2025 USA-NA-AIO Round 2, Problem 3, Part 20

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Part 20 (5 points, non-coding task)

In the loss function, we introduced a crutial learnable parameter τ, called temperature.

Let us explore some properties of τ.

Let $z_0 > z_1 > \cdots > z_{N-1}$.

Define

$$f_i = \frac{\exp\left(z_i/\tau\right)}{\sum_{j=0}^{N-1} \exp\left(z_j/\tau\right)}.$$

Do the following analysis. Reasoning is required.

1. Compute

$$\lim_{\tau \to 0^+} f_i.$$

2. Compute

$$\lim_{\tau \to \infty} f_i.$$

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Misplaced '#'

1. We have

$$
\begin{aligned}
\lim_{\tau \to 0^+} f_i &= \lim_{\tau \to 0^+} \frac{\exp\left(z_i/\tau\right)}{\sum_{j=0}^{N-1} \exp\left(z_j/\tau\right)} \\
&= \lim_{\tau \to 0^+} \frac{\exp\left((z_i - z_0)/\tau\right)}{\sum_{j=0}^{N-1} \exp\left((z_j - z_0)/\tau\right)} \\
&= \boxed{\begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{if } i \neq 0 \end{cases}}.
\end{aligned}
$$

2. Compute

$$
\begin{aligned}
\lim_{\tau \to \infty} f_i &= \lim_{\tau \to \infty} \frac{\exp\left(z_i/\tau\right)}{\sum_{j=0}^{N-1} \exp\left(z_j/\tau\right)} \\
&= \frac{1}{\sum_{j=0}^{N-1} 1} \\
&= \boxed{\frac{1}{N}}.
\end{aligned}
$$

"" END OF THIS PART ""

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