1. **Problem** Prove that conditional independence of A and B given C implies the following product rule: $P(AB \mid C) = P(A \mid C)\, P(B \mid C)$ that is show

$$P(A \mid BC) = P(A \mid C) \Rightarrow P(AB \mid C) = P(A \mid C)\, P(B \mid C)$$

2. **Problem** Given binary random variables A,B,C,D and
   A and B are conditionally independent given C and
   A and D are independent (unconditionally), and
   B and D are independent (unconditionally), and

$$\begin{aligned} P(A \mid C) &= 1/4 \\ P(B \mid C) &= 1/5 \\ P(DC) &= 1/6 \\ P(A \mid \neg C) &= 3/4 \\ P(B \mid \neg C) &= 2/5 \\ P(D \neg C) &= 1/3 \end{aligned}$$

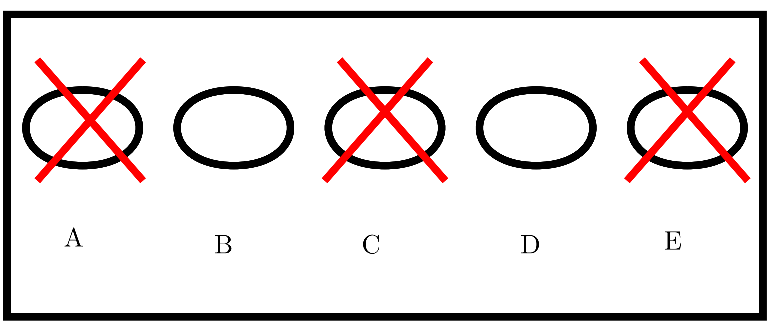
   calculate

$$P(\neg ABD)$$

3. **Problem (EM algorithm, file "hmm14.pdf" pp. 14–17)** You have a black box that produces random characters in the range $A, \ldots, E$ (including A and E). From the manual you know the distribution, but only up to a parameter:

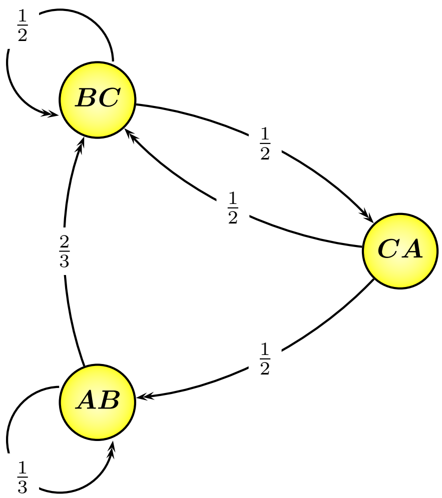
$$\begin{aligned} P(A) &= \mu \\ P(B) &= \frac{1}{2} - \mu \\ P(C) &= 2\mu \\ P(D) &= \mu \\ P(E) &= \frac{1}{2} - 3\mu \end{aligned}$$

   Also the machine lights are broken, so you can only see B and D outputs. After performing 40 experiments: you have seen B 10 times, and D 10 times.



   Estimate μ using Maximum Likelihood.

4. **Problem** Given the following finite state machine:



where observations in states are
state AB observations A and B with probabilities $\frac{1}{2}$,
state BC observations B and C with probabilities $\frac{1}{2}$,
state CA observations A and C with probabilities $\frac{1}{2}$.

and initial probabilities are

$$P(x_0) = \begin{cases} \frac{1}{2}, & x_0 = AB, \\ \frac{1}{2}, & x_0 = AC. \end{cases}$$

Part 1: use Viterbi algorithm to calculate the Most Probable Path that produced a sequence of observations $ABCA$.

Part 2: calculate the distribution of x_3 (4^{th} step), given the sequence of observations $ABCA$.

5. **Problem (random walk)**

Transition model

$$x_{t+1} = \begin{cases} x_t - 1, & \frac{1}{5}, \\ x_t, & \frac{3}{5}, \\ x_t + 1, & \frac{1}{5}. \end{cases}$$

Sensor model:

$$e_{t+1} = \begin{cases} x_t - 1, & \frac{1}{6}, \\ x_t, & \frac{2}{3}, \\ x_t + 1, & \frac{1}{6}. \end{cases}$$

Initial position:

$$x_0 = \begin{cases} -1, & \frac{1}{3}, \\ 0, & \frac{1}{3}, \\ +1, & \frac{1}{3}. \end{cases}$$

calculate the distribution of x_3 (4^{th} step), given the sequence of observations $-1, 2, 2, 1$.

6. **Problem** Read the problem, build Bayesian network (by hand), draw it (specify CPTs) and calculate the following probability

$$P(Fraud|Travelling\ AND\ HasComputer)$$

Show work – just a number will give you maximum 2 points (given it's the correct answer). On the other hand it's OK to leave your answer unsimplified (that is there are only numbers and arithmetic operations).

Problem statement: this is a very simplified fraud detection problem. We'll be dealing with 5 facts:

- Travelling (T) – whether the cardholder is travelling
- HasComputer (H) – whether the cardholder owns a computer
- Abroad (A) – whether the purchase is made abroad
- InternetPurchase (I) – whether it is an internet transaction
- Fraud (F) – whether the transaction is fraudulent

The following should be used to build the network (by hand):

- average cardholder is travelling 10% of the time
- 60% of the population own computers
- the chance of an abroad purchase is 10% while travelling and only 2% otherwise
- those who own a computer tend to make 50% of theirs purchases on internet, those who don't – only 5%.
- the chance of a fraudulent purchase is
  - inside US purchase and card was swiped – 1/100
  - inside US purchase and over internet – 1/50
  - abroad purchase and card was swiped – 1/40
  - abroad purchase and over internet – 1/20

7. **Problem** Consider a finite state machine from problem 4. Tool that is used to measure the current state is as follows: with probability 7/8 it will work as in problem 4, but with probability 1/8 it freezes and displays the measurement from the immediately previuos state.

Convert the problem so that one may use HMM solution. Specify all states of the new state machine and transition probabilities. If transition matrix is too big show only some values and explain how to calculate them. The description should be complete enough for someone to fill in the whole matrix.