

Important Formulae in Physics, Chemistry and Math for JEE

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1 Physics

1. Energy of a Wave(for a length dx at x):

$$dU = \frac{1}{2}T \left(\frac{\partial y}{\partial x} \right)^2 dx \quad (1.1)$$

$$= dK = \frac{1}{2}\mu v^2 \left(\frac{\partial y}{\partial x} \right)^2 dx = \frac{1}{2}\mu v_y^2 dx \quad (1.2)$$

2. Energy for a Wavelength worth of length:

$$\int_0^\lambda 2 \cdot \frac{1}{2} \mu \omega^2 \|\psi\|^2 dx \quad (1.3)$$

$$\equiv \int_0^\lambda 2 \cdot \frac{1}{2} \mu \omega^2 \|A(\sin(kx - \omega t))\|^2 dx \Big|_{t=0} \quad (1.4)$$

$$\mu \omega^2 A^2 \int_0^\lambda (\sin^2(kx)) dx = \boxed{\mu \omega^2 A^2 \frac{\lambda}{2}} \quad (1.5)$$

3. Poisson's ratio:

$$\nu = -\frac{d\epsilon_{\text{trans}}}{d\epsilon_{\text{axial}}} \quad (1.6)$$

4. Young Laplace Equation: For any general surface there exist two radii of curvature R_1 and R_2 . The Young Laplace equation relates the pressure difference across the surface solely due to surface tension:

$$\Delta p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (1.7)$$

- The excess pressure due to a spherical bubble is $\Delta p = \frac{2\sigma}{r}$
- The excess pressure due to a cylindrical bubble is $\Delta p = \frac{\sigma}{r}$

5. Some Fundamental Values:

(a) $hc = 1240 \text{ eV nm}$

(b) $2.303 \frac{RT}{F} = 0.06$

6. Dipoles:

(a) Potential due to electric dipole:

$$\Psi(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^3} \quad (1.8)$$

(b) Field due to electric dipole:

$$E_r = \frac{1}{4\pi\epsilon_0} \frac{2p \cos(\theta)}{r^3} \quad (1.9)$$

$$E_\theta = \frac{1}{4\pi\epsilon_0} \frac{p \sin(\theta)}{r^3} \quad (1.10)$$

(c) Field due to magnetic dipole:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\frac{3\mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3} \right) \quad (1.11)$$

(d) Field due to magnetic dipole of radius a on its axis:

$$B(z) = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{\frac{3}{2}}} \quad (1.12)$$

7. Moseley's Law:

$$\sqrt{\nu} = a(Z - b) \quad (1.13)$$

8. For a radially symmetric medium, the modification $r\mu \sin(\theta)$ is used instead of $\mu \sin(\theta)$. (It just works, idk why)

9. Relation between Q and E_{th} :

$$E_{th} = \left(1 + \frac{m}{M}\right) Q \quad (1.14)$$

where m is the mass of incoming particle while M is the mass of the particle at rest. (*Can be generalised by using $\frac{M+m}{M}$ and then interpreting some kind of total mass)

10. When moving up the plane, the slope of surface of the liquid in container is:

$$\tan(\theta) = \frac{a + g \sin(\alpha)}{g \cos(\alpha)} \quad (1.15)$$

11. Magnetic field due to a moving charge q :

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \vec{r})}{r^3} \quad (1.16)$$

12. In a perfectly elastic collision, the two particles exchange their vector momenta.

13. Displacement current:

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt} \quad (1.17)$$

14. In free expansion of any gas(including non-ideal ones), ΔU is conserved. Thus only for an ideal gas is ΔT also equal to 0.

15. For diffraction, minima at:

$$\theta = \frac{n\lambda}{a}$$

where $n \neq 0$, λ is the wavelength and a is the width of the slit

16. Angle of deflection for scattering:

$$\phi = \tan^{-1} \left(\frac{|k|}{bv^2} \right) \quad (1.18)$$

where b is the distance of closest approach, v is the initial/final velocity(at $r \rightarrow \infty$), and the potential(potential per mass if you want to be picky) is of the form $V(r) = \frac{k}{r}$

17. Impedance in an AC circuit is calculated as in a DC circuit (i.e

$$Z_1 + Z_2$$

for series and

$$Z_1 \oplus Z_2 = \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)^{-1}$$

for parallel), using the fact that impedance for inductor is

$$Z_L = i\omega L$$

and for capacitor is

$$Z_C = \frac{1}{i\omega C} = -i \frac{1}{\omega C}$$

18. For an ideal gas, the Kinetic Energy per unit volume is:

$$\mathcal{T}_V = \frac{\rho C_v T}{M}$$

, and thus when applying Bernoulli's principle on the motion of an ideal gas use this in the \mathcal{T}_i part inside the container

19. Facts about a standing wave:

- Near the anti-node potential energy of the particles, when they reach their extreme position, is minimum
- Any two particles vibrates either in same phase or out of phase.
- Near the anti-node kinetic energy of the particles, when they crosses their mean position, is maximum.
- Any two particles having minimum separation $\frac{\lambda}{2}$ may vibrate with same amplitude.

20. For calculating flux, consider a SPHERE OR A CUBE as a bounding Gaussian surface.

21. For calculating acceleration when the magnitude of the velocity is constant, use:

$$a = v \frac{d\theta}{dt} \quad (1.19)$$

22. Force between two plates of the capacitor:

$$F = \frac{q^2}{2A\epsilon_0} = \frac{\sigma^2 A}{2\epsilon_0} \quad (1.20)$$

23. Power of a lens is $\frac{1}{f_{lens}}$ where $f_{lens} > 0$ for convex lens and $f_{lens} < 0$ for concave lens.

24. Power of a **mirror** is $-\frac{1}{f_{mirror}}$ where $f_{mirror} > 0$ for **convex mirror** and $f_{mirror} < 0$ for **concave mirror**.

25. If in confusion, try to find the wavelength of the sound signal first (which is independent of the velocity of the observer) and then reason out that if you even have to take the observer velocity in account when using the Doppler finally in the end.

26. Height of water level in a capillary:

$$h = \frac{2T}{R_m \rho g} \quad (1.21)$$

$$= \frac{2T \cos(\theta)}{r \rho g} \quad (1.22)$$

where R_m is the radius of the meniscus, r is the radius of the tube and 1.22 occurs when the tube is of sufficient length.

Corollary: When tube is of the sufficient length, $R_m = \frac{r}{\cos(\theta)}$

27. If the piston moves without acceleration, the pressure across both sides is constant and thus the process is isobaric

28. If in a YDSE question, the medium between the slits and the screen also has $\mu_m > 1$, and the glass inserted is of index μ , use

$$\left(\frac{\mu}{\mu_m} - 1 \right) t$$

to calculate the path difference

29. If a question asks about moving a charge/mass away from the centre of a symmetric distribution, they are asking you to move it along an axis that preserves the symmetry.
30. Intensity of an electromagnetic wave:

$$I = (\text{Energy Density})(\text{Speed of the radiation}) = \left(\frac{1}{2} \epsilon_0 E^2 \right) (c) \quad (1.23)$$

31. Range of communication:

$$D = \sqrt{2Rh_t} + \sqrt{2Rh_r} \quad (1.24)$$

where h_t is the height of the transmitter and h_r the height of the receiver.

32. For a body of liquid with height h , it is very useful to take the average pressure as $(\rho gh/2)$ and multiply it over the surface area in contact to get the force.

33. Centre of masses and Moment of inertia of different objects:

(a) Ring: $C=(0,0), I=MR^2$

(b)

34. Current due to a stream of electrons:

$$i = neAv_d \quad (1.25)$$

35. End correction in a resonance tube:

(a) For a tube open at one end:

$$e = 0.6r = 0.3D$$

(b) For a tube open at both ends:

$$e = 1.2r = 0.6D$$

36. End correction in potentiometer:

$$e = \left| \frac{(R_1 \ell_2) - (R_2 \ell_1)}{R_1 - R_2} \right| \quad (1.26)$$

37. Loss in kinetic energy due to collision:

$$\frac{1}{2}\mu v_{\text{rel}}^2 \quad (1.27)$$

38. Potential inside a uniformly charged non-conducting sphere at a distance x from the centre :

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{2R} \left(3 - \frac{x^2}{R^2}\right) \quad (1.28)$$

where Q is the total charge on the sphere, R is the radius of the sphere and $x < R$

39. d due to Fresnel biprism in YDSE:

$$d = 2a(\mu - 1)\alpha \quad (1.29)$$

40. Orbits:

(a) Semi-major axis:

$$a = \frac{r_{\text{min}} + r_{\text{max}}}{2} \quad (1.30)$$

(b) Eccentricity:

$$e = \frac{R - r}{R + r} \quad (1.31)$$

(c) The angular momentum is preserved (Kepler's second law), and thus $r_1 v_1 = r_2 v_2$ for any r_1 and r_2 on the orbit.

41. Bohr quantization condition is a statement about the TOTAL Angular momentum of the atomic system in question. So, for eg., if two electrons are present in the same orbit n , the sum of their angular momentum in the orbits will be $\frac{nh}{2\pi}$

42. Self energy for a hollow sphere is

$$V = \frac{Q^2}{8\pi\epsilon_0 R}$$

43. Self energy for a solid sphere is

$$V = \frac{3Q^2}{20\pi\epsilon_0 R}$$

44. If in a rotating sphere, cylinder, shell, ring etc. is purely rolling ($v_0 = \omega R$), then the velocity vector of any point on its circumference passes through the topmost point P.

45. Kepler's law is valid for any trajectory, even a purely radial one (where a becomes $\frac{R}{2}$ where R is the initial starting point)

2 Chemistry

1. Entropy Formulae

$$\Delta S_{sys} = nC_v \ln \left(\frac{T_2}{T_1} \right) + nR \ln \left(\frac{V_2}{V_1} \right) \quad (2.1)$$

$$\Delta S_{sys} = nC_p \ln \left(\frac{T_2}{T_1} \right) - nR \ln \left(\frac{p_2}{p_1} \right) \quad (2.2)$$

2. Gibbs Free Energy

$$dG = Vdp - SdT \quad (2.3)$$

3. Applications of definition of enthalpy:

$$\Delta H = \Delta U + \Delta(PV) = \Delta U + \Delta n_g RT \quad (2.4)$$

(Even applies to phase change reactions)

4. Degree of Unsaturation:

$$DU = (n_C - 1) - \left(\frac{n_H + n_X - n_N}{2} \right) \quad (2.5)$$

5. Interplanar crystal spacing of cubic crystal families ($h \ k \ l$) is defined as

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}. \quad (2.6)$$

6. Height of the HCP unit cell:

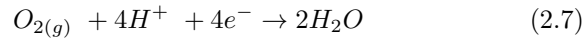
$$h_{hcp} = 4\sqrt{\frac{2}{3}}r$$

7. Base area of the HCP unit cell:

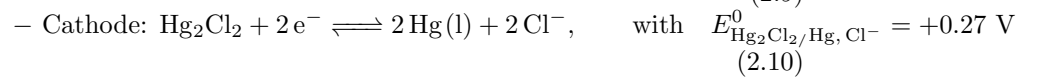
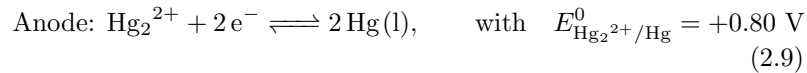
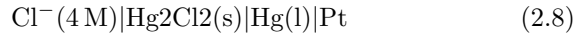
$$A_{hcp} = 6\sqrt{3}r^2$$

8. Electrode Reactions:

(a) Electrode reaction of reduction of Oxygen:



(b) Calomel electrode:



9. Formulae for conductivity:

(a)

$$R = \rho \frac{l}{A} \quad (2.11)$$

$$G = \frac{1}{R} = \frac{1}{\rho} \frac{A}{l} = \kappa \frac{A}{l} \quad (2.12)$$

$$G^* = \frac{l}{A} = R \times \kappa \quad (2.13)$$

$$\Lambda_m = \frac{\kappa}{C} \text{ (Unit: S cm}^2 \text{ mol}^{-1}) \quad (2.14)$$

$$\text{For unit l and A: } G = \frac{\kappa A}{l} = \kappa \quad (2.15)$$

$$\Rightarrow \Lambda_m = \kappa \quad (2.16)$$

$$\text{For molar volume: } \Lambda_m = \kappa \times V \quad (2.17)$$

G^* is calculated using an electrolyte with known conductivity

10. Anode is the terminal where oxidation occurs and where the *conventional current enters the circuit*; **in a battery or a source of DC, it is the *negative terminal* but in a passive load (e.g. an electrolytic cell), it is the *positive terminal*.**

11. The meaning of Van Der Waal's constants:

- a : It is the proportionality constant, and the $\left(\frac{n}{V}\right)^2$ comes from the fact that pressure depends on both force of repulsion and the frequency of collisions both of which are reduced by factors of concentration ($= \frac{n}{V}$)

- b

$$b = 4N_A V_{\text{molecule}}$$

System	Symmetries
Triclinic	None
Monoclinic	One C_2 axis
Orthorhombic	3 $\perp C_2$ axes
Rhombohedral	One C_3 axis
Tetragonal	One C_4 axis
Hexagonal	One C_6 axis
Cubic	Four C_3 axes in tetrahedral arrangement

12.

13. Closest distance between octahedral and tetrahedral void in fcc lattice:

$$d_{\text{oct to tet}} = \sqrt{\frac{3}{2}}r = \frac{\sqrt{3}}{4}a$$

14. The configuration for $[Cu(NH_3)_4]^{2+}$ is (showing copper electrons) $3d^8 4p_z^1$ while the electrons from (NH_3) (or any other SFL), go into the $d_{x^2-y^2}$, s , p_x and p_y orbitals

15. pH of a solution of salt of weak acid+strong base(all logs in base 10):

$$pH = \frac{pK_w + pK_a + \log(C)}{2} = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log(C) \quad (2.18)$$

16. pH of a solution of salt of weak base+strong acid(all logs in base 10):

$$pH = \frac{pK_w - pK_b - \log(C)}{2} = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log(C) \quad (2.19)$$

17. pH of a solution of salt of weak base+weak acid(all logs in base 10):

$$pH = \frac{pK_w + pK_a - pK_b}{2} = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b \quad (2.20)$$

18. β -D glucopyranose is more stable than α -D glucopyranose.

19. The chemical potential of the substance is equal to its Gibbs free Energy:

$$\mu^\circ(A) = \Delta G_A^\circ \quad (2.21)$$

20. For a liquid-gas equilibrium:

$$\mu_A^\circ(g) = \mu_A^\circ(l) + RT \ln(\chi_A) \quad (2.22)$$

χ_A is the mol fraction of A in the mixture (liquid phase)

21. For a solid-liquid equilibrium:

$$\mu_A^\circ(l) = \mu_A^\circ(s) + RT \ln(\chi_A) \quad (2.23)$$

where χ_A is same as above

22. To find the ebullioscopic constant rewrite 2.22 as:

$$\ln(\chi_A) = \frac{\mu_A^\circ(g) - \mu_A^\circ(l)}{RT} \quad (2.24)$$

$$= \frac{\Delta_{vap}G^\circ}{RT} \quad (2.25)$$

Differentiate 2.24 by temperature to get

$$\frac{d \ln(\chi_A)}{dT} = \frac{\partial \frac{\Delta G}{RT}}{\partial T} = -\frac{\Delta H}{RT^2} \quad (2.26)$$

$$\Rightarrow d \ln(\chi_A) = -\frac{\Delta H}{RT^2} dT \quad (2.27)$$

because

$$\frac{\Delta G}{T} = \frac{\Delta H}{T} - \Delta S.$$

Integrate this back using the limits T_b to $T_b + \Delta T$, to get:

$$\ln(\chi_A) = \frac{\Delta H}{R} \left(\frac{1}{T_b} - \frac{1}{T_b + \Delta T} \right) \approx \frac{\Delta H}{RT^2} \Delta T \quad (2.28)$$

Now, in a binary solution $\chi_A = 1 - \chi_B$. Assuming $\chi_B \ll 1$ (∴ Dilute condition)

$$\ln(1 - \chi_B) = -\chi_B = -\frac{\Delta H}{RT^2} \Delta T \quad (2.29)$$

which means:

$$\boxed{K_b = 1 / \frac{\Delta H}{RT^2} = \frac{RT^2}{\Delta H}} \quad (2.30)$$

23. Non-metals generally melt at a higher temperature than metals of the same group.
24. $[Co(NH_3)_4Cl_2]Cl$ is a diamagnetic compound.
25. Efficiency of an electrolytic cell:

$$\eta = \frac{\Delta G^\circ}{\Delta H^\circ} = 1 - T \frac{\Delta S^\circ}{\Delta H^\circ} \quad (2.31)$$

26. In the 3d series, when the lower elements (Sc, Ti, V) get oxidised, they almost always lose all of their 4s and 3d electrons and acquire the [Ar] noble gas configuration. Thus their oxides (like VO_2^+) are less oxidising than, say $Cr_2O_7^{2-}$ or MnO_4^- .
27. Di-carboxylic acids ($HOOC - (CH_2)_n - COOH$) on heating give different products:
 - (a) For $n = 0$ (Oxalic acid): Forms $HCOOH$ and CO_2
 - (b) For $n = 1$ (Malonic acid): Forms $CH_3 - COOH$ and CO_2
 - (c) For $n = 2, 3$ (Succinic, Glutaric acid): Forms the respective 5-membered and 6-membered ring anhydride
 - (d) For $n \geq 4$ (Adipic onwards): They start forming cyclic ketones
28. The Gold Number is the minimum weight (in milligrams) of a protective colloid required to prevent the coagulation of 10 ml of a standard hydro gold sol when 1 ml of a 10% sodium chloride solution is added to it
29. H_2O_2 oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium.

30. In determining reactivity with H_2/Pt or any other reagent with surface playing a dominant role, the least hindered compound will most easily adsorb and thus will react faster with such reagents

31. H_3PO_4 is used for elimination of alcohols.

32. Gases(ideal and real):

(a) Boyle's Temperature:

$$T_{bo} = \frac{a}{Rb} \quad (2.32)$$

(b) Critical Temperature:

$$T_c = \frac{8a}{27Rb} \quad (2.33)$$

(c) Critical Pressure:

$$P_c = \frac{a}{27b^2} \quad (2.34)$$

(d) Critical Volume:

$$V_c = 3b \quad (2.35)$$

33. $P_{\text{final}} = \sqrt{P_x^o \times P_y^o}$ for a very special case of minimum composition of one of the substances to have minimum mole fraction in vapour phase

34. As branching increases among isomeric alkanes, stability increases and hence heat of combustion decreases

35. If molality and molarity of any species in a solution is numerically the same, it is same for all the components and for it, numerical value of ml of solution = numerical value of gm of solvent

36. Rate of diffusion of a gas from an orifice:

$$r_{\text{eff}} = \frac{PA}{\sqrt{2\pi M_{\text{gas}}RT}} N_A \quad (2.36)$$

where P is the pressure of the container, A is the area of the orifice, M_{gas} is the molar mass of the gas.

37. Coagulating value= (millimoles of electrolyte used)/(volume of sol coagulated in L)

38. $\text{R} - \text{CONH}_2 + \text{HNO}_2 \rightarrow \text{R} - \text{COOH} + \text{N}_2$

39. Lyophilic sols are highly viscous

40. Reactions of MnO_4^- :

(a) In acidic medium: $\text{MnO}_4^- \rightarrow \text{Mn}^{2+}$ (Purple to pale-pink)

(b) In neutral medium: $\text{MnO}_4^- \rightarrow \text{MnO}_2$ (Purple to black-brown)

- (c) In basic medium: $\text{MnO}_4^- \rightarrow \text{MnO}_4^{2-}$ (Purple to Green)
41. Disiamylborane(Sia_2BH) is an organoborane used in organic synthesis. It is used for hydroboration-oxidation reactions of terminal alkynes, giving aldehydes via anti-Markovnikov hydration followed by tautomerization
 42. O_3 is diamagnetic.
 43. Coagulation value is the millimoles of an electrolyte that must be added to 1 L of a colloidal solution for complete coagulation.
 44. Beckmann rearrangement is the conversion of oxime to amide.
 45. DCC converts -OH into a good Leaving Group
 46. DIBAL-H is an electrophilic reducing reagent.
 47. $1\text{Debye} = \frac{1}{c} \times 10^{-21}\text{C} - \text{m} \approx 3.335 \times 10^{-30}\text{C} - \text{m}$
 48. Reactions of $\text{K}_2\text{Cr}_2\text{O}_7$:
 - (a) In acidic medium:
 49. Alum: $\text{KAl}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$
 50. Allenes with different groups on the two terminal carbons are optically active if there are an even number of double bonds in the allene, while those with odd number of double bonds are not.
 51. PCl_5 reacts with the carbonyl bond in a ketone to result in formation of geminal dichloride.
 52. Ketone + HNO_2/HCl forms the respective oxime.
 53. Hinsberg test: Benzene sulphonyl chloride is shaken with the sample with a strong base (KOH/NaOH) and the following observations occur:
 - (a) If there is a primary amine in the sample, it reacts to form a sulphonamide salt, which is soluble in water
 - (b) If there is a secondary amine in the sample, it reacts to form an insoluble sulphonamide
 - (c) If there is a tertiary amine in the sample, it doesn't react initially and stays insoluble. However, after adding a bit of dilute acid, it forms an ammonium salt and thus dissolves in the sample.
 54. $\text{P}_4 (\text{white}) + \text{NaOH} + 3\text{H}_2\text{O} \rightarrow \text{PH}_3 + \text{NaH}_2\text{PO}_2$
 55. To calculate the equivalent mass in a disproportionation reaction, calculate the equivalent masses in each of both the reduction and oxidation reactions, and then add them.

56. 231 for R=Me in amines and 213 for R=Et onwards for amines.

57. Entropy of mixing of ideal gases:

$$\Delta S = R \sum_{i=0}^n n_i \ln(\chi_i) = nR \sum_{i=0}^n \chi_i \ln(\chi_i) \quad (2.37)$$

58. $R - \text{CONH}_2 + \text{P}_2\text{O}_5 \rightarrow R - \text{C} \equiv \text{N}$

59. Moist Ag_2O is used for hydrolysis of SN_2 type.

3 Math

1. Triangle Inequalities:

$$|z_1 + z_2| \leq |z_1| + |z_2| \quad (3.1)$$

$$|z_1 - z_2| \geq ||z_1| - |z_2|| \quad (3.2)$$

2. BAC-CAB rule:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) \quad (3.3)$$

3. Wallis' Formula:

$$\int_0^{\frac{\pi}{2}} \sin^n(x) \cdot \cos^m(x) dx = \frac{[(n-1)(n-3)(n-5) \dots 2 \text{ or } 1][(m-1)(m-3)(m-5) \dots 2 \text{ or } 1]}{(m+n-0)(m+n-2)(m+n-4) \dots 2 \text{ or } 1} K \quad (3.4)$$

$$\text{where } K = \begin{cases} \frac{\pi}{2}, & \text{if } m \text{ and } n \text{ are even} \\ 1, & \text{otherwise} \end{cases}$$

4. Box product tricks:

(a)

$$\left[(\vec{a} \times \vec{b}) (\vec{b} \times \vec{c}) (\vec{c} \times \vec{a}) \right] = \left[\vec{a} \vec{b} \vec{c} \right]^2 \quad (3.5)$$

(b)

$$\left[\vec{a} \vec{b} \vec{c} \right]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix} \quad (3.6)$$

(c)

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{c} \cdot \vec{a})(\vec{d} \cdot \vec{b}) - (\vec{c} \cdot \vec{b})(\vec{d} \cdot \vec{a}) \quad (3.7)$$

5. Complex Numbers:

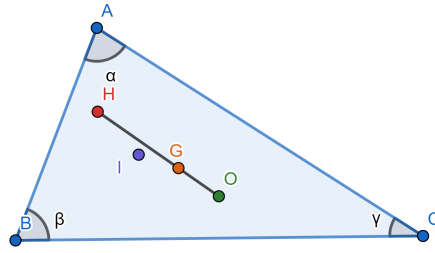
- (a) Expansion of $|1 - \alpha_r|$ and $|1 - \alpha_r|$ where α_r is the r^{th} root of unity:

$$|1 - \alpha_r| = \left| 1 - \cos\left(\frac{2\pi r}{n}\right) - i \sin\left(\frac{2\pi r}{n}\right) \right| = 2 \left| \sin\left(\frac{\pi r}{n}\right) \right| \quad (3.8)$$

*Use half angle formula and then that $|\cos(x)|$ has mod 1

6. Triangle Centers:

- (a) The centroid G lies on the line joining Orthocentre H and Circumcentre O :



- (b) Orthocentre H :

$$H = (\vec{a} + \vec{b} + \vec{c}) \quad (3.9)$$

- (c) The distance of orthocentre H from vertex A is:

$$HA = 2R \cos(A) \quad (3.10)$$

The distance of orthocentre H from side $a (= BC)$ is (where D is the foot of \perp from A):

$$HD = 2R \cos(B) \cos(C) \quad (3.11)$$

7. For angles A, B, C of a triangle:

$$\cos^2(A) + \cos^2(B) + \cos^2(C) = 1 - 2 \cos(A) \cos(B) \cos(C) \quad (3.12)$$

8. Taylor series of $\tan(x)$:

$$\tan(x) \approx x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots \quad (3.13)$$

9.

$$\text{The coefficient of } x^r \text{ in } (1 - x)^{-n} \text{ is } \binom{n+r-1}{r} \quad (3.14)$$

10. But for choosing, you use

$$\binom{n+r-1}{r-1}$$

11. Lagrange Interpolation: For a set of $k + 1$ data points $(x_0, y_0) \dots (x_k, y_k)$ where no two x_i, x_j are the same, there exists a *Interpolation Polynomial of Lagrange Form*:

$$L(x) := \sum_{j=0}^k y_j \ell_j(x) \quad (3.15)$$

where each ℓ_i is the Lagrange basis:

$$\ell_j(x) := \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m} = \frac{(x - x_0)}{(x_j - x_0)} \dots \frac{(x - x_{j-1})}{(x_j - x_{j-1})} \frac{(x - x_{j+1})}{(x_j - x_{j+1})} \dots \frac{(x - x_k)}{(x_j - x_k)} \quad (3.16)$$

12.
 - Blank plane is one region
 - Every line adds a region to the plane
 - Every point of intersection adds one more region
13. If A is a skew-symmetric matrix of odd dimension, its determinant is always 0
14. If stuck in a problem where Lagrange Mean Value theorem looks likely, take the function $g(x) = f(x) - x$ and then try the properties on it.
15. For a system of 3×3 linear equations:
- Unique solution if $\Delta \neq 0$
 - No solution if $\Delta = 0$ but at least one of Δ_1, Δ_2 and Δ_3 is non zero
 - Infinite solutions if $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

16. Sophie-Germain's Identity:

$$a^4 + 4b^4 = (a^2 + 2b^2 - 2ab)(a^2 + 2b^2 + 2ab) \quad (3.17)$$

17. BB^T is a symmetric matrix
18. if $P(z_1)$ and $Q(z_2)$ are two points on the circle $|z| = r$, the complex number representing the intersection of tangents is

$$\frac{2z_1 z_2}{z_1 + z_2}$$

19. To count the number of rectangles in a regular polygon, find the number of diagonals passing through the centre
20. If there is a regular n-gon on a unit circle the product of lengths

$$(A_1 A_2)(A_1 A_3)(A_1 A_4) \dots (A_1 A_n) = n$$

21. In the previous point, remember that if n is even then there exists a body diagonal too.

22. For an ellipse at the origin, a tangent with slope m is:

$$y = mx \pm \sqrt{a^2m^2 + b^2} \quad (3.18)$$

And for a hyperbola it is:

$$y = mx \pm \sqrt{a^2m^2 - b^2} \quad (3.19)$$

23. REMEMBER: If asked for non consecutive in combinatorics question, think "gaps"

24. Think rotation of planes, think of angles between the normals of the plane

25. Anytime you encounter an integral which involves something like $\frac{x^2 - 1}{x^2 + 1}$ try putting in $t = x + \frac{1}{x}$. for eg.:

$$\begin{aligned} I &= \int_1^{\sqrt{2}+1} \frac{x^2 - 1}{x^2 + 1} \frac{1}{\sqrt{1 + x^4}} dx \\ &= \int_1^{\sqrt{2}+1} \frac{(1 - \frac{1}{x^2})dx}{(x + \frac{1}{x})\sqrt{x^2 + \frac{1}{x^2}}} \end{aligned}$$

26. Stewart's theorem is a powerful theorem for Solutions of Triangles, and can be easily used to derive much stronger results than the hard to remember m-n rule. The theorem:

If in a $\triangle ABC$, a point D divides BC in m and n , then (3.20)

$$\boxed{b^2m + c^2n = a(l^2 + mn)} \quad (3.21)$$

27. The foot of the perpendicular from any focus of the ellipse to any tangent to the ellipse must lie on the auxiliary circle of the ellipse

28. The foot of perpendicular from the focus on any tangent to the parabola must lie on the tangent at the vertex to the parabola which in a way can be thought as the auxiliary circle of the parabola

29. If there is a differential equation like

$$\frac{dy}{dx} + P(x)y = 0, \quad (3.22)$$

it is linear i.e. for solutions y_1 and y_2 , $\lambda y_1 + \mu y_2$ is also a solution for $\lambda, \mu \in \mathbb{R}$. However if there is a non homogeneous part and the equation now becomes

$$\frac{dy}{dx} + P(x)y = Q(x), \quad (3.23)$$

and y_1 and y_2 are again some solutions, $\lambda y_1 + \mu y_2$ is not a solution for general λ and μ (Check by putting y_1 and y_2 into the Diff Eq 3.23 then checking $\lambda y_1 + \mu y_2$ doesn't satisfy 3.23 unless $\lambda + \mu = 1$)

30. If some integral with trig functions isn't working, try substitution by $\tan(\frac{x}{2})$

31. If there is a problem asking for min/max anywhere, and things look hopeless, try Cauchy-Schwarz inequality:

$$|\langle \mathbf{u}, \mathbf{v} \rangle|^2 \leq \langle \mathbf{u}, \mathbf{u} \rangle \cdot \langle \mathbf{v}, \mathbf{v} \rangle \quad (3.24)$$

where $\langle \cdot, \cdot \rangle$ is the inner product and \mathbf{u} and \mathbf{v} are vectors, which basically means list of numbers of any kind

32. For summation of $\sum_{r=1}^n 2^r \tan(2^r \theta)$, remember:

$$\tan(\theta) = \cot(\theta) - 2 \cot(2\theta) \quad (3.25)$$

$$\tan(2\theta) = \cot(2\theta) - 4 \cot(4\theta) \quad (3.26)$$

$$\vdots \quad (3.27)$$

$$\tan(2^n \theta) = \cot(2^n \theta) - 2^{n+1} \cot(2^{n+1} \theta) \quad (3.28)$$

33. Adjoint properties:

$$(a) \operatorname{adj}(A) = |A| A^{-1}, \Rightarrow |\operatorname{adj}(A)| = |A|^{n-1}$$

$$(b) \operatorname{adj}(\operatorname{adj}(A)) = |A|^{n-2} A$$

34. Number of surjective/onto functions $f : A \rightarrow B$ with $|A| = m$ and $|B| = n$ ($m > n$) is:

$$n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \binom{n}{3}(n-3)^m + \dots + (-1)^r \binom{n}{r} + \dots \quad (3.29)$$

using the principle of inclusion and exclusion

35. Equation of tangent to parabola:

$$ty = x + at^2 \quad (3.30)$$

or

$$y = mx + \frac{a}{m} \quad (3.31)$$

36. The product of the perpendiculars on the tangent from the foci is equal to b^2 .

37. The reflection of point A on the angle bisectors BE and CF lies on the side BC

38. SOT Formulas:

(a) Length of the median AD of a triangle:

$$l_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2} \quad (3.32)$$

(b) Length of the internal angle bisector of triangle from A:

$$d_a^2 = \frac{bc}{(b+c)^2} ((b+c)^2 - a^2) \quad (3.33)$$

39. For a chord of a conic section $S=0$, with its mid point at (h,k) , the formula is:

$$T = S_1 \quad (3.34)$$

40. $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

41.

$$\neg(\forall x)P(x) \Leftrightarrow (\exists x)(\neg P(x)); \quad (3.35)$$

$$\neg(\exists x)P(x) \Leftrightarrow (\forall x)(\neg P(x)). \quad (3.36)$$

42. For a system of quadratic equations which has one common root α

$$a_1x^2 + b_1x + c_1 = 0 \quad (3.37)$$

$$a_2x^2 + b_2x + c_2 = 0 \quad (3.38)$$

We can put in α^2 and α respectively to form

$$a_1\alpha^2 + b_1\alpha + c_1 = 0 \quad (3.39)$$

$$a_2\alpha^2 + b_2\alpha + c_2 = 0 \quad (3.40)$$

and then using the cross product formula, we write:

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad (3.41)$$

and thus equate the values of α from the equations to get:

$$\alpha = \frac{(b_1c_2 - b_2c_1)}{(c_1a_2 - c_2a_1)} = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)} \quad (3.42)$$

$$\Rightarrow (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1)^2 \quad (3.43)$$

$$\Rightarrow \boxed{\Delta_{bc}\Delta_{ab} = \Delta_{ca}^2} \quad (3.44)$$

43. If thinking about shortest distance where lines are acting as a constraint, then think about the mirror images of the points about those lines

44. If A is a symmetric matrix, then A^{-1} is a symmetric matrix too.
45. If three circles are tangent to a straight line with the smaller circle contained within the bigger two, the relation between their radii is:

$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} \quad (3.45)$$

46. If a question asks for collinearity of points on a biquadratic or higher curve, differentiate them twice (so that it gets rid of linear and lower terms), and then declare that they have to have real and distinct roots. Use the determinant to find the condition.

47. If the eccentricity of the hyperbola is $\sqrt{1 + \frac{b^2}{a^2}}$, then the eccentricity of the conjugate hyperbola is $\sqrt{1 + \frac{a^2}{b^2}}$

48. Sum of the sine of an AP of n terms of angles where first term is a and common difference is d :

$$\sin(a) + \sin(a + d) + \cdots + \sin(a + (n - 1)d) = \frac{\sin(\frac{nd}{2})}{\sin(\frac{d}{2})} \sin(\frac{2a + (n - 1)d}{2}) \quad (3.46)$$

! Similarly for cosine, we can write:

$$\cos(a) + \cos(a + d) + \cdots + \cos(a + (n - 1)d) = \frac{\sin(\frac{nd}{2})}{\sin(\frac{d}{2})} \cos(\frac{2a + (n - 1)d}{2}) \quad (3.47)$$

49. For a point on the ellipse, the normal at that point acts as the angle bisector between the lines joining the point and the two foci. i.e. PG is the angle bisector of PS and PS'

50. For a conic of the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\Delta := \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}. \quad (3.48)$$

If $\Delta = 0$, the conic is two straight lines. Then:

- $h^2 - ab = 0$ implies perpendicular lines
- $h^2 - ab < 0$ implies imaginary lines

51. If there is a comparison of $\sum_{cyclic} a^2$ and $\prod_{cyclic} a$, try to use the sum of the reciprocals to get an inequality and then use that in the expansion of
- $$\left(\sum_{cyclic} a \right)^2$$

52. For the mid-point V of a focal chord of a standard parabola:

$$PV^2 = AV^2 + 3AS^2 \quad (3.49)$$

where P is one of the ends of the focal chord, A is the vertex and S is the focus

53. if N is expressed in the form $N = p_1^{n_1} \times p_2^{n_2} \times p_3^{n_3} \dots p_k^{n_k}$ then the number of ways in which N can be expressed as a product of two co-prime numbers is 2^{k-1} where k is the number of its prime factors.

54. Radius of curvature:

- The radius of curvature ρ is:

$$\rho = \left| \frac{ds}{d\theta} \right| \quad (3.50)$$

- For a function $y = f(x)$, the radius of curvature ρ is:

$$\rho = \left| \frac{(1 + y'^2)^{\frac{3}{2}}}{y''} \right| \quad (3.51)$$

- For a point on the ellipse $P(a \cos(\theta), b \sin(\theta))$, ρ is:

$$\rho = \frac{(b \cos^2(\theta) + a \sin^2(\theta))^{\frac{3}{2}}}{ab} \quad (3.52)$$

- Thus, the maximum radius of curvature (minimum curvature) is at $(0, b)$ $\rho = \frac{a^2}{b}$ and the minimum radius of curvature (maximum curvature) is at $(a, 0)$, $\rho = \frac{b^2}{a}$

55. For a series in \tan and powers of 2, remember the following for telescoping:

$$\tan(\theta) = \cot(\theta) - 2 \cot(2\theta) \quad (3.53)$$

56. A ray passing through one of the focus of an ellipse from the inside passes through the other focus on reflection

57. A useful identity for \sec and \csc

$$a^2 \csc^2(\theta) + b^2 \sec^2(\theta) = (a \cot(\theta) - b \tan(\theta))^2 + (a + b)^2 \quad (3.54)$$