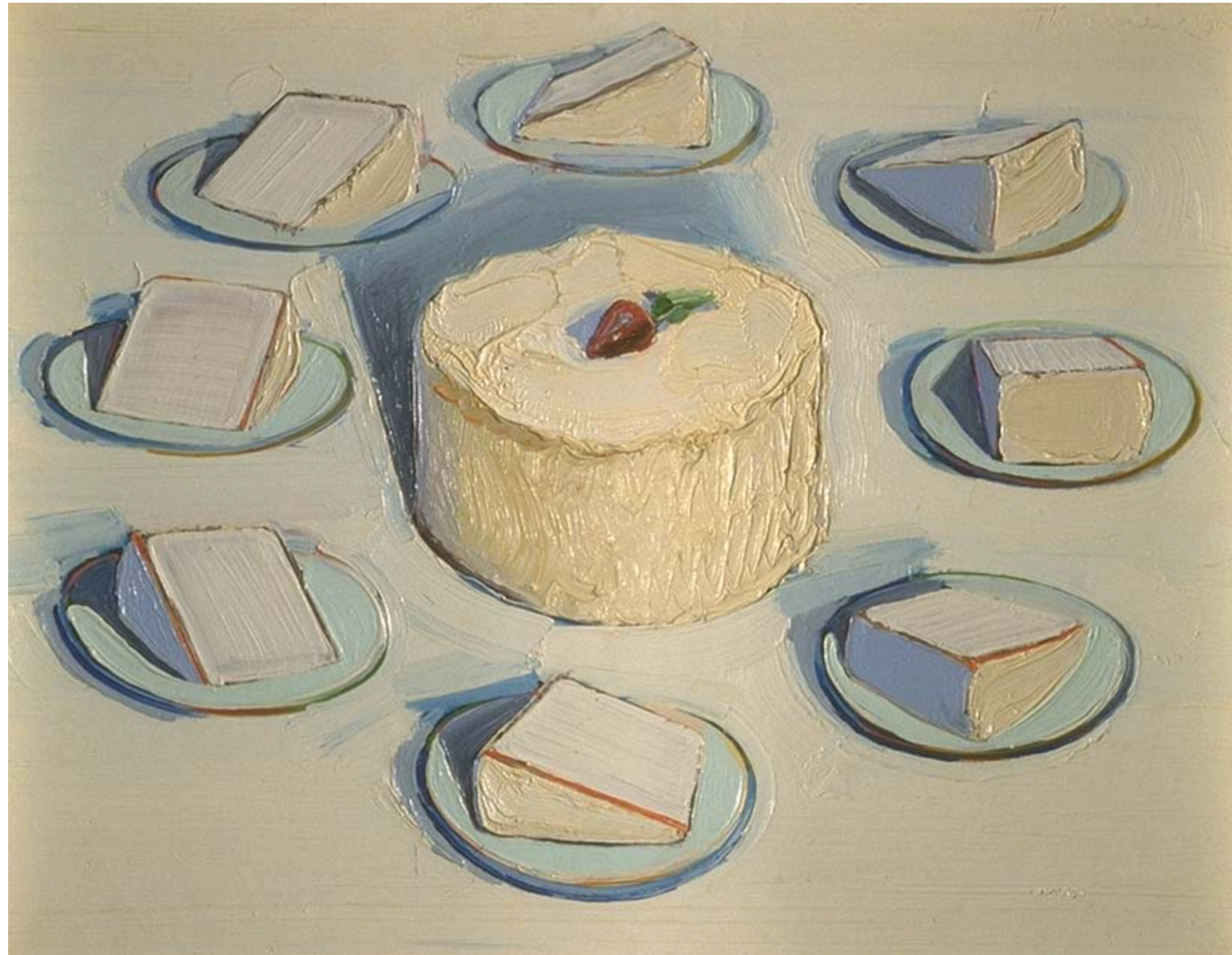


A warm-up exercise



PC: Wayne Thiebaud *Around the Cake*

How many cuts do you need for cut-choose? Moving knife?
Trimming method? Successive pair?

How many cuts do you need?

Recall:

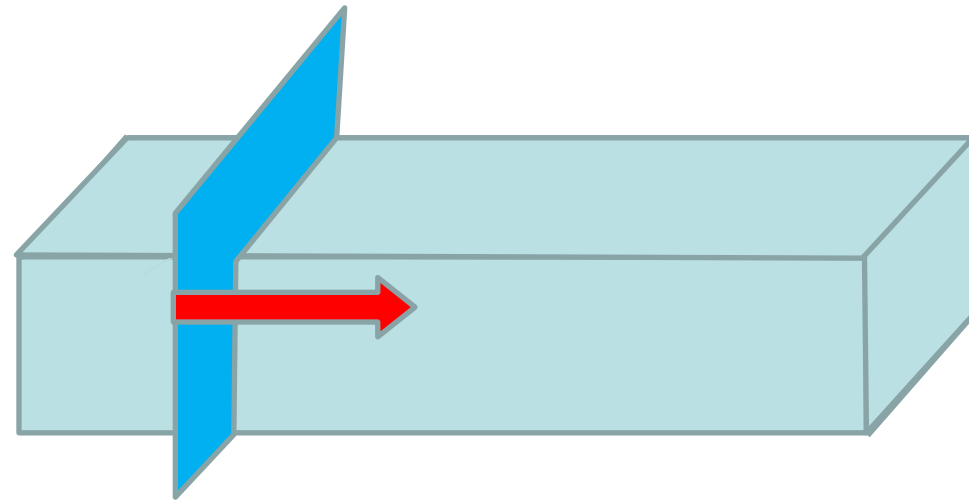
simple fair division for n players

Let S be the cake to be divided. Let P_1, P_2, \dots, P_n be n players with measures u_1, u_2, \dots, u_n . We want to divide S into s_1, s_2, \dots, s_n such that $u_i(s_i)$ is greater or equal to $1/n$.

Need:

extend moving knife, trimming method, and successive pair method to n players.

The moving knife



- Referee slides the knife from left to right
- Any one who thinks the left piece has reached $\frac{1}{3}$ of the cake says “stop” and gets the left piece
- The other two players use cut and choose method for the remaining piece.

The trimming method

Goal: where to cut without moving the knife.

- P1 cuts a slice of size $1/3$ from the cake
- The cut slice is passed to P2. If she values it more than $1/3$, then she trims it so the reduced value is exactly $1/3$.
- The slice (whether trimmed or not) is passed to P3. She takes it, if she considers it at least $1/3$ of the cake. Otherwise the slice is given to the last player who cuts it. The player receiving this piece drops out.
- Use cut and choose on the remaining portion of the cake.

The trimming method

Goal: where to cut without moving the knife.

- How many cuts?

First round of passing:

Second round of passing:

...

Last round (cut and choose):

Total =

The successive pair method

Key: two at a time

- Cut and choose between P1 and P2
- P1 and P2 each cut their share into three equal pieces
- P3 chooses one piece from both P1 and P2

Fewer cuts with Divide-Conquer

Idea: split into two groups based on satisfaction

Divide

In the first round of trimming method for n players,
satisfied with a piece: 1 player

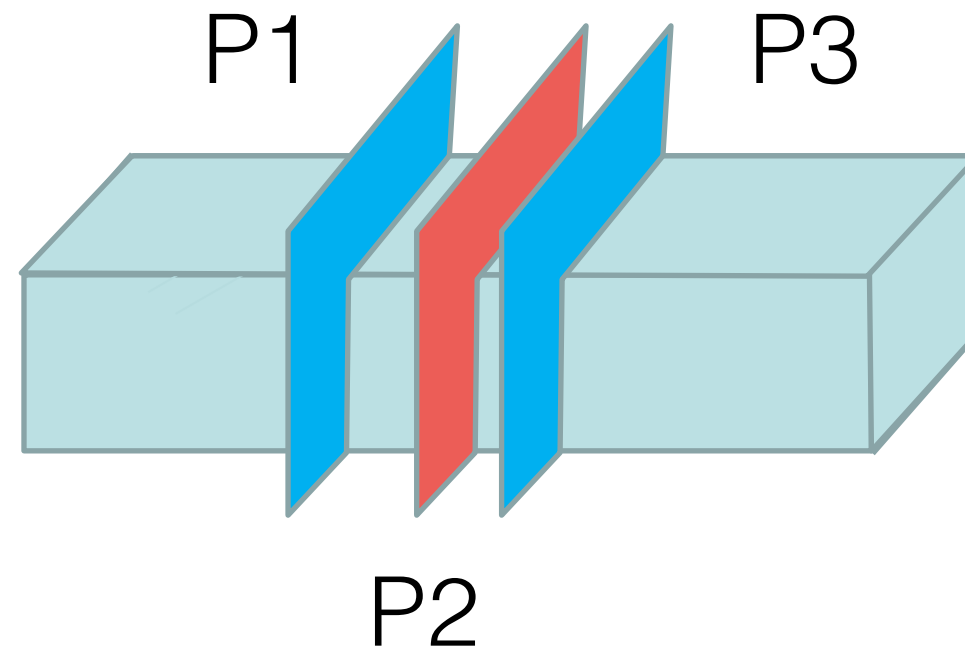
satisfied with the remaining: $n-1$ players

Conquer: solve $n-1$ players

Can the players be split differently?

Want: groups size roughly the same

Divide-Conquer when $n=4$



- Three of the four players make parallel cuts so that the left piece is $1/2$ of the cake.
- The non-cutter chooses the larger one defined by the middle cut.
- Two groups:
- Use cut-choose on each group.

Divide-Conquer when $n=5$



Hint: divide into 2:3

How many cuts do you need?

# of players	Method	# of cuts
1	no division is needed	
2	cut and choose	
3	use 2 cuts to reduce to 2-1	
4	use 3 cuts to reduce to 2-2	
5	use 4 cuts to reduce to 2-3	
6	use 5 cuts to reduce to 3-3	
7	use 6 cuts to reduce to 3-4	
8	use 7 cuts to reduce to 4-4	

Summary

- Algorithm for n players: moving knife, trimming, successive pair
- Number of cuts needed for each algorithm
- Divide and Conquer reduces number of cuts
- Key: $N \rightarrow N-1$ (induction); $N \rightarrow N/2$ (DC)

To do:

- A worksheet on induction

Reference:

Cake-Cutting Algorithms: be fair if you can by Robertson and Webb [Chapter 2]