2025 USA-NA-AIO Round 1, Problem 3, Part 13

USAAIO Mar 2025

Part 13 (5 points, non-coding task)

In this part, you are asked to compute $\nabla_\beta^2 L(\beta)$ and express your solutions in two forms. Reasoning is not required.

1. Write $\nabla_\beta^2 L(\beta)$ in the following summation form:

$$\nabla_\beta L(\beta) = \sum_{n=0}^{N-1} \cdots .$$

2. Denote

$$\mathbf{Z} = \begin{bmatrix} z_0 & 0 & \cdots & 0 \\ 0 & z_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & z_{N-1} \end{bmatrix}.$$

Write $\nabla_\beta^2 L(\beta)$ in terms of $\mathbf{X}$, $\mathbf{Z}$ with matrix operations (the summation symbol is not allowed).

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Misplaced '#'

We have

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$$\nabla_\beta^2 L(\beta) = \boxed{\sum_{n=0}^{N-1} \sigma\left(\mathbf{x}^{(n),\top}\beta\right)\left(1-\sigma\left(\mathbf{x}^{(n),\top}\beta\right)\right)\mathbf{x}^{(n)}\mathbf{x}^{(n)^\top}}$$
$$= \boxed{\mathbf{X}^\top\mathbf{Z}\mathbf{X}}.$$

""" END OF THIS PART """

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I believe the answer to the second part should be

$$\mathbf{X}^\top\mathbf{Z}(I-\mathbf{Z})\mathbf{X}.$$

(I is the identity matrix).

If not, where does the $1-\sigma(\ldots)$ come from?

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