We want to find the underlying topics of a document, and word-topic associations. Our model:

- Select document length: $N_d \sim Poisson(\xi)$
- Select topic mixture $\theta_d \sim Dir(\alpha)$
- For i from 1 to N_d:
  - Select topic $z_{i,d} \sim Multi(\theta_d)$
  - Select word $w_{i,d} \sim p(w_i|z_{i,d}, \beta_z)$ (i.e. according to topic-specific parameters)

The goal is then to recover the parameters α, β, θ, z. We break this down into two problems:

1) *Inference*: Given a new document and known α, β, discover θ and z.
2) *Learning*: Given a corpus of documents, estimate appropriate α, β.

We can use estimation methods to achieve this, as follows:

```
Infer(Document d, α, β):
  Do:
    # Update word-topic associations
    For n from 1 to N_d:
      For i from 1 to K:
        # See below for definition of Ψ
        φ_{n,i}^{(new)} ← β_{i,w_n} × exp(Ψ(γ_i^{(old)}) − Ψ(Σ_j^K γ_j^{(old)}))
      Normalize(φ_n^{(new)})

    # Update topic mixture (here all topics at once)
    γ^{(new)} ← α + Σ_n^N φ_n^{(new)}
  Until convergence
  Return φ, γ

Learn(Corpus c):
  Do:
    # Expectation step
    For document d in c:
      φ^{(d)}, γ^{(d)} ← Infer(d, α^{(old)}, β^{(old)})
    # Maximization step
    β^{(new)} ← Σ_d Σ_n φ_{n,i}^{(d)} w_{d,n}
    α^{(new)} ← Newton-Raphson(α^{(old)}) #Linear time due to special struct.
  Until convergence
  Return α, β

#Abramowitz, Stegun. Handbook of Mathematical functions, p. 259
Ψ(x):
  γ ← 0.57721566 # Euler-Mascheroni constant
  x' ← x − 1
  sum ← 0
  For n from 1 to many: # Depends on rate of convergence
    sum += x' / (n * (n + x'))
  return -γ + sum
```