We defined $\tan\theta = \frac{\sin\theta}{\cos\theta}$. The following questions help you think a little more about tan. Yes, summer is not too far away. :)

1. What is the period of $\tan x$

   The period of $\tan x$ is π. (1pt)

2. For what values of x is $\tan x$ positive? Negative? Zero? (1pt)

   $\tan x$ is 0 when $x = k\pi$ for any integer k; $\tan x$ is positive at $(0 + k\pi, \frac{\pi}{2} + k\pi)$ for any integer k; $\tan x$ is negative at $(-\frac{\pi}{2} + k\pi, 0 + k\pi)$ for any integer k.

3. Where does $\tan x$ have vertical asymptotes? (1pt)

   $\tan x$ has vertical asymptotes at $\frac{\pi}{2} + k\pi$ for any integer k.

4. Does $\lim_{x\to\frac{\pi}{2}} \tan x$ exist? Why or why not? (1pt)

   No it doesn't. Because $\tan x$ goes to positive infinity as x goes to $\frac{\pi}{2}$.

5. Does $\tan(|x|) = |\tan(x)|$? Why or why not? (1pt)

   No. $\tan(|\frac{3\pi}{4}|)$ is negative, but $|\tan(\frac{3\pi}{4})|$ is positive.

6. Show $\tan^2 x + 1 = \sec^2 x$. (3pt)

   This is equivalent to show $\frac{\sin^2(x)}{\cos^2(x)} + 1 = \frac{1}{\cos^2(x)}$ (1pt). Rewrite the left hand side, we have $\frac{\sin^2(x)+\cos^2(x)}{\cos^2(x)}$ (1pt) . Since $\sin^2(x) + \cos^2(x) = 1$, we obtained the right hand side which is $\frac{1}{\cos^2(x)}$ (1pt).

7. Show $\cos x \tan x \csc x = 1$. (2pt)

   Note that the left hand side can be written as $\cos x \frac{\sin x}{\cos x} \frac{1}{\sin(x)}$ (1pt). Simplify the fraction we get 1 (1pt).