### Week 5 Confidence interval

### 1. Introduction to confidence interval

Stat 140 - 04

Mount Holyoke College

1. Questions? (15 minutes)

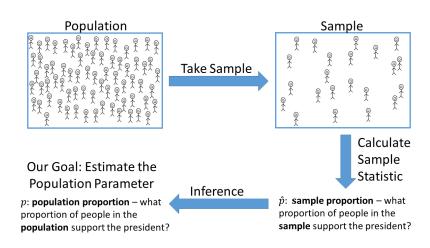
2. Last week

3. Today: confidence interva

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I recently received a gift of 100,000 M&M's. I would like to know what proportion of these M&M's are blue?



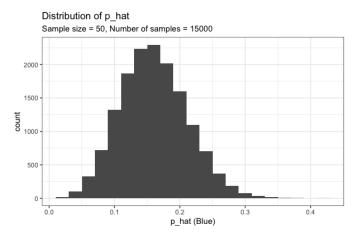
Sample once: 10 out of 50 M&M's are blue.

$$\hat{p} = 0.2, \qquad p = ?$$

We use the statistic from a sample as a **point estimate** for a population parameter.

Point estimates will not match population parameters exactly, but they are our best guess, given the data.

# Sample many times



A sampling distribution is the distribution of sample statistics computed for different samples of the same size from the same population.

A sampling distribution shows us how the sample statistic varies from sample to sample

#### Center

If samples are randomly selected, the sampling distribution will be centered around the population parameter.

# Spread

The spread of the distribution measures how much the statistic varies from sample to sample. Also known as **Standard Error** 

## Shape

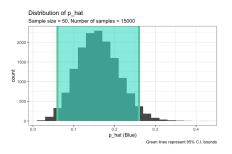
For most of the statistics we consider, if the sample size is large enough the sampling distribution will be symmetric and bell-shaped.

For a sufficiently large sample size, the distribution of sample proportion or sample mean is normal.

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 $\mathcal{N}$  (Population Parameter, Standard Error)

Green area = middle 95% of the values of sample statistics



Between what two values would you say 95% of sample statistics (proportion) lie?

$$P(? < \hat{p} < ?) = .95$$

1. Questions? (15 minutes)

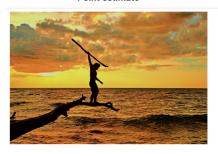
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3. Today: confidence interval

An interval estimate gives a range of plausible values for a population parameter.

### Point vs Interval

Point estimate



Confidence interval



One common form for an interval estimate is

# Point Estimate $\pm$ Margin of Error (ME)

where the margin of error reflects the precision of the sample statistic as a point estimate for the parameter.

### General Election: Trump vs. Biden (4-Way)



Top Battlegrounds: Trump vs. Biden

RCP Electoral Map | No Toss Up Map | RCP Senate Map | Senate NTU Map | RCP House Map | Latest Polls

Polling Data										
Poli	Date	Sample	MoE	Biden (D)	Trump (R)	Spread				
RCP Average	10/16 - 10/26			50.6	43.5	Biden +7.1				
Emerson	10/25 - 10/26	1121 LV	2.8	50	45	Biden +5				
IBD/TIPP	10/22 - 10/26	970 LV	3.2	50	46	Biden +4				
Rasmussen Reports	10/22 - 10/26	1500 LV	2.5	49	47	Biden +2				
JTN/RMG Research*	10/23 - 10/24	1842 LV	2.8	51	44	Biden +7				
CNBC	10/21 - 10/24	800 RV	3.5	51	40	Biden +11				
Economist/YouGov	10/18 - 10/20	1344 LV	3.2	52	43	Biden +9				
Reuters/Ipsos	10/16 - 10/20	949 LV	3.6	51	42	Biden +9				
Quinnipiac	10/16 - 10/19	1426 LV	2.6	51	41	Biden +10				
	All General Elect	tion: Trump vs. Bi	den Polling	Data						

Why is the margin of error smaller for the Emerson poll than the IBD/TIPP poll?

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Using the Emerson poll, calculate an interval estimate for the proportion of registered voters who planned to vote for Biden.

$$50 \pm 2.8 = (47.2, 52.8)$$

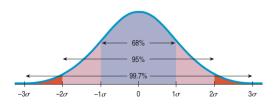
The 2020 presidential election already happened, so this is one of the rare situations in which we actually know the true population parameter, p!

In the actual election, 51% voted for Biden.

How would you find the margin of error to ensure that intervals of the form

# Point Estimate $\pm$ Margin of Error (ME)

would capture the parameter for 95% of all samples? (Hint: remember the normal distribution and 95% rule)



Using the central limit theorem, 95% confidence interval can be estimated using

# Point Estimate $\pm$ 2 Standard Error (SE)

#### Poll question

A survey of 1,502 Americans in January 2012 found that 86% consider the economy a "top priority" for the president and congress this year.

The standard error for this statistic is 0.01. What is the 95% confidence interval for the true proportion of all Americans that considered the economy a "top priority" at that time?

- **1**. (0.85, 0.87)
- 2. (0.84, 0.88)
- **3**. (0.82, 0.90)

In general, a confidence interval is of the form:

### Point Estimate $\pm$ Critical value $\times$ SE

Critical value depends on

- ▶ Desired level of confidence (e.g. 90%, 95%, 99%)
- Appropriate sampling distribution (so far we have only learned normal distribution)