

MATH 351 Assignment 2 Exercise 1

Steven Sharp

Every mathematical statement, such as $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, should be part of a sentence. Even equations need punctuation!

The notation $\lim_{x \rightarrow a} f(x) = L$ means that for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \varepsilon$. An incorrect way to typeset this definition is

$$\forall(\varepsilon > 0)\exists(\delta > 0) \ni (|x - a| < \delta \implies |f(x) - L| < \varepsilon).$$

The symbols \forall , \exists , \ni , and \implies should be used only in the context of the mathematical subject of formal logic and should not replace the words “for all”, “there exists”, and “such that”, and “implies”.

One of your instructor’s favorite mathematical statements is

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n,$$

otherwise known as Stirling’s formula. As an example, $100!$ is approximately equal to $\sqrt{200\pi}(100/e)^{100} \approx 9.32 \times 10^{157}$.

The following is true:

$$\begin{aligned} \left| \int_1^a \frac{\sin x}{x} dx \right| &\leq \int_1^a \left| \frac{\sin x}{x} \right| dx \\ &\leq \int_1^a \frac{1}{x} dx \\ &= \ln a. \end{aligned}$$

After first simplifying using the exponential and the natural log functions, L’Hôpital’s rule can be used to evaluate $\lim_{x \rightarrow 2^-} (4 - x)^{1/(2-x)}$.

Take $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. The inner product of \mathbf{x} and \mathbf{y} is defined by $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^\top \mathbf{y}$. It follows that

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x}^\top \mathbf{x} = \|\mathbf{x}\|^2,$$

which is always a non-negative real number.
