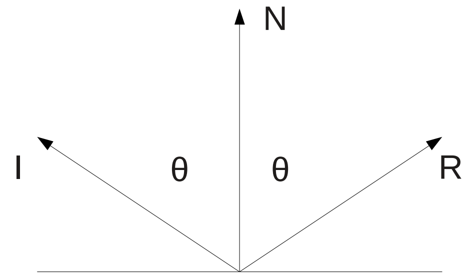
Geometry for recursive Ray Tracing

Reflection direction



Incoming ray: $Q + t\,D$
Incident ray: $I = -D$
Normal: N
Calculate reflection direction: R

All three vectors lie in a plane so

$$R = \alpha I + \beta N\,,$$

and in fact $\alpha = -1$.
Equal angles implies $I \cdot N = R \cdot N$
Solving:

$$I \cdot N = (-I + \beta N) \cdot N$$

$$\beta = 2\frac{I \cdot N}{N \cdot N}\text{, so}$$

$$R = -I + 2\frac{I \cdot N}{N \cdot N} N$$

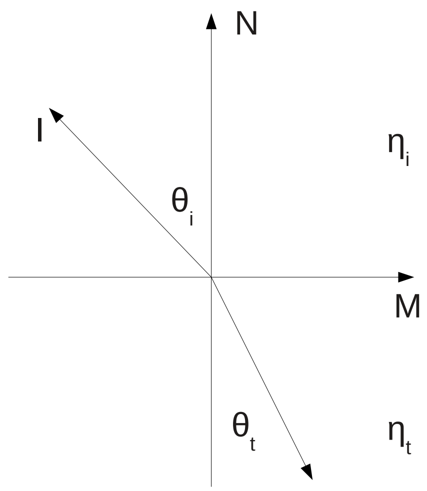
Assuming $N \cdot N = 1$, and $I = -D$:

$$R = D - 2(D \cdot N)\,N$$

Snell's Law

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

Transmission direction



N and M are an orthonormal basis
All vectors are planar, so

$$I = N\cos\theta_i - M\sin\theta_i$$

$$T = -N\cos\theta_t + M\sin\theta_t$$

Solve I for M

$$M = \frac{N\cos\theta_i - I}{\sin\theta_i}$$

From Snell's Law and trig identities

$$\sin\theta_t = \eta_{it}\sin\theta_i \quad \text{where} \quad \eta_{it} = \eta_i/\eta_t$$

$$\begin{aligned}
\cos^2\theta_t &= 1 - \sin^2\theta_t \\
&= 1 - \eta_{it}^2 \sin^2\theta_i \\
&= 1 - \eta_{it}^2 \left(1 - \cos^2\theta_i\right) \\
&= 1 - \eta_{it}^2 \left(1 - (I \cdot N)^2\right) \\
\cos\theta_t &= \sqrt{\left(1 - \eta_{it}^2\left(1 - (I \cdot N)^2\right)\right)}
\end{aligned}$$

Substituting into T equation:

$$T = -N\cos\theta_t + \eta_{it}\frac{\sin\theta_i}{\sin\theta_i}\left(N\cos\theta_i - I\right)$$

$$T = \eta_{it}\left(N(N \cdot I) - I\right) - N\sqrt{1 - \eta_{it}^2\left(1 - (I \cdot N)^2\right)}$$

Total Internal Reflection

If the sqrt argument is negative, the there is no solution
Meaning there is no transmission.

Implementation considerations for direction calculations:

Assume a transparent surface is embedded in air. (If you wish.)
With index-of-refraction η_{obj}
And ray with direction D .
When a ray hits a transparent surface, determine:

entering or leaving
index of refraction ratio
proper normal for reflection and transmission direction calculations
If $D \cdot N < 0$: // Entering obj

$\eta_r = 1/\eta_{obj}$ and $\bar{N} = N$

else: // Leaving obj

$\eta_r = \eta_{obj}/1$ and $\bar{N} = -N$

Then calculate (using $\bar{N}$)

Reflection direction: $R = D - 2(D \cdot \bar{N})\bar{N}$
Transmission direction: $T = \eta_{it}(\bar{N}(\bar{N} \cdot I) - I) \; - \bar{N}\sqrt{1 - \eta_{it}^2(1 - (I \cdot \bar{N})^2)}$ (if it exists)

Beer's Law:

The rate at which the energy of a light ray is absorbed is proportional to the energy of the ray.

Applied to transmission through a homogeneous material.

$I(t)$: the amount of light (energy) available at distance t into a material
$I'(t)$: how it changes (i.e., is absorbed) at point t . (This is what derivatives **are**).
$I'(t) = -\kappa I(t)$: Beer's law in equation form (a differential equation.)
$I(0) = I$: The initial (boundary) condition – the light available at the surface ($t=0$)
$I(t) = I e^{-\kappa t}$: The solution
Proof:

$$I'(t) = -\kappa I e^{-\kappa t} = -\kappa I(t)$$
$$I(0) = I$$