

1 Review

Fundamental Theorem of Calculus I: If f is continuous on the interval $[a, b]$ and F is an antiderivative of f , then

Fundamental Theorem of Calculus II: If f is continuous on an interval, then for x and a in that interval

Important: Remember that these theorems are named/numbered differently in the textbook.

2 u-Substitution

So far, to find antiderivatives we have only had limited techniques, such as

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Chain Rule: If f and g are differentiable functions, then

Question Have we found an antiderivative for some function?

Question What does that tell us about the following integral?

$$\int f'(g(x))g'(x) dx =$$

What we've done is called **u-substitution**.

The general idea, is that if g' is continuous on $[a, b]$ and f is continuous on the range of g , then to integrate

$$\int f'(g(x))g'(x) dx,$$

we define u as follows.

$$\begin{aligned} u &= \\ \frac{du}{dx} &= \\ \int f'(g(x))g'(x) dx &= \end{aligned}$$

Note that if we are computing $\int_a^b f'(g(x))g'(x) dx$, then we can either see that this equals

$$f(g(x)) \Big|_a^b \text{ or } f(u) \Big|_{g(a)}^{g(b)}.$$

Examples Evaluate each of the following.

1. $\int_2^3 2x \sin(x^2) dx$

2. $\int_0^1 x(x^2 + 1)^4 dx$

3. $\int \sin^2 x dx$

4. $\int \frac{x}{x+1} dx$

Exercises Evaluate each of the following integrals using u-substitution.

1. $\int \tan^2 x \sec^2 x dx$

2. $\int_0^1 \frac{x}{\sqrt{x^2 + 1}} dx$

3. $\int \frac{dx}{3x+1}$

6. $\int \frac{\ln x}{x} dx$

4. $\int_0^1 \frac{dx}{e^{3x}}$

7. $\int_0^\pi \cos^2 x \sin x \, dx$

5. $\int \frac{e^x}{1+e^{2x}} dx$

8. $\int \cos^2 x \sin^3 x \, dx$

$$9. \int_0^3 x e^{-x^2} dx$$

$$12. \int \frac{2x}{\sqrt{1-x^4}} dx$$

$$10. \int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

$$13. \int \frac{dx}{\sqrt{x}(1+\sqrt{x})} dx$$

$$11. \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

$$14. \int \sec x dx = \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$