Unit 6: CLT based inference

3. CLT based inference

Stat 140 - 02

Mount Holyoke College

Dr. Shan Shan

Slides posted at *http://sshanshans.github.io/stat140*

1. From yesterday: CLT based confidence interval

2. CLT based hypothesis testing

- Mean

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

- Two mean

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- Mean of paired data

$$\bar{x}_{\textsf{diff}} \pm t^* \sqrt{\frac{s_{\textsf{diff}}^2}{n_{\textsf{diff}}}}$$

- Mean

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

- Two mean

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- Mean of paired data

$$\bar{x}_{\mathsf{diff}} \pm t^* \sqrt{\frac{s_{\mathsf{diff}}^2}{n_{\mathsf{diff}}}}$$

- Proportion

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- Two proportions

$$\hat{p}_1 - \hat{p}_2 \pm$$

$$z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

From $N(0,1)$ or t_{n-1}

$$\textbf{Point Estimate} \pm Z^*/T^* \times \textbf{SE}$$

From original data

Standard Error

- Students were given words to memorize, then randomly assigned to take either a 90 min nap, or a caffeine pill. 2 $1/2$ hours later, they were tested on their recall ability.
- Variables
  - Explanatory variable: sleep or caffeine
  - Response variable: number of words recalled
- Is sleep or caffeine better for memory?

Mednick, Cai, Kanady, and Drummond (2008). "Comparing the benefits of caffeine, naps and placebo on verbal, motor and perceptual memory," Behavioral Brain Research, 193, 79-86.

Poll question

What is the parameter of interest in the sleep versus caffeine experiment?

a. Proportion
b. Difference in proportions
c. Mean
d. Difference in means
e. Mean of Paired data

- Let μ_s and μ_c be the mean number of words recalled after sleeping and after caffeine.
- Is there a difference in average word recall between sleep and caffeine?

Poll question

What are the null and alternative hypotheses?

a. $H_0 : \mu_s \neq \mu_c, H_A : \mu_s = \mu_c$
b. $H_0 : \mu_s = \mu_c, H_A : \mu_s \neq \mu_c$
c. $H_0 : \mu_s \neq \mu_c, H_A : \mu_s > \mu_c$
d. $H_0 : \mu_s = \mu_c, H_A : \mu_s > \mu_c$
e. $H_0 : \mu_s = \mu_c, H_A : \mu_s < \mu_c$

Note: the following two sets of hypotheses are equivalent, and can be used interchangeably:

$$H_0 : \mu_1 = \mu_2 \qquad\qquad H_0 : \mu_1 - \mu_2 = 0$$
$$H_A : \mu_1 \neq \mu_2 \qquad\qquad H_A : \mu_1 - \mu_2 \neq 0$$

The p-value is the chance of obtaining a sample statistic as extreme (or more extreme) than the observed sample statistic, if the null hypothesis is true

The p-value can be calculated as the proportion of statistics in a sampling distribution that are as extreme (or more extreme) than the observed sample statistic

Let's find it together
`http://www.lock5stat.com/StatKey/randomization_1_quant_1_cat/randomization_1_quant_1_cat.html`

- If the p-value is small
  - **Reject** H_0
  - the sample would be extreme if H_0 were true
  - the results are statistically significant
  - we have evidence for H_A
- If the p-value is not small
  - **Do not reject** H_0
  - the sample would not be too extreme if H_0 were true
  - the results are not statistically significant
  - the test is inconclusive; either H_0 or H_A may be true

How small?

The significance level, α, is the threshold below which the p-value is deemed small enough to reject the null hypothesis

Often $= 0.05$ by default, unless otherwise specified In short

If p-value $< \alpha$, reject H_0.
If p-value $\geq \alpha$, fail to reject H_0.