On Chubanov's method for Linear Programming
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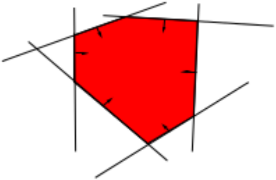
The Problem

Determining the feasibility of the systems of the form

$$Ax = b, Cx \leq d$$

in $\mathbb{R}^n$, with A an $m \times n$ matrix, C an $l \times n$ matrix, $b \in \mathbb{R}^n$, and $d \in \mathbb{R}^l$, where the elements of A, b, C, and d are integers, or determine if the system has no integer solutions.
This is the *Linear Feasiblity Problem (LPP)*, and has been used to model transportation problems, airline scheduling, networks etc.



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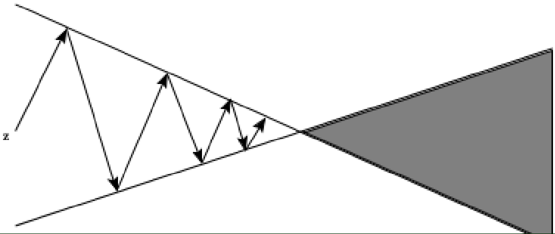

The Chubanov Divide and Conquer algorithm

Given a current guess z, a radius r and an error bound $\epsilon > 0$, the algorithm will either:

- Find an ϵ approximate solution $x^* \in$ ball $B(z, r)$ to the system, i.e some x^* such that $Ax^* = b, Cx^* \leq d + \epsilon\mathbf{1}$

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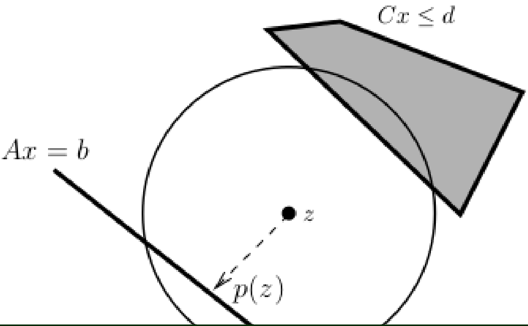

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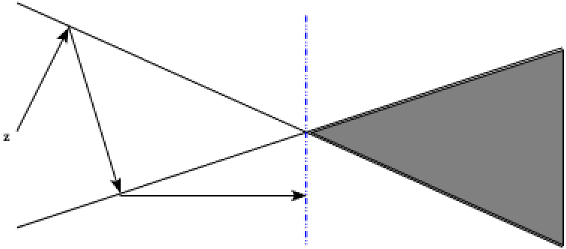

Extensions and Results

Theory:

- Chubanov's algorithm either returns a feasible solution or determines that no feasible solution exist.
- We use $D\&C$ to determine the feasibility of strict LFP's.

Practical Numerical Analysis:

- Despite its theoretical advantages, Chubanov's Relaxtion method appears to be practically much slower than the original relaxation method.

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Thank You!