

Suppose a man moves along a line with acceleration $\frac{F}{m} = 1\text{m/s}^2$. Suppose he is standing 2 meters to the right of a reference point at time 0 seconds, moving to the left at 1m/s.

1. Let's set up an initial value problem to find expressions for his position and velocity.

Let $s(t)$ denote the position of the man at time t , where t is measured in seconds and s is measured in meters. Let $s'(t)$ denote the velocity of the man at time t . Assume right is positive.

$$\begin{aligned}s''(t) &= 1 \\ s(0) &= 2 \\ s'(0) &= -2\end{aligned}$$

To find the solution for this initial value problem, we first antidifferentiate $s''(t)$. We have $s'(t) = t + b$. Plugging in $s'(0) = -2$, we have $b = -2$ and thus $s'(t) = t - 2$.

Next antidifferentiate again for $s'(t)$. We then have $s(t) = \frac{t^2}{2} - 2t + c$. Plugging in $s(0) = 2$, we have $c = 2$. Therefore, $s(t) = \frac{t^2}{2} - 2t + 2$.

2. Describe the motion of this man.

- At time 0 seconds, the man is 2 meters to the right of the reference point. It is moving to the left at a speed of 2 m/s.
- The velocity increases throughout the entire motion, since the acceleration is a positive constant.
- The man continues moves to the left with decreasing speed for 2 seconds.
- At time 2 seconds, the man comes to an instantaneous stop right at the reference point.
- Following that, the man moves to the right with increasing speed.
- The object passes the reference point at time 2 seconds.

In general, when you are asked to describe the motion on an object, follow the template below.

- Note the object's initial position, speed, and direction of motion.
- Next, note how long it moves in the initial direction, and whether it is slowing down or speeding up.
- Then write down every point at which it stops instantaneously, the subsequent direction of motion, and whether it is speeding up or slowing down following the instantaneous halt.
- Lastly, note all the times it passes the reference point (position 0), if at all.