
CS241 Homework 5

Transformations as 4 plane equations: A 4x4 matrix, when used as a transformation, can be considered as evaluating four plane equations. This can lead to alternate ways of understanding the effects of a transformation, and can form the basis for creating a transformation with desired effects. That is, when transforming a point V by a matrix P , the evaluation of

$$V' = PV$$

can be viewed as four plane equation evaluations:

$$x' = P_1 V, \quad y' = P_2 V, \quad z' = P_3 V, \quad \text{and} \quad w' = P_4 V,$$

where each P_i is a single row of P and $V' = (x', y', z', w')^T$.

Now, remembering that plane equation evaluations measure (scaled signed) distance from a plane, you can fully describe the effect of P by describing the effect of each plane equation P_i .

Problem: Use this thought process to derive the perspective transformation from class.

Hints:

1. The defining feature of the perspective transformation is that the homogeneous coordinate w' must measure distance from the plane defined by the eye, at $(0,0,0)$ and the view direction $(0,0,1)$. Use this to define P_4 .
2. The z'/w' coordinate measures distance from the front clipping plane at $z=d$ in the view direction $(0,0,1)$. Use this to define P_3 . Choose a scale factor that gives $z'/w' = 1$ at the far clipping plane.
3. Similarly for P_1 for x'/w' and P_2 for y'/w' .
4. Determine an over-all scale factor for the matrix which yields the perspective transformation of class.