An introduction to fair division and cake-cutting algorithms
Group projects

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In the first lecture, we examined the fair division problem which guarantees each player at least $1/n$ of the cake. From this simple beginning, a large number of interesting variations on the theme can be explored. Herein lies the goal of the group projects. There will be four projects on fair division. You will be working in pairs (sometimes triples) on these projects, and give a 15-minute presentation on the last day. The goal isn't to discover a whole new theory and try to prove it, but to have fun exploring on your own.

I would like to begin with a bird's eye view on where the group projects lie within the subject of fair division. So let's take a closer look at the basic question we asked before,

How to cut a cake fairly?

Cake is a metaphor of *desirable goods*. What if we want to divide unwanted objects (like house chores) or a mixture of goods and burdens? You will explore this theme in Project 1. Cake is also a *divisible good*, that is an object you can divide in smaller pieces. But in real life, many things aren't divisible. In Project 2, you will help resolve a divorce settlement on indivisible goods. You have probably known very well that "fairness" is a difficult thing to define. In today's class we looked at simple fair division and next week we will learn about envy-free division. Can you think of other fairness criteria? In Project 3, you will define what an exact division means. The last project looks at online division where players can arrive late and leave early. As you see, all projects are rooted in real-life applications and are very open-ended. So you can come up with a mathematical model you like and choose what problems you want to work on.

The four projects are described in the following.

1. The dirty work problem

   In real life, bigger is not always better. For example, if we are dividing unpleasant house chores, then we prefer a small portion. Can you formulate a fair division problem where each player wants the smallest fair and try to solve it? What fairness criteria do you want to use?

2. Negotiating to settlement in divorce

   Cake-cutting algorithms are not generally applicable to real-world situations involving indivisible goods, whose value is destroyed if they are divided. People see this conflict arise very often in splitting up property in divorce. Can you simulate a fair division problem where players want to fairly divide a set of property that involve indivisible goods?

3. Exact division

   We have seen in cut and choose that it's better to be the chooser than the cutter. Can you come up with a fairness criteria that balance the asymmetry? Can you formulate a fair division problem that uses the criteria you defined, and try to solve it?

4. Online division

   In traditional cake-cutting setting, all players are available at the time of the division. However, in real life, people may show up late or leave early. Can you simulate a fair division problem where players arrive and depart during the process of dividing a resource?

What problems should we think about? The answer lies in the other half of the basic question, "how to cut". The word cut refers to whether a procedure is continuous or finite. It also suggests to look at the number of cuts required in each algorithm. The question "how?" shows we are not only interested in the theoretical existence, but finding a procedure for it. Here are some other questions you might want to work on.

1. What fairness criteria do you want to use?
2. Can you modify the algorithms we have learned in class to apply to your problem?
3. How many cuts do you need?
4. Are you happy with the process and the result?
5. Are there any tricks or concerns?