An airplane, flying at 450 km/hr at a constant altitude of 5 km is approaching a camera mounted on the ground. Let θ be the angle of elevation above the groundd at which the camera is pointed.

1. Draw a well labeled picture of this situation and define all variables you will be using.

   The figure was drawn in class and omitted here. Suppose the camera is at point c, and the plane is vertically above point B. Let x be the distance between B and C.

2. When $\theta = \pi/3$, how fast does the camera have to rotate in order to keep the plane in view?

   We know that $\frac{dx}{dt}$ is -450 km/hr, and want to find out what is $\frac{d\theta}{dt}$. To answer this, we need to understand how x is related to θ.

   First we know, $\tan\theta = 5/x$.

   Differentiating $\tan\theta = 5/x$ with respect to t and with chain rule, we have

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = -5x^{-2}\frac{dx}{dt}$$

   We want to calculate $\frac{d\theta}{dt}$ when θ is $\frac{\pi}{3}$. At that moment $\cos\theta = 1/2$, $\tan\theta = \sqrt{3}$, so $x = \frac{5}{\sqrt{3}}$.
   Substituting gives

$$\frac{1}{(1/2)^2}\frac{d\theta}{dt} = -5(\frac{5}{\sqrt{3}})^{-2} \times (-450),$$

$$\frac{d\theta}{dt} = -20(\frac{5}{\sqrt{3}})^{-2} \times (-450) \text{ radians/hr}$$

   So the camera needs to rotate $-20(\frac{5}{\sqrt{3}})^{-2} \times (-450)$ radians per hour.