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## Orbital Perturbation Correction Analysis

### Introduction

We conducted an initial  $\Delta V$  analysis to determine the cost of correcting orbital perturbation for a collection of four objects, each with various initial orbits that lend themselves to specific types of perturbations. We chose one satellite in LEO and one in GEO to highlight the differences in effects on the COEs for the two regimes. Additionally, we chose one satellite in a Molniya orbit to provide an example of a spacecraft that uses orbital perturbation to its advantage, and an extra object in a sun synchronous orbit to compare with the Molniya orbit.

For simplicity, the following assumptions were made for every object:

- The area of each object is static and does not change throughout the orbit
- The area of each object is the same for both drag and SRP calculations
- The mass of each object is static and remains at its maximum value with full propellant

We used the Gaussian variation of parameters method to integrate COEs over a period of 3 days, 55 days for LEO/sun-synchronous and 3 years for GEO/Molniya to show the short and long term effects of orbital perturbations. The following perturbations were considered for each object:

- Atmospheric drag using an exponential model for atmospheric density
- Solar gravity and radiation pressure using a solar flux varying with days since aphelion
- Oblateness of the Earth with consideration of J2, J3, J4, J5 and J6

All other perturbations were excluded for simplicity. SRP calculations did not distinguish between umbra and penumbra. Additionally, atmospheric drag was not included for the GEO and sun-synchronous orbits due to its negligible effect at those altitudes. For the Molniya object, atmospheric drag was included only when the object's altitude was below 1000 km.

Lambert's transfers were utilized to correct for perturbations of LEO objects over a course of 3 days with a transfer at the end of each day. The propagation of LEO objects also included an exit condition that would end the propagation if the altitude of the object fell below 100 km. This serves as the explanation why the LEO objects only propagate 55 days, being the amount of time where altitude fell below 100 km if no correction actions were taken.

## Selected Objects

### LEO

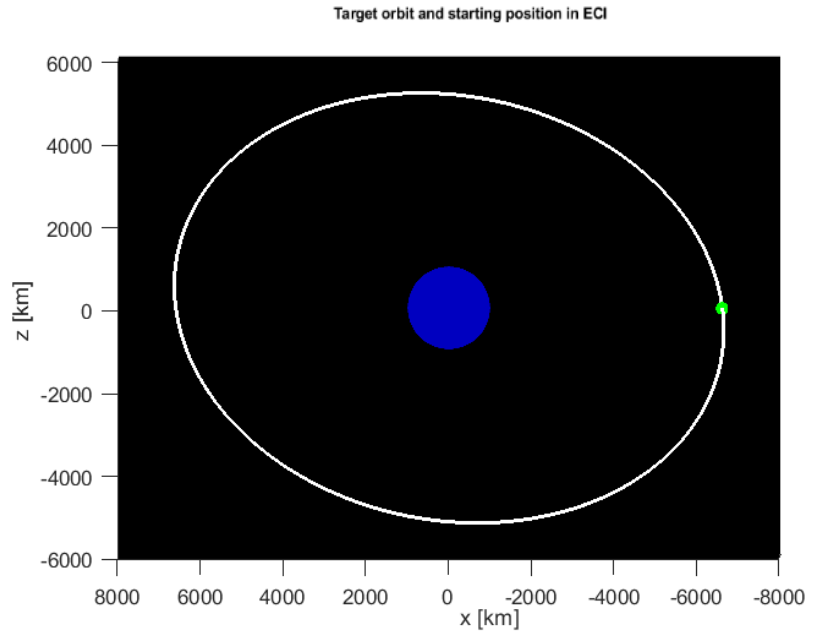
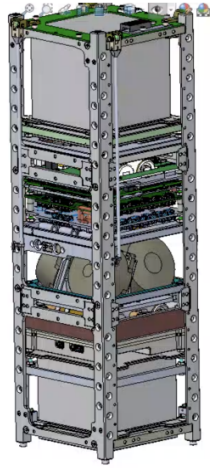


Figure 1: Light-1 Diagram [1] and Non-perturbed Orbit

Light-1 initial parameters			
$e$	0.0008998	$\Omega$	$188.9524^\circ$
$h$	$51,674 \text{ km/s}^2$	$\omega$	$143.5522^\circ$
$i$	$51.6362^\circ$	$\Theta$	$216.5481^\circ$
$m$	1.5 kg	$A$	$0.012 \text{ m}^2$
$C_d$	2.2	$C_r$	1.2
TLE	1 51509U 98067TG 22337.88714517 .00120346 00000-0 53609-3 0 9990 2 51509 51.6362 188.9524 0008998 143.5522 216.6095 15.83367809 47479		
Epoch	3 December 2022		

GEO

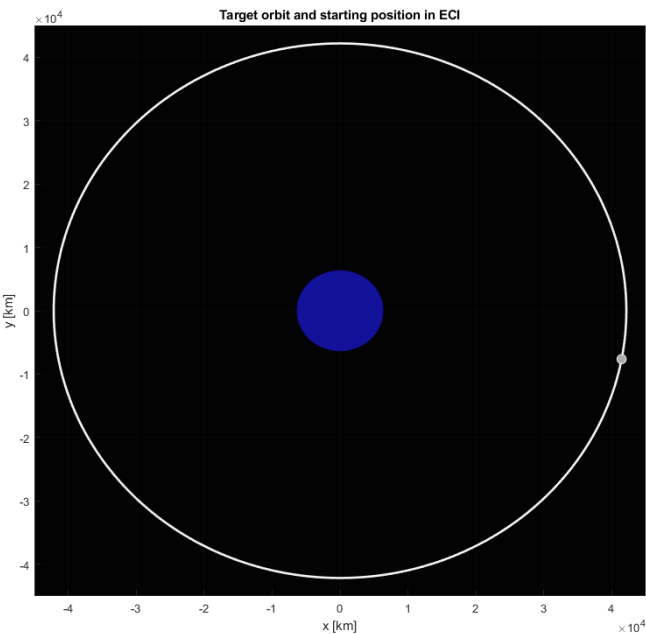


Figure 2: GOES 18 Diagram [2] and Non-perturbed Orbit

GOES 18 initial parameters			
$e$	0.0001839	$\Omega$	142.99°
$h$	129,640 km/s <sup>2</sup>	$\omega$	104.40°
$i$	0.0254°	$\Theta$	102.19°
$m$	5192 kg	$A$	34.16 m <sup>2</sup>
$C_d$	2.2	$C_r$	1.2
TLE	1 51850U 22021A 22336.79858193 .00000101 00000+0 00000+0 0 9992 2 51850 0.0254 142.9864 0001839 104.3997 334.7920 1.00271308 2822		
Epoch	2 December 2022		

SPECIALITY ORBIT 1

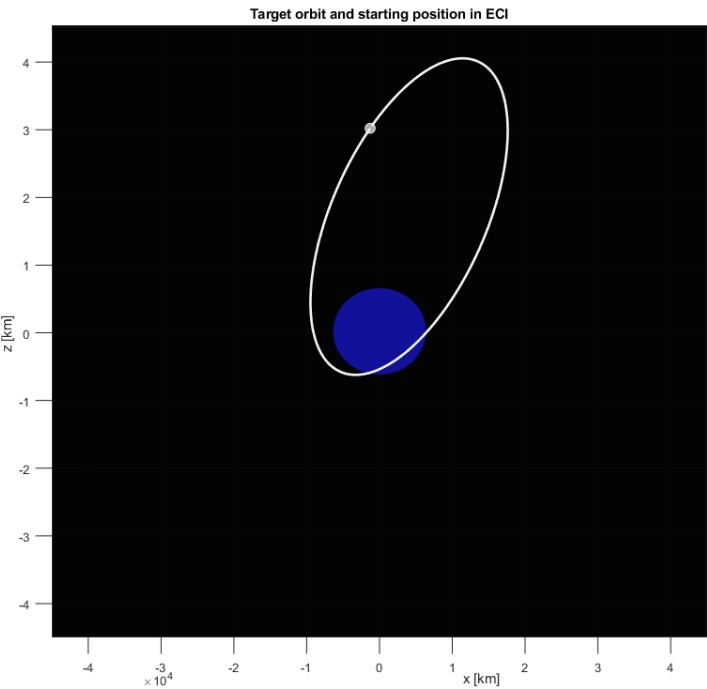


Figure 3: Molniya 3-50 Diagram [3] and Non-perturbed Orbit

Molniya 3-50 initial parameters			
$e$	0.7335	$\Omega$	$308.57^\circ$
$h$	69,922 km/s <sup>2</sup>	$\omega$	$282.48^\circ$
$i$	$63.28^\circ$	$\Theta$	$199.52^\circ$
$m$	1740 kg	$A$	17.85 m <sup>2</sup>
$C_d$	2.2	$C_r$	1.2
TLE	1 25847U 99036A 22337.96206502 -.00000410 00000+0 -59866-4 0 9994 2 25847 63.2833 308.5722 7335374 282.4803 10.7916 2.00663043171512		
Epoch	3 December 2022		

SPECIALITY ORBIT 2

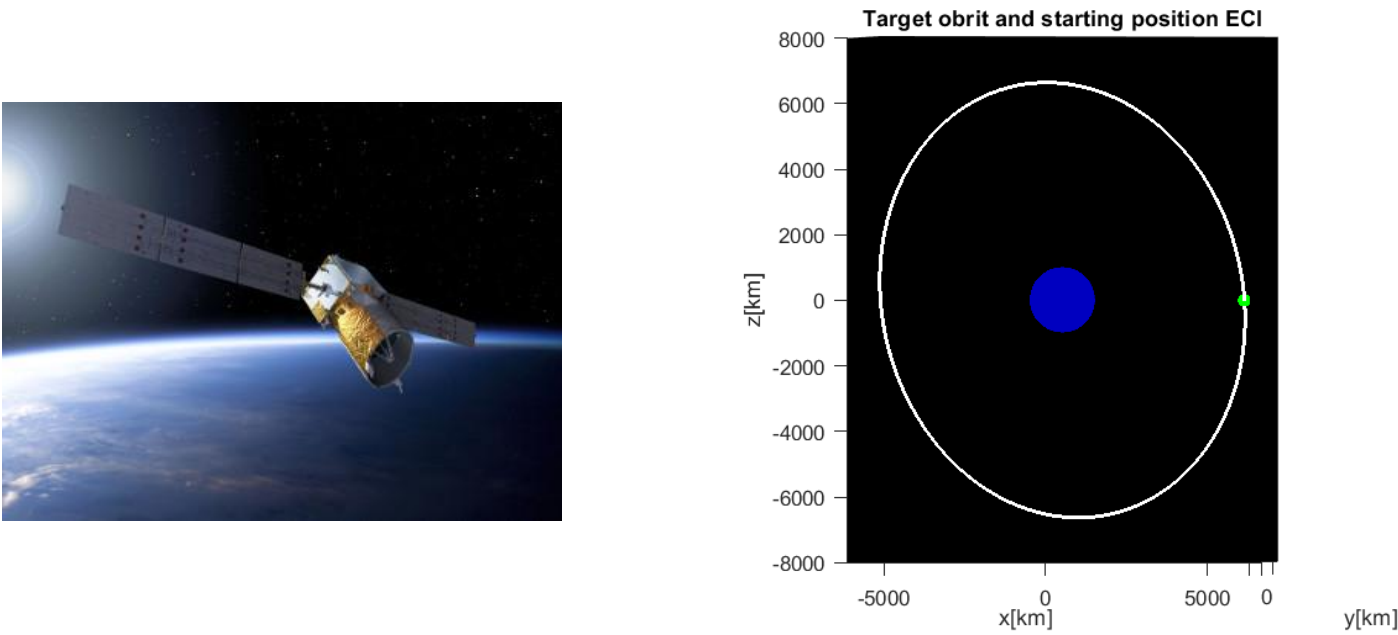
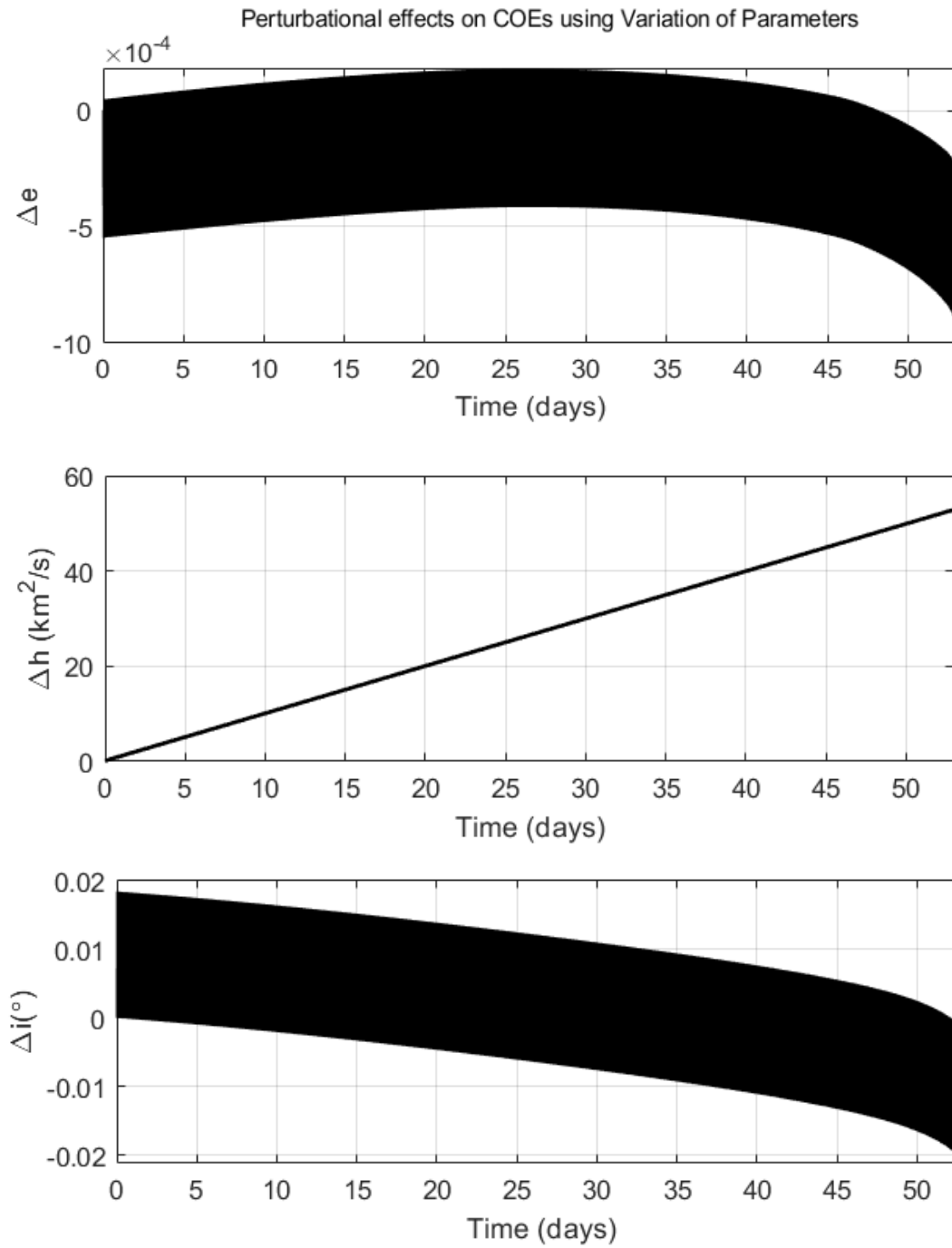


Figure 4: AEOLUS Diagram [4] and Non-perturbed Orbit

AEOLUS initial parameters			
$e$	0.0013318	$\Omega$	$336.5172^{\circ}$
$h$	51,635 km/s <sup>2</sup>	$\omega$	$90.8371^{\circ}$
$i$	$96.7321^{\circ}$	$\Theta$	$269.2902^{\circ}$
$m$	1336 kg	$A$	3.8 m <sup>2</sup>
$C_d$	2.2	$C_r$	1.2
TLE	1 43600U 18066A 22333.28074184 .00083235 42413-5 32542-3 0 9990 2 43600 96.7321 336.5172 0013318 90.8371 269.4428 15.86952408247268		
Epoch	29 November 2022		

## Results

### LEO - Light-1 LONG TERM



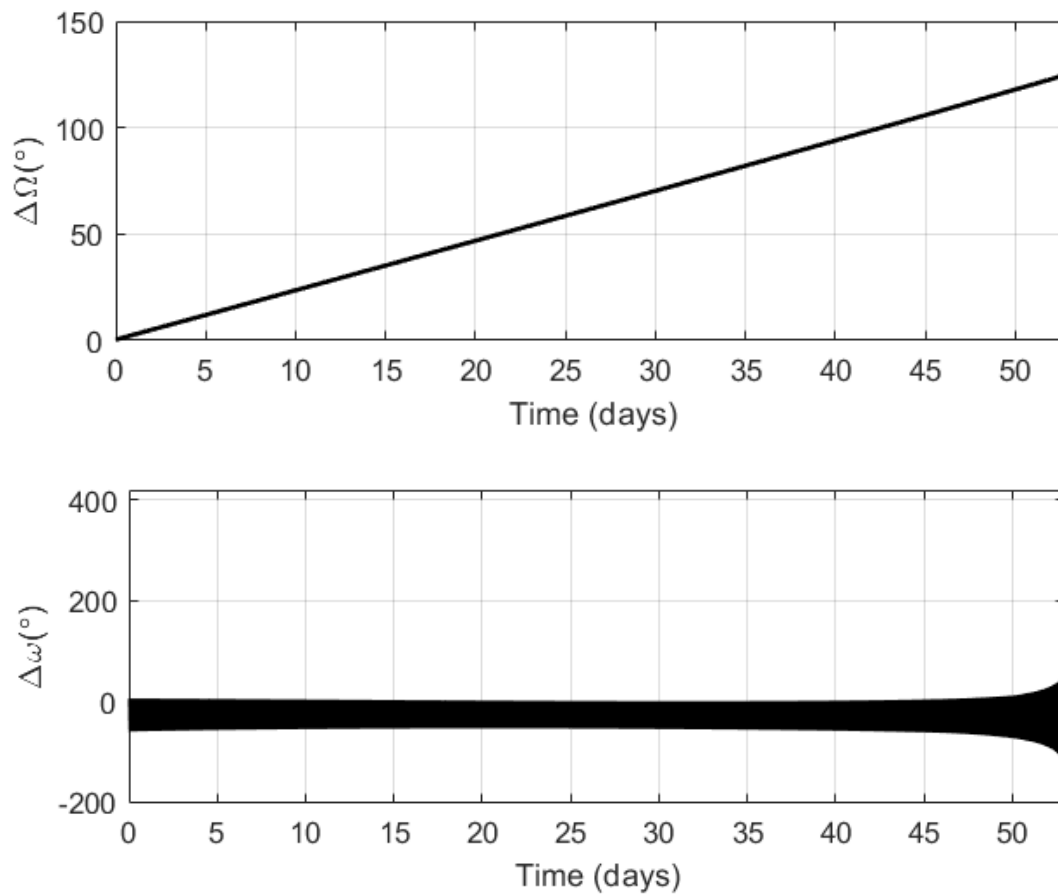


Figure 5: Long term perturbations effects on LEO object

Light-1 $\Delta V$ Correction			
$dt$	433.2050 sec	$\Delta V$	17.8724 km/s

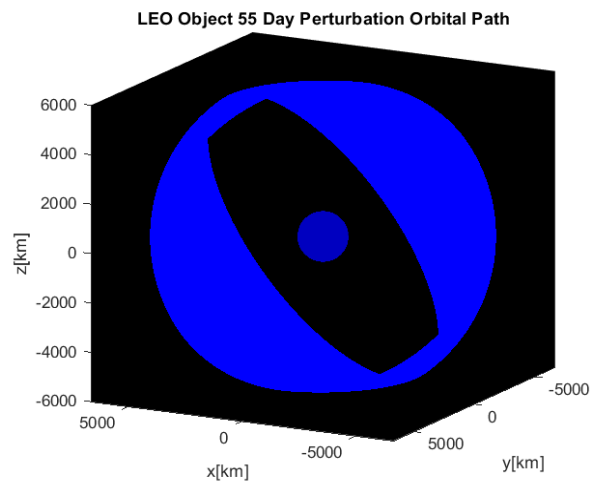
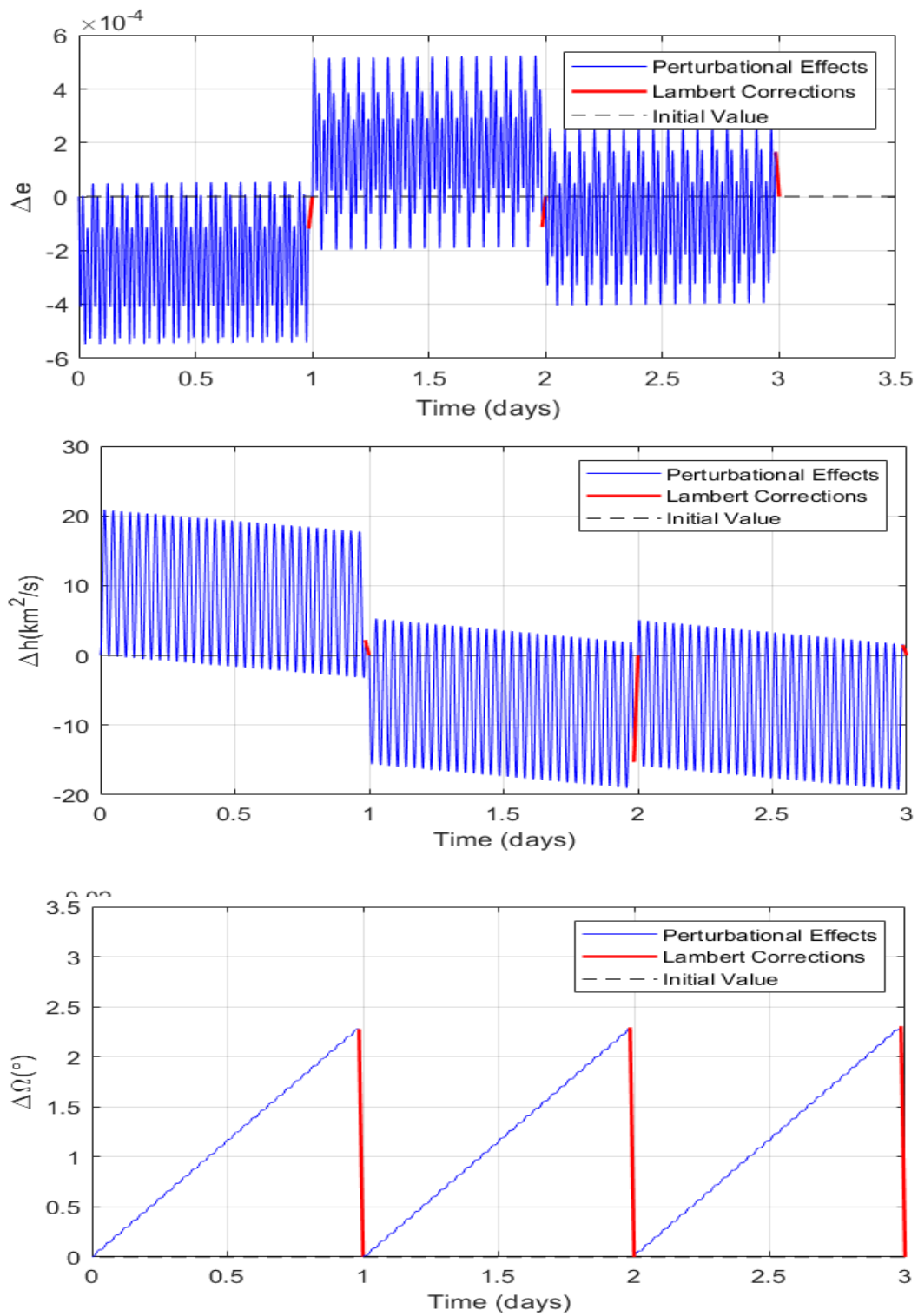


Figure 6: ECI Light-1 Orbital Path

## LEO - Light-1 SHORT TERM

Perturbational effects on COEs using Variation of Parameters





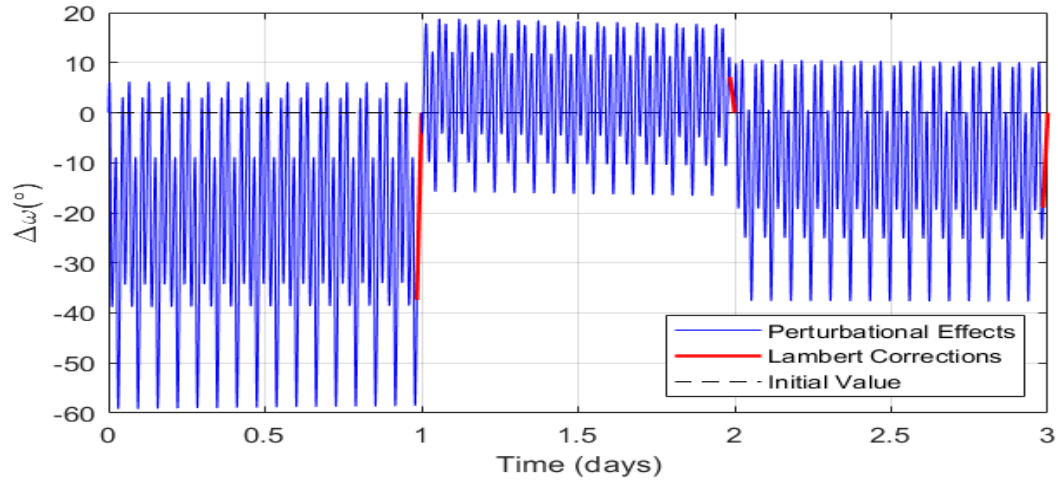


Figure 7: Short term perturbations effects on LEO object & DV Correction

Light-1 DeltaV Correction			
$dt1$	1286.5 sec	DV1	0.7585 km/s
$dt2$	1299.8 sec	DV2	0.4364 km/s
$dt3$	1316.1 sec	DV3	0.4711 km/s

It is observed that this means of correction takes far less DV than in the previous case of waiting for the object to fall below 100 km in altitude. This makes sense as the transfer is having to correct for less drift of COES.

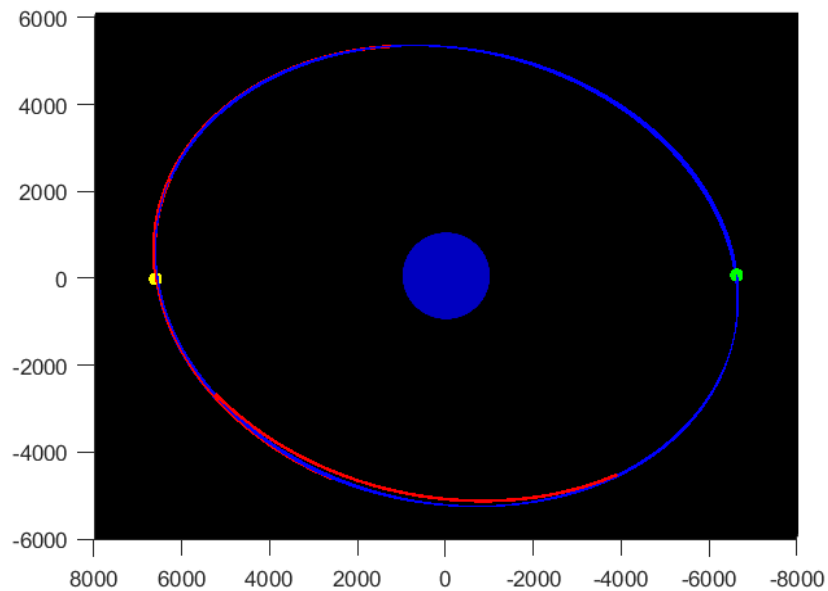
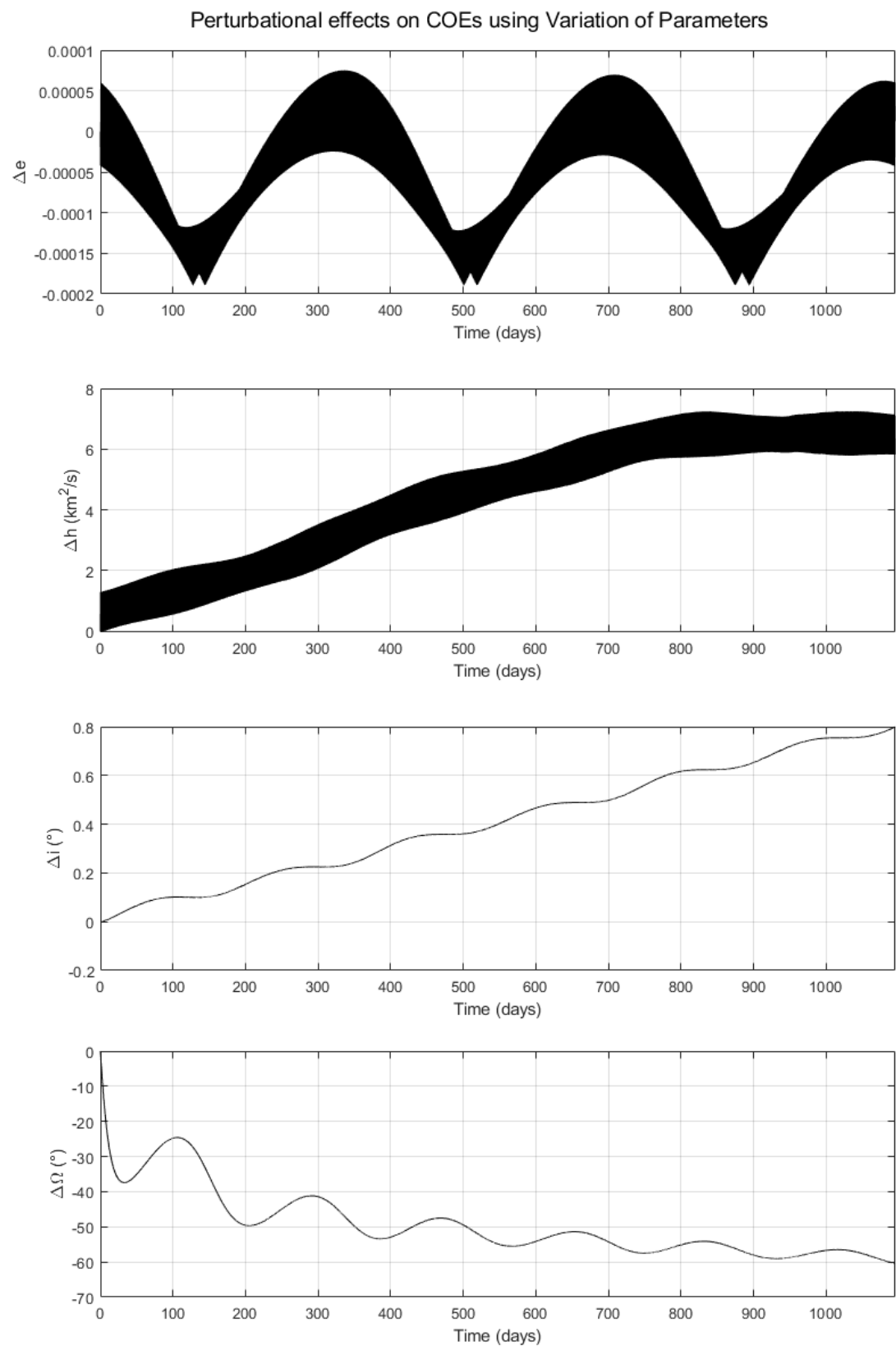


Figure 8: Light-1 Perturbations with Corrections (Blue-Perts/Red-Lamberts/Green-TStart/Yellow-TStop)

**GEO - GOES 18 LONG TERM**



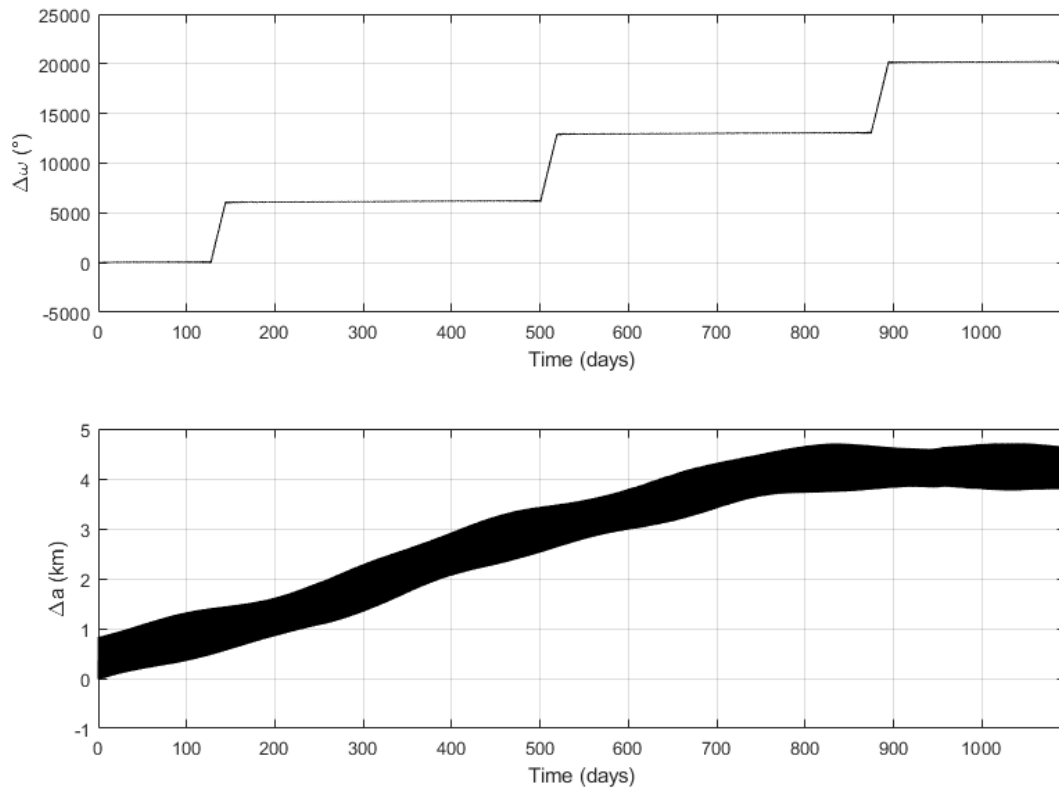


Figure 9: Long term perturbation effects on GOES-1

The GEO object shows low variability in some parameters and extreme variability in others. First, the semi-major axis has minimal deviation with a maximum change of less than 5 km compared to an absolute value of 42,170 km. The eccentricity changes significantly relative to its initial value, but still remains well within circular approximation, increasing to no more than 0.0001 and decreasing to 0 with a period of almost exactly one year, corresponding to major contributions from solar gravity and solar radiation pressure.

The change in inclination, on the other hand, is significant and correction would require about 14.3 m/s of  $\Delta V$  each year to return to  $0^\circ$  inclination given the apparent rate of change of approximately  $0.27^\circ$  per year. It is assumed that correction of the other orbital parameters would be unnecessary due either to their minimal deviation or meaninglessness for a near-circular orbit in the case of argument of perigee and RAAN.

This loss of meaning when defining argument of perigee for a circular orbit is especially apparent in the plot, as it nominally oscillates by around  $15^\circ$  per period as shown in the short term results, but when the eccentricity approaches 0 each year, the argument of perigee increases rapidly by hundreds of degrees per day since the tiniest perturbation for an orbit that close to circular results in a huge offset of the argument of perigee. However, the RAAN stays about the same because the inclination is never close enough to  $0^\circ$  to result in the same behavior.

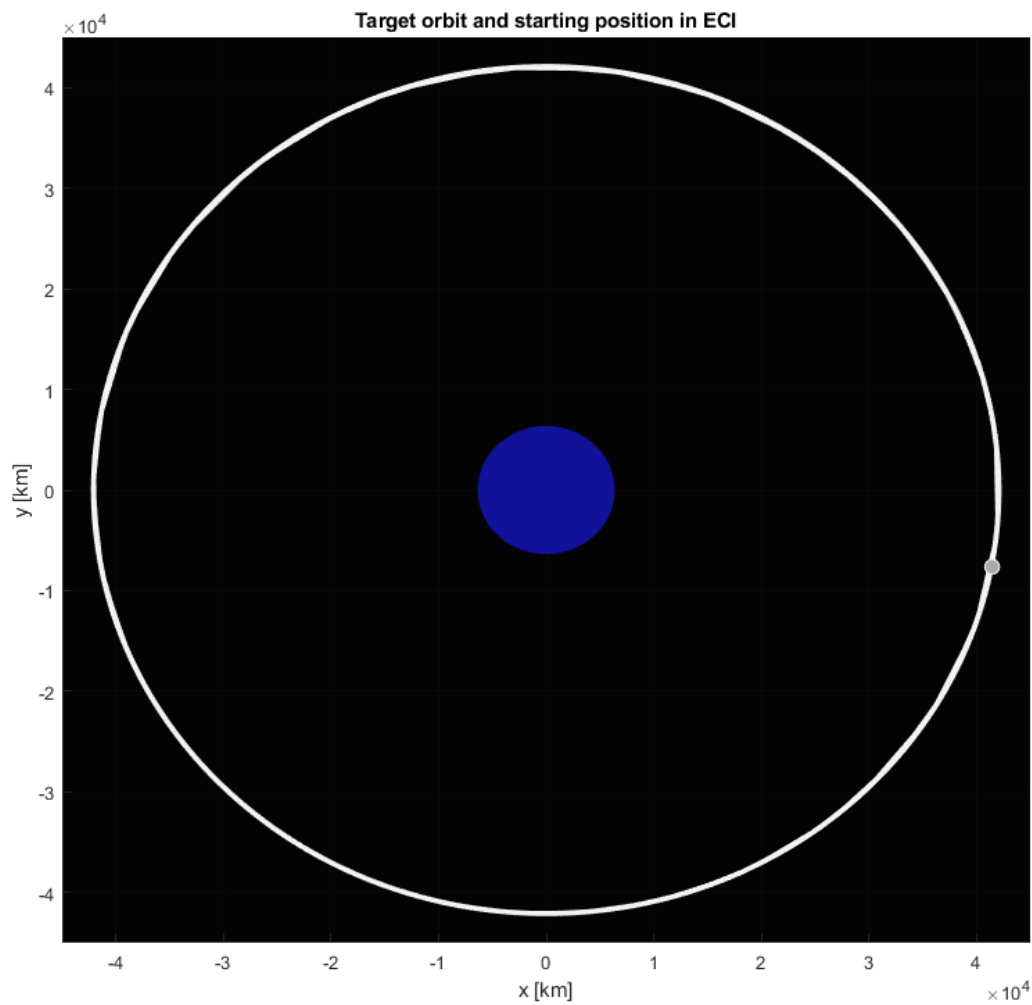
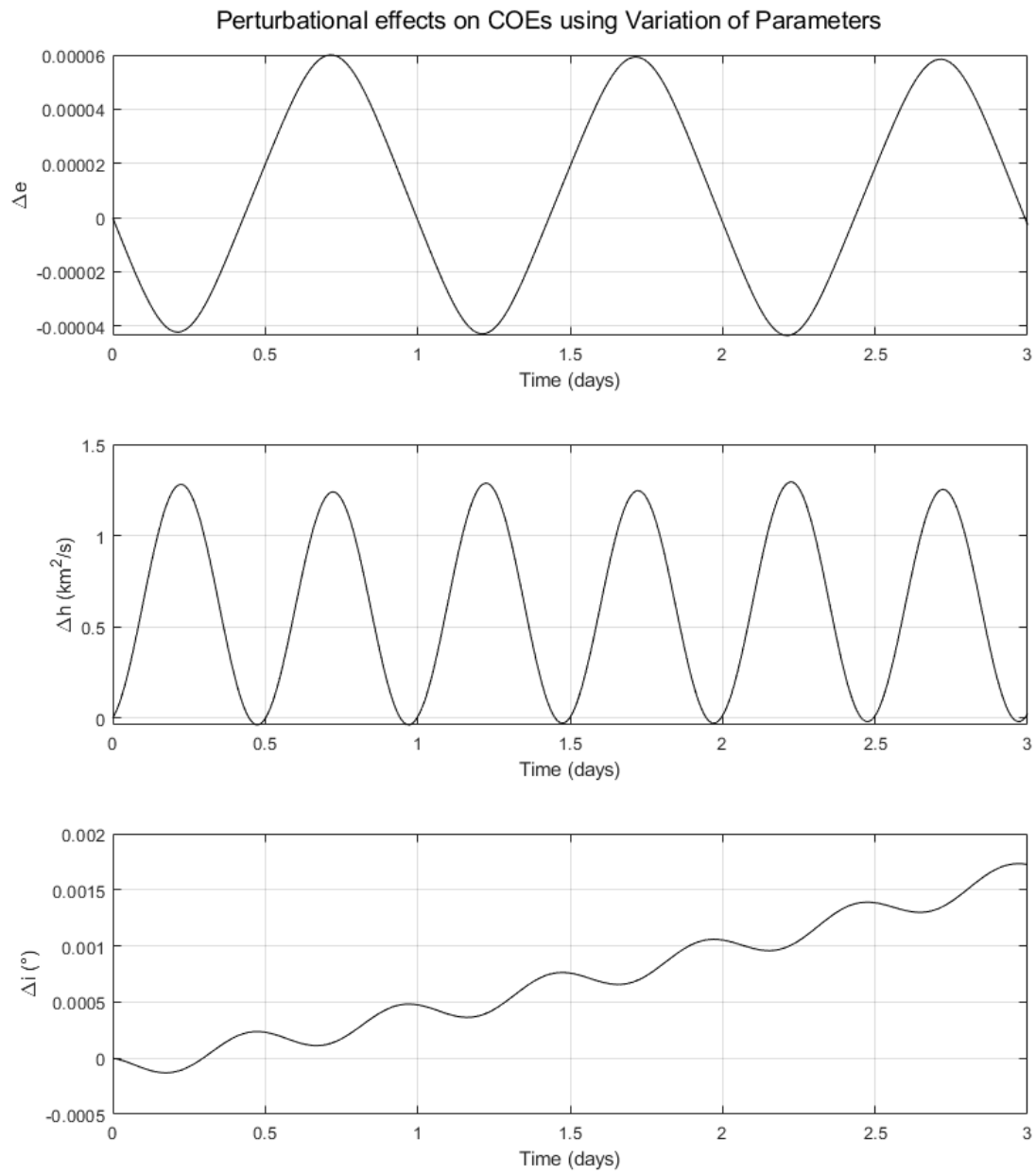


Figure 10: Long term GOES 18 orbital path

## GEO - GOES 18 SHORT TERM



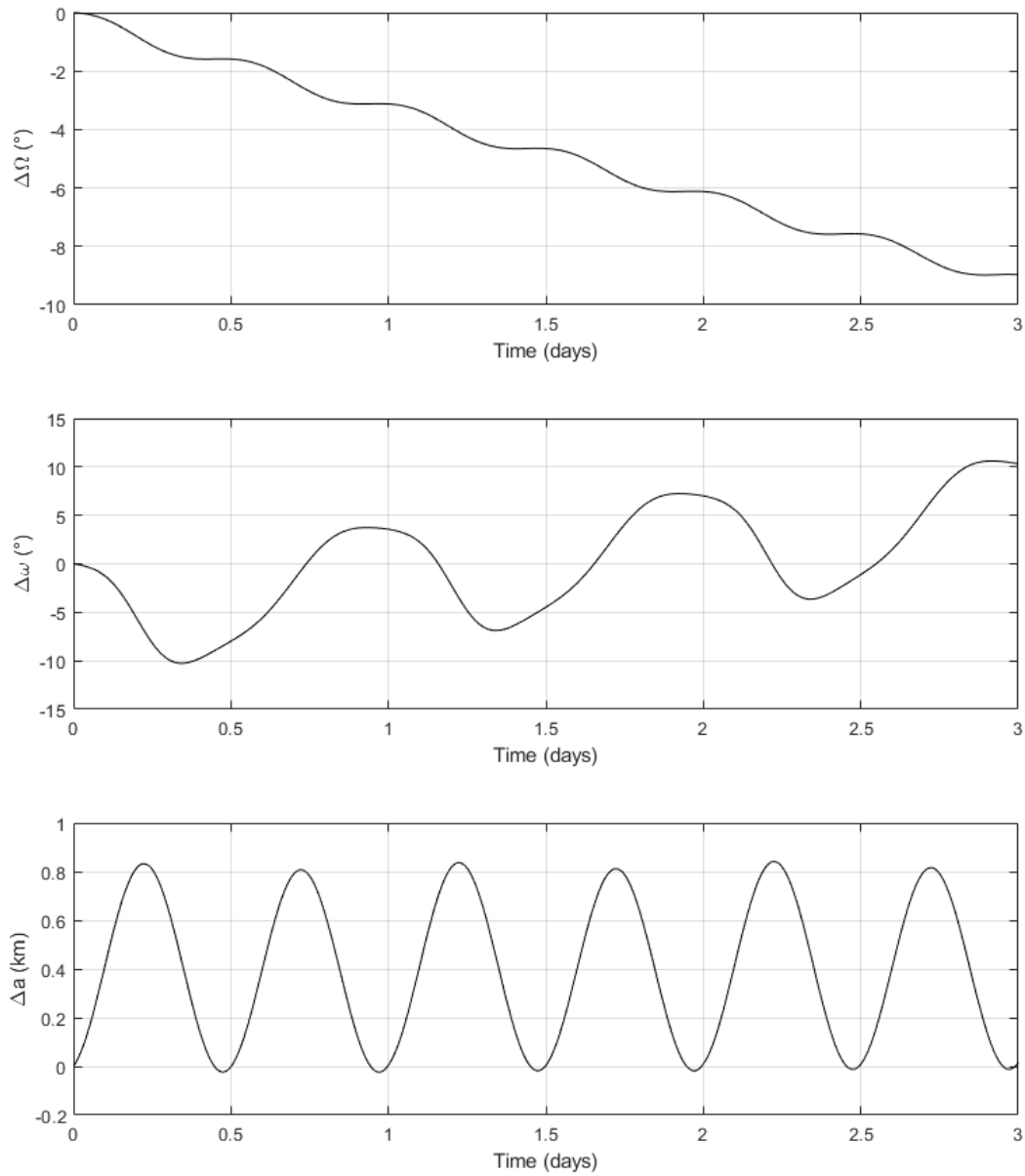
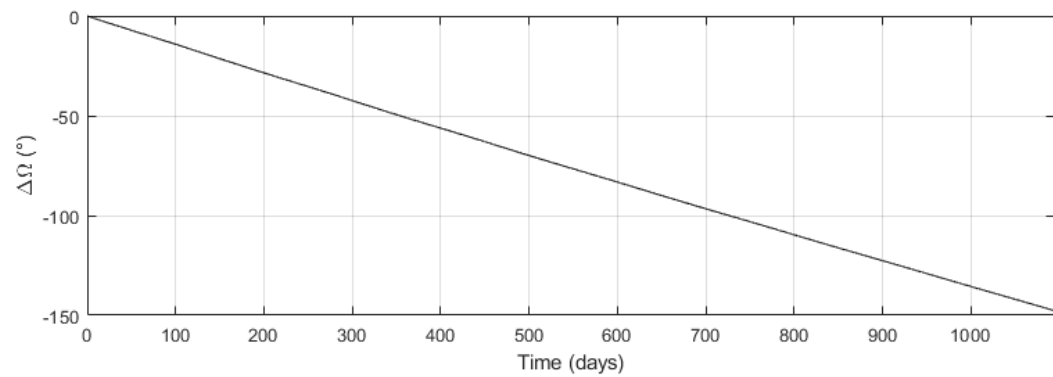
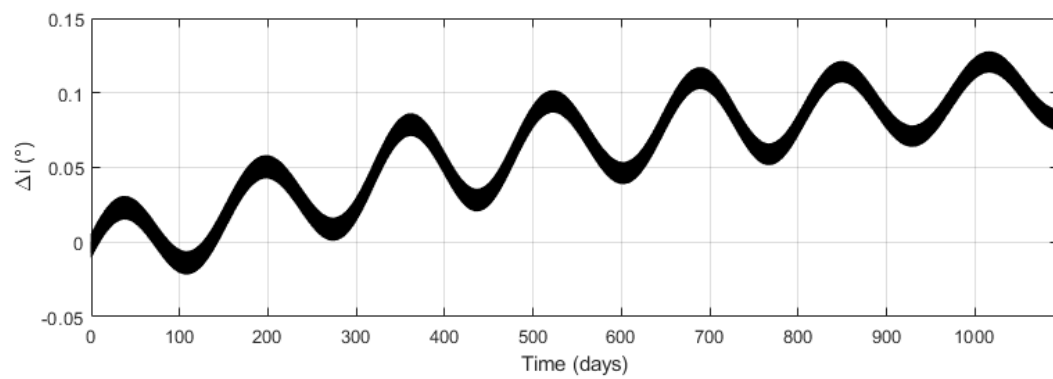
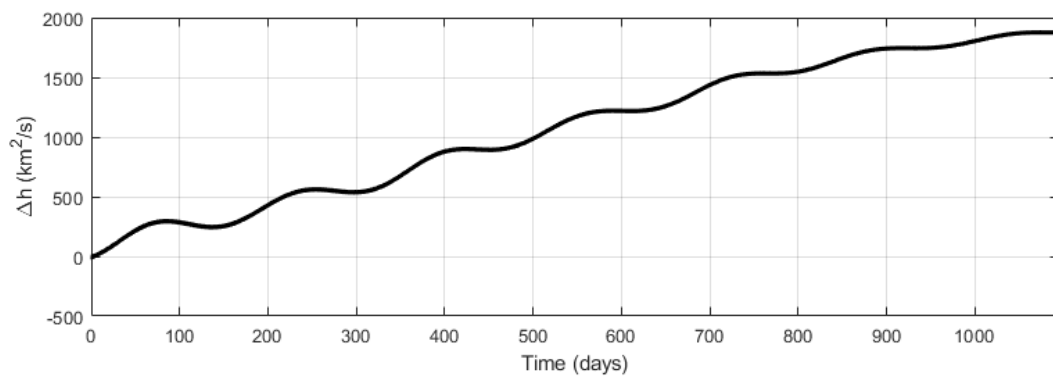
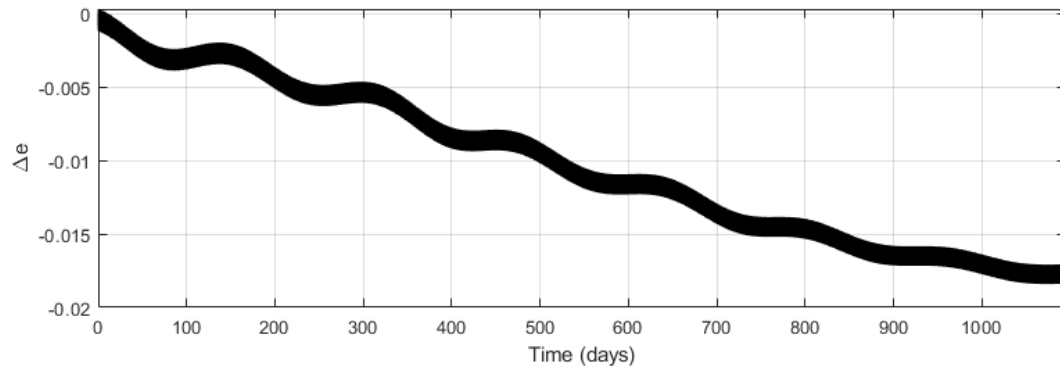


Figure 11: Short term perturbation effects on GOES-1

The short term results actually show the maximum change in RAAN over the entire three year period of the long term results, decreasing by about  $3^{\circ}$  per day, which diminishes significantly over the first 30 days of the simulation and never reaches such a level again. This is possibly indicative of future issues with RAAN change in the following simulations.

## SPECIALITY ORBIT 1 - Molniya 3-50 LONG TERM

Perturbational effects on COEs using Variation of Parameters



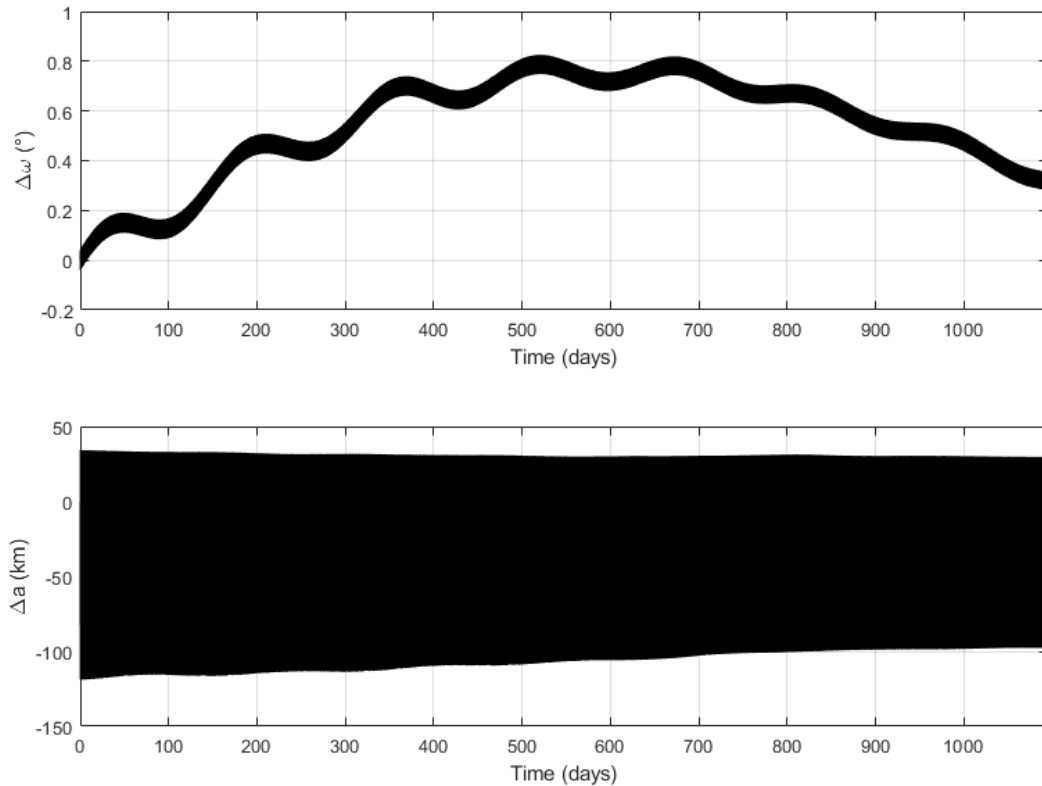


Figure 12: Long term perturbation effects on Molniya 3-50

The Molniya orbit is characterized by the use of the ‘magic’ inclination of  $63.43^\circ$ , which neutralizes change in inclination or argument of perigee due to Earth’s oblateness, and the high eccentricity allows the satellite to zip by one hemisphere at perigee and spend hours with relatively little change in position over the other hemisphere at apogee. This allows the satellite to act like a geostationary object for certain latitudes without having to reach geostationary altitude. In fact, three Molniya satellites could continually monitor a single point on the Earth [5].

The long term simulation results are consistent with this characterization, as the inclination changes periodically but does not significantly deviate from its initial value, and the argument of perigee similarly shows tiny periodic variations that eventually return to zero after several more years. The most likely suspects for these variations are solar gravity and SRP, as they are periodic over a very long time span and the effect of drag should be minimal considering the short duration spent near perigee.

Additionally, the eccentricity and semi-major axis are affected far more than GEO, with eccentricity moving by almost 0.02 and semi-major axis oscillating over 100 km compared to an absolute value of 26,500 km. The most significant orbital element change was related to RAAN as the fixed inclination leads to the right ascension decreasing by a relatively constant value of about  $0.14^\circ$  per day. This eventually affects which areas of the northern hemisphere the satellite observes at apogee because it differs from the Earth’s orbital rotation of  $0.986^\circ$  per day.



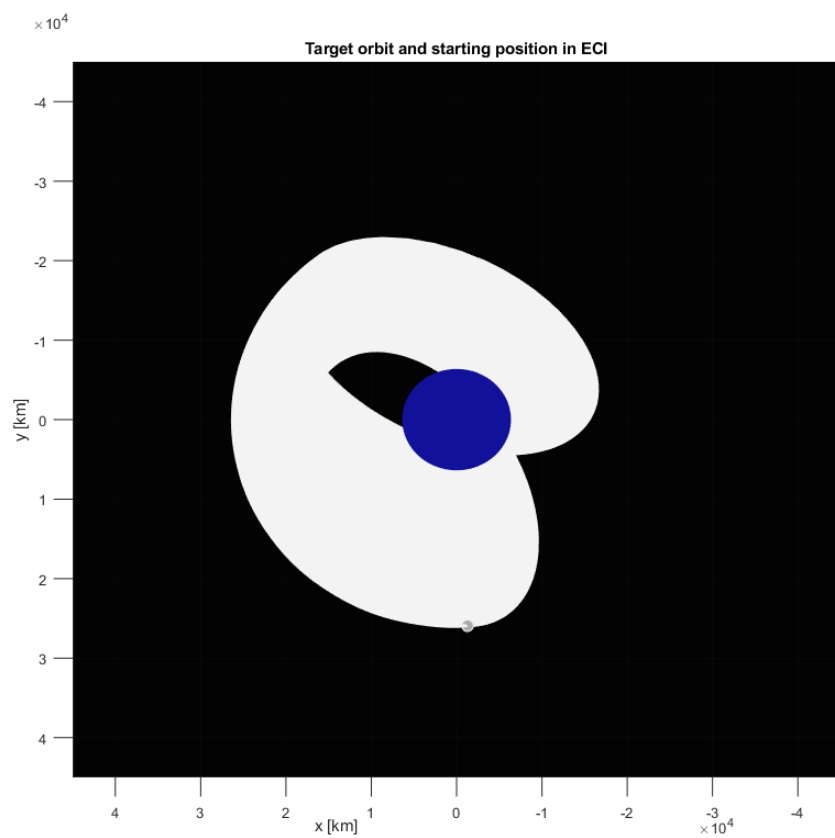
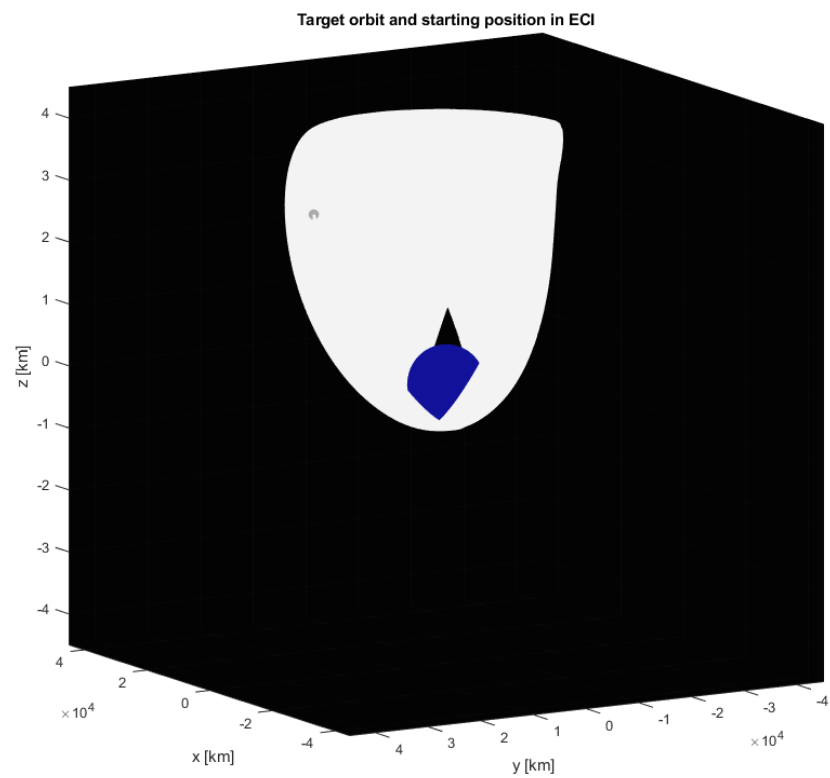
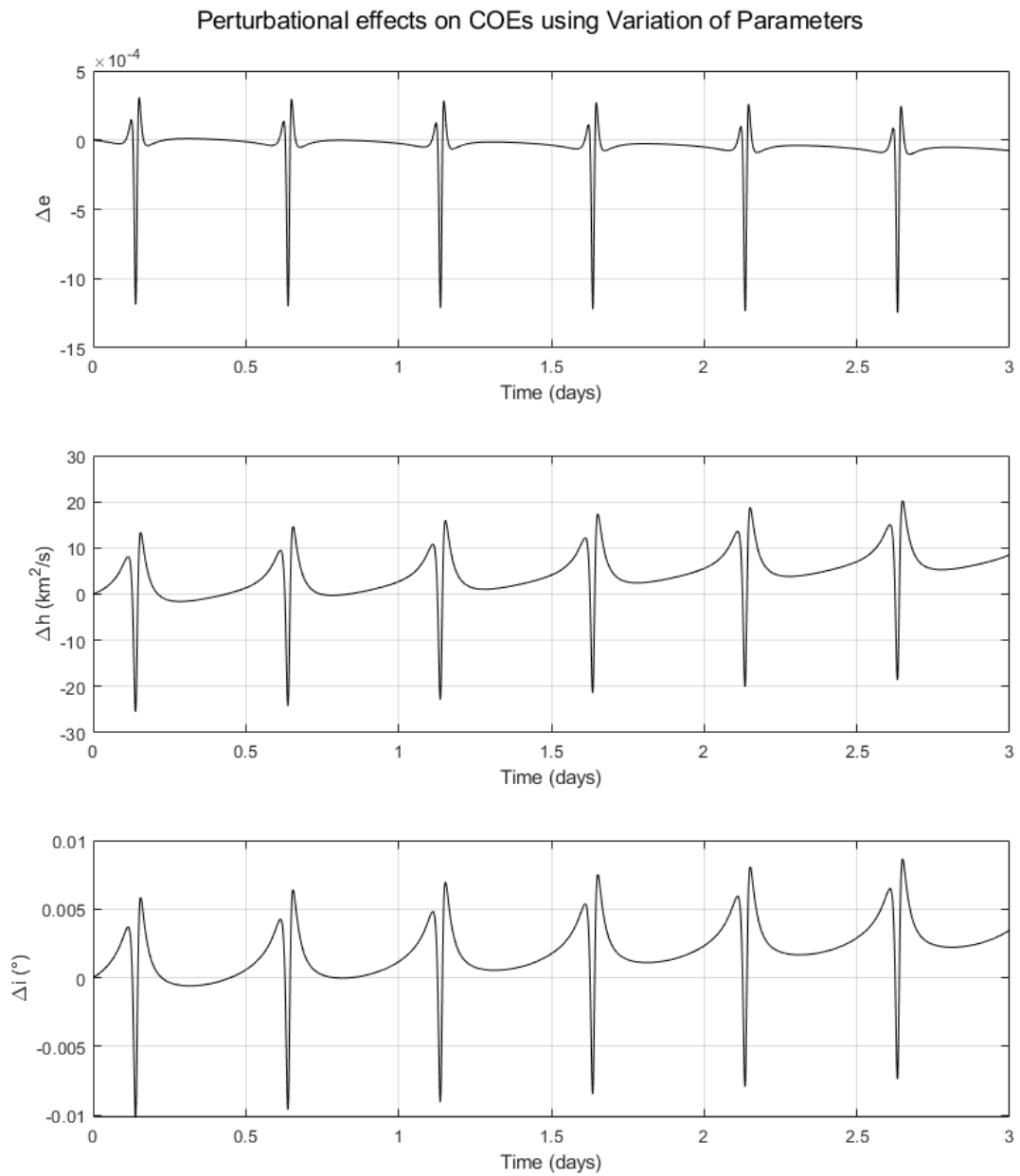


Figure 13: Long term Molniya 3-50 orbital path

## SPECIALITY ORBIT 1 - Molniya 3-50 SHORT TERM



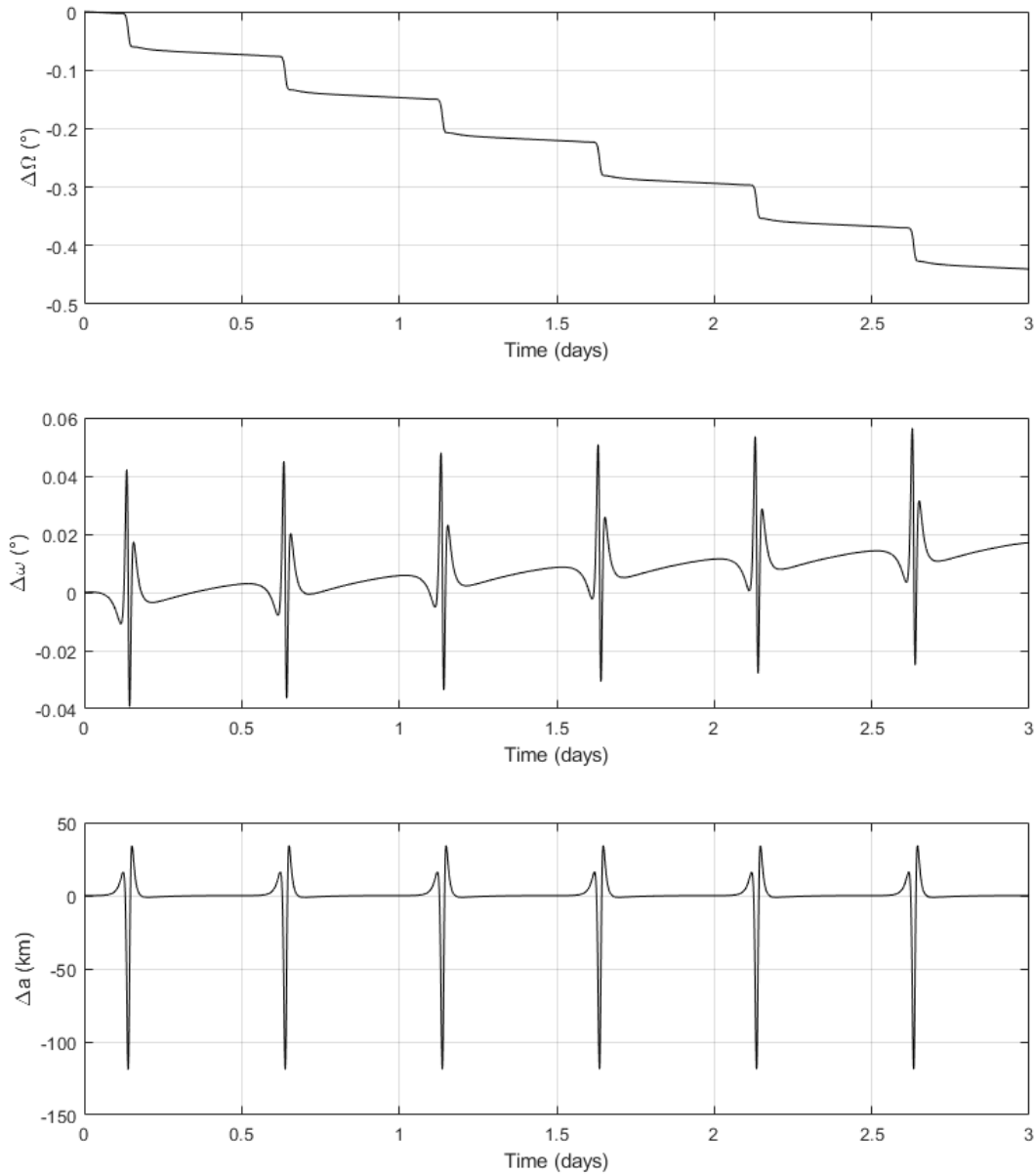
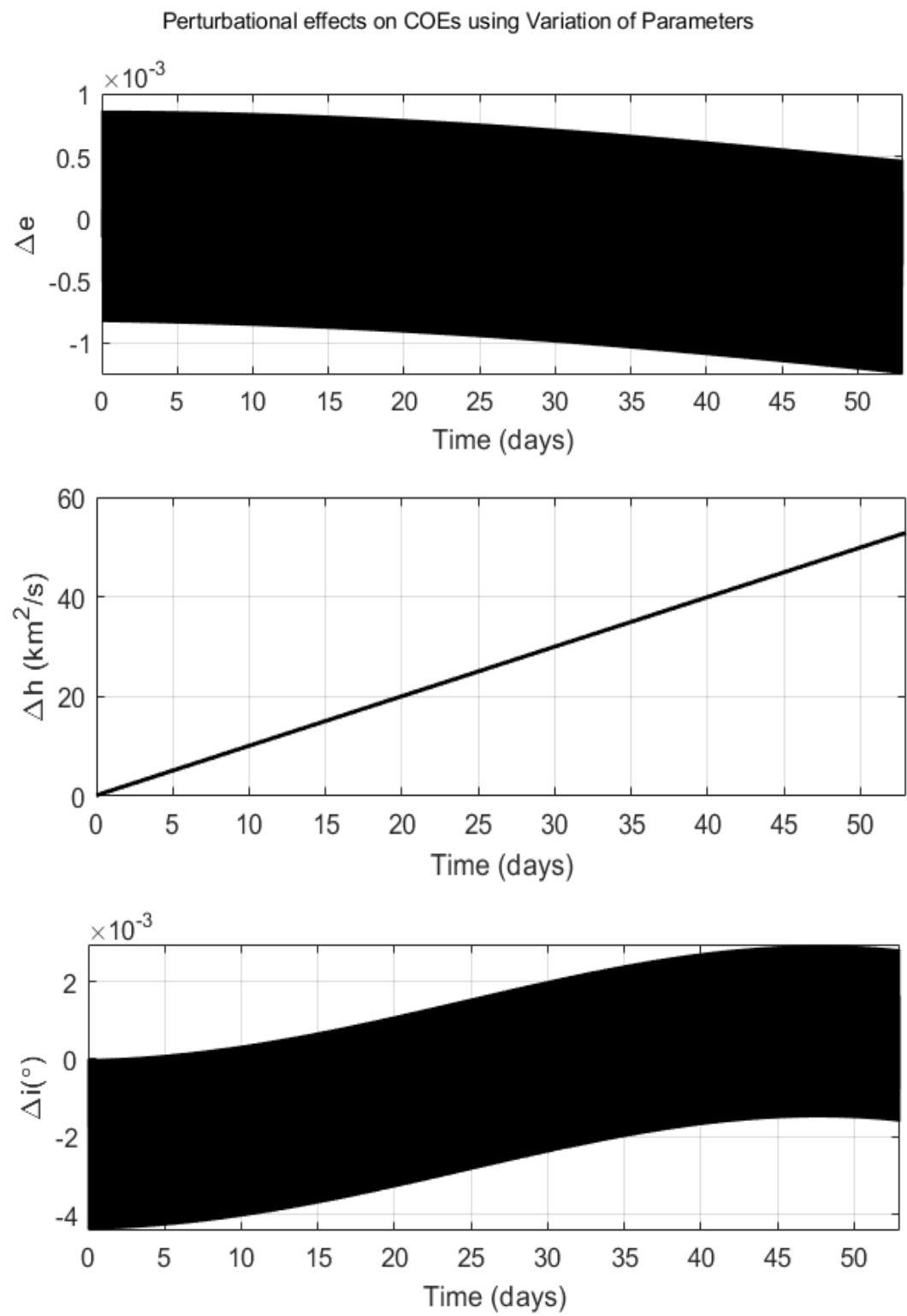


Figure 14: Short term perturbation effects on Molniya 3-50

The effect of atmospheric drag is apparent as short spikes of extreme variation as the object passes over perigee at high speed. This accounts for much of the variation in RAAN as the rate of change is much lower near apogee. While not related in any way, the plot for argument of perigee is visually similar to a cardiogram.

Interestingly, despite the simulation model for RAAN being verified for individual perturbations, the rate of RAAN change observed here,  $-0.14^\circ$  per day, is about half of the expected value of  $-0.3^\circ$  per day. This discrepancy is similar to that in the upcoming sun-synch orbit and is suggestive of either interference between perturbations or an unknown error in the simulation.

SPECIALITY ORBIT 2 - Aeolus LONG TERM



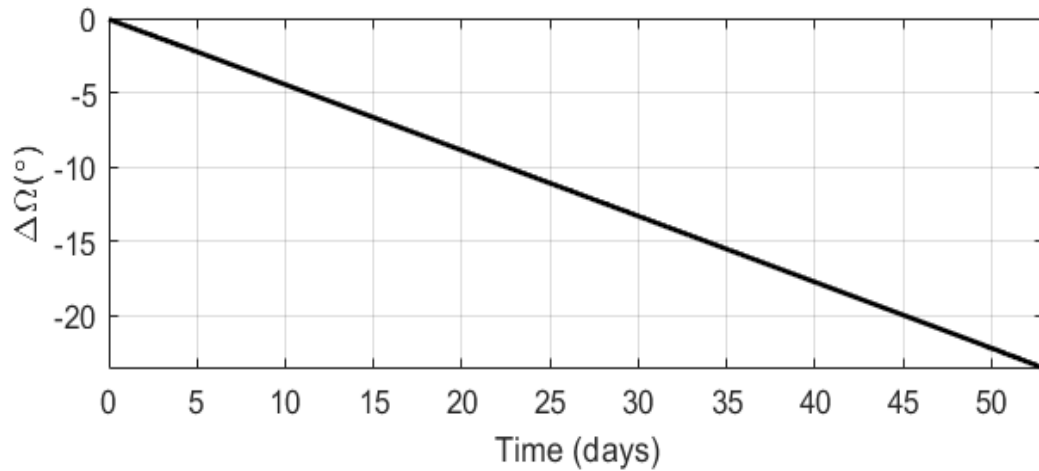


Figure 15: Perturbation effects on Aeolus

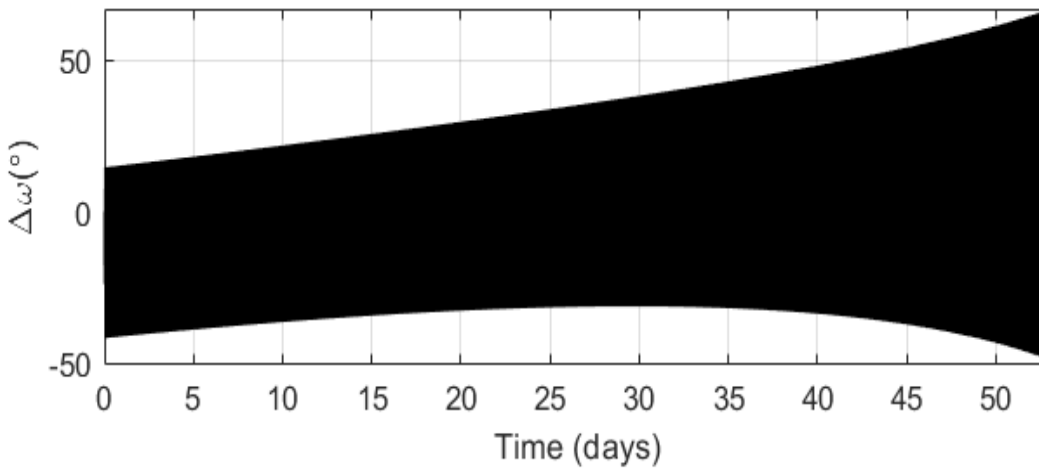
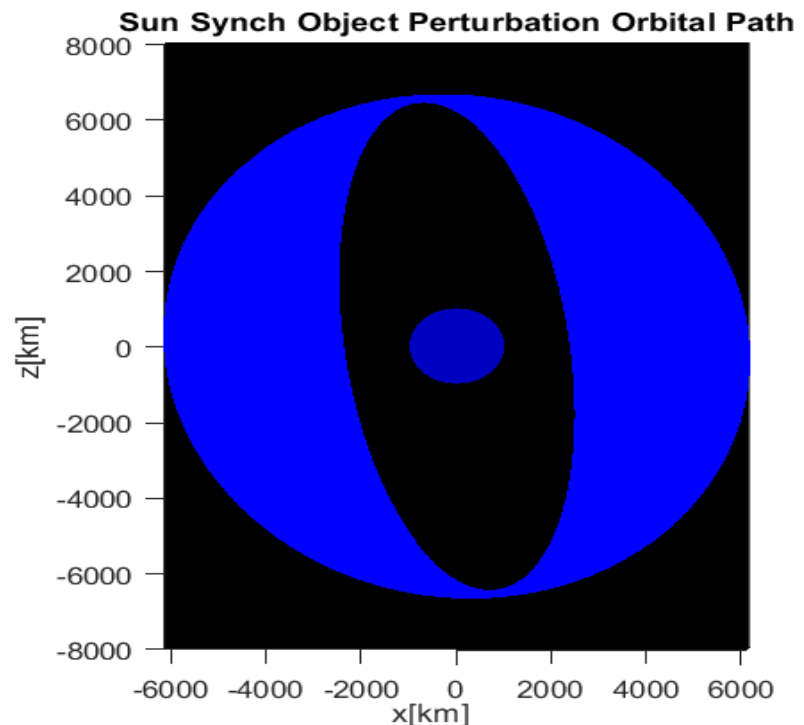


Figure 16: Aeolus Perturbation Path

The expectation for a sun-synch orbit is that the rate of change for the RAAN matches that of the earth's motion. This found to be around  $0.986^{\circ}/\text{day}$ . The rate as provided by the propagation method used however produces a plot that suggests the rate of change is around  $0.4^{\circ}/\text{day}$  for the Aeolus satellite. We know this is indeed a sun-synch object so the question then becomes why are the results obtained from this propagation method not matching the expected values. The assumed reason being that since this object has a LEO orbit the drag model used is the cause.



## Allocated Work

Steven:

- Selection of GEO and Molniya satellites
- Formulation of drag-excluded VoP propagation method
- Propagation of GEO and Molniya orbits (Part I & III)
- Creation of GEO and Molniya related plots
- GEO and Molniya trend discussion
- Report Introduction

Quintin:

- Selection of LEO and Sun-synch satellites
- Formulation of drag-included VoP propagation method
- Propagation of LEO orbit for long term effect and DV correction calculation (Part I)
- Propagation of LEO orbit for short term effect and sustained DV corrections (Part II)
- Propagation of Aeolus sun-synch orbit
- LEO and Aeolus trend discussion
- Creation of LEO and Aeolus related plots

Both:

- Selection of propagation method
- Determining assumptions/simplifications of each object

X\_ Quintin Cockrell

X\_ Steven Sharp

## References

- [1] ICHEP 2020 (28 July 2020 - 6 August 2020): Contribution List - Indico.  
<https://indico.cern.ch/event/868940/contributions>
- [2] NOAA Satellites, Public domain, via Wikimedia Commons
- [3] "Molniya-3," astronautix.com
- [4] "ADM-Aeolus." *ESA*, [https://www.esa.int/ESA\\_Multimedia/Images/2011/03/ADM-Aeolus3](https://www.esa.int/ESA_Multimedia/Images/2011/03/ADM-Aeolus3).
- [5] Kidder, S. Q., and T. H. Vonder Haar, 1990: On the use of satellites in Molniya orbits for meteorological observation of middle and high latitudes. *Journal of Atmospheric and Oceanic Technology*, 7, 517-522.