2} - \frac{1}{u}\,du$ $= \frac{1}{(\ln 2)(u-e^2)}\,du$

$= \lim_{b\to\infty} \frac{1}{(\ln 2)e^2} \ln|u-e^2| - \ln|u| \Big|_{1+e^2}^{2^b+e^2}$ $\frac{1}{u(u-e^2)} = \frac{A}{u} + \frac{B}{u-e^2}$

$1 = A(u-e^2) + B(u)$

$= (A+B)u - Ae^2$

$= \lim_{b\to\infty} \frac{1}{(\ln 2)e^2}\Big(\ln 2^b - \ln(2^b+e^2)$ $1 = -Ae^2$ $A+B=0$

$A = -\frac{1}{e^2}$ $B = \frac{1}{e^2}$

$+ \ln(1+e^2)\Big)$

$= \frac{1}{(\ln 2)e^2}\left(\ln(1+e^2) + \lim_{b\to\infty} \ln\left(\frac{2^b}{2^b+e^2}\right)\right)$

$= \frac{1}{(\ln 2)e^2}\left(\ln(1+e^2) + \ln(1)\right)$

$= \boxed{\frac{\ln(1+e^2)}{(\ln 2)\,e^2}}$

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(4(b)) $\int_1^\infty \frac{1+\sin\theta}{\theta^2}\,d\theta$

$-1 \le \sin\theta \le 1$

$0 \le \sin\theta + 1 \le 2$

$0 \le \frac{\sin\theta+1}{\theta^2} \le \frac{2}{\theta^2}$

$0 \le \int_1^\infty \frac{\sin\theta+1}{\theta^2}\,d\theta \le \int_1^\infty \frac{2}{\theta^2}\,d\theta = 2$

by page 2 # 2

So $\int_1^\infty \frac{\sin\theta+1}{\theta^2}\,d\theta$ converges.

(c) $\int_2^\infty \frac{dx}{x(\ln(x))^2} = \lim_{b\to\infty}\int_2^b \frac{dx}{x(\ln(x))^2}$

$= \lim_{b\to\infty}\left(-\frac{1}{\ln x}\Big|_2^b\right)$

$= \lim_{b\to\infty}\left(\frac{1}{\ln 2} - \frac{1}{\ln b}\right)$

$= \frac{1}{\ln 2}$ so converges

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14(d) $\int_1^2 \frac{dx}{x(\ln(x))^2} = \lim_{b\to 1} \int_b^2 \frac{dx}{x(\ln(x))^2}$

note discontinuity at $x=0$

$= \lim_{b\to 1} \left(-\frac{1}{\ln x} \Big|_b^2 \right)$

$= \lim_{b\to 1} \left(\frac{1}{\ln(b)} - \frac{1}{\ln(2)} \right)$

$= \infty$ diverges

(e) $\int_0^1 \frac{dx}{x(\ln(x))^2} = \int_0^{1/2} \frac{dx}{x(\ln(x))^2} + \int_{1/2}^1 \frac{dx}{x(\ln(x))^2}$

$= \lim_{b\to 0} \int_b^{1/2} \frac{1}{x(\ln(x))^2}\,dx + \lim_{c\to 1} \int_{1/2}^c \frac{1}{x(\ln(x))^2}\,dx$

$= \lim_{b\to 0} \left(-\frac{1}{\ln(x)} \Big|_b^{1/2} \right) + \lim_{c\to 1} \left(-\frac{1}{\ln(x)} \Big|_{1/2}^c \right)$

$= \lim_{b\to 0} \left(\frac{1}{\ln b} - \frac{1}{\ln(1/2)} \right) + \lim_{c\to 1} \left(\frac{1}{\ln(1/2)} - \frac{1}{\ln(c)} \right)$

$= \frac{1}{\ln(1/2)} + \infty$ diverges

So $\int_0^1 \frac{dx}{x(\ln(x))^2}$ diverges.

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