CS241 Homework 2

Homogeneous coordinates: Homogeneous coordinates are called such because that word means ''all of the same or similar kind or nature''. This problem should make sense of that designation by showing that any point, transformation matrix, or plane equation has a multitude of representations, all scalar multiples of each other. For each of the following, let $P=(x,y,z,w)$ be a homogeneous point representing the "real" point $(x/w,y/w,z/w)$. Let M be any 4x4 transformation matrix, and $Q=[A,B,C,D]$ be the coefficients of any plane equation. Show the following:

1. Any scalar multiple of P, say kP, represents the same real point as P.
2. M will transform P and any scalar multiple kP to different homogeneous points that represent the same real point.
3. M and any scalar multiple of M will transform P and any scalar multiple of P to a multitude of homogeneous points, all of which represent the same real point.
4. If $Q=[A,B,C,D]$ represents the plane of all points (x,y,z) which satisfy $Ax+By+Cz+D=0$ then any non-zero scalar multiple of Q represents the same plane. That is, a point is on Q if and only if it is on kQ.
5. **Why is this statement FALSE when it is so similar to the preceding statements:** If P is in the solution set of that plane equation, (i.e., P is on the plane), then any non-zero scalar multiple of P satisfies the plane equation.
6. So... In light the failure in (5), change the definition of the plane represented by Q to be the solution set of this new equation $Ax+By+Cz+Dw=0$. Now, show that both (4) and (5) hold true with this new definition of a plane equation.

Hint: *Yes, most of this problem really is as easy as showing that factors of k in both a numerator and denominator cancel out. Nevertheless, all these facts are worth noticing and understanding.*