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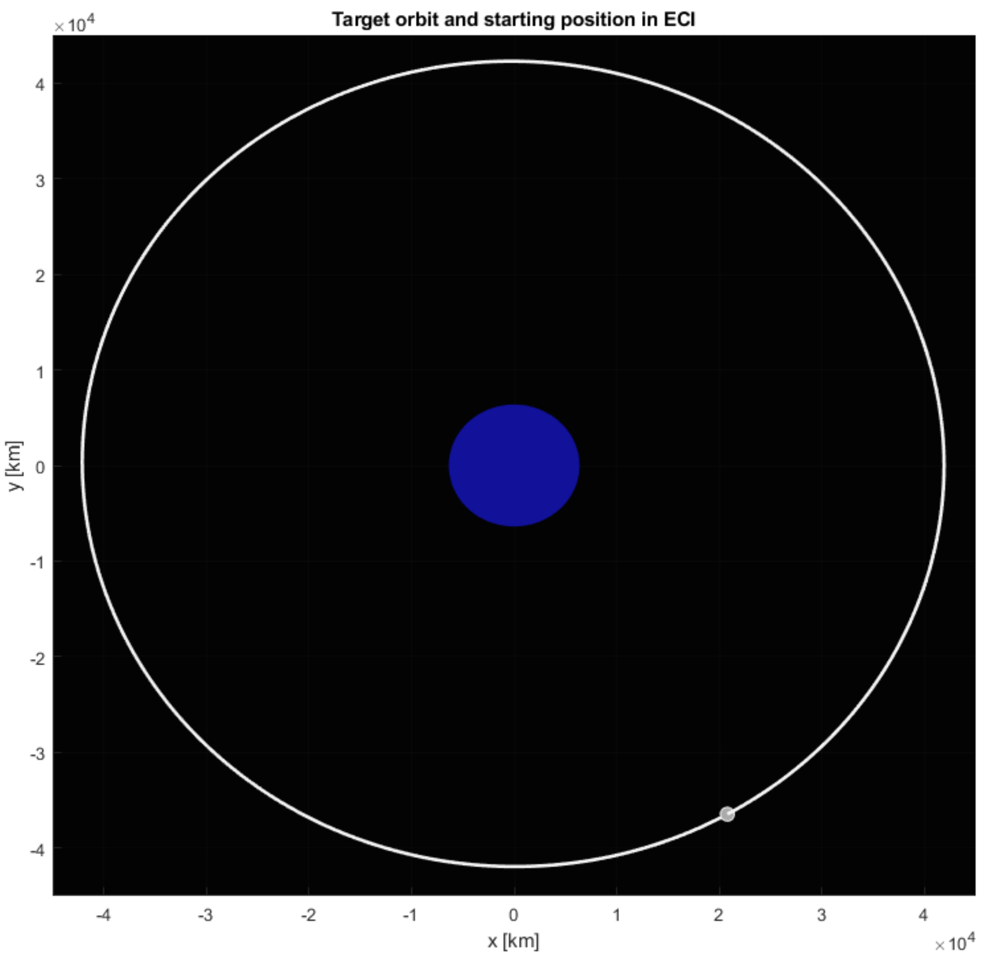
Rendezvous Flight Plan for a GEO Satellite Service Mission

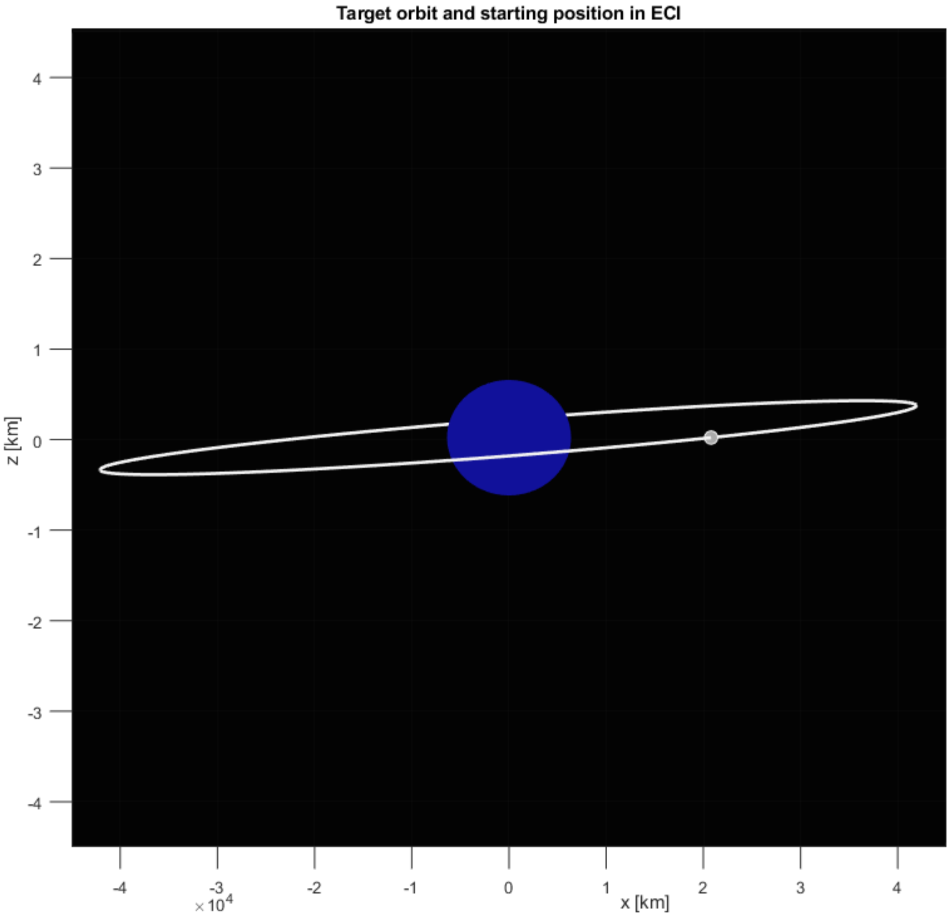
Introduction

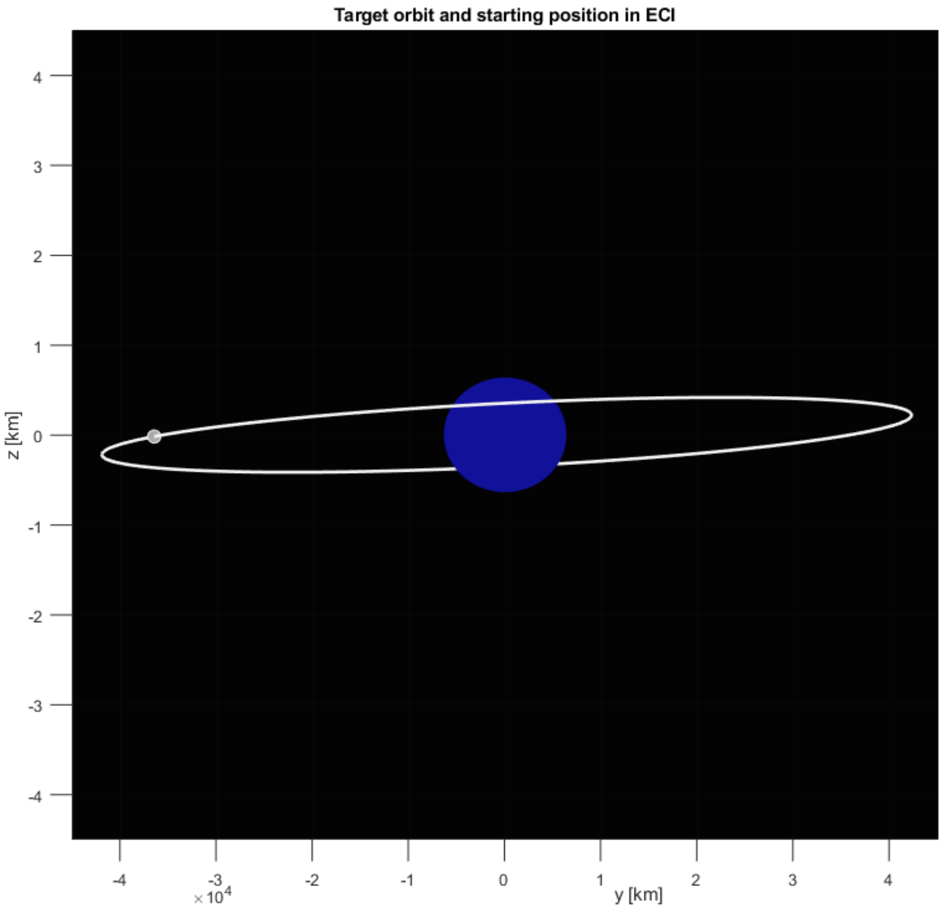
The Lockheed-Martin Advanced Extremely High Frequency 6 satellite has suffered a catastrophic failure and needs service to restore functionality and complete the satellite constellation. This satellite was selected for the upcoming NASA rendezvous mission because it has a near circular orbit that allows the Clohessy-Wiltshire equations to be applicable, but still has some eccentricity so there will be a small, but not insignificant, difference between the Clohessy-Wiltshire equations and the elliptical proximity equations.

The orbital elements of the AEHF-6 satellite, henceforth referred to as the 'target,' at the start of the mission are as follows:

e	0.0046256	Ω	301.9920°
h	129,646.85 km²/s	ω	357.6804°
i	5.6966°	Θ	0°




Target orbit and starting position in ECI
z [km]
x [km]
×10^4
4
3
2
1
0
-1
-2
-3
-4


Target orbit and starting position in ECI
z [km]
y [km]
×10^4
4
3
2
1
0
-1
-2
-3
-4

Mission Overview

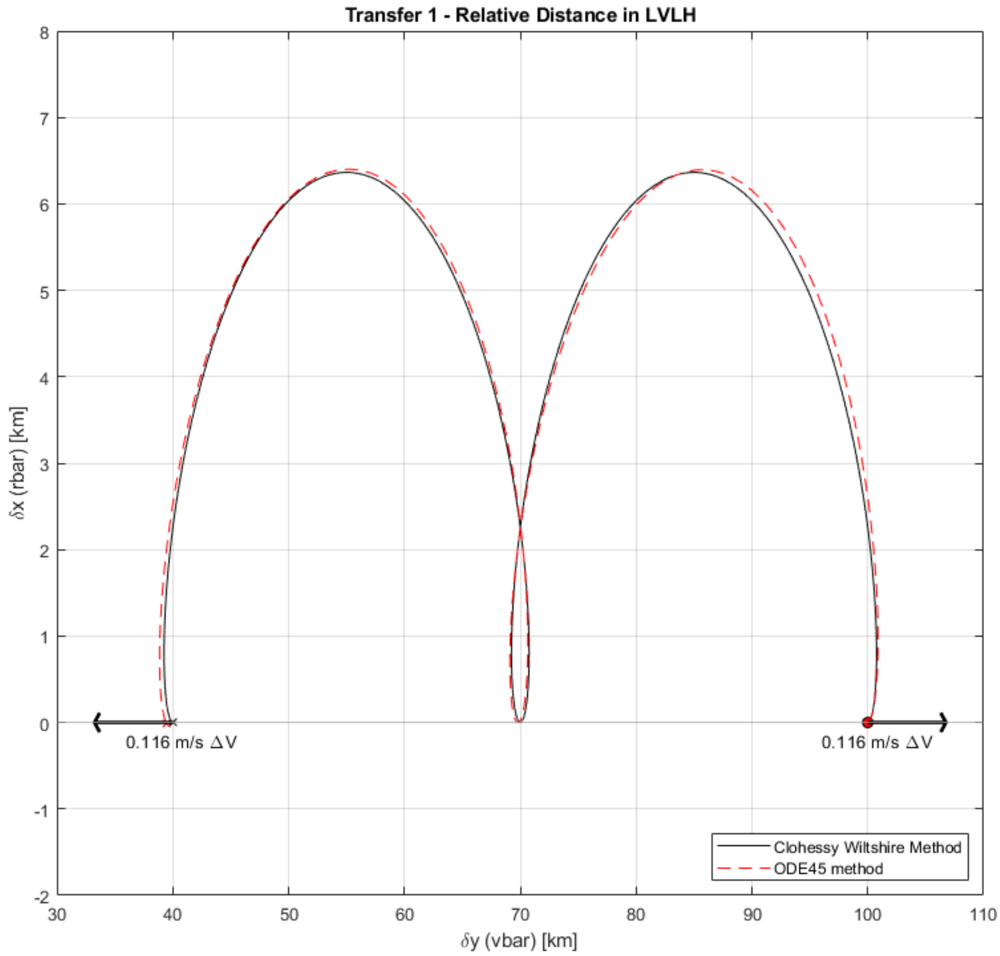
The flight plan for this mission is a series of 6 transfers with intermediate holds, with the chaser starting 100 km downrange in front of the target. The first transfer consists of an impulsive hop along the vbar which is allowed to repeat once before the second impulse, leaving the chaser 40 km downrange in front of the target. The chaser holds its position in vbar station keeping for one period of the target. The second transfer places the chaser into a 40 km x 20 km football orbit around the target for one period before returning to the vbar, before again holding position for one period of the target. The third transfer consists of an impulsive hop along the vbar followed by two smaller hops generated by reducing, but not eliminating, the relative velocity at each vbar intersect. The chaser is now 1 km downrange in front of the target and holds for one more period of the target. The final three transfers are continuous thrust vbar approaches from 1000 to 300 m, 300 m to 20 m, and 20 m to rendezvous, each followed by a 10½ hour vbar station keeping hold.

Vbar approach is considered necessary at relative distances less than 1 km for sensor calibration and accuracy and plume impingement avoidance. The relative velocities of the final three transfers are 0.5, 0.2, and 0.02 m/s respectively; these were chosen to minimize collision risk but also keep a reasonably short transfer time.

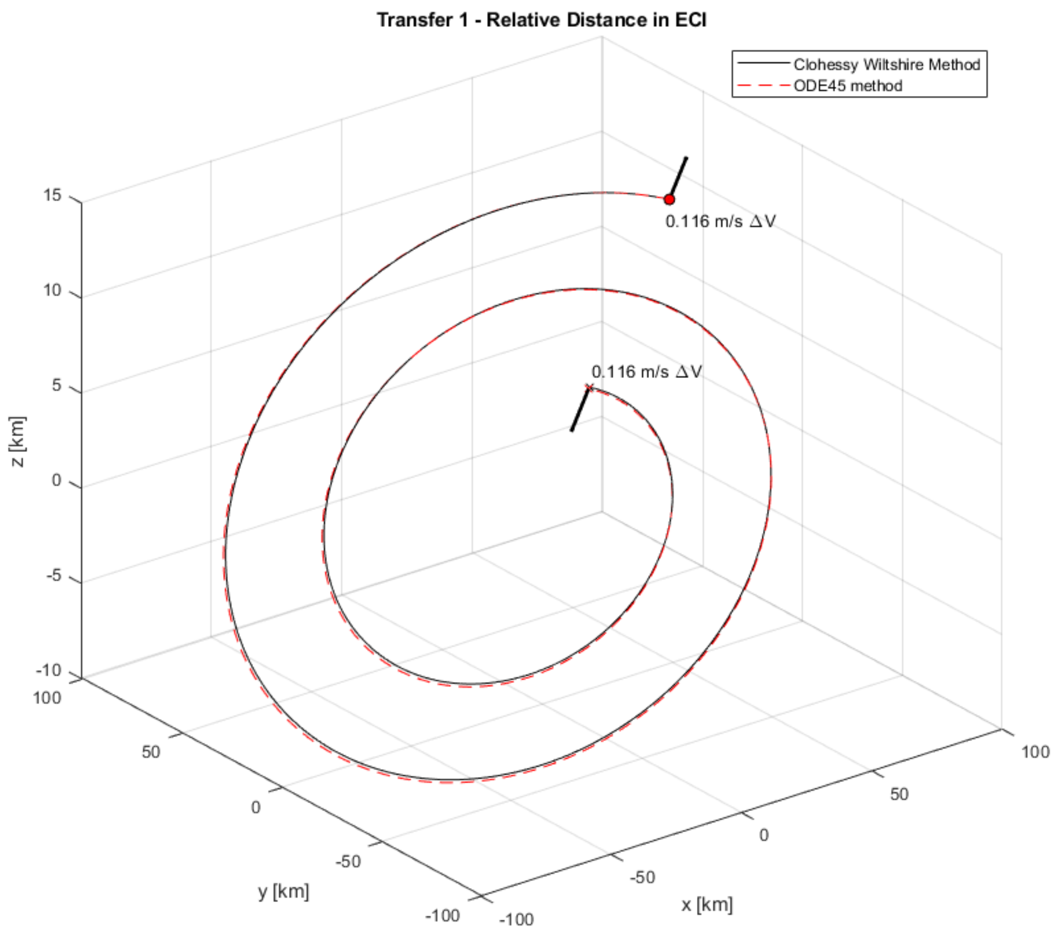
Transfer	ΔV (m/s)	Total ΔV (m/s)	Time	Total Time
Transfer 1	0.2320	0.2320	01:23:51:04	T+ 01:23:51:04
	Hold 1		00:23:56:04	T+ 02:23:47:08
Transfer 2	0.0029	0.2349	00:23:56:04	T+ 03:23:43:12
	Hold 2		00:23:56:04	T+ 04:23:39:16
Transfer 3	0.0032	0.2381	02:23:48:12	T+ 07:23:27:28
	Hold 3		00:23:56:04	T+ 08:23:23:32
Transfer 4	701	701.2381	00:00:23:20	T+ 08:23:46:52
	Hold 4		00:10:31:34	T+ 09:10:18:26
Transfer 5	280.4	981.6381	00:00:23:20	T+ 09:10:41:46
	Hold 5		00:10:31:34	T+ 09:21:13:20
Rendezvous	20.004	1001.6421	00:02:46:40	T+ 10:00:00:00
Total	1001.6 m/s		10:00:00:00	

Transfer 1			
Transfer Time	01:23:51:04	Starting MET	T+ 00:00:00:00
Hold Time	00:23:56:04	Ending MET	T+ 02:23:47:08
Total Time	02:23:47:08	ΔV	0.2320 m/s
Start of Transfer 1		End of Transfer 1 Hold	
Target Absolute Position in ECI (km)	20786.0312 -36465.6816 -168.6287	Target Absolute Position in ECI (km)	20724.904 -36500.4245 -175.6364
Chaser Relative Position in LVLH (km)	0 100 0	Chaser Relative Position in LVLH (km)	0 40 0

The chaser begins the mission at a relative distance of 100 km from the target, entirely in the vbar direction. At MET 00:00:00:00, the chaser performs an impulsive maneuver of 0.116 m/s in the positive vbar direction to transfer into a lower orbit. The chaser coasts on this new orbit for two periods before performing another impulsive maneuver of 0.116 m/s in the negative vbar direction to match velocities with the target. The chaser then holds at this relative position for one period of the target, which is just shy of 24 hours. The non-circular ODE45 method for this transfer ended up 541.2 m closer to the target on the vbar than the Clohessy-Wiltshire method, an error of 1.35%.



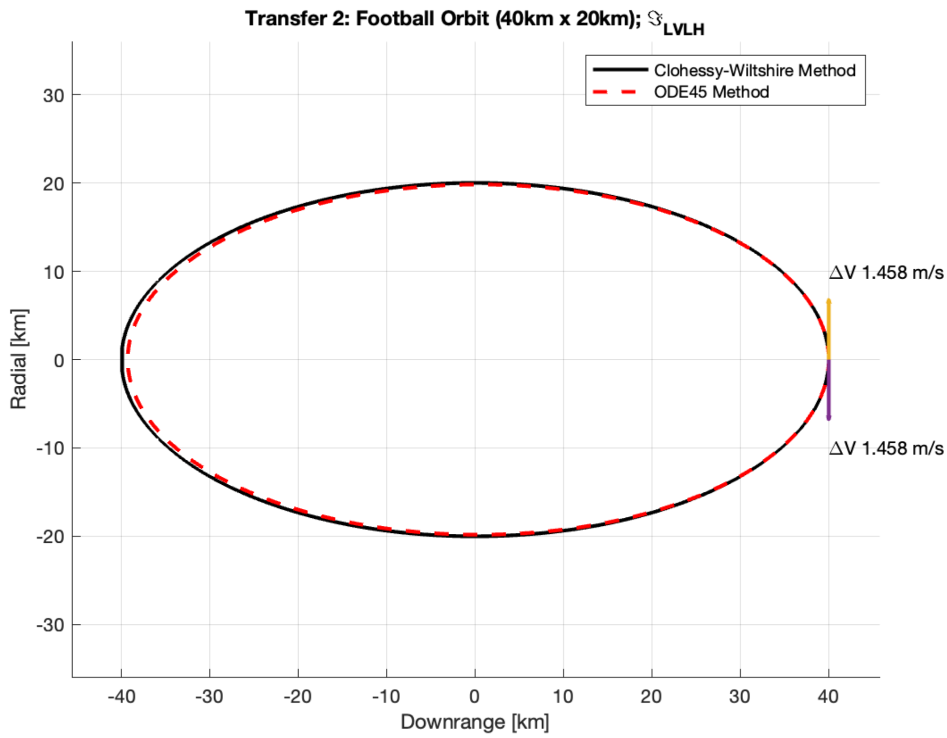
Transfer 1 - Relative Distance in LVLH
δx (rbar) [km]
δy (vbar) [km]
0.116 m/s ΔV
0.116 m/s ΔV
Clohessy Wiltshire Method
ODE45 method

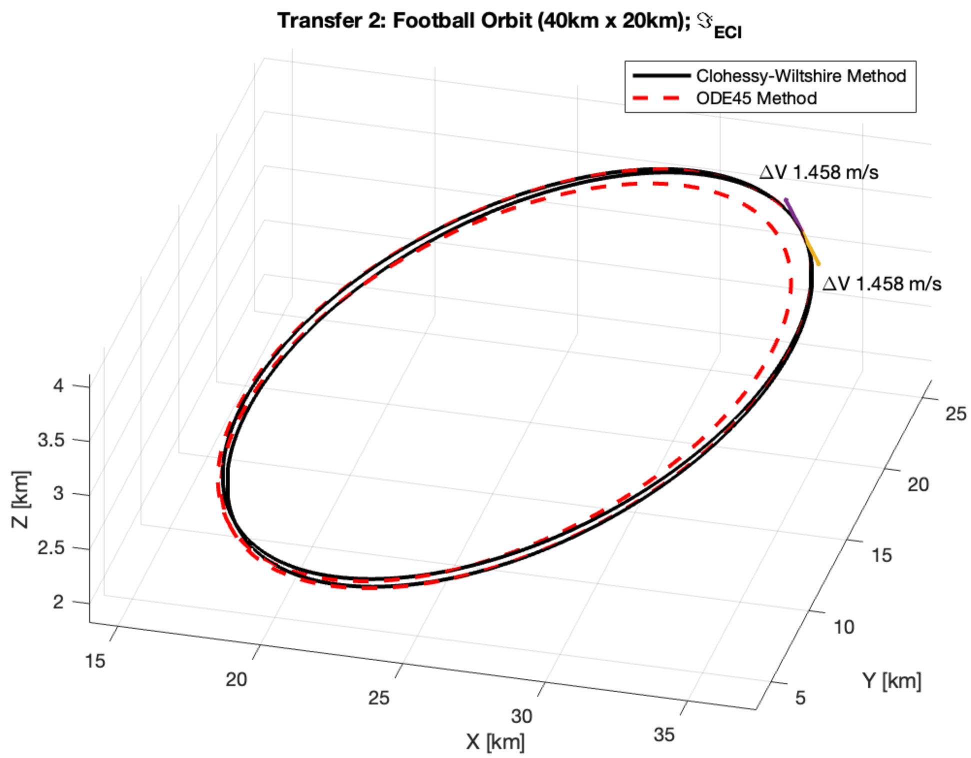


Transfer 1 - Relative Distance in ECI
Clohessy Wiltshire Method
ODE45 method
0.116 m/s ΔV
0.116 m/s ΔV
z [km]
y [km]
x [km]

Transfer 2			
Transfer Time	00:23:56:04	Starting MET	T+ 02:23:47:08
Hold Time	00:23:56:04	Ending MET	T+ 04:23:39:16
Total Time	01:23:52:08	ΔV	0.00292 m/s
Start of Transfer 2		End of Transfer 2 Hold	
Target Absolute Position in ECI (km)	20724.904 -36500.4245 -175.6364	Target Absolute Position in ECI (km)	20648.82680 -36543.47610 -184.34757
Chaser Relative Position in LVLH (km)	0 40 0	Chaser Relative Position in LVLH (km)	0 40 0

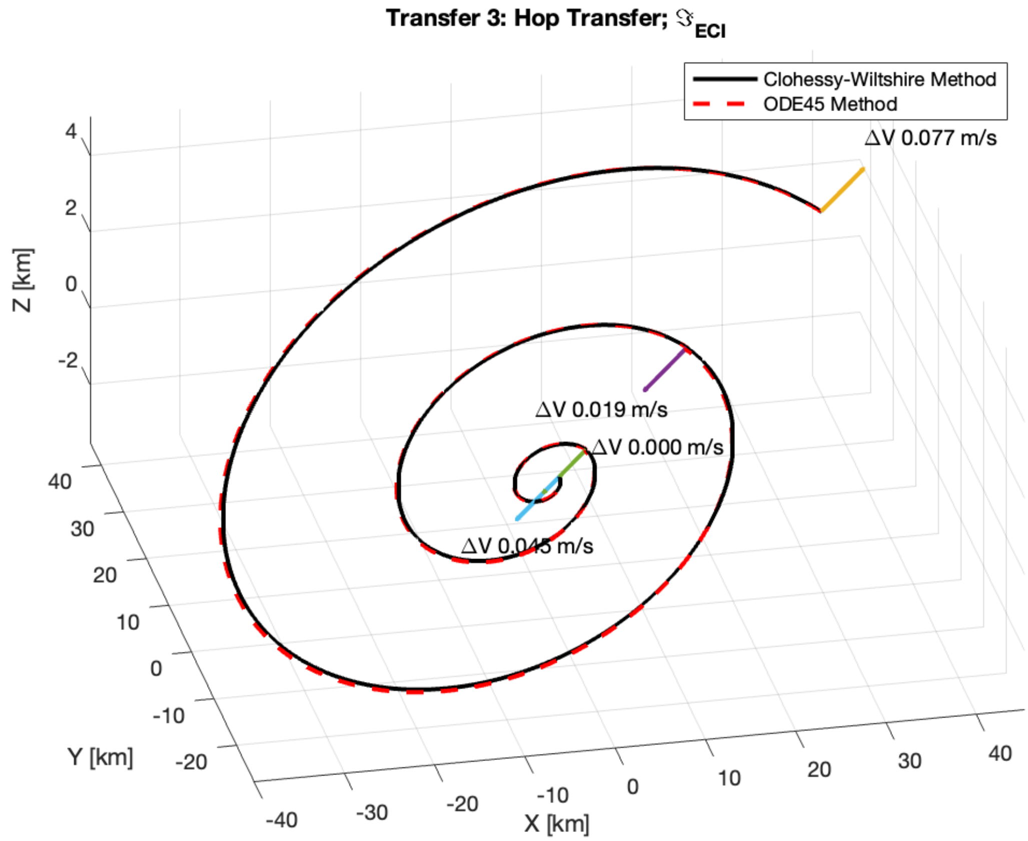
Following Transfer 1, the chaser now lies 40km downrange of the target in the Local-Vertical-Local-Horizontal frame (i.e., from the perspective of the target) with no relative velocity. As part of the mission, the chaser is to observe the target from all directions from at least 20km away but no farther than 40km away; the required distances are a function of the sensor resolution and range onboard the chaser. A simple way to "orbit" the target is to burn into a football orbit; this changes the eccentricity of the chaser's orbit and, from the perspective of the target, is seen to rotate around the target. To accomplish this, the chaser is to burn radially to its orbit in the amount of 1.458 m/s. This will put the chaser in the football orbit and, for a period, will observe the target from all directions. Once a period is over and the target and chaser have returned to their initial relative positions, the chaser will burn for 1.458 m/s again in the opposite radial direction. This will move the chaser out of the football orbit and back into the same orbit as its chaser at a relative position of 40 km downrange and a relative velocity of 0 m/s. The chaser requires 2.9169 m/s to move into and out of the football orbit. Interestingly, assumptions made by Clohessy-Wiltshire pose a large difference when compared to the iterative differential method. At its max, the difference in the methods shows a 0.8km, or 1%, difference in the downrange direction in the LVLH frame across the major diameter of the football orbit.

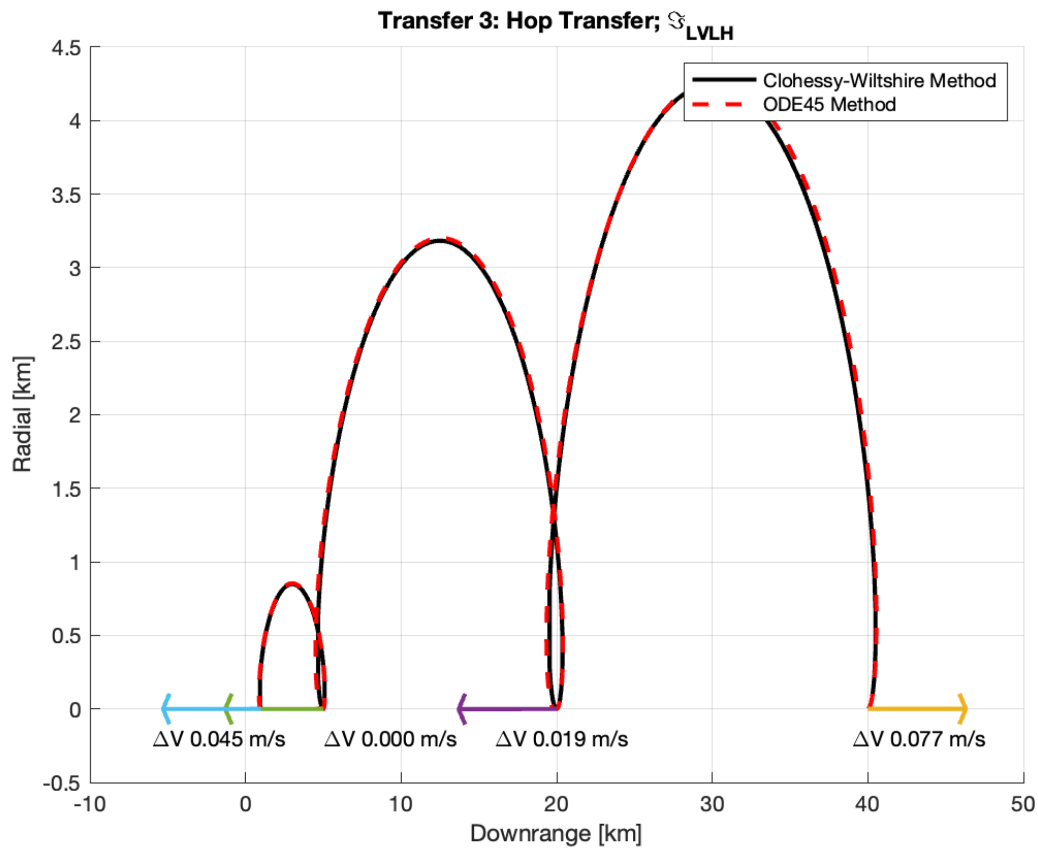

Transfer 2: Football Orbit (40km x 20km); $\Im_{LVLH}$
Clohessy-Wiltshire Method
ODE45 Method
ΔV 1.458 m/s
ΔV 1.458 m/s
Radial [km]
Downrange [km]
30
20
10
0
-10
-20
-30
-40
-30
-20
-10
0
10
20
30
40


Transfer 2: Football Orbit (40km x 20km); $\Im_{ECI}$
Clohessy-Wiltshire Method
ODE45 Method
ΔV 1.458 m/s
ΔV 1.458 m/s
Z [km]
4
3.5
3
2.5
2
X [km]
15
20
25
30
35
Y [km]
5
10
15
20
25

Transfer 3			
Transfer Time	02:23:48:12	Starting MET	T+ 04:23:39:16
Hold Time	00:23:56:04	Ending MET	T+ 08:23:23:32
Total Time	03:23:44:16	ΔV	0.1547 m/s
Start of Transfer 3		End of Transfer 3 Hold	
Target Absolute Position in ECI (km)	20648.82680 -36543.47610 -184.34757	Target Absolute Position in ECI (km)	20496.40264 -36629.10112 -201.76748
Chaser Relative Position in LVLH (km)	0 40 0	Chaser Relative Position in LVLH (km)	0 1 0

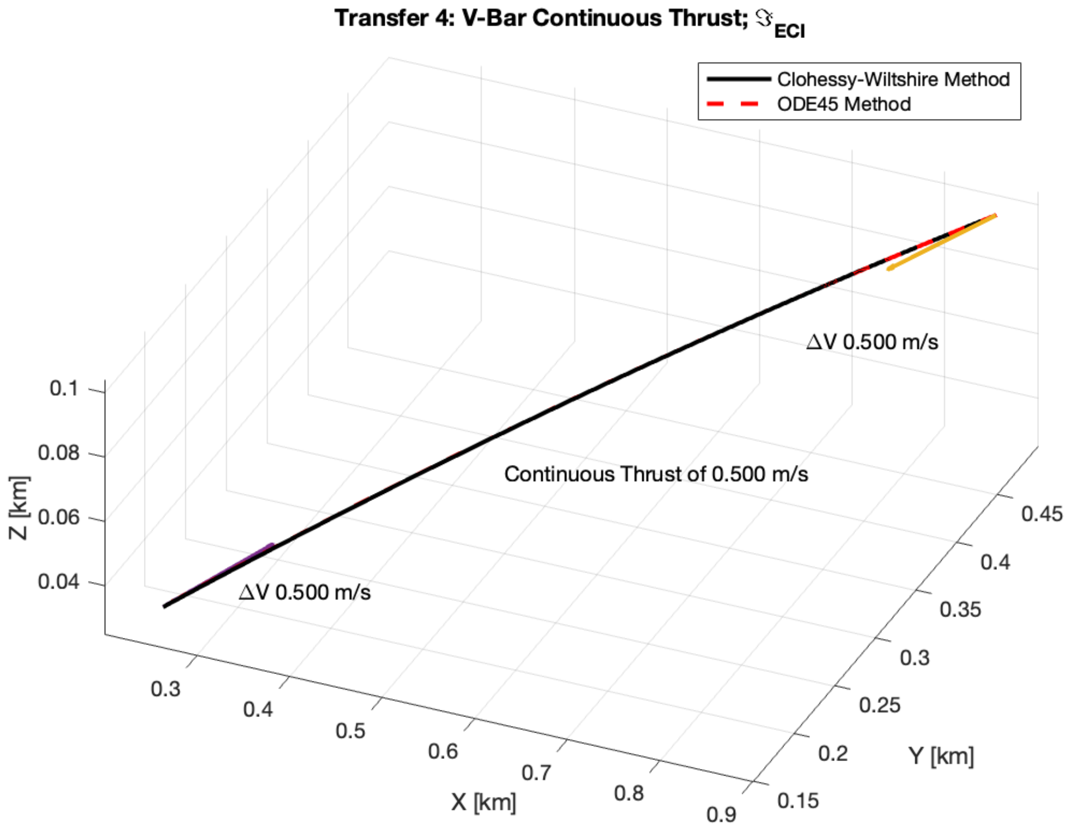
After Transfer 3, the chaser remains in the same orbit as the target 40 km downrange and with no relative velocity in the LVLH frame. From here, the chaser is to move towards the target, specifically to 1 km downrange with no relative velocity. The chaser will accomplish this via 3 hops. Each hop transfer will move the chaser in a higher altitude temporarily such that the target moves underneath it and when the chaser intercepts the original orbit, the downrange distance will decrease. The first hop will move the chaser from 40 km downrange to 20 km downrange; the second to move it from 20 km downrange to 5 km downrange; and the final from 5 km downrange to 1 km downrange in the LVLH frame. Hops are performed by performing an impulsive maneuver in the downrange direction proportional to the amount of distance it will cover. Because the three hops are of different lengths, the chaser's relative velocity will be different. Instead of setting the relative velocity of the chaser to 0 at the end of each hop and re-burning to initiate the next hop, a short optimization was performed to burn the difference required. As a consequence, 4 impulse burns were performed, totaling 0.1547 m/s. This includes initiating each hop and setting the chaser relative velocity to 0 at the end of the final hop. Because the hop transfer works by waiting for the target to come back to its original position within its orbit, each hop takes 1 period to complete. The total transfer time, as such, takes 3 periods, or just under 3 days. Including the 1 period hold after the transfer is complete, the full transfer and hold takes just under 4 days at 3 days 23 hours. The triple hop maneuver using the ODE method puts the chaser at a relative position 0.9582 km downrange of the target, resulting in a 4.18% error between the ODE and Clohessy-Wiltshire methods. The burns in ECI are shown to be in the radial direction, as well as in the LVLH frame. The resulting ECI and LVLH plots show the football orbit.


Transfer 3: Hop Transfer; $\Im_{ECI}$
Clohessy-Wiltshire Method
ODE45 Method
ΔV 0.077 m/s
ΔV 0.019 m/s
ΔV 0.000 m/s
ΔV 0.045 m/s
Z [km]
Y [km]
X [km]

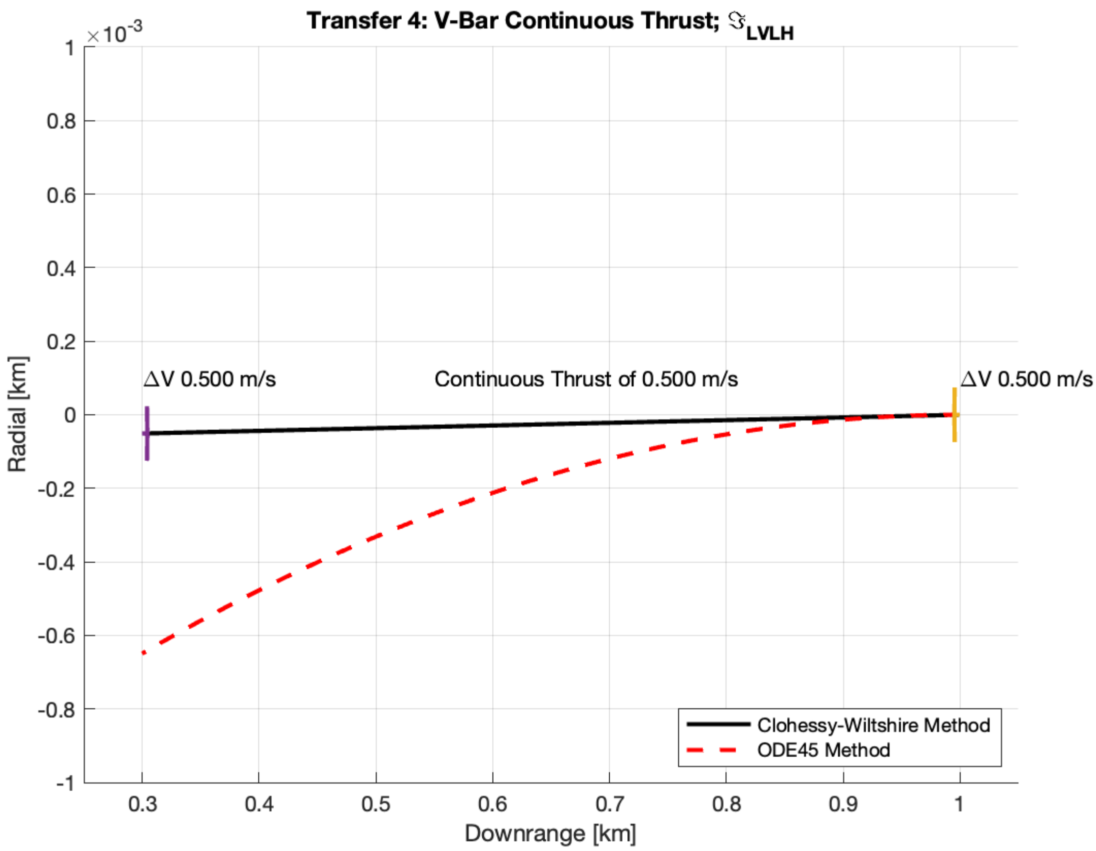

Transfer 3: Hop Transfer; $\Im_{LVLH}$
Clohessy-Wiltshire Method
ODE45 Method
ΔV 0.045 m/s
ΔV 0.000 m/s
ΔV 0.019 m/s
ΔV 0.077 m/s
Radial [km]
Downrange [km]

Transfer 4			
Transfer Time	00:00:23:20	Starting MET	T+ 08:23:23:32
Hold Time	00:10:31:34	Ending MET	T+ 09:10:18:26
Total Time	00:10:54:54	ΔV	701.0 m/s
Start of Transfer 4		End of Transfer 4 Hold	
Target Absolute Position in ECI (km)	20496.40264 -36629.10112 -201.76748	Target Absolute Position in ECI (km)	26508.13709 32643.48571 3967.66135
Chaser Relative Position in LVLH (m)	0 1000 0	Chaser Relative Position in LVLH (m)	0 300 0

Following the triple-hop-maneuver, the chaser is now at 1000 m downrange of the target with no relative velocity. Here, the chaser's next objective is to approach the target along the v-bar direction. This will allow the chaser to take continuous measurements and observations of the target as it closes the distance. To perform this maneuver, the chaser will start a v-bar approach, that is, continuously thrusting along the v-bar direction to simultaneously stay on the same orbit as the target while coming closer. It does this by instantaneously performing a burn in the v-bar direction and continually thrusting until the target is reached where the chaser will instantaneously burn again in the opposite direction to come back to 0 relative velocity. In the case of the fourth transfer, the chaser will burn at a constant velocity of 0.5 m/s. Over the course of the 700 m it must close, in addition to the instantaneous burns at the start and end of the maneuver, the total delta-V for this transfer comes out to 701 m/s. While an expensive burn, the chaser must spend at least 700 m/s to approach 300 m downrange of the target from 1000 m as the total cost is directly related to the constant velocity and burn time; only the transfer time will increase (while lowering the instantaneous burn costs and vise versa). Following this maneuver, the transfer takes 23 minutes. In addition to a 7 hour hold time to confirm observations and measurements, the total time for the transfer and hold comes out to just under 11 hours at 10 hours and 54 minutes. The ODE method for the v-bar approach puts the chaser just over 300 m downrange of the target but lowers its altitude by 60 cm. This puts the error between the ODE and Clohessy-Wiltshire methods at 0.2%. The hops are shown to have the chaser converge on the target in the ECI plot, an expected result of the hop transfers. The direction of the burns in the ECI plot are also consistent with what's required of a hop transfer by burning along the v-bar.



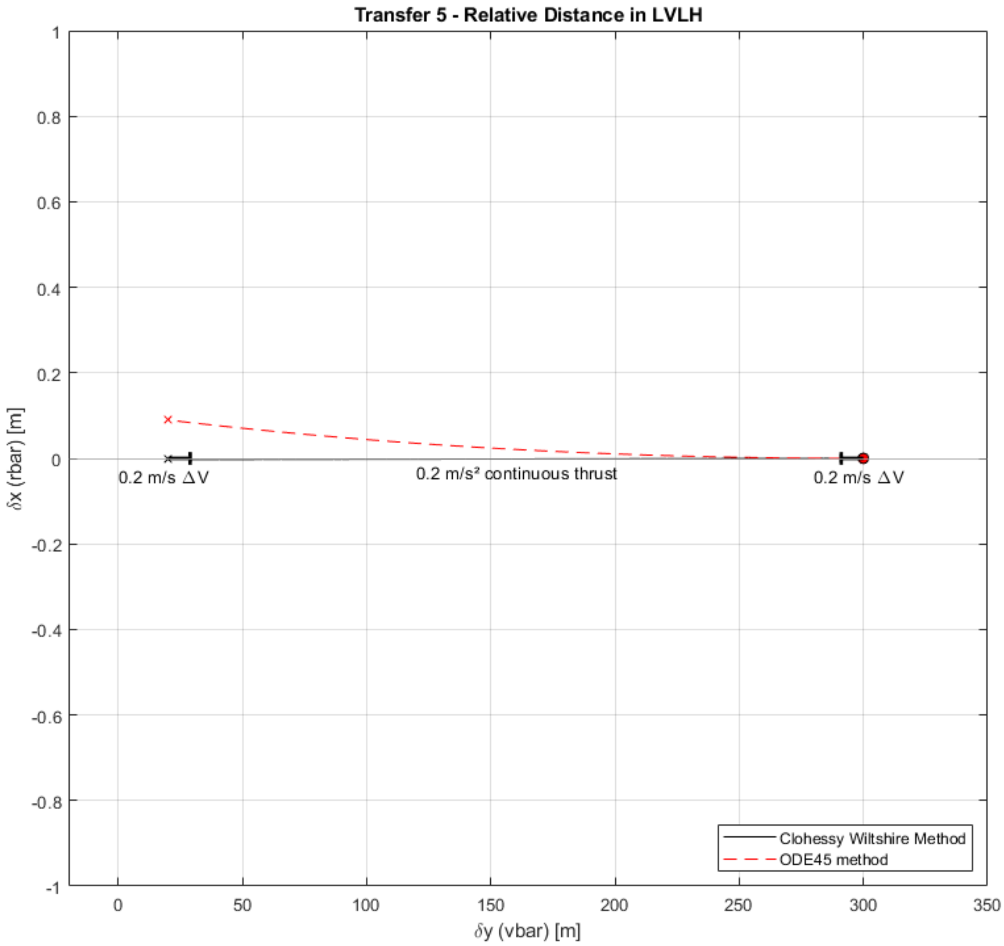
Transfer 4: V-Bar Continuous Thrust; $\Im_{ECI}$
Clohessy-Wiltshire Method
ODE45 Method
ΔV 0.500 m/s
Continuous Thrust of 0.500 m/s
ΔV 0.500 m/s
Z [km]
0.1
0.08
0.06
0.04
X [km]
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Y [km]
0.15
0.2
0.25
0.3
0.35
0.4
0.45



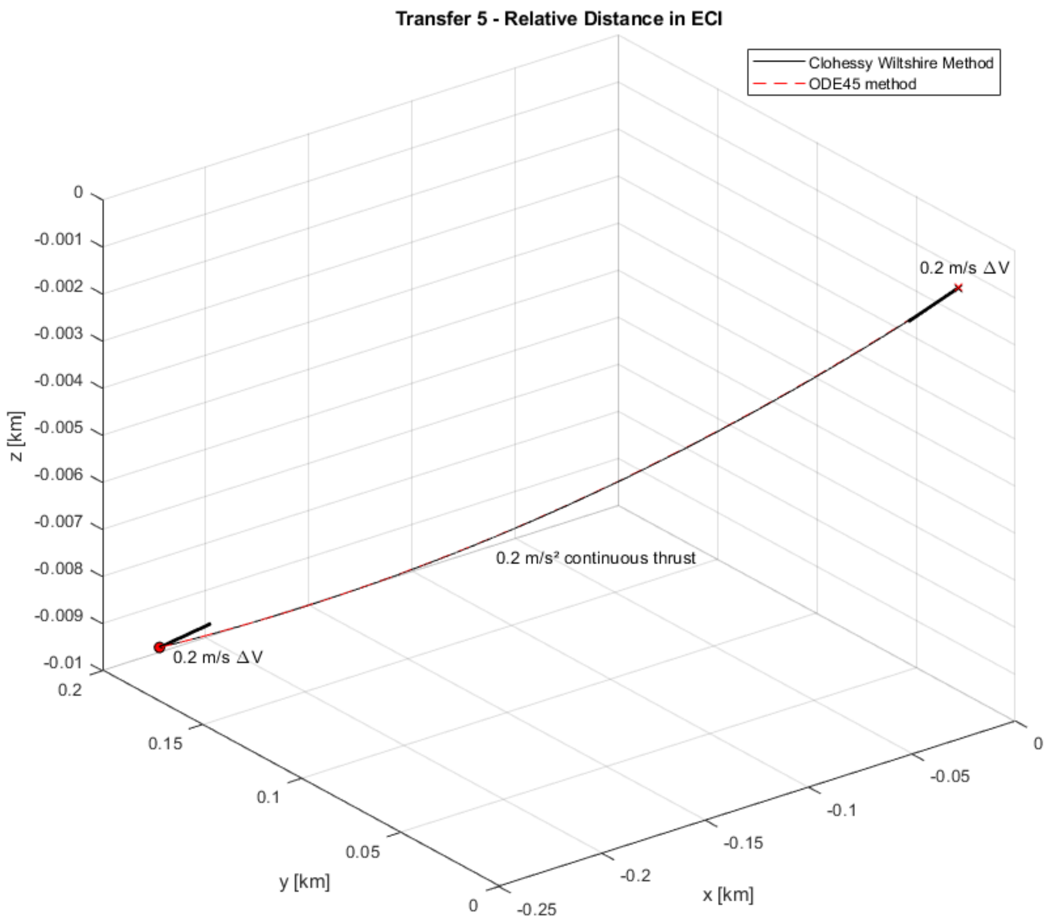
Transfer 4: V-Bar Continuous Thrust; $\Im_{LVLH}$
$\times 10^{-3}$
ΔV 0.500 m/s
Continuous Thrust of 0.500 m/s
ΔV 0.500 m/s
Clohessy-Wiltshire Method
ODE45 Method
Radial [km]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Downrange [km]
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Transfer 5			
Transfer Time	00:00:23:20	Starting MET	T+ 09:10:18:26
Hold Time	00:10:31:34	Ending MET	T+ 09:21:13:20
Total Time	00:10:54:54	ΔV	280.4 m/s
Start of Transfer 5		End of Transfer 5 Hold	
Target Absolute Position in ECI (km)	26508.13709 32643.48571 3967.66135	Target Absolute Position in ECI (km)	-34787.0766 -23437.4766 -4181.5308
Chaser Relative Position in LVLH (m)	0 300 0	Chaser Relative Position in LVLH (m)	0 20 0

The second vbar approach starts 300 km downrange in front of the target. The chaser performs an impulsive maneuver of 0.2 m/s in the negative rbar direction to initiate the approach, then accelerates continuously in the negative vbar direction at 0.2 m/s² until it reaches 20 km downrange of the target, at which point it performs another impulsive maneuver of 0.2 m/s in the positive rbar direction to match velocities with the target. The ODE45 method ended at the same point on the vbar as the Clohessy-Wiltshire method, but was 8.97 cm away from the vbar along the positive rbar direction, an error of just 0.45%.



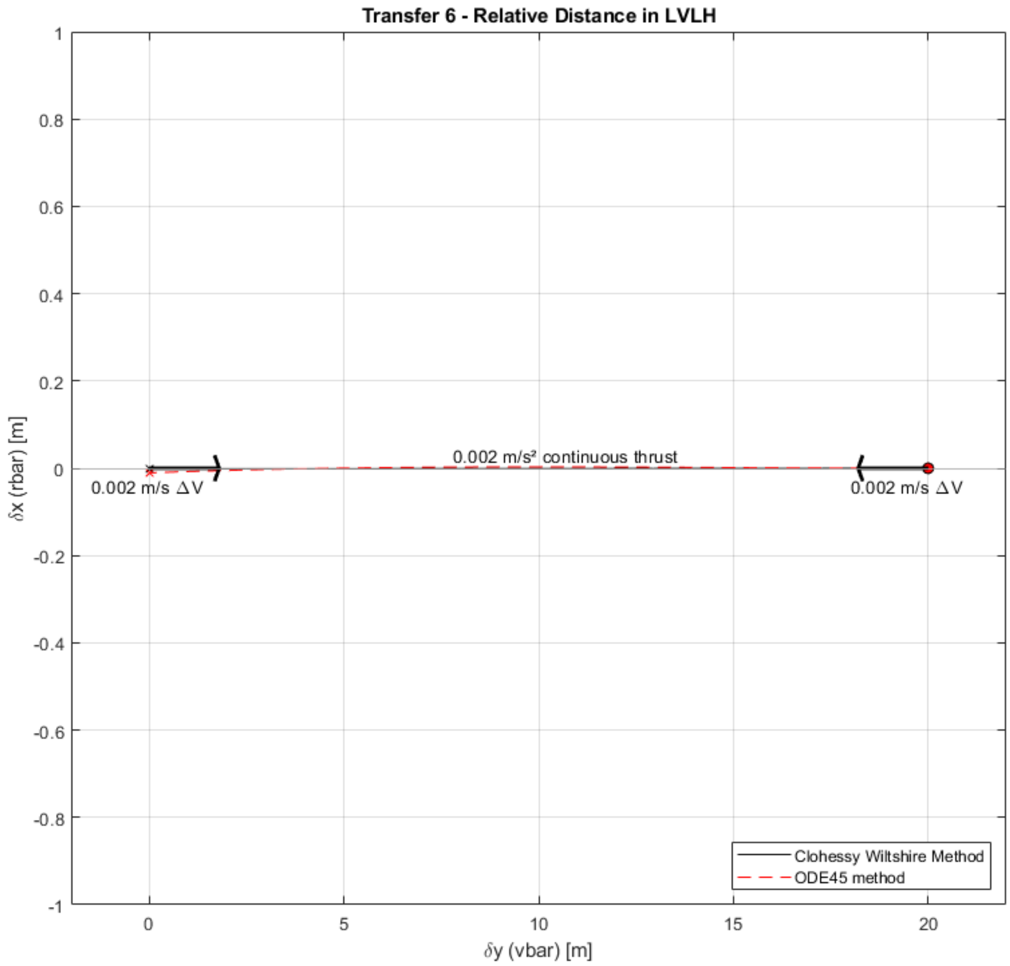
Transfer 5 - Relative Distance in LVLH
δx (rbar) [m]
δy (vbar) [m]
0.2 m/s ΔV
0.2 m/s² continuous thrust
0.2 m/s ΔV
Clohessy Wiltshire Method
ODE45 method

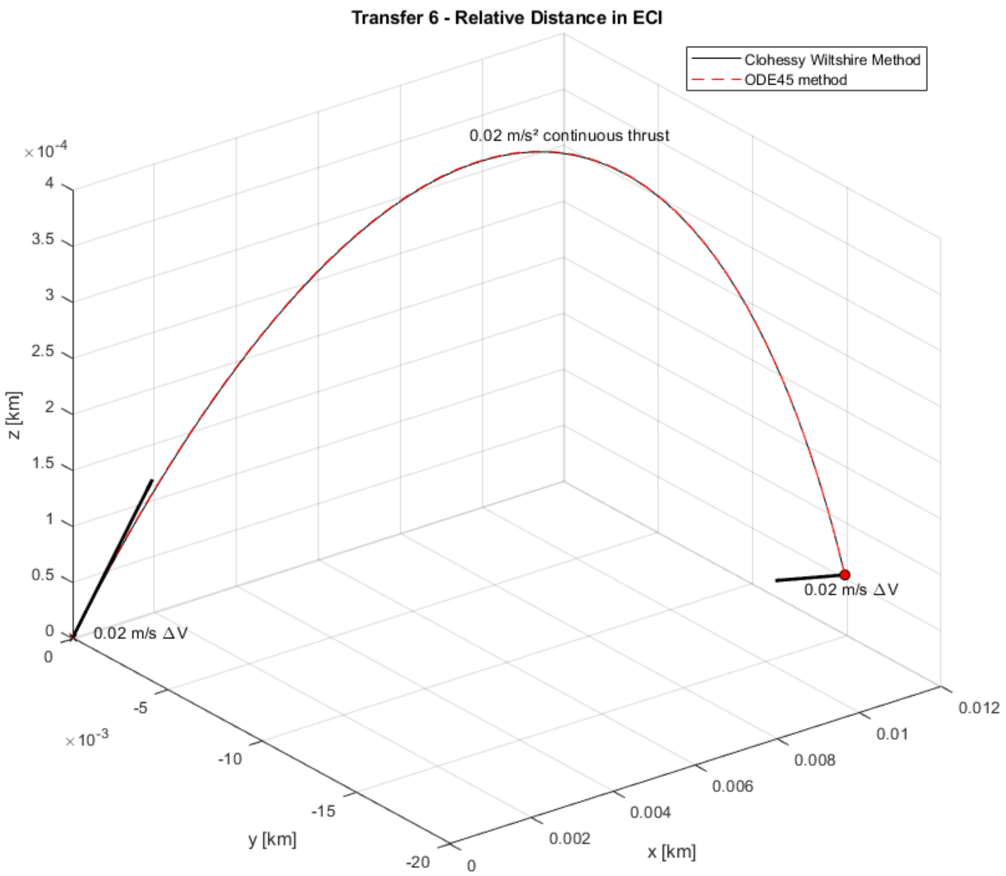


Transfer 5 - Relative Distance in ECI
Clohessy Wiltshire Method
ODE45 method
0.2 m/s ΔV
0.2 m/s² continuous thrust
0.2 m/s ΔV
z [km]
y [km]
x [km]

Transfer 6			
Transfer Time	00:02:46:40	Starting MET	T+ 09:21:13:20
Hold Time	N/A	Ending MET	T+ 10:00:00:00
Total Time	00:02:46:40	ΔV	20.004 m/s
Start of Transfer 6		End of Transfer 6	
Target Absolute Position in ECI (km)	-34787.0766 -23437.4766 -4181.5308	Target Absolute Position in ECI (km)	-9938.5991 -40725.5241 -2993.1550
Chaser Relative Position in LVLH (m)	0 20 0	Chaser Relative Position in LVLH (m)	0 0 0

The final vbar approach starts 20 km downrange in front of the target. The chaser performs an impulsive maneuver of 0.02 m/s in the negative rbar direction to initiate the approach, then accelerates continuously in the negative vbar direction at 0.02 m/s^2 until it reaches the target, at which point it performs another impulsive maneuver of 0.02 m/s in the positive rbar direction to match velocities with the target. The ODE45 method ended 1.06 cm away from the target in the negative rbar direction and 0.38 cm away from the target in the positive vbar direction, for a total distance of 1.13 cm.


Transfer 6 - Relative Distance in LVLH
0.002 m/s² continuous thrust
0.002 m/s ΔV
0.002 m/s ΔV
Clohessy Wiltshire Method
ODE45 method
δx (rbar) [m]
δy (vbar) [m]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20


Transfer 6 - Relative Distance in ECI
Clohessy Wiltshire Method
ODE45 method
0.02 m/s² continuous thrust
0.02 m/s ΔV
0.02 m/s ΔV
z [km]
×10^-4
y [km]
×10^-3
x [km]
4
3.5
3
2.5
2
1.5
1
0.5
0
0
-5
-10
-15
-20
0
0.002
0.004
0.006
0.008
0.01
0.012

Appendix

Full State Vectors				
Transfer and Hold	Target Absolute Position in ECI (km)	Target Absolute Velocity in ECI (km)	Chaser Relative Position in LVLH (km)	Chaser Relative Velocity in LVLH (km)
Pre Transfer 1	20786.0312 -36465.6816 -168.6287	2.6733 1.5165 0.3063	0 100 0	0 0 0
Post Transfer 1	20724.904 -36500.4245 -175.6364	2.67330 1.51650 0.30630	0 40 0	0 0 0
Post Transfer 2	20648.82680 -36543.47610 -184.34757	2.67647 1.51090 0.30627	0 40 0	0 0 0
Post Transfer 3	20496.40264 -36629.10112 -201.76748	2.68278 1.49968 0.30621	0 0.1 0	0 0 0
Post Transfer 4	26508.13709 32643.48571 3967.66135	-2.36736 1.95136 -0.09715	0 0.3 0	0 0 0
Post Transfer 5	-34787.0766 -23437.4766 -4181.5308	1.7293 -2.5434 0.0119	0 0.02 0	0 0 0
Post Transfer 6	-9938.5991 -40725.5241 -2993.1550	2.9884 -0.7349 0.2140	0 0 0	0 0 0