Perspective

Perspective requires a division

The perspective projection of a point (x,y,z) onto a viewplane z=1
By similar triangles: $(x', y')=(x/z, y/z)$
But we don't want z/z to lose all depth information

Homogeneous coordinates

Use 4D points (x,y,z,w) to represent 3D points like this
$(x, y, z, w) \sim (x/w, y/w, z/w)$ if $w \neq 0$
For non-zero scalars:
$(sx, sy, sz, sw) \sim (sx/sw, sy/sw, sz/sw) \sim (x/w, y/w, z/w)$
We can interpret w=0 as:
points at infinity., or
vectors (directions)
We can rig w to contain a useful quantity for perspective

Perspective projection transformation

View frustum is specified by
r_x : half width to viewing distance ratio
r_y : half height to viewing distance ratio
f : distance to view plane
b : distance to far plane
Projection matrix:

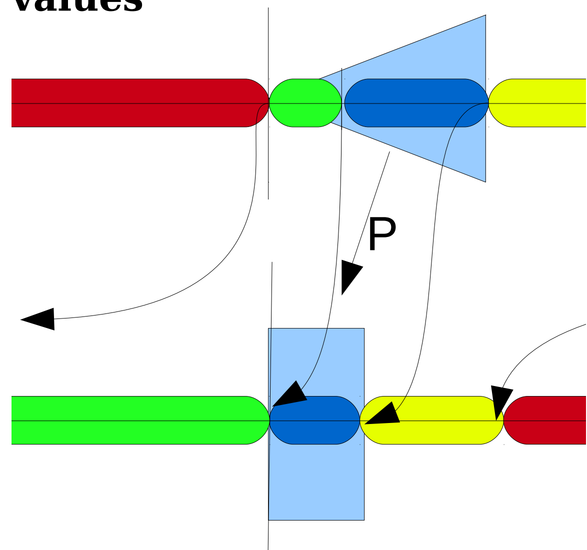
$$\begin{bmatrix} \frac{1}{r_x} & 0 & 0 & 0 \\ 0 & \frac{1}{r_y} & 0 & 0 \\ 0 & 0 & \frac{b}{b-f} & -\frac{fb}{b-f} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Some View Frustum calculations:

Corners and edges
Front corners are $(\pm w, \pm h, d, 1)$,
Corners at z depth are $E = (\pm wz/d, \pm hz/d, z, 1)$
Transform to $PE = (\pm z, \pm z, ?, z) \Rightarrow (\pm 1, \pm 1, ?)$
Points on z axis, different depths:
$P(0,0,0,1)^T = (0,0,-?,0)^T \Rightarrow \infty(-Z)$ (Eye goes to -infinity on z axis)
$P(0,0,f,1)^T = (0,0,0,0)^T \Rightarrow (0,0,0)$ (Front CP goes to 0)
$P(0,0,b,1)^T = (0,0,b,b)^T \Rightarrow (0,0,1)$ (Back CP goes to 1)
$P(0,0,1,0)^T = (0,0,b/(b-f),1)^T \Rightarrow (0,0,b/(b-f))$ (Infinity to vanishing point)

Perspective Transformation's effect on depth values

Eye → -infinity on z axis
Eye:front → -infinity:0
front:back → 0:1
back:infinity → 1:some z
-infinity:eye → some z : infinity



Perspective transformation's effect on ranges of z values.

Homogeneous coordinate facts

(x, y, z, w) when $w \neq 1$ is a "finite" point

Homogeneous division gives the associated 3D point: $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

$(x, y, z, 0)$ has multiple related interpretations:

vector (x, y, z)

point at ∞ in direction (x, y, z)

intersection of parallel lines in direction (x, y, z)

vanishing point in direction (x, y, z)

These facts can be seen by trying:

Translate (x, y, z, w) for both $w=0$ and $w \neq 0$

Consider $\lim_{w \rightarrow 0} (x, y, z, w)$

Some notes about Projection coordinate systems

In truth, this C.S. is most useful **before** the homogeneous division.

That is given a point (x,y,z,w) we have stated the bounds of Projection space as

$$-1 \leq \frac{x}{w} \leq +1$$

$$-1 \leq \frac{y}{w} \leq +1$$

$$0 \leq \frac{z}{w} \leq +1$$

but consider that (-x,-y,-z,-w) also satisfies these bounds.

The division by w loses (sign) info! We can't allow that!

This works:

$$-w \leq x \leq +w$$
$$-w \leq y \leq +w$$
$$0 \leq z \leq +w$$

Eye position and plane

P is perspective matrix, M is any viewing matrix

M : maps world → eye coordinate systems

P : maps eye → projection coordinate systems

so the inverses map as follows

M^{-1} : maps eye → world coordinate systems

P^{-1} : maps projection → eye coordinate systems

Eye Position:

Eye position in eye C.S. is the origin $(0,0,0,1)^T$

so $M^{-1}(0,0,0,1)^T$ is the eye position in world C.S.,

and the eye position in projection C.S. is $P(0,0,0,1)^T = (0,0,-1,0)^T$,

which is infinity along the -z axis.

Eye Plane:

Eye plane in eye CS is $z=0$. In plane equation form this is

$$[0 \ \ 0 \ \ 1 \ \ 0][x \ y \ z \ 1]^T \ = \ 0$$

A slight manipulation with the identity matrix (in the form $M M^{-1}$) gives

$$[0 \ \ 0 \ \ 1 \ \ 0]M \ \ M^{-1}[x \ y \ z \ 1]^T \ = \ 0$$

which can be interpreted as a plane equation where

$$M^{-1}[x \ y \ z \ 1]^T$$

is a point in world coordinates, and so

$$[0 \ \ 0 \ \ 1 \ \ 0]M$$

must be a plane **IN WORLD COORDS**

Stereo projections

Require a shear before projection:

x_e is the eye's offset in the x direction

x_s is the screen center's offset in the x direction

$$\begin{bmatrix} 1 & 0 & \frac{x_e - x_s}{d} & -x_e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ways to present stereo images:

Side-by-side
- cross-eyed
- wall-eyed aka parallel-eyed
- end-on-mirror
- double mirror system
- head mounted display
- stereo viewer aka transparency viewer

3D glasses
- prismatic & self-masking crossview glasses
- red-blue glasses (actually red-cyan) aka anaglyph
- linearly polarized glasses and a silvered screen
- circularly polarized glasses
- flicker with synchronized glasses aka alternate frame sequencing
- interference filter (infitec) glasses
- ColorCode 3D

other
- lenticular sheet
- parallax barrier
- varifocal lens