Random-Contact Memory-Decay Independent Cascades

Social Networks Final Project Proposal

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1 INTRODUCTION

The widespread adoption of social media has drastically changed not only the speed and frequency of our communication, but also the regularity of unintentional exposure to other opinions, brands, and products through our own social networks. The latter concept emerges during the large amount of time spent browsing on social networks such as Facebook, Twitter, or Instagram seeing other people's conversations, posts about their opinions, recommendations, or product advertisements. These two channels of communication drive the dynamics of influence through social networks.

The ability of social networks to carry information quickly, even without direct communication, makes them ripe for companies or organizations who want to spread their products, brands, or ideas effectively. For entities to effectively maximize their influence, a representative model of the dynamics in the social network is essential.

Formally, the problem introduced by Kempe et al. [9] attempts to maximize the expected number of total vertices activated (i.e., total influence) given k initially active nodes and an influence graph $G = (V, E)$ where for each $(u, v) \in E$ there is a propagation probability. This is of course a hard optimization problem to solve. In fact, it is NP-hard. Kempe et al. proved the first optimization guarantees of the influence maximization problem under certain propagation models [9]. Recently there has been extensive research into models of influence and paradigms for influence maximization in social networks [5–7, 11, 14, 17]. Many of them add to the ideas in [9] and pursue similar guarantees for influence maximization.

Following the introduction of the influence maximization problem, many extended the concept and propagation models to competing influence in networks with practical applications to diffusion of conflicting opinions, containing misinformation, and promoting competing products [1, 3, 4, 8, 12, 13, 15, 18].

We will propose a new *Random-Contact Memory-Decay Independent Cascade (RCMD-IC)* model that we believe represents the dynamics in social networks particularly well, and would create a more representative framework to model the timeliness, frequency, and memorable elements of competing influence.

2 RELATED WORK

2.1 Early Influence Models

When Kempe et al. [9] introduced their influence maximization framework, they suggested two general models for the spread of influence over a network: *Linear Threshold (LT) model* and an *Independent Cascade (IC) model.*

2.1.1 Independent Cascade Model. The model is initialized with a set of *active* nodes S_0 which attempt to *infect* or *influence* nodes connected on outgoing edges in discrete time steps. Formally, the active node $v \in S_0$ has one chance to infect each $u \in N_{out}(v)$. Infection occurs with probability $p_{v,u}$. If v is successful, u will attempt to infect $w \in N_{out}(u)$ in the next period. If v does not succeed, then u will stay uninfected for subsequent rounds. A succinct expression for the probability of infection in the next time period follows given a set of active nodes S:

$$P(\text{v is active}) = 1 - \prod_{u \in N_{in}(v) \cap S} (1 - p_{u,v})$$

The important properties of this model are its monotonicity and submodularity which guarantee a $(1 - 1/e)$ approximation to the maximum influence under a greedy optimization algorithm. The proof in [9] considers all outcomes of influence along edges as a deterministic probability space over possible graphs where a live edge path from S_0 to a node indicates that node's activation. The number of activated nodes for a point in the space can be expressed as the size of the union of reachable nodes from a seed set. The expected number of activated nodes is equivalent to the weighted sum of activated nodes for all points in the space weighted by their probability of occurrence. Since cardinalities of unions and weighted sums are submodular, it follows that the IC model is also submodular.

2.2 Discrete Time-Dependent Influence

There have recently been many attempts to capture the effect of time in IC models of influence. Research into time dependent models revealed many papers since 2012 implemented disjoint parts of our concept.

2.2.1 Independent Cascade Model with Meeting Events (IC-M). Wei Chen et al. [5] proposed a new model that captured time-delay between activation and influence and took into account the time-critical nature in the case of viral marketing for example. The model introduces meeting probabilities $m_{u,v}$ for all $(u, v) \in E$. The dynamics of the model are very similar to IC. At each time step $1 \leq t \leq \tau$ where τ is the deadline, u is given a chance to meet v with probability $m_{u,v}$. If they meet, u attempts to activate v with probability $p_{u,v}$ only once. Once all active nodes meet with their neighbors, the process halts. Following from this definition of diffusion the probability any edge (u, v) is activated between time t' and t where $t' < t$ can be expressed by

$$p_{u,v}(1 - (1 - m_{u,v})^{t-t'-1})$$

since $(1 - (1 - m_{u,v})^{t-t'-1})$ is the probability of at least 1 meeting. Wei Chen et al. showed that their model has the same greedy optimization guarantees using similar proof methodologies to [9], but defines reachability in terms of live edge paths with less than τ hops to enforce a time constraint. They also derive a recursive expression for the activation probability at any time t which can be computed efficiently within an in-arborescence [1] graph using dynamic programming.

2.2.2 Continuously activated and Time-restricted IC (CT-IC). W. Lee et al. [11] propose a different variant of the original IC model, claiming that past cascade models are "too ideal to be applied to the real-world viral marketing applications." They note that people continuously attempt to activate neighbors and the process stops when there are no more activations.

As the name of their model suggests, active nodes continue to attempt activation in every timestep. One timestep after u is activated, it attempts to active v with probability $p_0(u, v)$. For each timestep t the CT-IC model defines

$$p_t(u, v) = p_0(u, v) \cdot f_{u,v}(t)$$

where $f_{u,v} : \mathbb{N}_0 \to \mathbb{R}_0^+$ is non-increasing and $f_{u,v}(0) = 1$.

Similar to the IC-M model in [5] activation stops at a time constraint T. Again, there is a closed form expression for probability of activation from a neighbor u. The probability that v is activated exactly at $t_u + t$ is

$$p_{t_u+t}(u, v) = p_{t-1}(u, v) \prod_{i=0}^{t-2} p_i(u, v) \tag{1}$$

As in the IC-M model, computing these probabilities is difficult, but restricting the graph to an arborescence or only simple paths in a directed graph allows for more tractable solutions. Submodularity and monotonicity are also proven for the CT-IC model by creating a discrete probability space where the times of activation are decided previously.

2.3 Continuous Time-Dependent Influence

2.3.1 Continuously Time IC (CT-IC). Rodriguez et al. [7] examine the influence maximization problem with their model proposed in [6]. It is essentially a continuous version of the discrete IC-M where the delay probability is drawn from a continuous distribution. They define $f(t_v|t_u; \alpha_{u,v})$ as the conditional probability of node u activating v, where $t_v > t_u$ are transmission times and $\alpha_{u,v}$ is a pairwise transmission rate. In [7] they consider

$$f(t_v|t_u; \alpha_{u,v}) \propto e^{\alpha_{u,v}(t_v - t_u)}$$

Their problem aligns with the original problem by maximizing $\sigma(A; T) = \mathbb{E}[N(A; T)]$ where $N(A; T)$ is the number of infected nodes up to time T given source nodes A. Rodriguez et al. shows that the process can be modeled with a Continuous Time Markov Chain (CTMC) and proves its submodularity using similar techniques to discrete influence models over a probability space defined by small Δt.

2.3.2 Time Varying IC (TV-IC). The last variation relevant to our model is the TV-IC model proposed by Ohsaka et al. in [17] in which they introduce the concept of time decaying phenomenon. By estimating average edge probabilities in real networks, they found that these probabilities themselves decayed throughout the propagation process. In other words, the novelty of ideas, rumors, or products decay overtime. Their model includes a sift invariant time delay $f_{u,v}(t|t_u) = f_{u,v}(t - t_u)$ similar to the CT-IC model, but adds another term such that when the influence reaches v it is activated with probability $p_{u,v}(t)$. Hence,

$$Pr[\text{v activated at time } t|u \text{ active at } t_u] = p_{u,v}(t) f_{u,v}(t - t_u).$$

This model can be easily reduced to past models by defining $p_{u,v}(t)$ to be a constant or $p_{u,v}(T) = 0$ for a deadline T. In this case, $p_{u,v}(t) : \mathbb{R}^+ \to [0, 1]$ is necessarily a non-increasing function. Ohsaka et al. shows that it is essential for time decay influence probabilities to be non-increasing to guarantee submodularity.

3 MOTIVATION

All the models we have covered brought up valid elements missing from IC models, however, we believe that none of them quite capture the dynamics in social networks. While the CT-IC model is one of the building blocks of our motivation, the example used to justify the CT-IC model was slightly flawed. W. Lee et al. [11] considered this example:

> Suppose you buy a new product and write a positive post about it in your Facebook wall. The post appears to your friends and persuades them to have a positive opinion, which may lead them to buy it. The important thing here is that when revisiting your wall later, your friends may be persuaded to buy the product although they were not persuaded before. In other words, your positive post will have continuous influence on your friends.

While we believe that an attempted influence continues past the initial attempt, it seems doubtful that someone would continually look back at posts and finally be persuaded later on. Furthermore, this model leaves no room for multiple unique attempts by the same person to persuade you which seems contradictory to the name of the model.

Consider the example where you buy a new product and mention it on Facebook and Twitter. Friend A doesn't go on Facebook or Twitter often and misses your post, but you mentioned it to him at work. He thinks about it the rest of the day but eventually forgets that you mentioned it. Friend B lives next-door and goes on social media all the time. You tell him about the product. Two days later Friend B sees the link for the product on both sites and decides to buy.

In this more realistic example, there are many more dynamics occurring: multiple random and concurrent influence attempts as well as decaying influence of past attempts. Many factors play into these elements. For example, people that interact more frequently are more likely to have multiple influence attempts. People with larger numbers of friends may have the opposite effect with diluted news feeds. Lastly, there may be differences between influential friends and memorable influencers (i.e., celebrity endorsements or established brands) or influence attempts (i.e., funny ads) which might not be very convincing, but will stay in your memory longer.

These elements become more crucial in the presence of competing campaigns. When businesses attempt to solve the maximum influence problem, getting there first and often [2], and in a memorable way [10, 16] is key to capturing business and creating brand

[1] a directed tree where all edges point into the root

awareness. We believe that these are essential elements of social network dynamics and should be captured in order to achieve effective influence maximization.

4 MODEL PROPOSAL

4.1 Model Definition

We now describe our extension to the IC model, the *Random-Contact Memory-Decay Independent Cascade (RCMD-IC)* model. The social network we will examine is based off a graph $G = (V, E)$ where the vertices V are the entities in the network and directed edges E represent relationships or connections between entities. Each edge $(u, v) \in E$ is associated with an *influence probability* $p_{u,v}$. For simplicity, we will keep $p_{u,v}$ constant. We also define for each edge a *contact probability* $c_{u,v}$ defined by $c : E \longrightarrow [0, 1]$ [2]. Finally, we define a *memory influence probability* $m_{u,v}(t)$ for each edge defined by $m : \mathbb{Z}^+ \longrightarrow [0, 1]$, also a non-increasing function.

The diffusion process in the RCMD-IC unfolds in discrete time steps. An initial seed set $S \subseteq V$ is activated at $t = 0$. For each $t \geq 1$ any node u activated in any previous period has contact with any of its inactive neighbors $v \in N_{out}(u)$ with probability $c_{u,v}$. If there is no contact, u will not interact with v any further in the current time period. If they come into contact, u will attempt to influence v. Whether u succeeds or not, the unique attempt by v will continue to attempt to influence u in future periods based on v's memory. Therefore, the attempt will activate v with probability

$$p_{u,v}(t) * m_{u,v}(t - t_c)$$

where t_c is the time of contact. When $t < 0$ $m_{u,v}(t) = 0$ and $m_{u,v}(0) = 1$ since memory will not effect the probability in the time period of contact or before. Once a node becomes active it will remain active. Eventually, we can force the process to stop at time T, a deadline constraint.

Our model can be reduced to past variants of IC with specific function definitions. For example, if we take $p_{u,v}(t)$ to be constant and we set $c_{u,v} = 1$ we can reduce to the CT-IC model.

See appendix A for a simple visual example of the dynamics.

4.2 Computing Influence Probabilities

The diffusion process of our model unfortunately introduces a possibly more challenging problem of computing influence due to the multiple influence attempts allowed for each neighbor. Consider a simple example where u is activated at time t_u and $v \in N_{out}(u)$. Let $P_{t_u+3|t_u}$ be the probability v is activated by u by time $t_u + 3$. We have

$$\begin{aligned} P_{t_u+3|t_u} &= 1 - (1 - [c_{u,v}p_{u,v} \\ &+ c_{u,v}(1 - p_{u,v})p_{u,v}m_{u,v}(1)]) \\ &\times (1 - c_{u,v}p_{u,v}) \end{aligned}$$

The first and last terms represent contact and activation in the same period, while the second term represents contact and failure to activate with a memory activation in the next period. We can generalize this for the probability activated by a deadline constraint $T > t_u$:

$$\begin{aligned} P_{T|t_u} = 1 - \prod_{s=t_u+1}^{T-1} \Bigg(1 - \Bigg(&c_{u,v}p_{u,v} \\ &+ c_{u,v}p_{u,v} \sum_{i=s}^{T-2} \left[m_{u,v}(i - s + 1) \prod_{k=s}^{i} (1 - p_{u,v}m_{u,v}(i - k)) \right] \Bigg) \Bigg) \end{aligned}$$

In the case when $s = T - 1$ the summation evaluates to 0 and only $(1 - c_{u,v}p_{u,v})$ remains for that term. Just as other work has done, we plan to derive activation probabilities for any node at any time t on an arborescence with a recursive function. This will allow for easier computation of influence probabilities. Using $P_{t|t'}$ derived above we have included a sketch of this expression below. Given $ap(v, t)$ is the activation probability of u at t and a seed set S we have the base cases

$$ap(v, t) = \begin{cases} 1 & v \in S \text{ and } t = 0 \\ 0 & v \notin S \text{ and } t = 0 \\ 0 & v \in S \text{ and } t \geq 1 \end{cases}$$

For all other cases we have that

$$\begin{aligned} ap(v, t) = &\prod_{u \in N_{in}(v)} \left(1 - \sum_{t'=0}^{t-2} ap(u, t')(1 - P_{t-1|t'})\right) \\ &- \prod_{u \in N_{in}(v)} \left(1 - \sum_{t'=0}^{t-1} ap(u, t')(1 - P_{t|t'})\right) \end{aligned}$$

4.3 Potential Model Properties

A consequence of this model that we hope holds true during further research is submodularity. This would imply that under a greedy maximization procedure we can achieve (1-1/e) optimality when choosing seeds.

Theorem 4.1. *The influence function $\sigma_T(\cdot)$ is monotone and submodular for all $t \geq 1$*

Proof Sketch. We can reduce the RCMD-IC model to the CT-IC model by reconstructing a new graph and make use of the monotone and submodular properties of the CT-IC model. W. Lee et al. [11] prove these properties using the following process [3]. First, decide t for each $(u, v) \in E$ before the activation process starts. Choosing t is represented by the function $h : E \longrightarrow \mathbb{N}$. Create a graph $G' = (V, E, h)$ with weight $h(u, v)$ for $(u, v) \in E$. Similar to other influence process proofs we now have chosen influence deterministically since $v \in V$ is active at time t if and only if there exists $u \in S$ and a path from u to v in G' whose length is equal to or less than t. Furthermore, each $h(u, v)$ is drawn from a predetermined density function that we displayed in (1) based on $p_0(u, v)$ and the function $f_{u,v}(t)$.

Consider an active node u and a neighbor v in the RCMD-IC model with deadline T where $u \in S$. We can reconstruct our edge by first deleting the original edge (red) and adding c_t nodes for contact in each possible time period and d_t for $t \geq 2$ to act as dummy nodes to reach the contact nodes. Using this set-up we can run the CT-IC model on this graph and it will be equivalent to the RCMD-IC. To

[2] This is identical to the meeting probability seen in [5] but we named it differently to avoid confusion with 'memory' probability

[3] Version with full proof: http://dm.postech.ac.kr/techreport/TechReport-POSTECH-CSE-2012-02-CT-IC.pdf

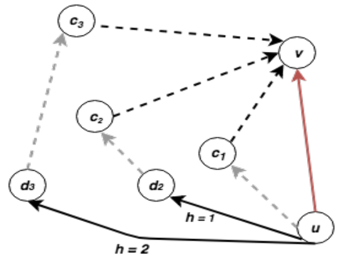

Figure 1: Newly constructed edges between u and v

see this we must define parameters for each edge. First, we define each dark solid edge to have a deterministic $h(u, d_t) = t - 1$. Each gray edge will act as a random contact in a specific period, so we define the weight distribution as

$$h(u, c_t) = \begin{cases} 0 & \text{with prob. } c_{u,v} \\ T & \text{with prob. } 1 - c_{u,v} \end{cases}$$

Therefore, if there is no contact, the weight is T and by definition there cannot be influence before the deadline T along this path[4]. Finally, the dark dashed edges (c_t, v), we can assign:

$$p_0(c_t, v) = p_{u,v}$$
$$f_{c_t,v}(t) = m_{u,v}(t)$$

Given these definitions, if there is a path from u to v this new graph with weight less than or equal to t, then v is activated by u within t in the RCMD-IC model.

Continuing the proof in [11], if we take $\sigma_h(S, t)$ to be the cardinality of the union of activated nodes by time t given a draw of h, we can define influence on G as:

$$\sigma(S, t) = \sum_h Pr(h) \cdot \sigma_h(S, t)$$

Since $\sigma(S, t)$ is a weighted sum of submodular functions, we can conclude that $\sigma(S, t)$ is submodular as well. □

5 FUTURE RESEARCH PLAN

5.1 Model Properties and Computation

Building upon the work in this proposal we would like to formalize the proofs for the properties we have discussed (i.e. submodularity) which we believe set a strong foundation for future work.

5.2 Data Experiments

We plan to utilize common network datasets from Stanford Network Analysis Project (SNAP) to explore the dynamics of RCMD-IC compared to similar IC models with a range of graph parameters. Specifically, we will explore the extension of our model to competitive influence where there may be competing seed sets S and S'. Under what types of conditions does our model behave like others? If it doesn't, can we explain why?

5.3 Influence Maximization

If our research produces promising results, it will open possibilities for the development of influence maximization algorithms for the RCMD-IC model. This research would include establishing efficient algorithms for influence probability computation and greedy algorithms for influence optimization in our new framework. Specifically, we will also pursue methods to optimize under competing influence and extend results from [12] to RCMD-IC.

A EXAMPLE SCENARIOS

Here we provide a visualization of our model's dynamics. Consider one inactive node v and two active nodes $u1$ and $u2$ at $t = 0$ with incoming edges to v. $u1$ is a user of Product1 and $u2$ is a user of competing Product2. We fix the periods in which v has contact with the other nodes ($u1$ at $t = 1$ and $u2$ at $t = 3, 5$). To illustrate the importance of memory, we set the same influence probabilities, but different memory functions:

$$p_{u_1,v} = p_{u_2,v} = 0.25$$
$$m_{u_1,v} = exp(-0.5t)$$
$$m_{u_2,v} = exp(-t)$$

Using these parameters, v will forget u_2's influence attempts at twice the rate of u_1.

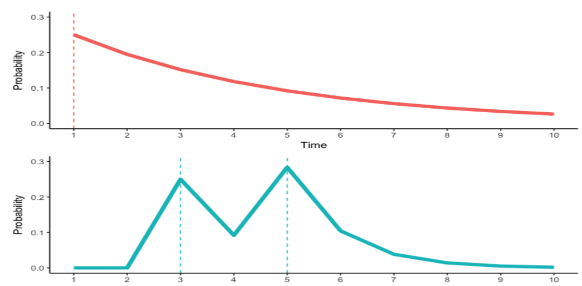

Figure 2: Individual influence probabilities given $u1$ (top) has contact at $t = 1$ or $u2$ (bottom) has contact at $t = 3$ and $t = 5$

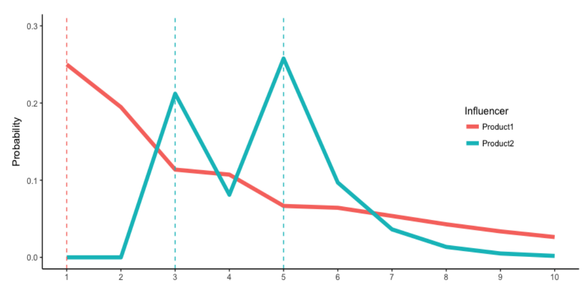

Figure 3: Influence probability by $u1$ and $u2$ for $1 \leq t \leq 10$

Since a more memorable attempt from $u1$ occurred first, $u2$ does not have a significant advantage in influencing v even in the short

[4] To generalize this we could also use ∞

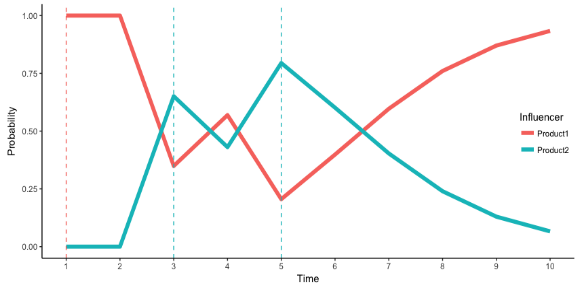

Figure 4: Probability of adopting Product1 or Product2 given influence at time t for $1 \leq t \leq 10$

period where $u2$ and v are in contact twice. Eventually $u1$'s first attempt outlasts the two attempts by $u2$. If v is somehow not activated until $t = 8$ or $t = 9$, it will likely be by $u1$.

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