2025 USA-NA-AIO Round 1, Problem 2, Part 2

USAAIO

Mar 2025

Part 2 (10 points, non-coding task)

Define $\nabla_{\mathbf{z}} f(\mathbf{z})$ to be the gradient of function f with respect to vector/matrix $\mathbf{z}$.

Compute the following gradients. Reasoning is required.

1. $\nabla_{\mathbf{x}} y$.

   The final answer should be in a matrix form.

2. $\nabla_{\mathbf{W}} y$.

   The final answer should be in an element-wise form.

3. $\nabla_{\mathbf{b}} y$.

   The final answer should be in a matrix form.

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Misplaced '#'

1. Since $\mathbf{y} \in \mathbb{R}^M$ and $\mathbf{x} \in \mathbb{R}^N$, $\nabla_{\mathbf{x}} \mathbf{y} \in \mathbb{R}^{M \times N}$.

We have

$$\begin{aligned}\frac{\partial y_m}{\partial x_n} &= \frac{\partial \left(\sum_{i=0}^{N-1} w_{mi} x_i + b_m\right)}{\partial x_n} \\ &= w_{mn}.\end{aligned}$$

Therefore,

$$\boxed{\nabla_{\mathbf{x}} \mathbf{y} = \mathbf{W}}.$$

2. Since $\mathbf{y} \in \mathbb{R}^M$ and $\mathbf{W} \in \mathbb{R}^{M \times N}$, $\nabla_{\mathbf{W}} \mathbf{y} \in \mathbb{R}^{M \times M \times N}$.

We have

$$\begin{aligned}\frac{\partial y_m}{\partial w_{kn}} &= \frac{\partial \left(\sum_{i=0}^{N-1} w_{mi} x_i + b_m\right)}{\partial w_{kn}} \\ &= \boxed{x_n \delta_{mk}}.\end{aligned}$$

3. Since $\mathbf{y} \in \mathbb{R}^M$ and $\mathbf{b} \in \mathbb{R}^M$, $\nabla_{\mathbf{b}} \mathbf{y} \in \mathbb{R}^{M \times M}$.

We have

$$\begin{aligned}\frac{\partial y_m}{\partial w_k} &= \frac{\partial \left(\sum_{i=0}^{N-1} w_{mi} x_i + b_m\right)}{\partial b_k} \\ &= \delta_{mk}.\end{aligned}$$

Therefore,

$$\boxed{\nabla_{\mathbf{b}} \mathbf{y} = \mathbf{I}_{M \times M}}.$$

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""" END OF THIS PART """

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2025 USA-NA-AIO Round 1, Problem 3, Part 15	1	Mar 2025
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Topic	Replies	Activity
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