The following questions help you think a little more about $\cos(x)$.
If $g(x) = \cos(x)$,

1. Use the definition of the derivative to show that $g'(x) = -\sin(x)$. [4pt]

   From the definition of derivatives, we have

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} \quad (1) \\ &= \lim_{h\to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \quad (1) \\ &= \cos(x) \lim_{h\to 0} \frac{\cos(h) - 1}{h} - \sin(x) \lim_{h\to 0} \frac{\sin(h)}{h} \quad (1) \\ &= -\sin(x) \end{aligned}$$

   since $\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$ and $\lim_{h\to 0} \frac{\sin(h)}{h}$ is 1. (1)

2. True of False: $g''(x) = -g(x)$. Explain your answer. [1pt]

   True, since $g''(x) = -\cos(x) = -g(x)$.

3. Graph $y = \cos(x)$ on $[-2\pi, 2\pi]$.

4. Why is $\cos(x)$ not invertible on $[-2\pi, 2\pi]$.? [1pt]

   $\cos(x)$ is not one-to-one on $[-2\pi, 2\pi]$.

5. What is the simplest domain on which $\cos(x)$ is invertible? [1pt]

   $\cos(x)$ is invertible on $[0, \pi]$.

6. Let's call the inverse of $\cos(x)$ on that domain $\cos^{-1}(x)$, or $\arccos x$. What is the domain of $\cos^{-1}(x)$? The range? [1pt]

   The domain of $\cos^{-1}(x)$ is $[-1, 1]$, and the range is $[0, \pi]$.

7. What is $\cos(\cos^{-1} x)$? For which values of x is that true? [1pt]

   $\cos(\cos^{-1} x)$ is x for x in $[-1, 1]$.

8. What is $\cos^{-1}(\cos x)$? For which values of x is that true? [1pt]

   $\cos^{-1}(\cos x)$ is x for x in $[0, \pi]$.

9. Sketch a graph of $y = \cos^{-1} x$.