# Important Formulae in Physics, Chemistry and Math for JEE

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## 1 Physics

1. Energy of a Wave(for a length $dx$ at $x$):

$$dU = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 dx \tag{1.1}$$

$$= dK = \frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 dx = \frac{1}{2}\mu v_y^2 dx \tag{1.2}$$

2. Energy for a Wavelength worth of length:

$$\int_0^\lambda 2\cdot\frac{1}{2}\mu\omega^2\|\psi\|^2 dx \tag{1.3}$$

$$\equiv \int_0^\lambda 2\cdot\frac{1}{2}\mu\omega^2\|A(\sin(kx-\omega t))\|^2 dx\Big|_{t=0} \tag{1.4}$$

$$\mu\omega^2A^2\int_0^\lambda(\sin^2(kx))dx = \boxed{\mu\omega^2A^2\frac{\lambda}{2}} \tag{1.5}$$

3. Poisson's ratio:

$$\nu = -\frac{d\epsilon_{\text{trans}}}{d\epsilon_{\text{axial}}} \tag{1.6}$$

4. Young Laplace Equation: For any general surface there exist two radii of curvature $R_1$ and $R_2$. The Young Laplace equation relates the pressure difference across the surface solely due to surface tension:

$$\Delta p = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \tag{1.7}$$

5. Some Fundamental Values:

   (a) $hc = 1240\ eV\ nm$

(b) $2.303\frac{RT}{F} = 0.06$

6. Dipoles:

   (a) Potential due to electric dipole:

$$\Psi(r) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} \tag{1.8}$$

   (b) Field due to electric dipole:

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos(\theta)}{r^3} \tag{1.9}$$

$$E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin(\theta)}{r^3} \tag{1.10}$$

   (c) Field due to magnetic dipole:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{r}(\mathbf{m}\cdot\mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3}\right) \tag{1.11}$$

   (d) Field due to magnetic dipole of radius $a$ on its axis:

$$B(z) = \frac{\mu_0 I a^2}{2(a^2+z^2)^{\frac{3}{2}}} \tag{1.12}$$

7. Moseley's Law:

$$\sqrt{\nu} = a(Z-b) \tag{1.13}$$

8. For a radially symmetric medium, the modification $r\mu\sin(\theta)$ is used instead of $\mu\sin(\theta)$. (It just works, idk why)

9. Relation between $Q$ and $E_{th}$:

$$E_{th} = \left(1 + \frac{m}{M}\right)Q \tag{1.14}$$

   where $m$ is the mass of incoming particle while $M$ is the mass of the particle at rest. (*Can be generalised by using $\frac{M+m}{M}$ and then interpreting some kind of total mass)

10. When moving up the plane, the slope of surface of the liquid in container is:

$$\tan(\theta) = \frac{a + g\sin(\alpha)}{g\cos(\alpha)} \tag{1.15}$$

11. Magnetic field due to a moving charge $q$ :

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} \tag{1.16}$$

12. In a perfectly elastic collision, the two particles exchange their vector momenta.

13. Displacement current:
$$i_d = \epsilon_0 \frac{d\Phi_E}{dt} \tag{1.17}$$

14. In free expansion of any gas(including non-ideal ones), $\Delta U$ is conserved. Thus only for an ideal gas is $\Delta T$ also equal to 0.

15. For diffraction, minima at:
$$\theta = \frac{n\lambda}{a}$$
where $n \neq 0$, $\lambda$ is the wavelength and $a$ is the width of the slit

16. Angle of deflection for scattering:
$$\phi = \tan^{-1}\left(\frac{|k|}{bv^2}\right) \tag{1.18}$$
where $b$ is the distance of closest approach, $v$ is the initial/final velocity(at $r \to \infty$), and the potential(potential per mass if you want to be picky) is of the form $V(r) = \frac{k}{r}$

17. Impedance in an AC circuit is calculated as in a DC circuit (i.e
$$Z_1 + Z_2$$
for series and
$$Z_1 \oplus Z_2 = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1}$$
for parallel), using the fact that impedance for inductor is
$$Z_L = i\omega L$$
and for capacitor is
$$Z_C = \frac{1}{i\omega C} = -i\frac{1}{\omega C}$$

18. For an ideal gas, the Kinetic Energy per unit volume is:
$$T_V = \frac{\rho C_v T}{M}$$
, and thus when applying Bernoulli's principle on the motion of an ideal gas use this in the $T_i$ part inside the container

19. Facts about a standing wave:
    - Near the anti-node potential energy of the particles, when they reach their extreme position, is minimum

- Any two particles vibrates either in same phase or out of phase.
- Near the anti-node kinetic energy of the particles, when they crosses their mean position, is maximum.
- Any two particles having minimum separation $\frac{\lambda}{2}$ may vibrate with same amplitude.

20. For calculating flux, consider a SPHERE OR A CUBE as a bounding Gaussian surface.

21. For calculating acceleration when the magnitude of the velocity is constant, use:
$$a = v\frac{d\theta}{dt} \tag{1.19}$$

22. Force between two plates of the capacitor:
$$F = \frac{q^2}{2A\epsilon_0} = \frac{\sigma^2 A}{2\epsilon_0} \tag{1.20}$$

23. Power of a lens is $\frac{1}{f_{lens}}$ where $f_{lens} > 0$ for convex lens and $f_{lens} < 0$ for concave lens.

24. Power of a **mirror** is $-\frac{1}{f_{mirror}}$ where $f_{mirror} > 0$ for **convex mirror** and $f_{mirror} < 0$ for **concave mirror**.

25. If in confusion, try to find the wavelength of the sound signal first(which is independent of the velocity of the observer) and then reason out that if you even have to take the observer velocity in account when using the Doppler finally in the end.

26. Height of water level in a capillary:
$$h = \frac{2T}{R_m \rho g} \tag{1.21}$$
$$= \frac{2T\cos(\theta)}{r\rho g} \tag{1.22}$$
where $R_m$ is the radius of the meniscus, $r$ is the radius of the tube and 1.22 occurs when the tube is of sufficient length.
**Corollary:** When tube is of the sufficient length, $R_m = \dfrac{r}{\cos(\theta)}$

27. If the piston moves without acceleration, the pressure across both sides is constant and thus the process is isobaric

28. If in a YDSE question, the medium between the slits and the screen also has $\mu_m > 1$, and the glass inserted is of index $\mu$, use
$$\left(\frac{\mu}{\mu_m} - 1\right)t$$
to calculate the path difference

29. If a question asks about moving a charge/mass away from the centre of a symmetric distribution, they are asking you to move it along an axis that preserves the symmetry.

30. Intensity of an electromagnetic wave:

$$I = \text{(Energy Density)(Speed of the radiation)} = \left(\frac{1}{2}\epsilon_0 E^2\right)(c) \tag{1.23}$$

31. Range of communication:

$$D = \sqrt{2Rh_t} + \sqrt{2Rh_r} \tag{1.24}$$

where $h_t$ is the height of the transmitter and $h_r$ the height of the receiver.

32. For a body of liquid with height $h$, it is very useful to take the average pressure as $(\rho g h/2)$ and multiply it over the surface area in contact to get the force.

33. Centre of masses and Moment of inertia of different objects:

    (a) Ring: C=(0,0),I $=MR^2$

    (b)

34. Current due to a stream of electrons:

$$i = neAv_d \tag{1.25}$$

# 2 Chemistry

1. Entropy Formulae

$$\Delta S_{sys} = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right) \tag{2.1}$$

$$\Delta S_{sys} = nC_p \ln\left(\frac{T_2}{T_1}\right) - nR\ln\left(\frac{p_2}{p_1}\right) \tag{2.2}$$

2. Gibbs Free Energy

$$dG = Vdp - SdT \tag{2.3}$$

3. Applications of definition of enthalpy:

$$\Delta H = \Delta U + \Delta(PV) = \Delta U + \Delta n_g RT \tag{2.4}$$

(Even applies to phase change reactions)

4. Degree of Unsaturation:

$$DU = (n_C - 1) - \left(\frac{n_H + n_X - n_N}{2}\right) \tag{2.5}$$

5. Interplanar crystal spacing of cubic crystal families ($h\ k\ l$) is defined as

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}. \tag{2.6}$$

6. Height of the HCP unit cell:

$$h_{hcp} = 4\sqrt{\frac{2}{3}}r$$

7. Base area of the HCP unit cell:

$$A_{hcp} = 6\sqrt{3}r^2$$

8. Electrode Reactions:

   (a) Electrode reaction of reduction of Oxygen:

   $$O_{2(g)}\ + 4H^+\ + 4e^- \rightarrow 2H_2O \tag{2.7}$$

   (b) Calomel electrode:

   $$\mathrm{Cl^-(4\,M)|Hg2Cl2(s)|Hg(l)|Pt} \tag{2.8}$$

   $$\text{Anode: } \mathrm{Hg_2}^{2+} + 2\,\mathrm{e^-} \rightleftharpoons 2\,\mathrm{Hg(l)}, \qquad \text{with} \quad E^0_{\mathrm{Hg_2}^{2+}/\mathrm{Hg}} = +0.80\ \mathrm{V} \tag{2.9}$$

   $$-\ \text{Cathode: } \mathrm{Hg_2Cl_2} + 2\,\mathrm{e^-} \rightleftharpoons 2\,\mathrm{Hg(l)} + 2\,\mathrm{Cl^-}, \qquad \text{with} \quad E^0_{\mathrm{Hg_2Cl_2/Hg,\,Cl^-}} = +0.27\ \mathrm{V} \tag{2.10}$$

9. Formulae for conductivity:

   (a)

   $$R = \rho\frac{l}{A} \tag{2.11}$$

   $$G = \frac{1}{R} = \frac{1}{\rho}\frac{A}{l} = \kappa\frac{A}{l} \tag{2.12}$$

   $$G* = \frac{l}{A} = R \times \kappa \tag{2.13}$$

   $$\Lambda_m = \frac{\kappa}{C}(\text{Unit: S cm}^2\text{ mol}^{-1}) \tag{2.14}$$

   $$\text{For unit l and A: } G = \frac{\kappa A}{l} = \kappa \tag{2.15}$$

   $$\Rightarrow \Lambda_m = \kappa \tag{2.16}$$

   $$\text{For molar volume: } \Lambda_m = \kappa \times V \tag{2.17}$$

   G* is calculated using an electrolyte with known conductivity

10. Anode is the terminal where oxidation occurs and where the *conventional current enters the circuit*; **in a battery or a source of DC, it is the *negative terminal*** *but* **in a passive load(e.g. an electrolytic cell), it is the *positive terminal.***

11. The meaning of Van Der Waal's constants:

    - $a$: It is the proportionality constant, and the $\left(\frac{n}{V}\right)^2$ comes from the fact that pressure depends on both force of repulsion and the frequency of collisions both of which are reduced by factors of concentration $(= \frac{n}{V})$
    - $b$

$$b = 4N_A V_{molecule}$$

| System | Symmetries |
|---|---|
| Triclinic | None |
| Monoclinic | One $C_2$ axis |
| Orthorhombic | $3 \perp C_2$ axes |
| Rhombohedral | One $C_3$ axis |
| Tetragonal | One $C_4$ axis |
| Hexagonal | One $C_6$ axis |
| Cubic | Four $C_3$ axes in tetrahedral arrangement |

12.

13. Closest distance between octahedral and tetrahedral void in fcc lattice:

$$d_{oct\ to\ tet} = \sqrt{\frac{3}{2}}r = \frac{\sqrt{3}}{4}a$$

14. The configuration for $[Cu(NH_3)_4]^{2+}$ is (showing copper electrons) $3d^8 4p_z^1$ while the electrons from $(NH_3)$(or any other SFL), go into the $d_{x^2-y^2}, s, p_x$ and $p_y$ orbitals

15. pH of a solution of salt of weak acid+strong base(all logs in base 10):

$$pH = \frac{pK_w + pK_a + \log(C)}{2} = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log(C) \tag{2.18}$$

16. pH of a solution of salt of weak base+strong acid(all logs in base 10):

$$pH = \frac{pK_w - pK_b - \log(C)}{2} = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log(C) \tag{2.19}$$

17. pH of a solution of salt of weak base+weak acid(all logs in base 10):

$$pH = \frac{pK_w + pK_a - pK_b}{2} = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b \tag{2.20}$$

18. $\beta$-D glucopyranose is more stable than $\alpha$-D glucopyranose.

19. The chemical potential of the substance is equal to its Gibbs free Energy:

$$\mu^\circ(A) = \Delta G^\circ_A \tag{2.21}$$

20. For a liquid-gas equilibrium:

$$\mu^\circ_A(g) = \mu^\circ_A(l) + RT\ln(\chi_A) \tag{2.22}$$

$\chi_A$ is the mol fraction of A in the mixture (liquid phase)

21. For a solid-liquid equilibrium:

$$\mu^\circ_A(l) = \mu^\circ_A(s) + RT\ln(\chi_A) \tag{2.23}$$

where $\chi_A$ is same as above

22. To find the ebullioscopic constant rewrite 2.22 as:

$$\ln(\chi_A) = \frac{\mu^\circ_A(g) - \mu^\circ_A(l)}{RT} \tag{2.24}$$

$$= \frac{\Delta_{vap}G^\circ}{RT} \tag{2.25}$$

Differentiate 2.24by temperature to get

$$\frac{d\ln(\chi_A)}{dT} = \frac{\partial \frac{\Delta G}{RT}}{\partial T} = -\frac{\Delta H}{RT^2} \tag{2.26}$$

$$\Rightarrow d\ln(\chi_A) = -\frac{\Delta H}{RT^2}dT \tag{2.27}$$

because

$$\frac{\Delta G}{T} = \frac{\Delta H}{T} - \Delta S.$$

Integrate this back using the limits $T_b$ to $T_b + \Delta T$, to get:

$$\ln(\chi_A) = \frac{\Delta H}{R}\left(\frac{1}{T_b} - \frac{1}{T_b + \Delta T}\right) \approx \frac{\Delta H}{RT^2}\Delta T \tag{2.28}$$

Now, in a binary solution $\chi_A = 1 - \chi_B$. Assuming $\chi_B << 1(\because$ Dilute condition)

$$\ln(1 - \chi_B) = -\chi_B = -\frac{\Delta H}{RT^2}\Delta T \tag{2.29}$$

which means:

$$\boxed{K_b = 1/\frac{\Delta H}{RT^2} = \frac{RT^2}{\Delta H}} \tag{2.30}$$

23. Non-metals generally melt at a higher temperature than metals of the same group.

24. $[Co(NH_3)_4Cl_2]Cl$ is a diamagnetic compound.

25. Efficiency of an electrolytic cell:

$$\eta = \frac{\Delta^\circ G}{\Delta^\circ H} = 1 - T\frac{\Delta^\circ S}{\Delta^\circ H} \tag{2.31}$$

26. In the 3d series, when the lower elements (Sc, Ti, V) get oxidised, they almost always lose all of their $4s$ and $3d$ electrons and acquire the $[Ar]$ noble gas configuration. Thus their oxides (like $VO_2^{+}$)are less oxidising than, say $Cr_2O_7^{2-}$ or $MnO_4^{-}$

27. Di-carboxylic acids($HOOC-(CH_2)_n-COOH$) on heating give different products:

    (a) For $n=0$(Oxalic acid): Forms $HCOOH$ and $CO_2$
    (b) For $n=1$(Malonic acid): Forms $CH_3-COOH$ and $CO_2$
    (c) For $n=2,3$(Succinic, Glutaric acid): Forms the respective 5-membered and 6-membered ring anhydride
    (d) For $n \geq 4$(Adipic onwards): They start forming cyclic ketones

28. The Gold Number is the minimum weight (in milligrams) of a protective colloid required to prevent the coagulation of 10 ml of a standard hydro gold sol when 1 ml of a 10% sodium chloride solution is added to it

29. $H_2O_2$ oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium.

30. In determining reactivity with $H_2$/Pt or any other reagent with surface playing a dominant role, the least hindered compound will most easily adsorb and thus will react faster with such reagents

31. $H_3PO_4$ is used for elimination of alcohols.

32. Gases(ideal and real):

    (a) Boyle's Temperature:
    $$T_{bo} = \frac{a}{Rb} \tag{2.32}$$
    (b) Critical Temperature:
    $$T_c = \frac{8a}{27Rb} \tag{2.33}$$
    (c) Critical Pressure:
    $$P_c = \frac{a}{27b^2} \tag{2.34}$$
    (d) Critical Volume:
    $$V_c = 3b \tag{2.35}$$

33. $P_{\text{final}} = \sqrt{P_x^o \times P_y^o}$ for a very special case of minimum composition of one of the substances to have minimum mole fraction in vapour phase

34. As branching increases among isomeric alkanes, stability increases and hence heat of combustion decreases

35. If molality and molarity of any species in a solution is numerically the same, it is same for all the components and for it,numerical value of ml of solution = numerical value of gm of solvent

36. Rate of diffusion of a gas from an orifice:

$$r_{\text{eff}} = \frac{PA}{\sqrt{2\pi M_{gas} RT}} N_A \tag{2.36}$$

where $P$ is the pressure of the container, $A$ is the area of the orifice, $M_{gas}$ is the molar mass of the gas.

# 3 Math

1. Triangle Inequalities:

$$|z_1 + z_2| \leq |z_1| + |z_2| \tag{3.1}$$

$$|z_1 - z_2| \geq ||z_1| - |z_2|| \tag{3.2}$$

2. BAC-CAB rule:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) \tag{3.3}$$

3. Wallis' Formula:

$$\int_0^{\frac{\pi}{2}} \sin^n(x) \cdot \cos^m(x) dx = \frac{[(n-1)(n-3)(n-5)\ldots 2 \ or \ 1][(m-1)(m-3)(m-5)\ldots 2 \ or \ 1]}{(m+n-0)(m+n-2)(m+n-4)\ldots 2 \ or \ 1} K \tag{3.4}$$

$$\text{where } K = \begin{cases} \frac{\pi}{2}, & \text{if } m \text{ and } n \text{ are even} \\ 1, & \text{otherwise} \end{cases} \tag{3.5}$$

4. Box product tricks:

(a)

$$\left[(\vec{a} \times \vec{b}) \ (\vec{b} \times \vec{c}) \ (\vec{c} \times \vec{a})\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2 \tag{3.6}$$

(b)

$$\left[\vec{a}\vec{b}\vec{c}\right]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix} \tag{3.7}$$

(c)

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{c} \cdot \vec{a})(\vec{d} \cdot \vec{b}) - (\vec{c} \cdot \vec{b})(\vec{d} \cdot \vec{a}) \tag{3.8}$$

5. Complex Numbers:

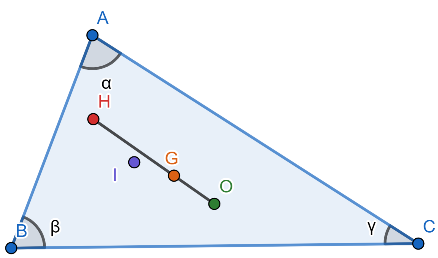
   (a) Expansion of $|1-\alpha_r|$ and $|1-\alpha_r|$ where $\alpha_r$ is the r$^{\text{th}}$ root of unity:

   $$|1-\alpha_r| = \left|1-\cos\left(\frac{2\pi r}{n}\right) - i\sin\left(\frac{2\pi r}{n}\right)\right| = 2\left|\sin\left(\frac{\pi r}{n}\right)\right| \tag{3.9}$$

   *Use half angle formula and then that $|cis(x)|$ has mod 1

6. Triangle Centers:

   (a) The centroid $G$ lies on the line joining Orthocentre $H$ and Circumcentre $O$:




   (b) Orthocentre $H$:

   $$H = \left(\vec{a}+\vec{b}+\vec{c}\right) \tag{3.10}$$

7. Taylor series of $\tan(x)$:

   $$\tan(x) \approx x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots \tag{3.11}$$

8. $$\text{The coefficient of } x^r \text{ in } (1-x)^{-n} \text{ is } \binom{n+r-1}{r} \tag{3.12}$$

9. But for choosing, you use

   $$\binom{n+r-1}{r-1}$$

10. Lagrange Interpolation: For a set of $k+1$ data points $(x_0, y_0)\dots(x_k, y_k)$ where no two $x_i, x_j$ are the same, there exists a *Interpolation Polynomial of Lagrange Form*:

    $$L(x) := \sum_{j=0}^{k} y_j \ell_j(x) \tag{3.13}$$

    where each $\ell_i$ is the Lagrange basis:

    $$\ell_j(x) := \prod_{\substack{0\le m\le k\\ m\ne j}} \frac{x-x_m}{x_j-x_m} = \frac{(x-x_0)}{(x_j-x_0)}\cdots\frac{(x-x_{j-1})}{(x_j-x_{j-1})}\frac{(x-x_{j+1})}{(x_j-x_{j+1})}\cdots\frac{(x-x_k)}{(x_j-x_k)} \tag{3.14}$$

11. • Blank plane is one region
    • Every line adds a region to the plane
    • Every point of intersection adds one more region
12. If $A$ is a skew-symmetric matrix of odd dimension, its determinant is always 0
13. If stuck in a problem where Lagrange Mean Value theorem looks likely, take the function $g(x) = f(x) - x$ and then try the properties on it.
14. For a system of $3 \times 3$ linear equations:
    - Unique solution if $\Delta \neq 0$
    - No solution if $\Delta = 0$ but at least one of $\Delta_1$, $\Delta_2$ and $\Delta_3$ is non zero
    - Infinite solutions if $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$
15. Sophie-Germain's Identity:
$$a^4 + 4b^4 = (a^2 + 2b^2 - 2ab)(a^2 + 2b^2 + 2ab) \tag{3.15}$$
16. $BB^T$ is a symmetric matrix
17. if $P(z_1)$ and $Q(z_2)$ are two points on the circle $|z| = r$, the complex number representing the intersection of tangents is
$$\frac{2z_1z_2}{z_1 + z_2}$$
18. To count the number of rectangles in a regular polygon, find the number of diagonals passing through the centre
19. If there is a regular n-gon on a unit circle the product of lengths
$$(A_1A_2)(A_1A_3)(A_1A_4)\dots(A_1A_n) = n$$
20. In the previous point, remember that if $n$ is even then there exists a body diagonal too.
21. For an ellipse at the origin, a tangent with slope $m$ is:
$$y = mx \pm \sqrt{a^2m^2 + b^2} \tag{3.16}$$
    And for a hyperbola it is:
$$y = mx \pm \sqrt{a^2m^2 - b^2} \tag{3.17}$$
22. REMEMBER: If asked for non consecutive in combinatorics question, think "gaps"

23. Think rotation of planes, think of angles between the normals of the plane

24. Anytime you encounter an integral which involves something like $\dfrac{x^2-1}{x^2+1}$ try putting in $t = x + \frac{1}{x}$. for eg.:

$$\begin{aligned} I &= \int_1^{\sqrt{2}+1} \frac{x^2-1}{x^2+1} \frac{1}{\sqrt{1+x^4}} dx \\ &= \int_1^{\sqrt{2}+1} \frac{(1-\frac{1}{x^2})dx}{(x+\frac{1}{x})\sqrt{x^2+\frac{1}{x^2}}} \end{aligned}$$

25. Stewart's theorem is an overpowered theorem for Solutions of Triangles, and can be easily used to derive much stronger results than the hard to remember m-n rule. The theorem:

$$\text{If in a } \Delta\text{ABC, a point } D \text{ divides } BC \text{ in } m \text{ and } n,\ then \tag{3.18}$$

$$\boxed{b^2m + c^2n = a(l^2 + mn)} \tag{3.19}$$

26. The foot of the perpendicular from any focus of the ellipse to any tangent to the ellipse must lie on the auxiliary circle of the ellipse

27. The foot of perpendicular from the focus on any tangent to the parabola must lie on the tangent at the vertex to the parabola which in a way can be thought as the auxiliary circle of the parabola

28. If there is a differential equation like

$$\frac{dy}{dx} + P(x)y = 0, \tag{3.20}$$

it is linear i.e. for solutions $y_1$ and $y_2$, $\lambda y_1 + \mu y_2$ is also a solution for $\lambda, \mu \in \mathbb{R}$. However if there is a non homogeneous part and the equation now becomes

$$\frac{dy}{dx} + P(x)y = Q(x), \tag{3.21}$$

and $y_1$ and $y_2$ are again some solutions, $\lambda y_1 + \mu y_2$ is not a solution for general $\lambda$ and $\mu$(Check by putting $y_1$ and $y_2$ and into the Diff Eq 3.21 then checking $\lambda y_1 + \mu y_2$ doesn't satisfy 3.21 unless $\lambda + \mu = 1$)

29. If some integral with trig functions isn't working, try substitution by $\tan(\frac{x}{2})$

30. If there is a problem asking for min/max anywhere, and things look hopeless, try Cauchy-Schwarz inequality:

$$|\langle \mathbf{u}, \mathbf{v} \rangle|^2 \leq \langle \mathbf{u}, \mathbf{u} \rangle . \langle \mathbf{v}, \mathbf{v} \rangle \tag{3.22}$$

where $\langle \cdot, \cdot \rangle$ is the inner product and $\mathbf{u}$ and $\mathbf{v}$ are vectors, which basically means list of numbers of any kind

31. For summation of $\sum_{r=1}^{n} 2^r \tan(2^r\theta)$, remember:

$$\tan(\theta) = \cot(\theta) - 2\cot(2\theta) \tag{3.23}$$
$$\tan(2\theta) = \cot(2\theta) - 4\cot(4\theta) \tag{3.24}$$
$$\vdots \tag{3.25}$$
$$\tan(2^n\theta) = \cot(2^n\theta) - 2^{n+1}\cot(2^{n+1}\theta) \tag{3.26}$$

32. Adjoint properties:
    (a) $\operatorname{adj}(A) = |A|A^{-1}, \Rightarrow |\operatorname{adj}(A)| = |A|^{n-1}$
    (b) $\operatorname{adj}(\operatorname{adj}(A)) = |A|^{n-2}A$

33. Number of surjective/onto functions $f : A \to B$ with $|A| = m$ and $|B| = n (m > n)$ is:

$$n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \binom{n}{3}(n-3)^m + \cdots + (-1)^r\binom{n}{r} + \ldots \tag{3.27}$$

    using the principle of inclusion and exclusion

34. Equation of tangent to parabola:

$$ty = x + at^2 \tag{3.28}$$

    or

$$y = mx + \frac{a}{m} \tag{3.29}$$

35. The product of the perpendiculars on the tangent from the foci is equal to $b^2$.

36. The reflection of point A on the angle bisectors BE and CF lies on the side BC

37. SOT Formulas:
    (a) Length of the median AD of a triangle:

$$l_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2} \tag{3.30}$$

    (b) Length of the internal angle bisector of triangle from A:

$$d_a^2 = \frac{bc}{(b+c)^2}\left((b+c)^2 - a^2\right) \tag{3.31}$$

38. For a chord of a conic section S=0, with its mid point at (h,k), the formula is:

$$T = S_1 \tag{3.32}$$

39. $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

40.
$$\neg(\forall x)P(x) \Leftrightarrow (\exists x)(\neg P(x)); \tag{3.33}$$
$$\neg(\exists x)P(x) \Leftrightarrow (\forall x)(\neg P(x)). \tag{3.34}$$

41. For a system of quadratic equations which has one common root $\alpha$

$$a_1x^2 + b_1x + c_1 = 0 \tag{3.35}$$
$$a_2x^2 + b_2x + c_2 = 0 \tag{3.36}$$

We can put in $\alpha^2$ and $\alpha$ respectively to form

$$a_1\alpha^2 + b_1\alpha + c_1 = 0 \tag{3.37}$$
$$a_2\alpha^2 + b_2\alpha + c_2 = 0 \tag{3.38}$$

and then using the cross product formula, we write:

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \tag{3.39}$$

and thus equate the values of $\alpha$ from the equations to get:

$$\alpha = \frac{(b_1c_2 - b_2c_1)}{(c_1a_2 - c_2a_1)} = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)} \tag{3.40}$$

$$\Rightarrow (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2 \tag{3.41}$$

$$\Rightarrow \boxed{\Delta_{bc}\Delta_{ab} = \Delta_{ca}^2} \tag{3.42}$$

42. If thinking about shortest distance where lines are acting as a constraint, then think about the mirror images of the points about those lines