

## Week 3: Basic regression

### 4. How useful is a linear model

Stat 140 - 04

Mount Holyoke College

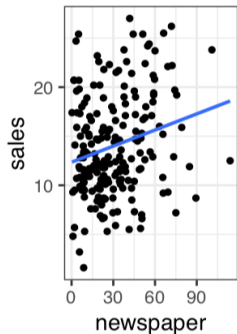
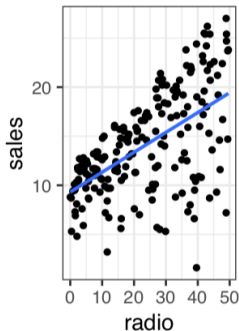
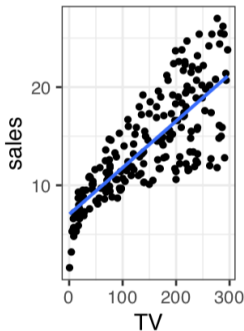
We have a data set of 200 markets, and we are interested in the relationship between sales and advertising budget. We look at the following variables.

- ▶ **sales** is a measure of sales volume in thousands of units
- ▶ **TV** is TV advertising budget
- ▶ **radio** is radio advertising budget
- ▶ **newspaper** is newspaper advertising budget

## Poll question

Which of these models would you prefer to use for predicting sales?

- a TV
- b radio
- c newspaper

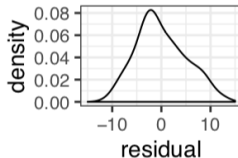
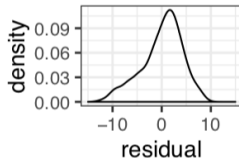
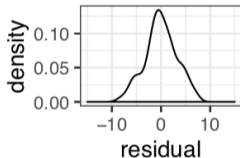
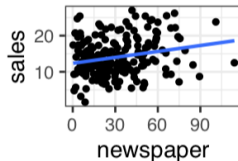
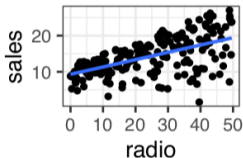
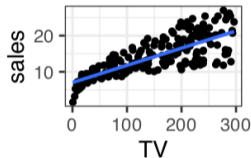


Being as specific and concrete as possible, write down a rule for selecting your preferred model

1. based only on **visual characteristics** of the plot.
2. based only on **a quantitative summary** of the data. You can describe how you would calculate your numeric summary of the data in a general sense; if you'd like you can write down a formula.

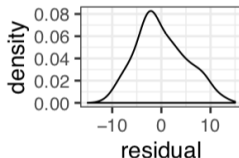
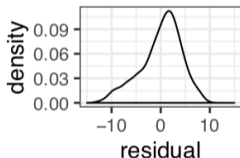
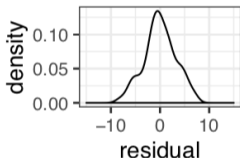
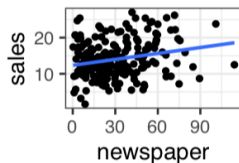
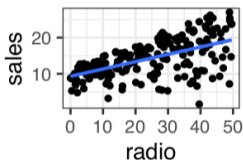
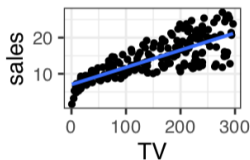
## Residuals:

- ▶  $e_i = y_i - \hat{y}_i$  (vertical distance between point and line)
- ▶ Smaller residuals mean the predictions were better.
- ▶ The key is to measure the spread of residuals.

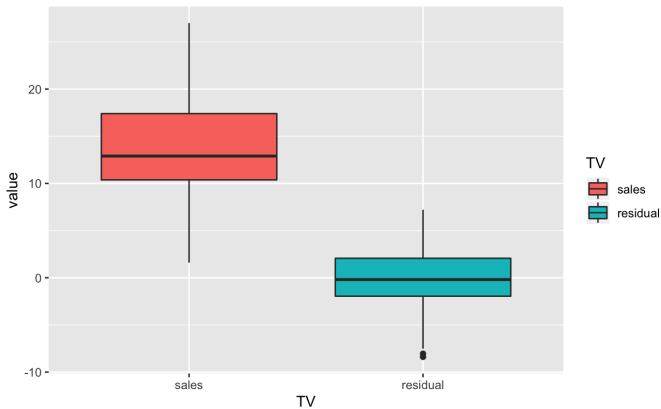


Measure spread of residuals with the standard deviation. We call this the **residual standard error**,  $s_{\text{RES}}$ .

- ▶ TV: 3.26
- ▶ radio: 4.28
- ▶ newspaper: 5.09



The variability in the residuals describes how much variation remains after using the model



Let's compute the reduction in variation.

$$\frac{s_{\text{sales}}^2 - s_{\text{RES}}^2}{s_{\text{sales}}^2} = 0.61$$

This number describes the amount of variation in the  $y$ -variable that is explained by the least squares line.

An value of 61% indicates that 61% of the variation in sales can be accounted for by the TV advertisement budget.



## Variation accounted by the model

- ▶ TV: 0.61
- ▶ radio: 0.33
- ▶ newspaper: 0.05

meaning,

- ▶ 61% of the variation in sales can be accounted for by the TV advertisement budget;
- ▶ 33% of the variation in sales can be accounted for by the radio advertisement budget;
- ▶ 5% of the variation in sales can be accounted for by the newspaper advertisement budget.

Statisticians found the variation accounted by the model can be computed by  $R^2$ , the **square of correlation**.

Square of the correlation coefficient  $R$ : between 0 and 1, closer to 1 is better.

$R^2$  describes the amount of variation in the  $y$ -variable that is explained by the least squares line.

```
linear_fit <- lm(Mortality ~ Calcium, data = mortality_water)
summary(linear_fit)
```

```
##
## Call:
## lm(formula = Mortality ~ Calcium, data = mortality_water)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -348.61 -114.52   -7.09  111.52  336.45
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1676.3556    29.2981  57.217 < 2e-16 ***
## Calcium     -3.2261     0.4847  -6.656 1.03e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 143 on 59 degrees of freedom
## Multiple R-squared:  0.4288, Adjusted R-squared:  0.4191
## F-statistic: 44.3 on 1 and 59 DF, p-value: 1.033e-08
```

*Annotations:*

- b0 intercept* points to the Intercept coefficient.
- b1 slope* points to the Calcium coefficient.
- Useful later* points to the p-value for the Calcium coefficient.
- R squared* points to the Multiple R-squared value.