HW#2

AHP 3. (a)

The person used the quotient rule instead of taking the individual derivative of the numerator and denominator

$$\lim_{x \to 0} \frac{\sin(x)}{x} \overset{LHP}{=} \lim_{x \to 0} \frac{\cos(x)}{1} = 1$$

AHP 3. (b)

LHP doesn't apply, it's not an indeterminant form

$$\lim_{x \to 0} \frac{\cos(x)}{x} = \frac{1}{0} \quad \text{limit undefined.}$$

AHP 3. (c)

1^{∞} is indeterminate value.

Observe the limit is defn of e.

let $y = \left(1 + \frac{1}{x}\right)^x$

Then $\ln y = x\left(\ln\left(1 + \frac{1}{x}\right)\right) = \dfrac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}}$.

Then $\displaystyle \lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} \overset{LHP}{=} \lim_{x \to \infty} \frac{\frac{1}{1+\frac{1}{x}} \cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}}$

$\displaystyle = \lim_{x \to \infty} \frac{1}{1 + \frac{1}{x}} = 1$ then $\displaystyle \lim_{x \to \infty} y = e$

HW # 1

AHP #2.

$$f(f^{-1}(x)) = x$$

Differentiate on both sides

$$\frac{d}{dx}[f(f^{-1}(x))] = \frac{d}{dx} x$$

$$LHS = f'(f^{-1}(x)) \cdot \frac{d}{dx}[f^{-1}(x)]$$

$$RHS = 1$$

Hence $$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$$