

Advanced Mechanics
Problem set #1

1. Prove:

a) $\sum_k (\epsilon_{ijk} \epsilon_{lmk}) = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}$

b) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$

c) $d/dt (\mathbf{r} \times (\mathbf{v} \times \mathbf{r})) = r^2 \mathbf{a} + (\mathbf{r} \cdot \mathbf{v})\mathbf{v} - (v^2 + \mathbf{r} \cdot \mathbf{a})\mathbf{r}$

2. Find the angle between the surfaces defined by $r^2 = 24$ and $x + y + z^2 = 20$ at $(2, 2, -4)$.

3. A particle moves in an orbit defined by $\mathbf{r} = A \sin(\omega t)\mathbf{i} + 3A \cos(\omega t)\mathbf{j}$.

a) Find \mathbf{v} and \mathbf{a} of the particle.

b) Find the speed of the particle at time, $t = 2\pi/\omega$.

c) Find the angle between \mathbf{v} and \mathbf{a} at time, $t = 2\pi/\omega$.