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AERO 560 – Advanced Spacecraft Dynamics and Control
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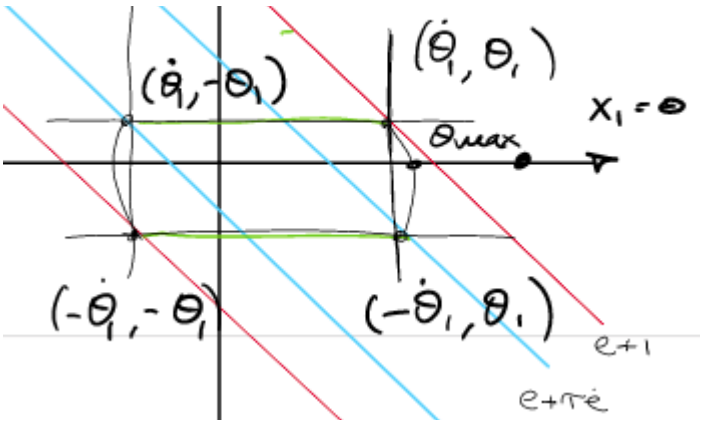
13 November 2022

4 – Bang-Bang Thrusters and Thruster Allocation

Problem 1.



$$\theta_1 + \tau\dot\theta_1 = \Delta_{on}$$
$$\theta_1 - \tau\dot\theta_1 = \Delta_{off}$$

$$\Delta_{on} + \Delta_{off} = \theta_1 + \tau\dot\theta_1 + \theta_1 - \tau\dot\theta_1$$
$$= 2\theta_1$$
$$\rightarrow \theta_1 = \frac{\Delta_{on} + \Delta_{off}}{2}$$

$$\Delta_{on} - \Delta_{off} = \theta_1 + \tau\dot\theta_1 - \theta_1 + \tau\dot\theta_1$$
$$= 2\tau\dot\theta_1$$
$$\rightarrow \dot\theta_1 = \frac{\Delta_{on} - \Delta_{off}}{2\tau}$$

$$\ddot\theta_1 = \frac{m}{J} \text{ when on, } 0 \text{ when off}$$

$$\Delta t_{off} = \frac{\theta_1 - (-\theta_1)}{\dot\theta_1} = 2\frac{\theta_1}{\dot\theta_1}$$

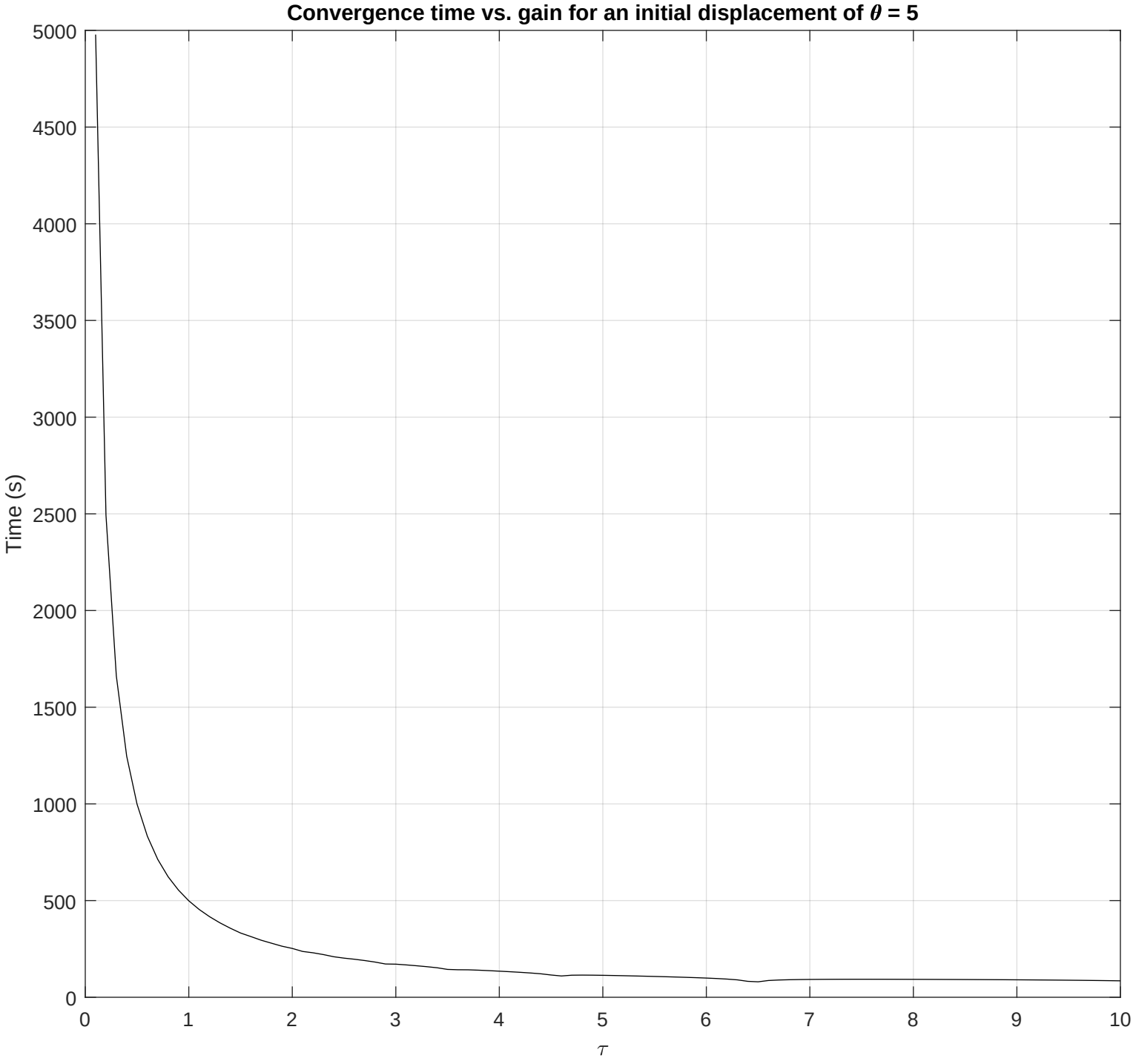
$$\Delta t_{on} = \frac{\dot\theta_1 - (-\dot\theta_1)}{\ddot\theta_1} = 2\frac{\dot\theta_1}{\ddot\theta_1}$$

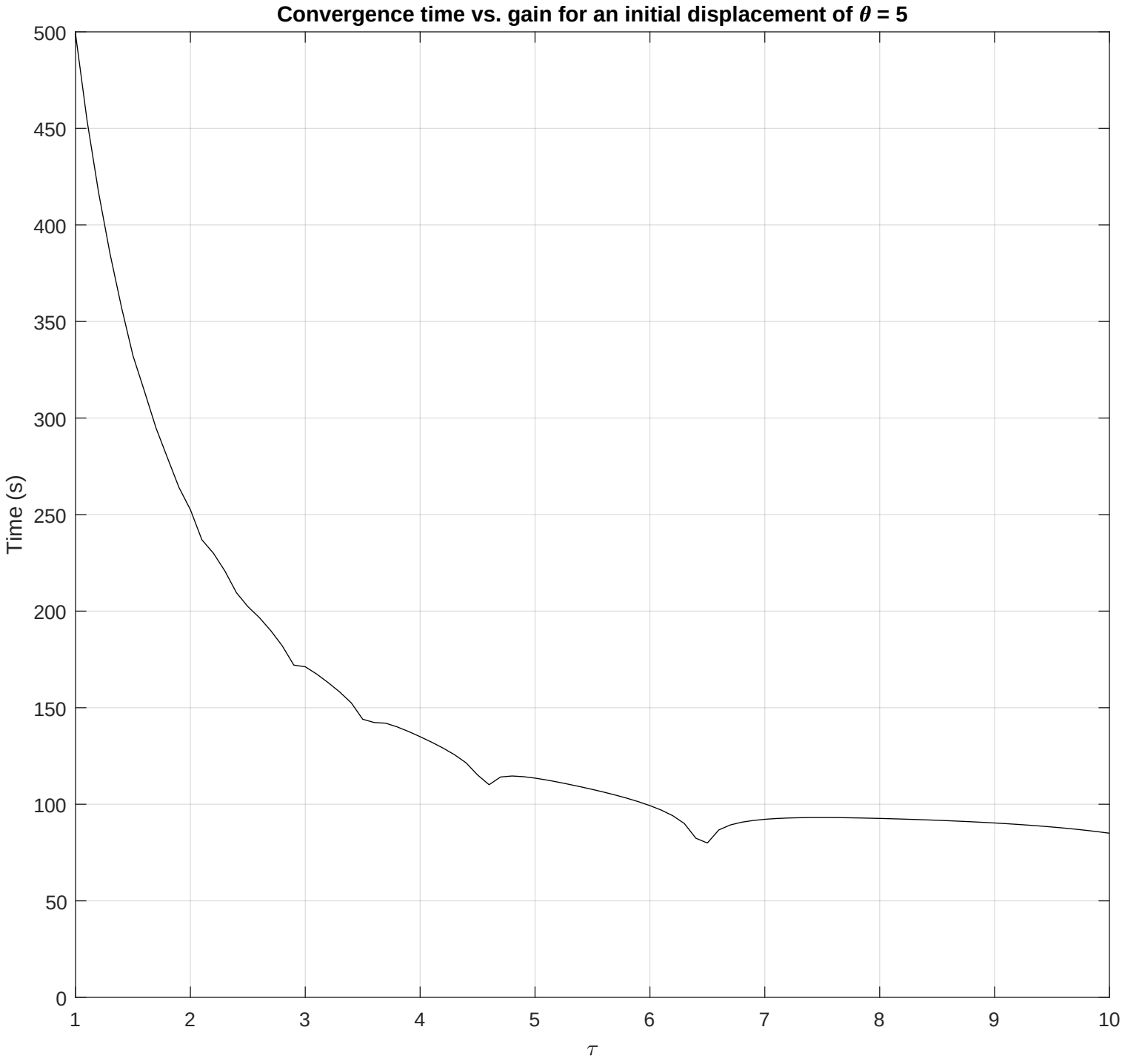
$$DC = \frac{\Delta t_{on}}{\Delta t_{on} + \Delta t_{off}}$$
$$= \frac{2\frac{\dot\theta_1}{\ddot\theta_1}}{2\frac{\dot\theta_1}{\ddot\theta_1} + 2\frac{\theta_1}{\dot\theta_1}}$$
$$= \frac{\dot\theta_1}{\dot\theta_1 + \frac{\theta_1\ddot\theta_1}{\dot\theta_1}}$$

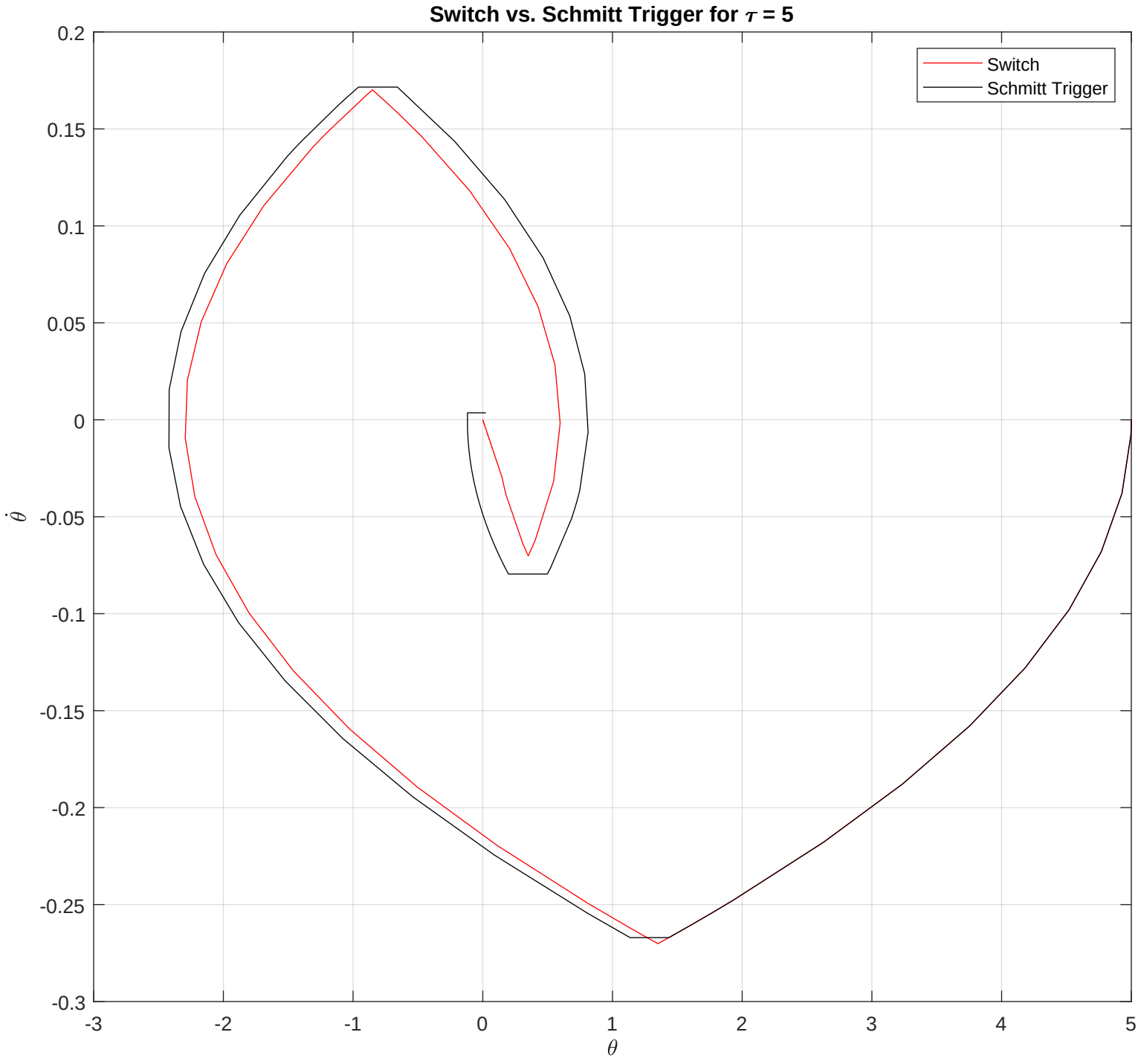
$$\rightarrow DC = \frac{\frac{\Delta_{on} - \Delta_{off}}{2\tau}}{\frac{\Delta_{on} - \Delta_{off}}{2\tau} + \frac{\frac{\Delta_{on} + \Delta_{off}}{2}\cdot\frac{m}{J}}{\frac{\Delta_{on} - \Delta_{off}}{2\tau}}}$$
$$= \frac{\Delta_{on} - \Delta_{off}}{\Delta_{on} - \Delta_{off} + 2\tau^2\frac{m}{J}\cdot\frac{\Delta_{on} + \Delta_{off}}{\Delta_{on} - \Delta_{off}}}$$
$$= \frac{(\Delta_{on} - \Delta_{off})^2}{(\Delta_{on} - \Delta_{off})^2 + 2\tau^2\frac{m}{J}\cdot(\Delta_{on} + \Delta_{off})}$$

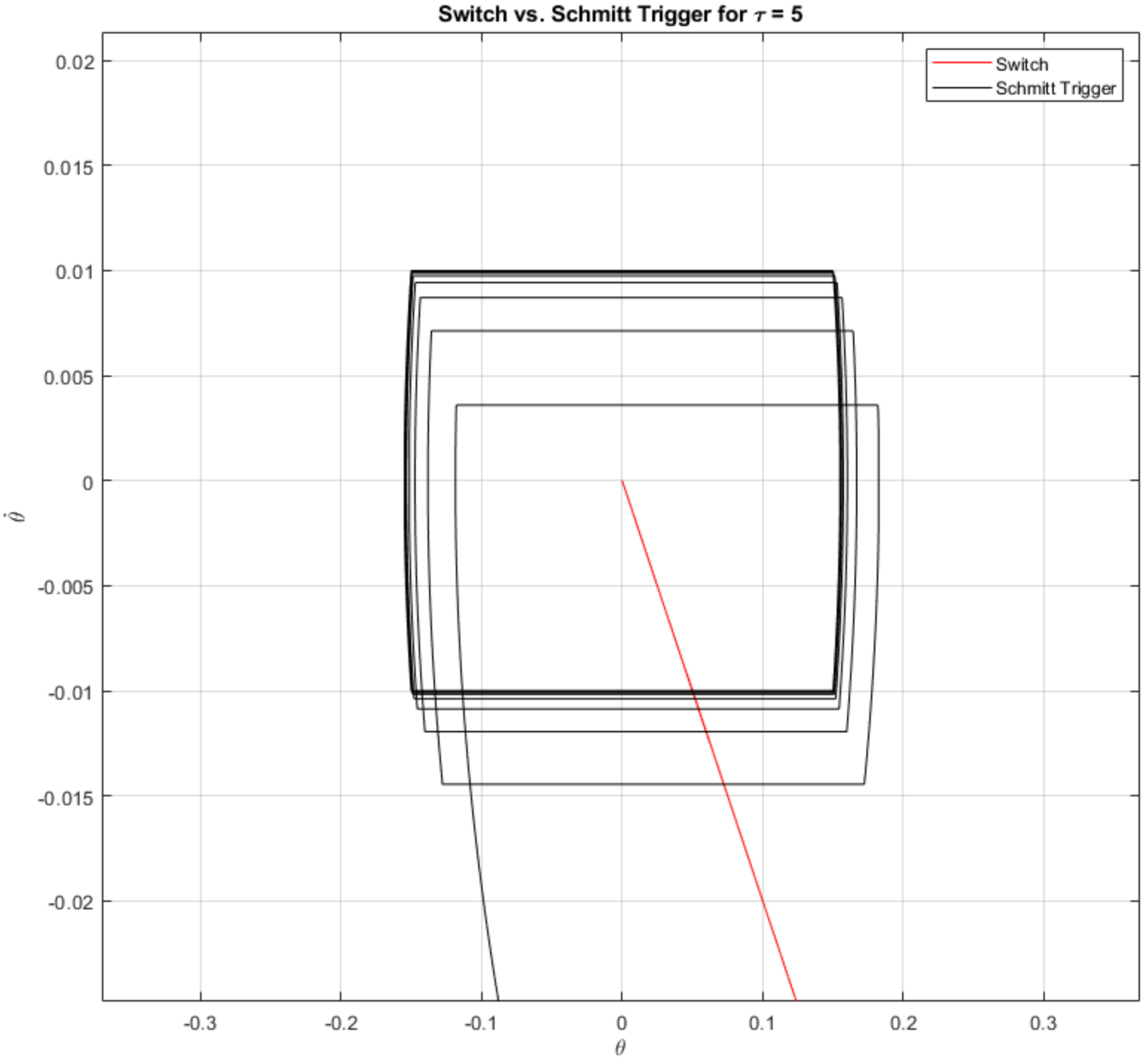
Problem 2.

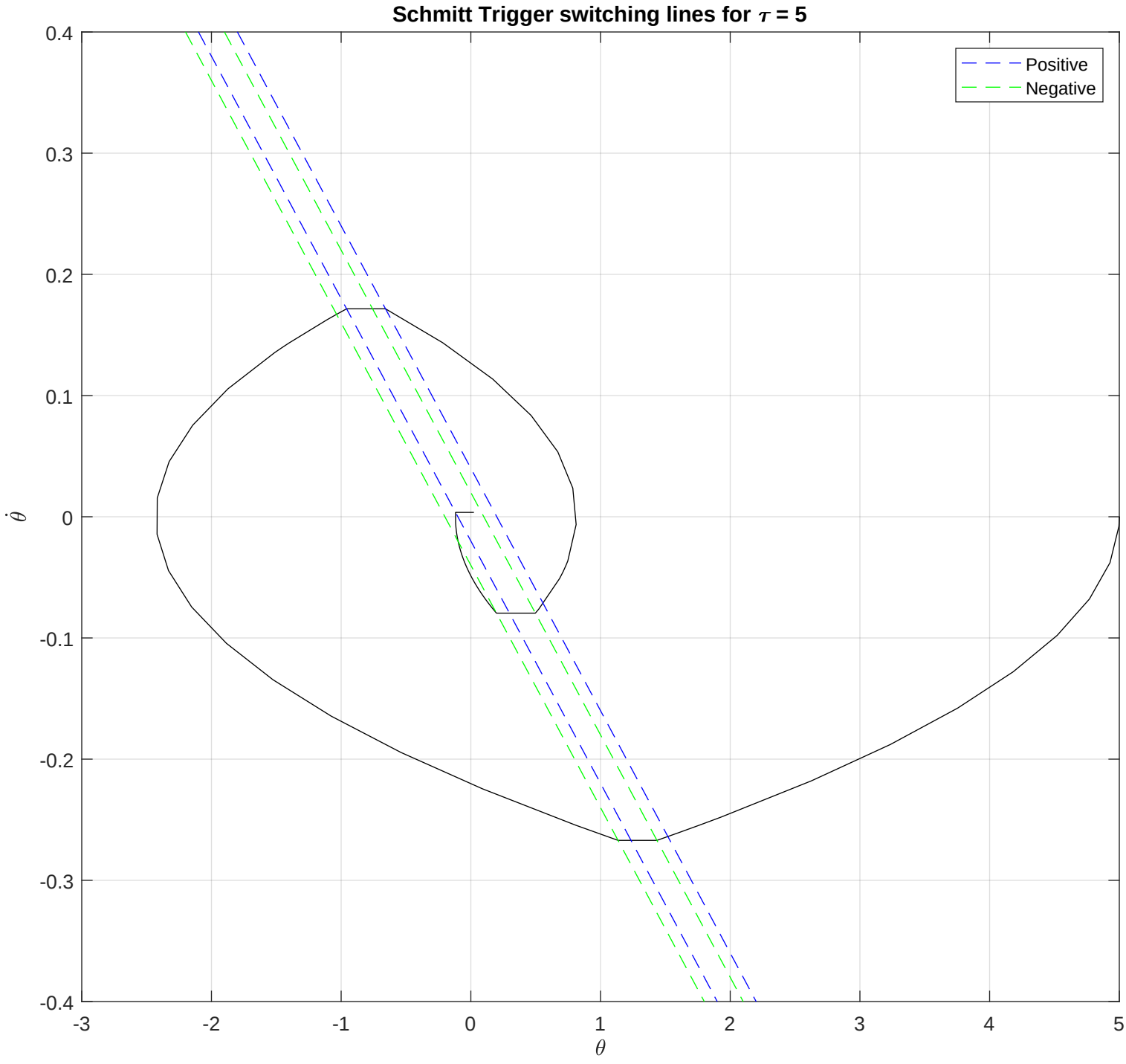
The convergence time is inversely proportional to τ with the inflection point at $\tau = 1$, and levels out to a minimum of about 90 seconds.

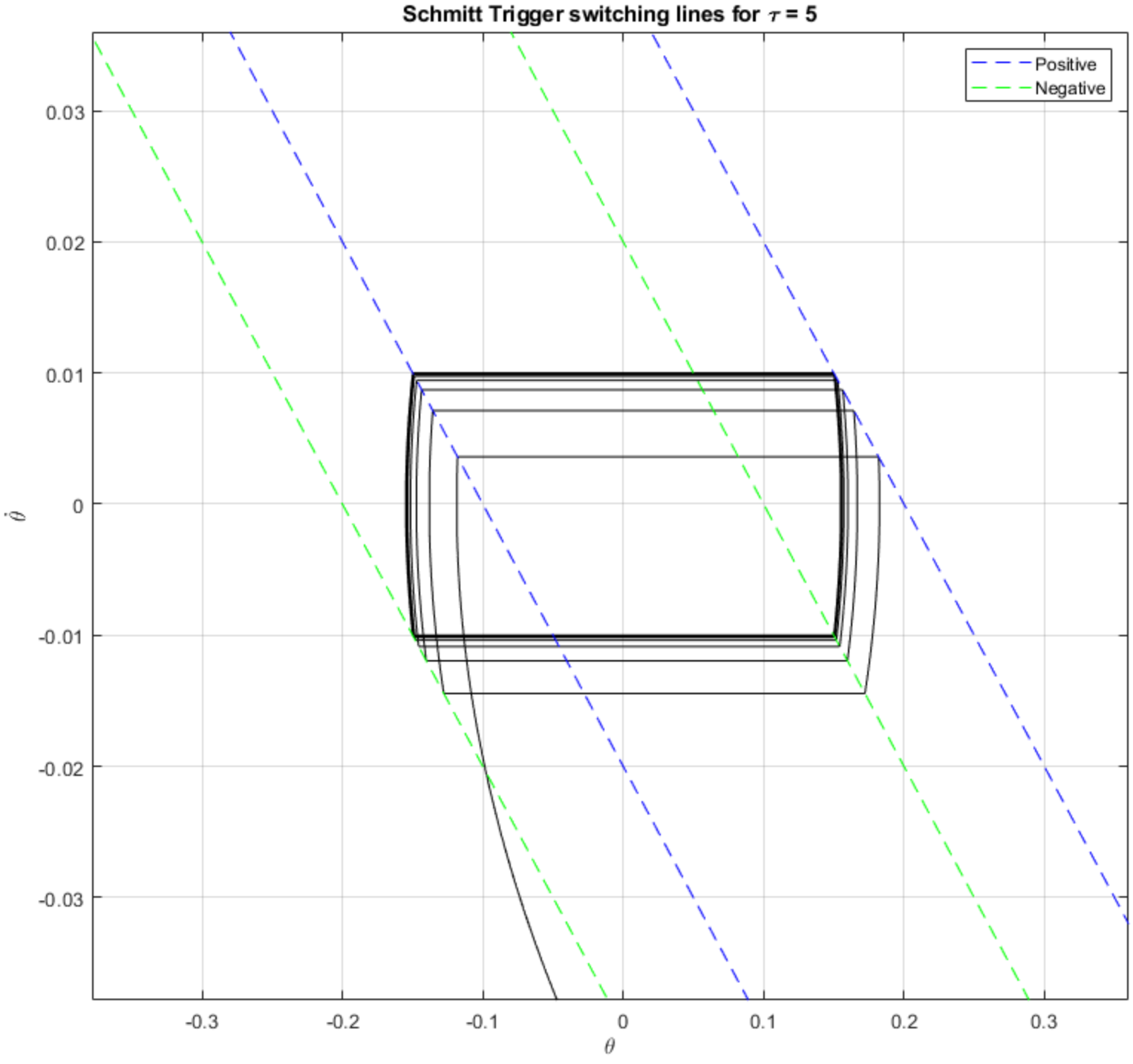

Convergence time vs. gain for an initial displacement of θ = 5
Time (s)
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
1
2
3
4
5
6
7
8
9
10
τ

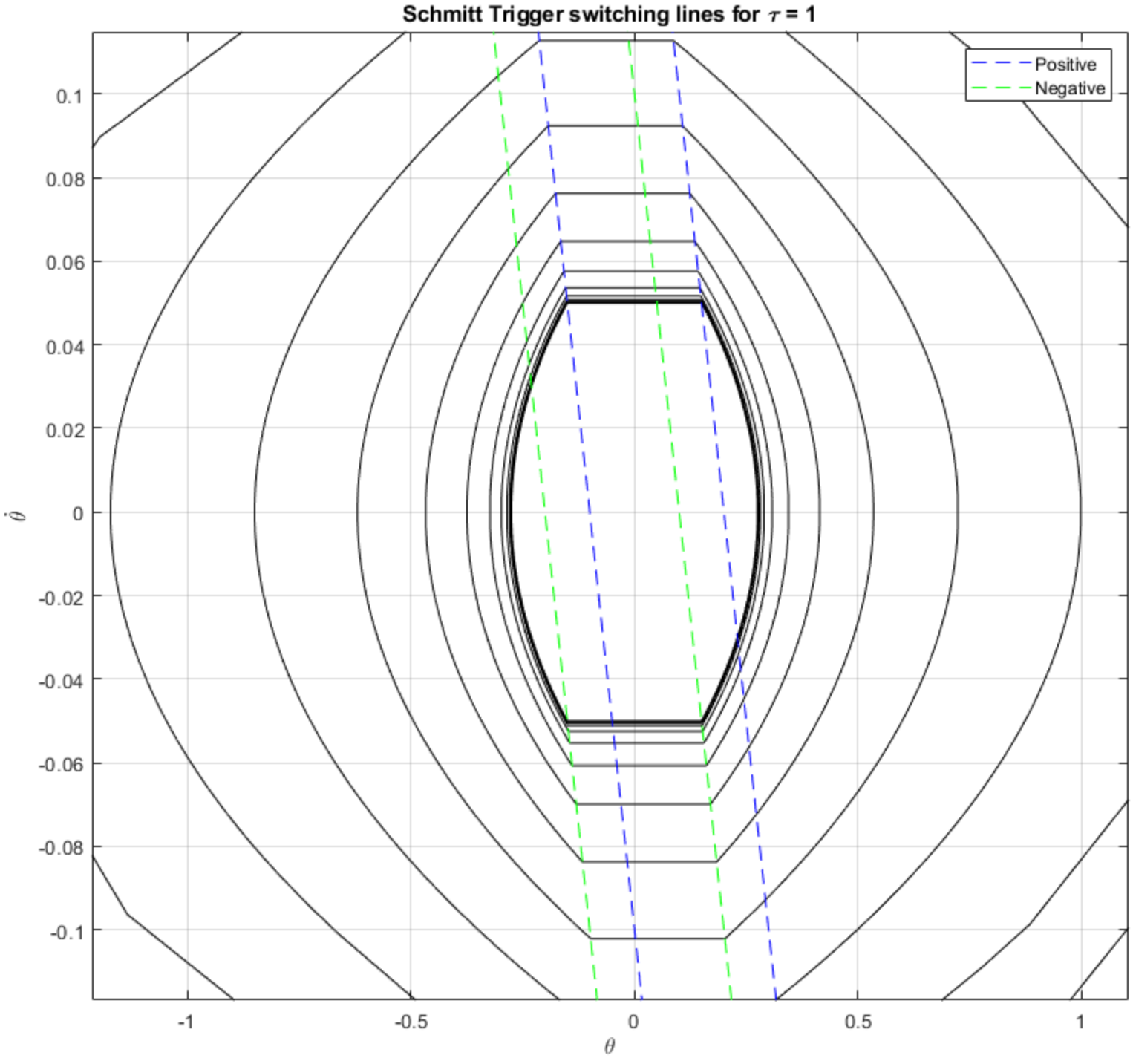

Convergence time vs. gain for an initial displacement of θ = 5
Time (s)
500
450
400
350
300
250
200
150
100
50
0
1
2
3
4
5
6
7
8
9
10
τ


Switch vs. Schmitt Trigger for τ = 5
Switch
Schmitt Trigger
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
$\dot{\theta}$
-3
-2
-1
0
1
2
3
4
5
θ

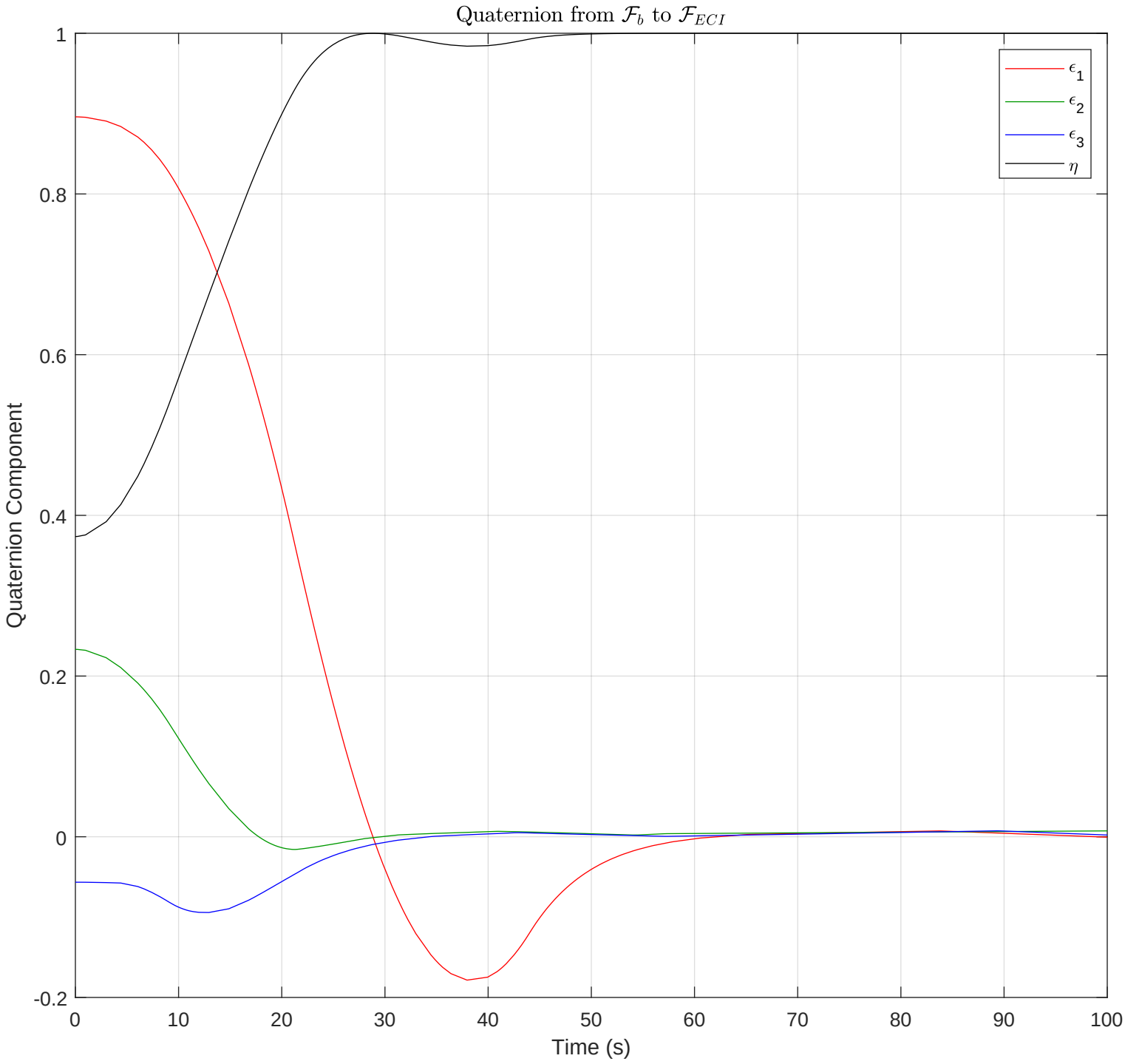

Switch vs. Schmitt Trigger for τ = 5
Switch
Schmitt Trigger
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
$\dot{\theta}$
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
θ

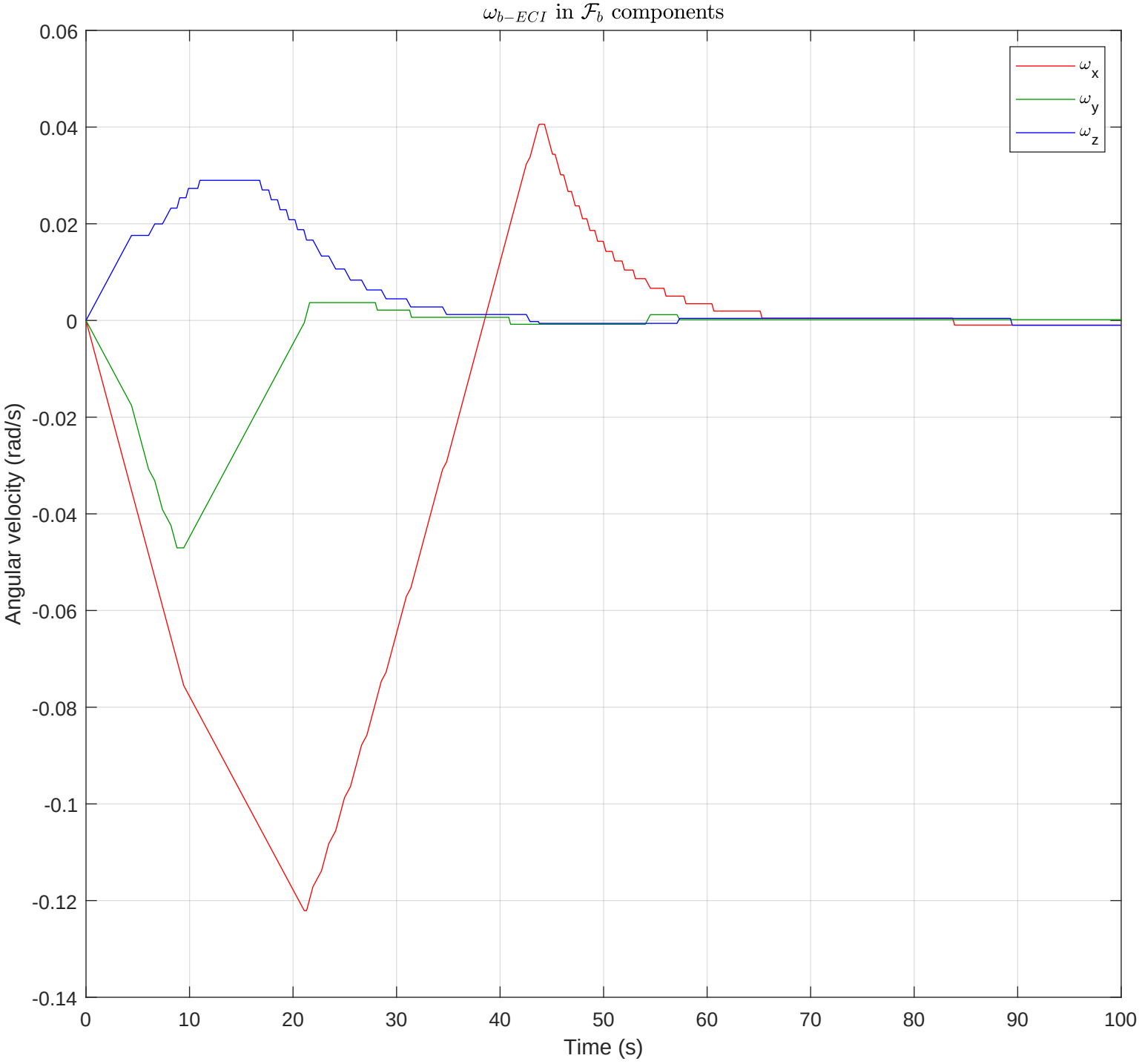

Schmitt Trigger switching lines for τ = 5
Positive
Negative
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
$\dot{\theta}$
-3
-2
-1
0
1
2
3
4
5
θ

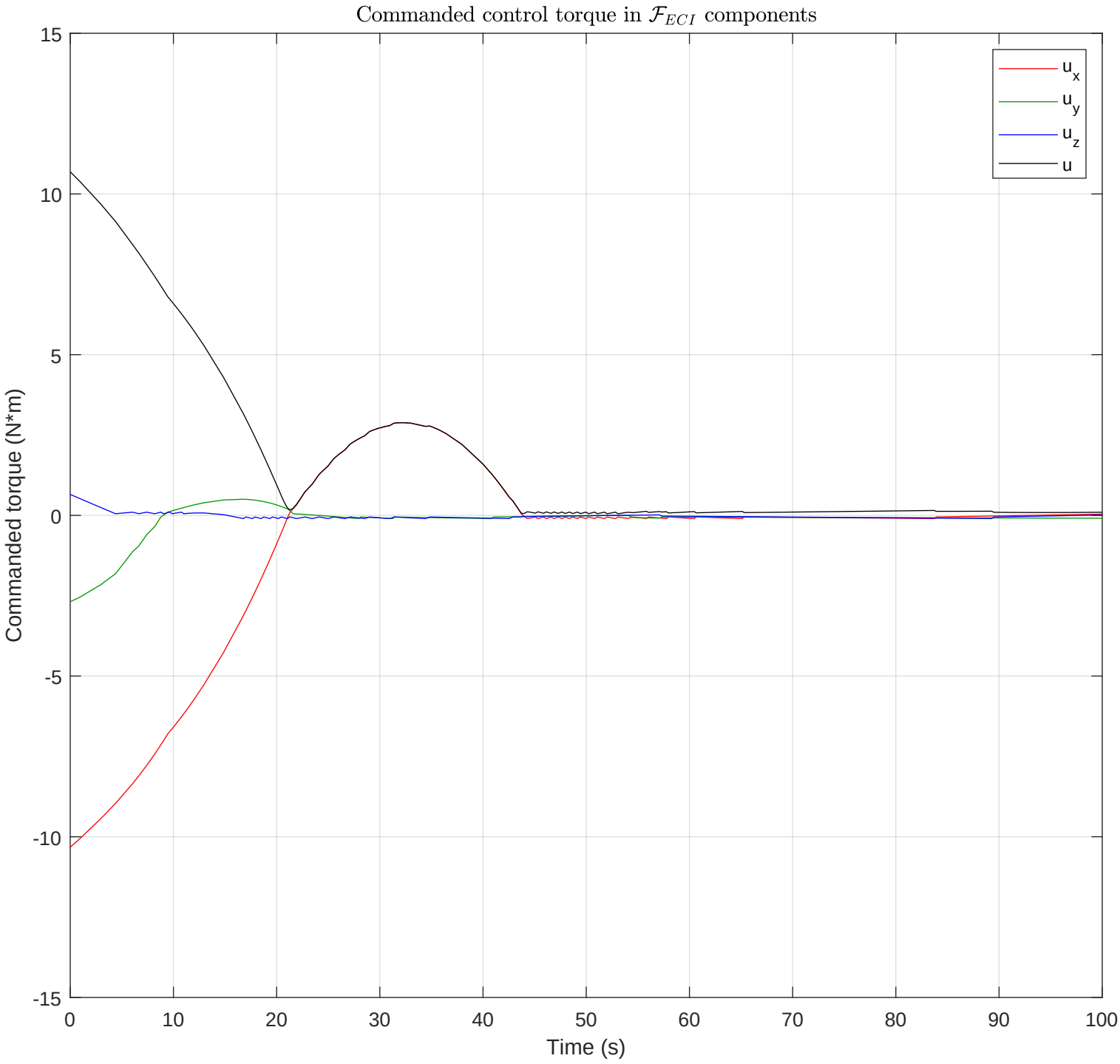

Schmitt Trigger switching lines for $\tau = 5$
Positive
Negative
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
$\dot{\theta}$
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
θ

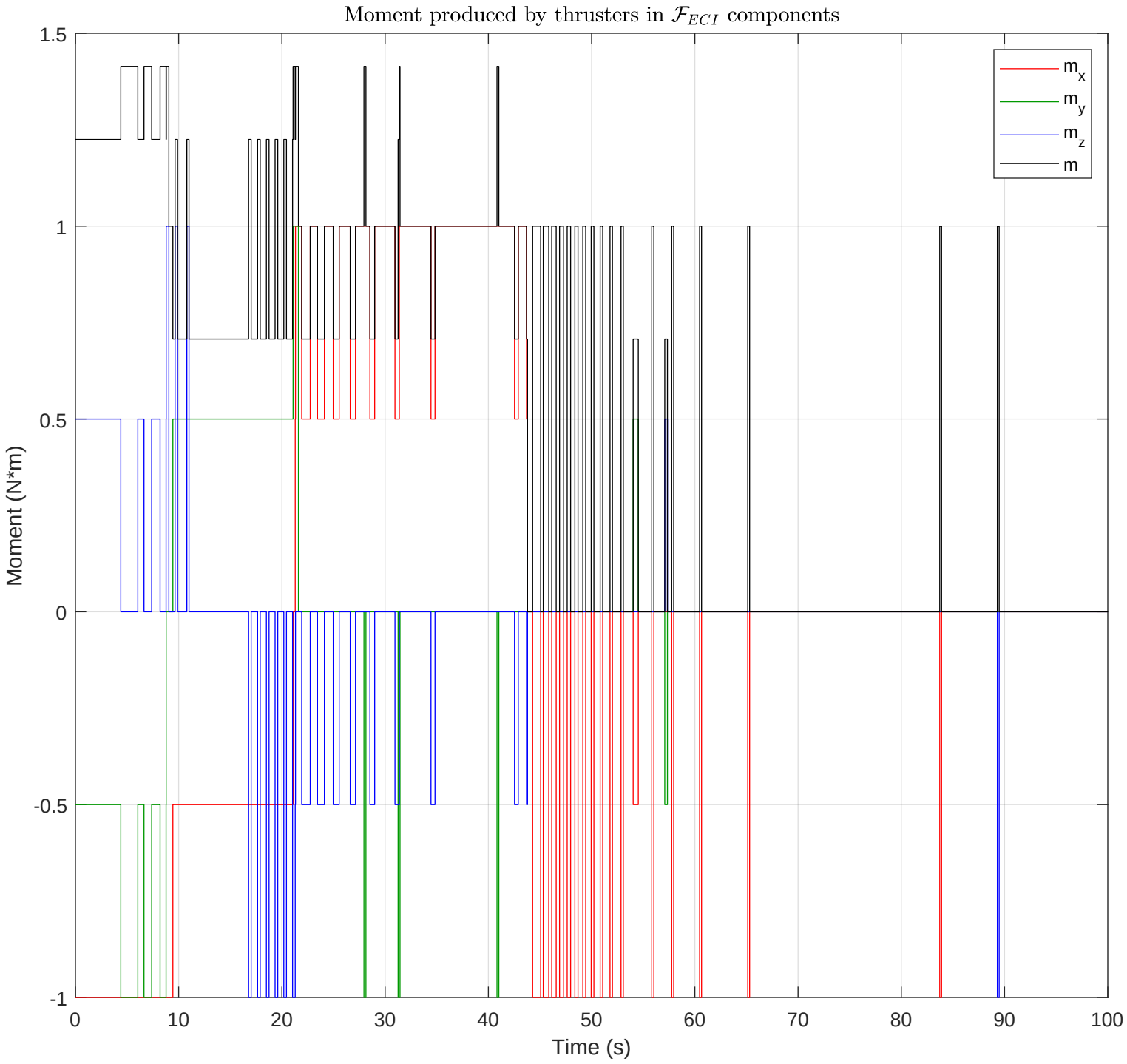

Schmitt Trigger switching lines for $\tau = 1$
Positive
Negative
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
$\dot{\theta}$
-1
-0.5
0
0.5
1
θ

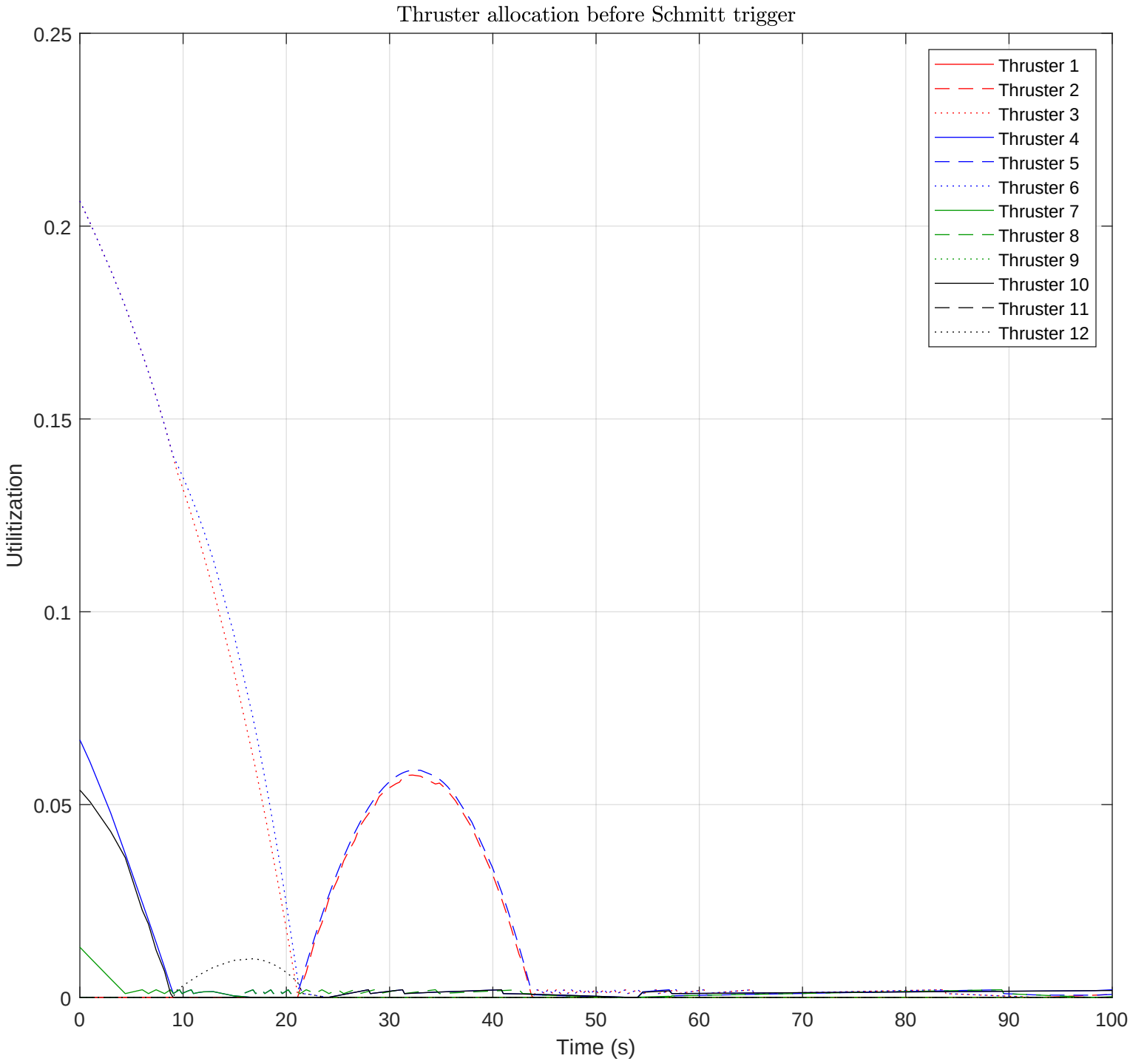
Problem 3.

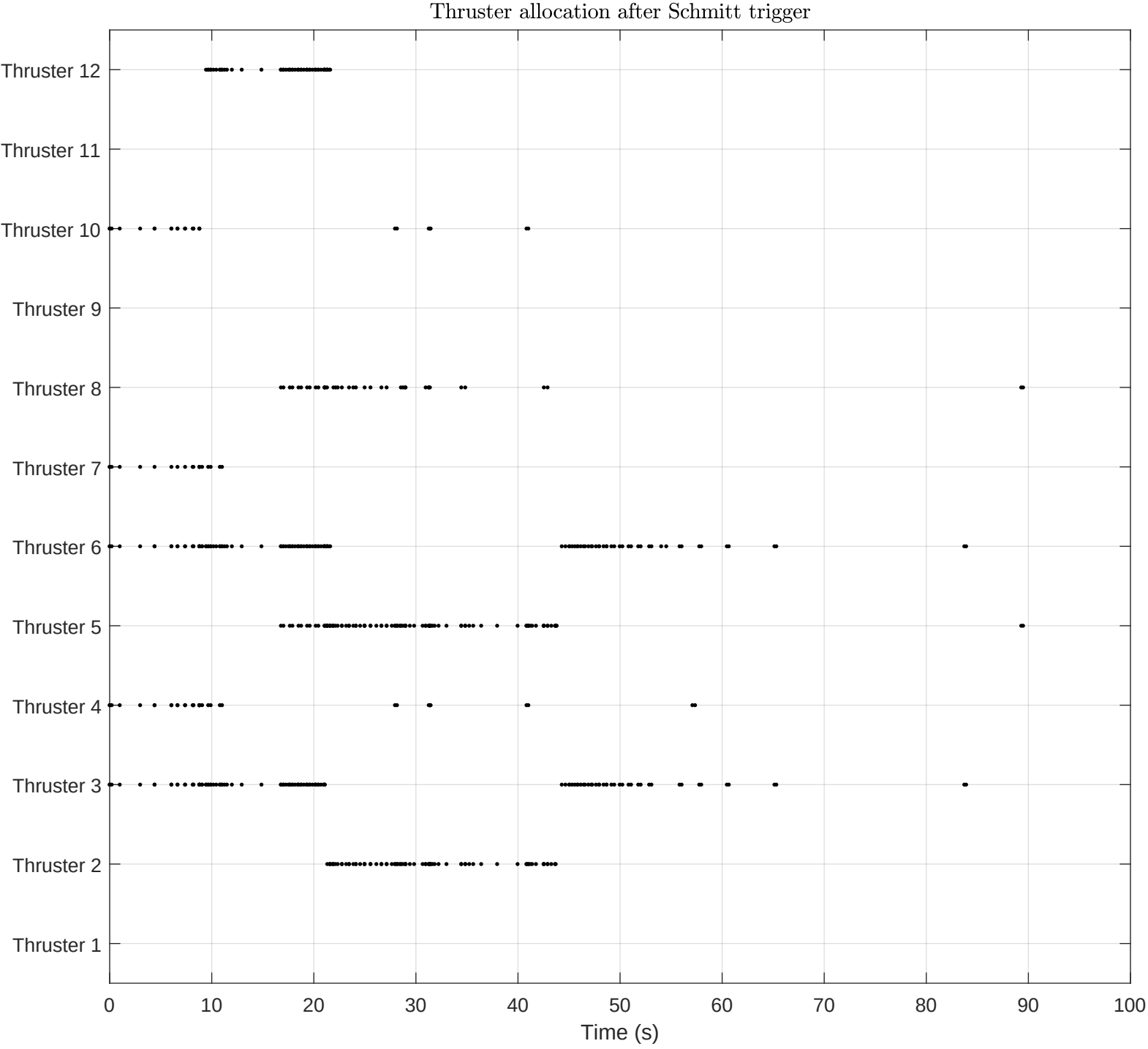

Quaternion from $\mathcal{F}_b$ to $\mathcal{F}_{ECI}$
Quaternion Component
Time (s)
ϵ_1
ϵ_2
ϵ_3
η
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60
70
80
90
100


ω_{b-ECI} in $\mathcal{F}_b$ components
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
Angular velocity (rad/s)
0
10
20
30
40
50
60
70
80
90
100
Time (s)
ω_x
ω_y
ω_z


Commanded control torque in $\mathcal{F}_{ECI}$ components
Commanded torque (N*m)
Time (s)
15
10
5
0
-5
-10
-15
0
10
20
30
40
50
60
70
80
90
100
u_x
u_y
u_z
u


Moment produced by thrusters in $\mathcal{F}_{ECI}$ components
Moment (N*m)
Time (s)
1.5
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80
90
100
m_x
m_y
m_z
m


Thruster allocation before Schmitt trigger
Utilitization
Time (s)
0
0.05
0.1
0.15
0.2
0.25
0
10
20
30
40
50
60
70
80
90
100
Thruster 1
Thruster 2
Thruster 3
Thruster 4
Thruster 5
Thruster 6
Thruster 7
Thruster 8
Thruster 9
Thruster 10
Thruster 11
Thruster 12


Thruster allocation after Schmitt trigger
Thruster 12
Thruster 11
Thruster 10
Thruster 9
Thruster 8
Thruster 7
Thruster 6
Thruster 5
Thruster 4
Thruster 3
Thruster 2
Thruster 1
0
10
20
30
40
50
60
70
80
90
100
Time (s)