A warm-up exercise

PC: Lewis Caroll

Once upon a time there was a cake to be divided “fairly” between the unicorn and the lion. How should this be done? Write down on a piece of paper, what does “fairness” mean to you?

The mathematics of sharing

- How to quantify “fairness”?

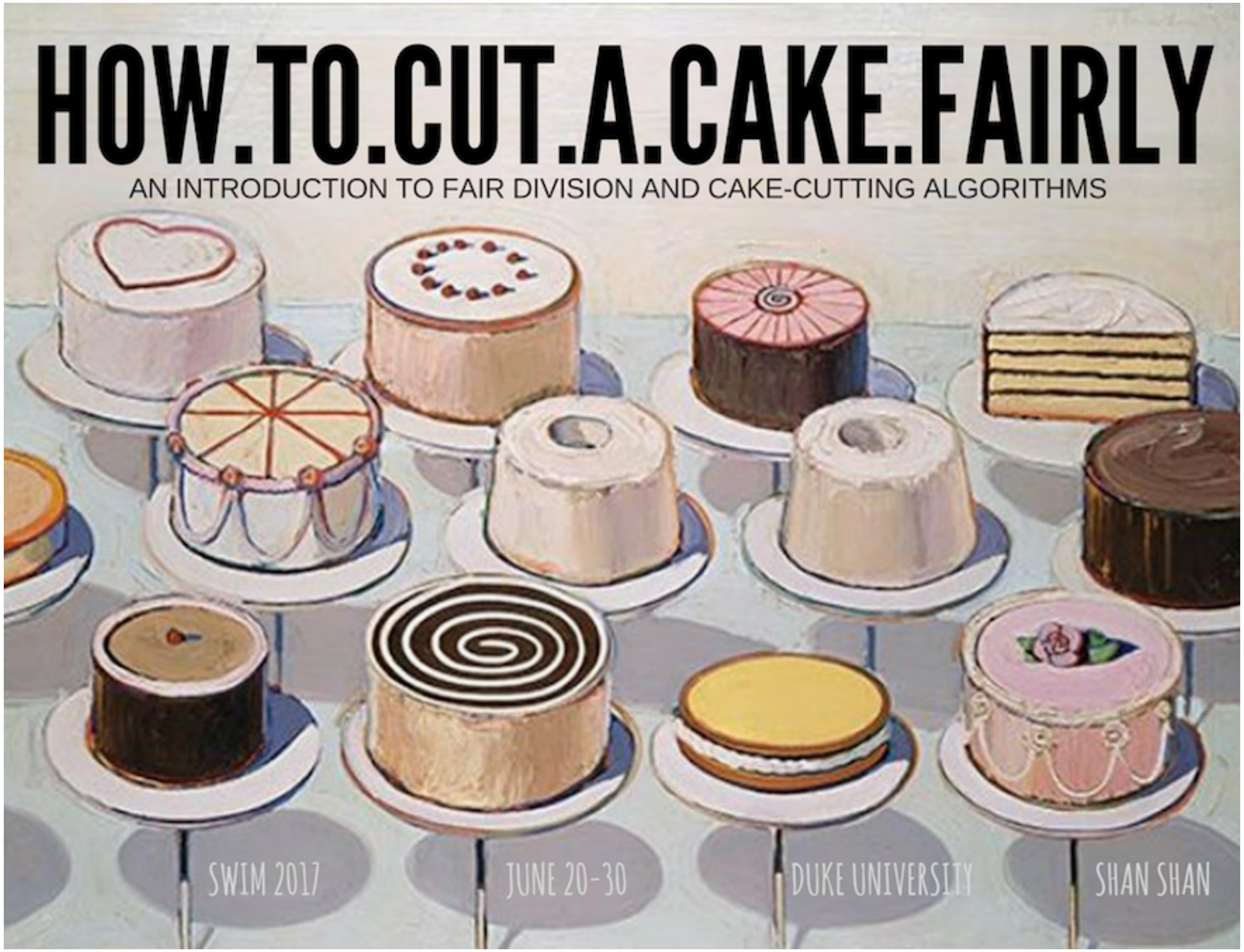
Use graph theory, complexity, number theory, combinatorial topology, etc.

- Day 1 - 3

Simple fair division: existence, algorithm and complexity

- Day 4 - 6

Some variations on the themes of fair division



course website
https://sshanshans.github.io/swim.html

Modeling the conflict

“a cake to be divided “fairly” between the horse and the lion”

- How should we model a cake?

- Unicorn and lion?

Modeling the conflict

“a cake to be divided “fairly” between the horse and the lion”

- Fairness is subject to each player’s own assessment.
- How to model a player’s assessment?

- One of the many ways to define “fairness”:

A mathematical translation

“a cake to be divided “fairly” between the horse and the lion”

- cake: S
- horse and lion: P1, P2
- assessment: u1, u2
- fairness: each player thinks she gets at least half of the cake

Simple division

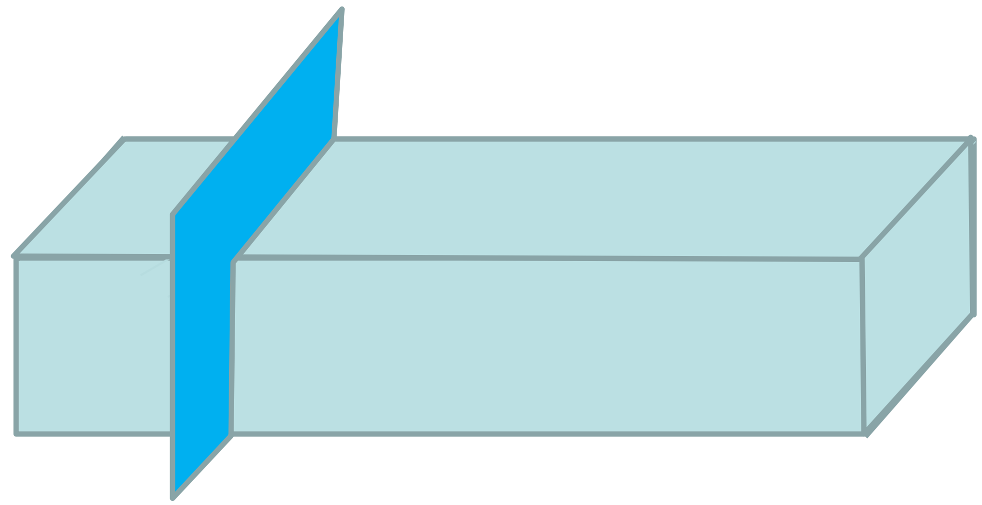
- Simple fair share

- Suppose the cake is now divided into two slices, s1, s2. The following table gives how the unicorn and the lion value each slice.

	s1	**s2**
Unicorn	0.3	0.7
Lion	0.6	0.4

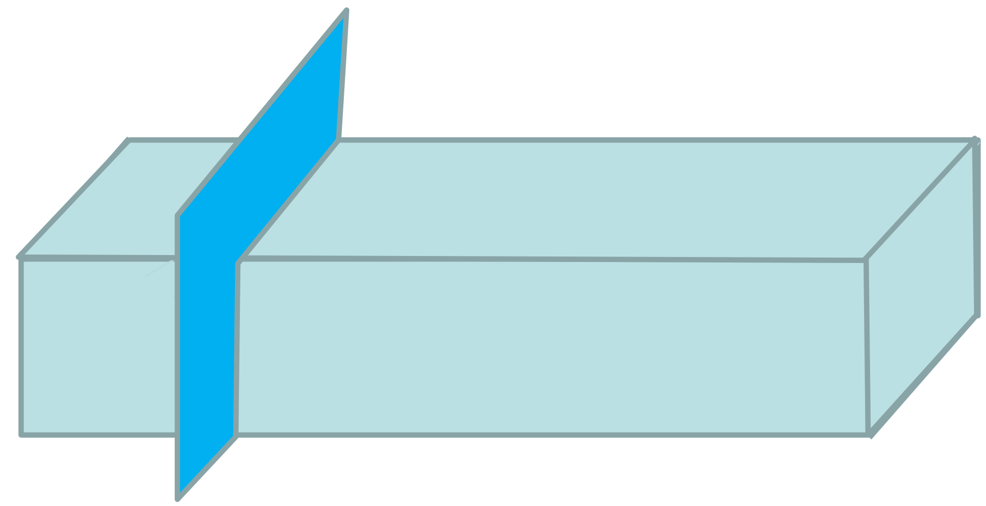
Which of two slices is a fair share to the horse? To the lion? Find a simple division using s1 and s2.

How to decide s1, s2?



I cut you choose

How to decide s1, s2?



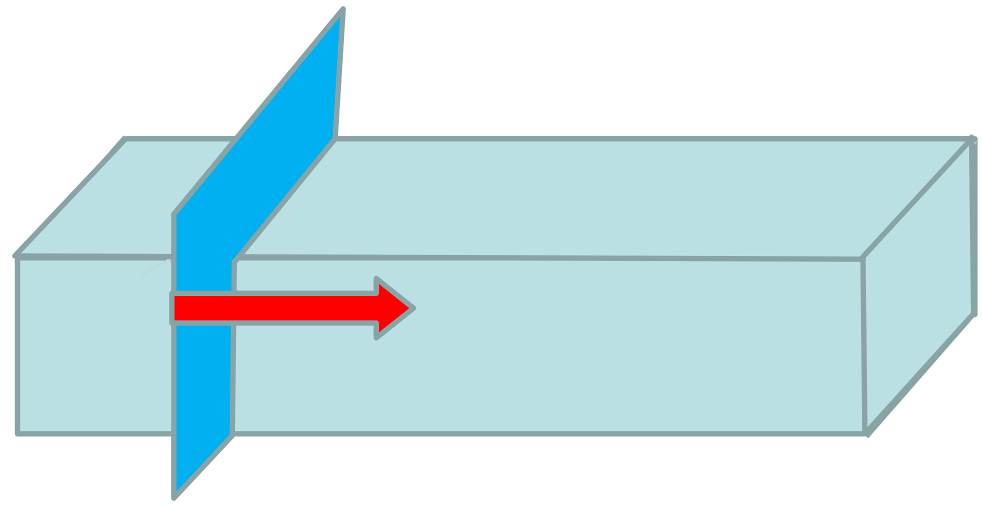
I cut you choose

- Why is the cutter guaranteed a fair share in this method? Why is the chooser guaranteed a fair share?
- Would you rather be the cutter or the chooser?
- Now try to come up with your own questions. What would happen if...?

The fox wants the cake too…

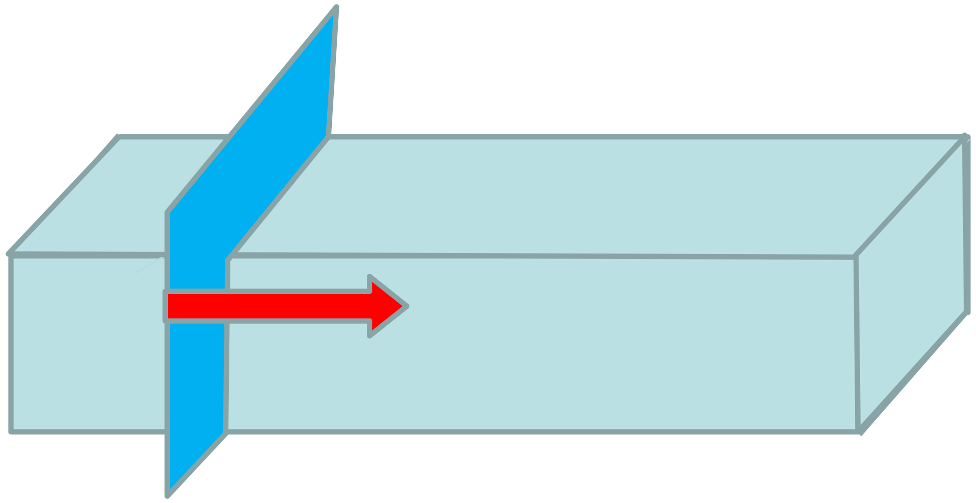
“a cake to be divided “fairly” among unicorn, lion and fox”

The moving knife



- Referee slides the knife from left to right
- Any one who thinks the left piece has reached 1/3 of the cake says “stop” and gets the left piece
- The other two players use cut and choose method for the remaining piece.

The moving knife



- Why is everyone guaranteed a fair share in this method?
- How does the moving knife algorithm differ from the cut-choose method?
- Can you think of any other interesting questions?

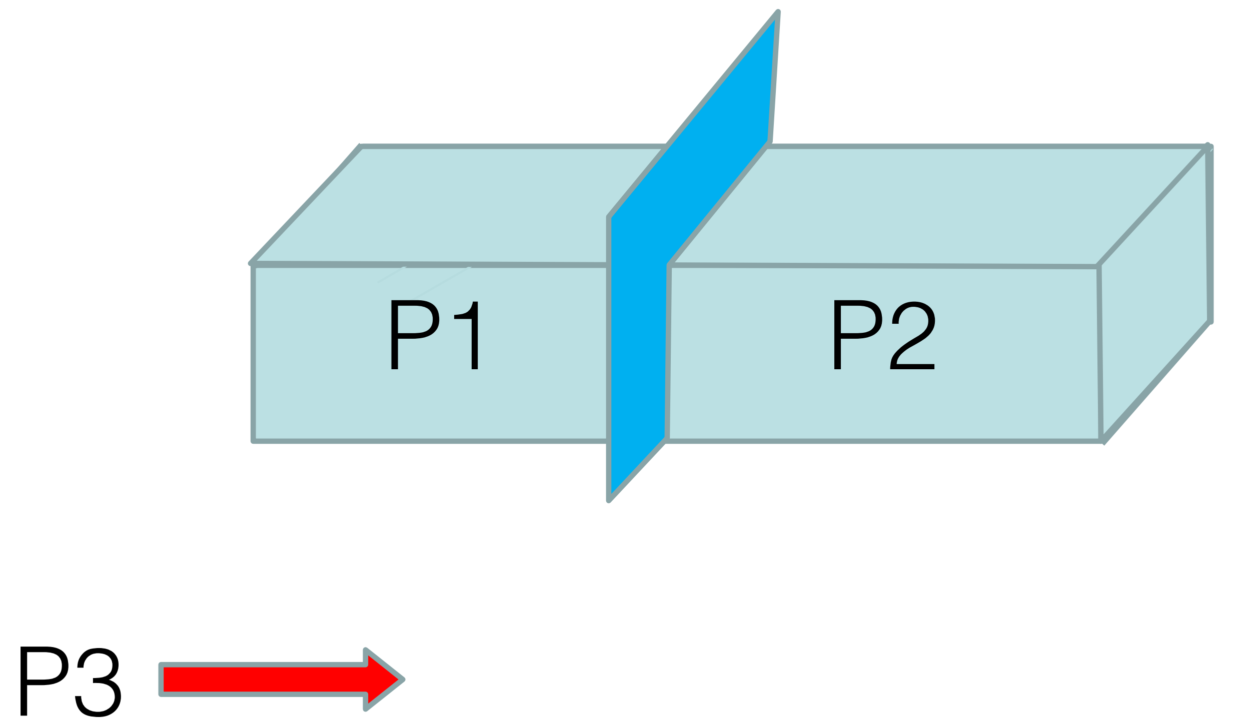
The trimming method

Goal: where to cut without moving the knife.

- P1 cuts a slice of size 1/3 from the cake
- The cut slice is passed to P2. If she values it more than 1/3, then she trims it so the reduced value is exactly 1/3.
- The slice (whether trimmed or not) is passed to P3. She takes it, if she considers it at least 1/3 of the cake. Otherwise the slice is given to the last player who cuts it. The player receiving this piece drops out.
- Use cut and choose on the remaining portion of the cake.

The successive pair method

Key: two at a time



- Cut and choose between P1 and P2
- P1 and P2 each cut their share into three equal pieces
- P3 chooses one piece from both P1 and P2

Summary

- Mathematical model of simple fair division
- Algorithm for two players: cut-choose
- Algorithms for three players: moving knife, trimming, successive pair
- Key: fairness is subject to each player’s assessment

To do:

- A worksheet on basic set notation.

Reference:

Cake-Cutting Algorithms: be fair if you can by Robertson and Webb [Chapter 1]