# Memorandum

To: Dr. Faysal Kolkailah
From: Bryce Ipson, Steven Sharp
Date: June 3, 2022
Subject: Material Evaluation for Life Support Oxygen Tanks

**Introduction:**

Spacecraft supporting biological experiments need a supply of oxygen, which requires storage in pressurized tanks. These tanks must be strong enough to withstand the extreme pressure variations of ambient pressure, launch, and the vacuum environment. We modeled an oxygen tank for a space station life support system as a thick-walled pressure vessel with closed ends. This tank will be attached to the exterior of an existing spacecraft that docks with the station, transfers the oxygen tank, and then returns to Earth. The launch vehicle has already been contracted for this mission.

There is a predetermined amount of space between the spacecraft and launch vehicle fairing, so we considered the size of the tank to be fixed. The space station has a maximum oxygen usage of 250 liters per day at 1 atm, and requires a supply for at least 365 days. The manufacturing department has requested that we determine whether an aluminum-lithium alloy that they overstocked would be a safe choice, given that the department only has enough material left to produce a 1 inch thick wall for the tank.

**Nomenclature:**

A - cross-sectional area ($in^2$)
d - diameter (in)
L - length (in)
p - pressure (psi)
r - radius (in)
t - thickness (in)
V - volume (in^3)

$\sigma$ - stress (psi)

Subscripts:

a - axial

h - hoop

i - inner

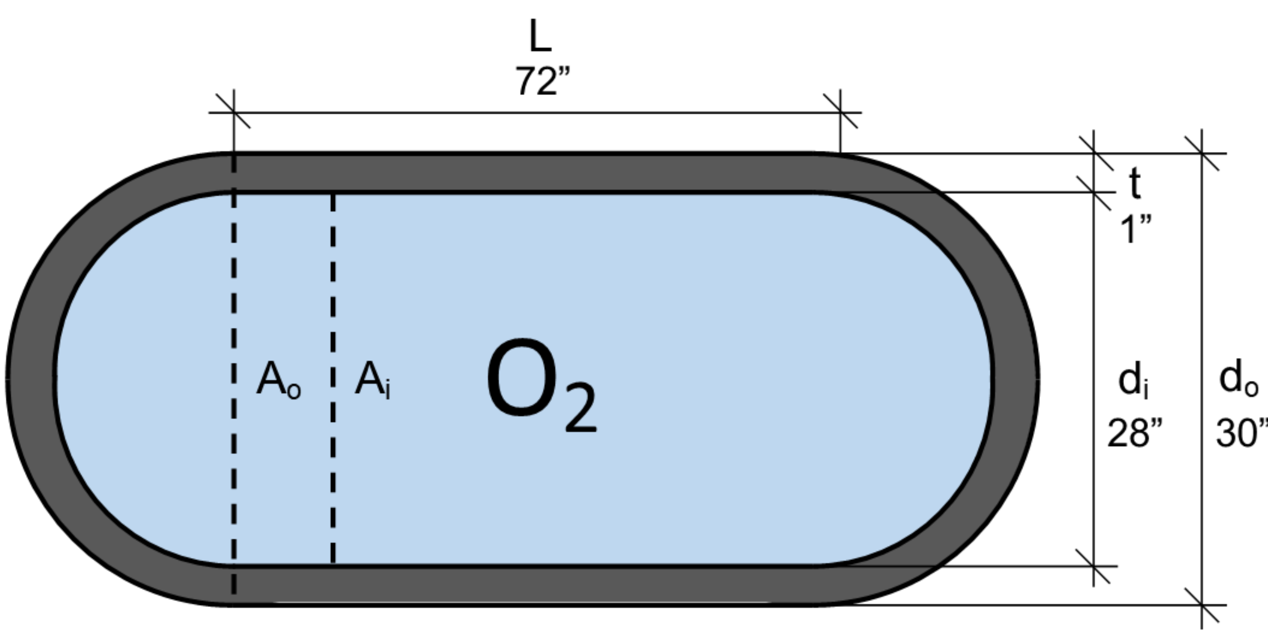
o - outer

r - radial

**Methodology:**

Failure of a tank with highly compressed oxygen would be extremely dangerous, with the possibility of explosion generating large amounts of orbital debris, contamination, and impulse applied to the space station. Therefore, we assume a factor of safety of 2.

The schematic for the pressure vessel is depicted below:




Our material candidate for the oxygen tanks is the same used for tanks on several other launch vehicles (such as the Falcon 9 rocket) [1]. The metal used there was an alloy of aluminum and lithium useful for both its strength and lightness. The approximate yield strength was given in a research article to be 65.2 ksi [1]. In a separate article, researchers found the Young's Modulus to be $15.23\times10^3$ ksi [2].

The volume of the tank is

$$\begin{aligned} V &= \pi r_i^2 L + \frac{4}{3}\pi r_i^3 \\ &= \pi\left( (14\,\mathrm{in})^2 (72\,\mathrm{in}) + \frac{4}{3}(14\,\mathrm{in})^3 \right) \\ &= 55828.2\,\mathrm{in}^3 \end{aligned}$$

Therefore, the required internal pressure for the station's maximum oxygen usage is

$$\begin{aligned} & p_1 V_1 = p_2 V_2 \\ \rightarrow p_i &= \frac{\left(250\,\frac{\mathrm{L}}{\mathrm{day}}\right)\left(61.0237\,\frac{\mathrm{in}^3}{\mathrm{L}}\right)(365\,\mathrm{days})}{55828.2\,\mathrm{in}^3}(14.696\,psi) \\ &= 1465.81\,\mathrm{psi} \end{aligned}$$

We compute the principal stresses at each altitude and determine whether the material fails under the loading condition. Our prediction is that the trend will show an increase in internal tensile forces as ambient pressures drop with an increase in altitude. In order to measure the decrease in ambient pressure, we are using the 1976 Standard Atmosphere model in a MATLAB coded function.

**Results:**

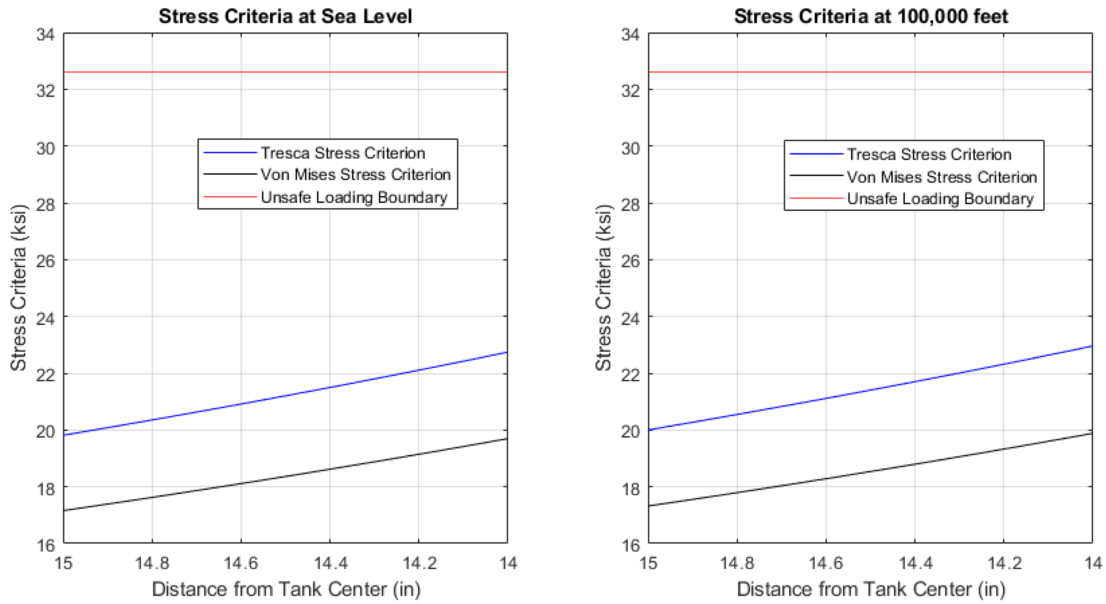



Figure 1: *Stress Criteria at Sea Level and Maximum Altitude*

We see that the stress criteria increases as we move from the exterior of the tank to the inner surface, due to the high internal pressure compared to the exterior pressure.

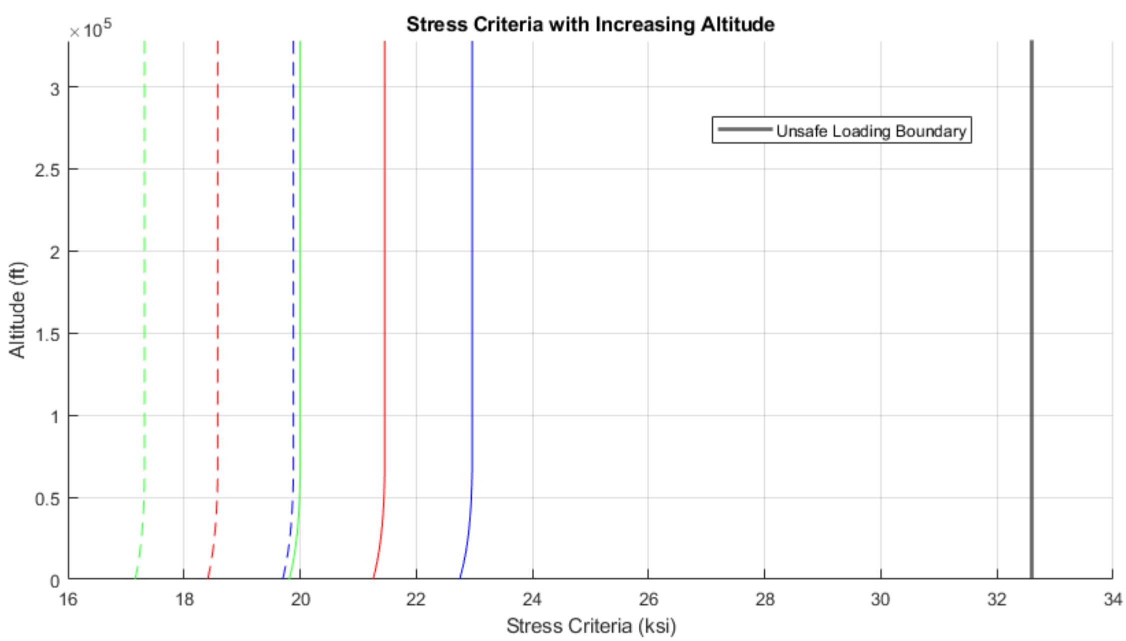



Figure 2: *Stress Criteria Atmospheric Profile*

In the above figure, blue lines are the stresses on the interior face of the tank wall, red lines are at the center of the wall thickness, and green lines are measured at the exterior face. The dashed lines represent the Von Mises stress criterion, while the solid lines represent the Tresca stress criteria.

There is a distinct lack of change in stress above 100,000 ft. This is caused by the external pressure diminishing to an extremely small value; it only decreases by a minimal amount after this. If a vessel can withstand the ambient pressure difference up to 100,000 feet, it can likely continue all the way into a vacuum environment.

The pressure vessel can handle an internal pressure of 1,500 psi, but what is the maximum pressure at which the pressure vessel does not exceed the factor of safety? Conservatively, at an internal pressure of 2,085 psi, the Tresca stress criterion just reaches the unsafe loading boundary at the interior surface:

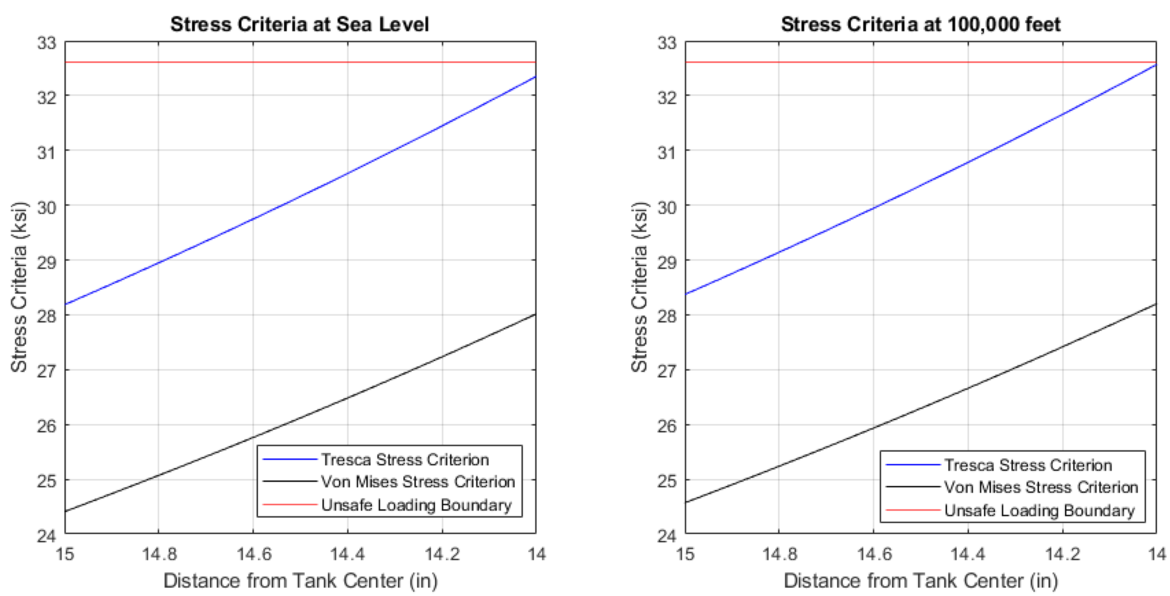



Figure 3: *Maximum Tresca Loading*

The least conservative value for the maximum internal pressure is the pressure that causes the Von Mises stress criterion to reach the unsafe loading boundary, which occurs at 2,410 psi:

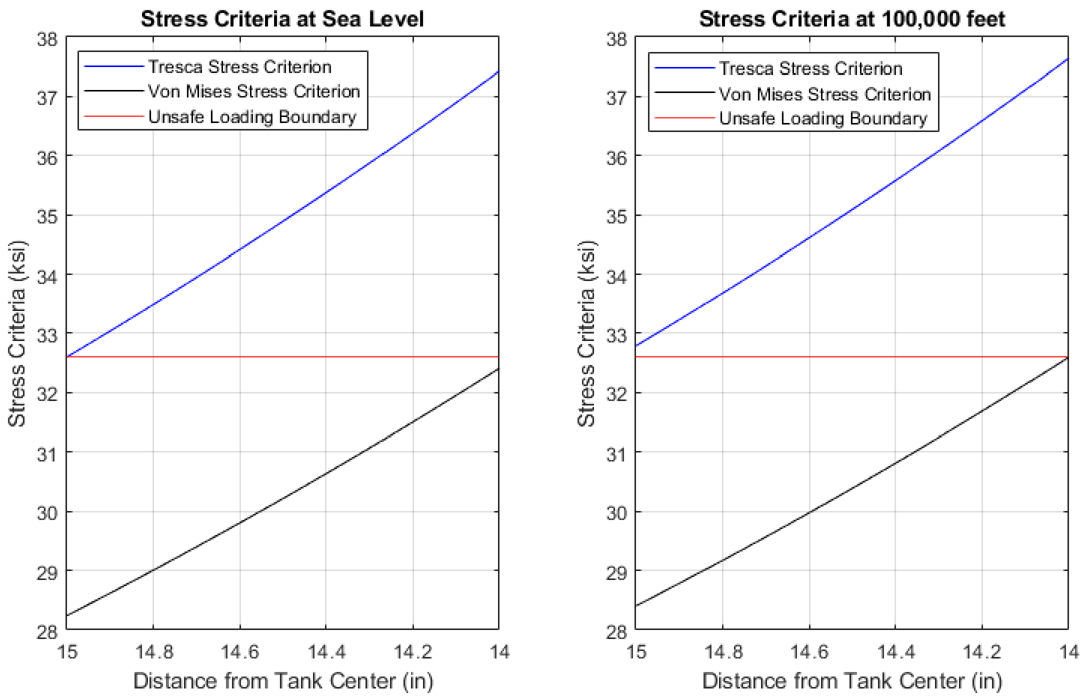



Figure 4: *Maximum Von Mises Loading*

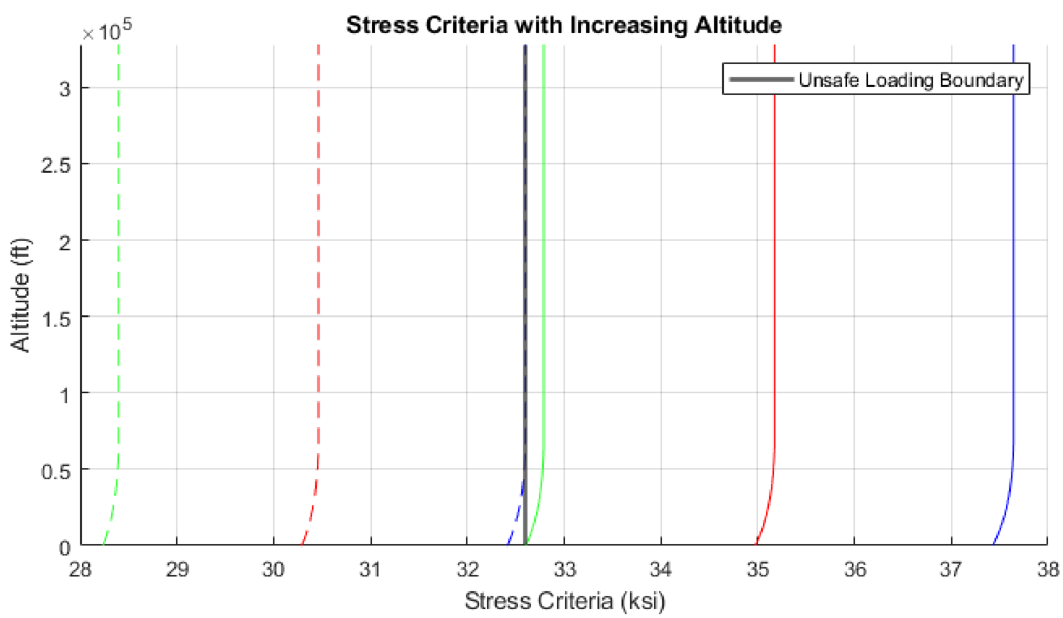



Figure 5: *Maximum Von Mises Loading versus Altitude*

## Conclusion:

The aluminum-lithium alloy will support the internal pressure required to supply the space station for a whole year, with material to spare even with a high factor of safety. Our choice of material is justified not only because of our research and what other professionals have to say but also due to its ability to fulfill our goals in this scenario. Aluminum-Lithium alloys appear to be very capable of withstanding the pressure change as our launch vehicle ascends.

In the realm of spacecraft design, there is often much more to consider when creating pressurized containers. Hypervelocity impacts, radiation, and atomic oxygen are just some of the factors which degrade materials launched into orbit. Some containers are also more complex and these need to be evaluated to determine points where increased stress may occur. However, when it is possible, we recommend simpler shapes and smart placement of the pressure vessels be implemented for the good of missions. A simple cylinder with spherical end caps by nature of its geometry is less susceptible to weak points for example and is thus a good choice for fluid tanks. Corners and edges such as that on a cylinder with flat ends should be avoided, as the actual stress in these locations is much higher than the modeled stress and the seam at the edges is often weaker than the bulk material.

Given more resources, we would also implement a launch profile and determine the dynamic pressure at each altitude for more accurate results. We could also use the temperature of the atmosphere and/or heating of the launch vehicle to add the effects of thermal expansion into our model.

## References:

[1] Niedzinski, Michael, “The Evolution of Constellium Al-Li Alloys for Space Launch and Crew Module Applications,” Light Metal Age, 11 Feb. 2019, https://www.lightmetalage.com/news/industry-news/aerospace/article-the-evolution-of-constellium-al-li-alloys-for-space-launch-and-crew-module-applications/

[2] Noble, B., Harris, S. J., and Dinsdale, K., “The elastic modulus of aluminium-lithium alloys,” *Journal of Materials Science*, Vol. 17, Feb. 1982, pp. 461 - 468, https://link.springer.com/article/10.1007/BF00591481#:~:text=The%20Young's%20moduli%20of%20the,GPa%20and%20105%20GPa%20respectively.

**Appendix**

General form of hoop and radial stress:

$$\sigma_h = A + \frac{B}{r^2}$$
$$\sigma_r = A - \frac{B}{r^2}$$

Boundary conditions:

$$\sigma_r(r_o) = -p_o$$
$$\sigma_r(r_i) = -p_i$$

Derivation of Lamé's equations:

$$A - \frac{B}{r_i^2} = p_i$$
$$\rightarrow B = (A - p_i) r_i^2$$
$$A - \frac{(A - p_i)}{r_o^2} = -p_o$$
$$A r_o^2 - A + p_i = -p_o r_o^2$$
$$A(r_o^2 - 1) = -p_o r_o^2 - p_i$$
$$\rightarrow A = \frac{p_o r_o^2 + p_i}{1 - r_o^2}$$

$$\rightarrow \sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2}$$
$$\rightarrow \sigma_h = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2}$$

Axial stress for a closed cylinder:

$$p_o A_o - p_i A_i + \sigma_a (A_o - A_i) = 0$$
$$\rightarrow \sigma_a = \frac{p_i A_i - p_o A_o}{A_o - A_i}$$
$$= \frac{p_i \pi r_i^2 - p_o \pi r_o^2}{\pi r_o^2 - \pi r_i^2}$$
$$= \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$