e. $p = e^{10}$, $p^* = 22000$

f. $p = 10^{\pi}$, $p^* = 1400$

g. $p = 8!$, $p^* = 39900$

h. $p = 9!$, $p^* = \sqrt{18\pi}(9/e)^9$

2. Find the largest interval in which p^* must lie to approximate p with relative error at most 10^{-4} for each value of p.

a. π b. e c. $\sqrt{2}$ d. $\sqrt[3]{7}$

3. Suppose p^* must approximate p with relative error at most 10^{-3}. Find the largest interval in which p^* must lie for each value of p.

a. 150 b. 900 c. 1500 d. 90

4. Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. (iv) Compute the relative errors in parts (ii) and (iii).

a. $\frac{4}{5} + \frac{1}{3}$

b. $\frac{4}{5} \cdot \frac{1}{3}$

c. $\left(\frac{1}{3} - \frac{3}{11}\right) + \frac{3}{20}$

d. $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

5. Use three-digit rounding arithmetic to perform the following calculations. Compute the absolute error and relative error with the exact value determined to at least five digits.

a. $133 + 0.921$

b. $133 - 0.499$

c. $(121 - 0.327) - 119$

d. $(121 - 119) - 0.327$

e. $\dfrac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$

f. $-10\pi + 6e - \dfrac{3}{62}$

g. $\left(\frac{2}{9}\right) \cdot \left(\frac{9}{7}\right)$

h. $\dfrac{\pi - \frac{22}{7}}{\frac{1}{17}}$

6. Repeat Exercise 5 using four-digit rounding arithmetic.

7. Repeat Exercise 5 using three-digit chopping arithmetic.

8. Repeat Exercise 5 using four-digit chopping arithmetic.

9. The first three nonzero terms of the Maclaurin series for the arctangent function are $x - (1/3)x^3 + (1/5)x^5$. Compute the absolute error and relative error in the following approximations of π using the polynomial in place of the arctangent:

a. $4\left[\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right)\right]$

b. $16\arctan\left(\frac{1}{5}\right) - 4\arctan\left(\frac{1}{239}\right)$

10. The number e can be defined by $e = \sum_{n=0}^{\infty}(1/n!)$, where $n! = n(n-1)\cdots 2\cdot 1$ for $n \neq 0$ and $0! = 1$. Compute the absolute error and relative error in the following approximations of e:

a. $\displaystyle\sum_{n=0}^{5} \frac{1}{n!}$

b. $\displaystyle\sum_{n=0}^{10} \frac{1}{n!}$

11. Let

$$f(x) = \frac{x\cos x - \sin x}{x - \sin x}.$$

a. Find $\lim_{x\to 0} f(x)$.

b. Use four-digit rounding arithmetic to evaluate $f(0.1)$.

c. Replace each trigonometric function with its third Maclaurin polynomial, and repeat part (b).

d. The actual value is $f(0.1) = -1.99899998$. Find the relative error for the values obtained in parts (b) and (c).

a. $e \approx \sum_{n=0}^{5} \frac{1}{n!}$ **b.** $e \approx \sum_{j=0}^{5} \frac{1}{(5-j)!}$

c. $e \approx \sum_{n=0}^{10} \frac{1}{n!}$ **d.** $e \approx \sum_{j=0}^{10} \frac{1}{(10-j)!}$

3. The Maclaurin series for the arctangent function converges for $-1 < x \leq 1$ and is given by

$$\arctan x = \lim_{n\to\infty} P_n(x) = \lim_{n\to\infty} \sum_{i=1}^{n} (-1)^{i+1} \frac{x^{2i-1}}{2i-1}.$$

a. Use the fact that $\tan \pi/4 = 1$ to determine the number of n terms of the series that need to be summed to ensure that $|4P_n(1) - \pi| < 10^{-3}$.

b. The C++ programming language requires the value of π to be within 10^{-10}. How many terms of the series would we need to sum to obtain this degree of accuracy?

4. Exercise 3 details a rather inefficient means of obtaining an approximation to π. The method can be improved substantially by observing that $\pi/4 = \arctan \frac{1}{2} + \arctan \frac{1}{3}$ and evaluating the series for the arctangent at $\frac{1}{2}$ and at $\frac{1}{3}$. Determine the number of terms that must be summed to ensure an approximation to π to within 10^{-3}.

5. Another formula for computing π can be deduced from the identity $\pi/4 = 4\arctan \frac{1}{5} - \arctan \frac{1}{239}$. Determine the number of terms that must be summed to ensure an approximation to π to within 10^{-3}.

6. Find the rates of convergence of the following sequences as $n \to \infty$.

a. $\lim_{n\to\infty} \sin \frac{1}{n} = 0$ **b.** $\lim_{n\to\infty} \sin \frac{1}{n^2} = 0$

c. $\lim_{n\to\infty} \left(\sin \frac{1}{n}\right)^2 = 0$ **d.** $\lim_{n\to\infty} [\ln(n+1) - \ln(n)] = 0$

7. Find the rates of convergence of the following functions as $h \to 0$.

a. $\lim_{h\to 0} \frac{\sin h}{h} = 1$ **b.** $\lim_{h\to 0} \frac{1-\cos h}{h} = 0$

c. $\lim_{h\to 0} \frac{\sin h - h\cos h}{h} = 0$ **d.** $\lim_{h\to 0} \frac{1-e^h}{h} = -1$

8. **a.** How many multiplications and additions are required to determine a sum of the form

$$\sum_{i=1}^{n} \sum_{j=1}^{i} a_i b_j?$$

b. Modify the sum in part (a) to an equivalent form that reduces the number of computations.

9. Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial, and let x_0 be given. Construct an algorithm to evaluate $P(x_0)$ using nested multiplication.

10. Example 5 of Section 1.2 gives alternative formulas for the roots x_1 and x_2 of $ax^2+bx+c=0$. Construct an algorithm with input a, b, c and output x_1, x_2 that computes the roots x_1 and x_2 (which may be equal or be complex conjugates) using the best formula for each root.

11. Construct an algorithm that has as input an integer $n \geq 1$, numbers $x_0, x_1, \ldots, x_n$, and a number x and that produces as output the product $(x - x_0)(x - x_1)\cdots(x - x_n)$.

12. Assume that

$$\frac{1-2x}{1-x+x^2} + \frac{2x-4x^3}{1-x^2+x^4} + \frac{4x^3-8x^7}{1-x^4+x^8} + \cdots = \frac{1+2x}{1+x+x^2},$$