Q2) The image is of size 512×512

Let the image be $x(n, m)$

$$\therefore x(n,m) = \begin{cases} 1 & \text{if } n = 256 \\ 0 & \text{otherwise} \end{cases}$$

Let $X(k, l)$ be the DFT of $x(n,m)$

$$\therefore X(k,l) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} x(n,m) e^{\frac{-2\pi j k n}{N}} e^{\frac{-2\pi j l m}{M}}$$

Now, $N = 512$ and $M = 512$

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$$\therefore X(k,l) = \sum_{n=0}^{511} \sum_{m=0}^{511} x(n,m) e^{\frac{-2\pi j k n}{512}} e^{\frac{-2\pi j l m}{512}}$$

$$\Rightarrow X(k,l) = e^{\frac{-2\pi j k \times 256}{512}} \sum_{m=0}^{511} x(256, m) e^{\frac{-2\pi j l m}{512}}$$

[Since $x(n,m) = 0 \ \forall \ n \neq 256$]

$$\Rightarrow X(R,l) = e^{-j\pi R} \sum_{m=0}^{511} 1.e^{-j\frac{2\pi ml}{512}}$$

[Since $x(256,m) = 1$]

$$\Rightarrow X(R,l) = (-1)^R \sum_{m=0}^{511} e^{-j\frac{2\pi ml}{512}}$$

Case 1: $l = 0$

When $l = 0$, $e^{-j\frac{2\pi ml}{512}} = 1$

$$\Rightarrow$$

$$X(R,0) = (-1)^R \cdot 512$$

Case 2: $l \neq 0$

$$X(k, l) = (-1)^k \sum_{n=0}^{511} \left(e^{\frac{-j2\pi l}{512}} \right)^n$$

Sum of G.P.

$$\Rightarrow X(R,\ell) = (-1)^R \times \left[\frac{1 - \left(e^{-j\frac{2\pi\ell}{512}}\right)^{512}}{1 - e^{-j\frac{2\pi\ell}{512}}} \right]$$

Numerator: $1 - \left(e^{-j\frac{2\pi\ell}{512}}\right)^{512}$

$$= 1 - e^{-j2\pi\ell}$$

$$= 1 - (1)^{\ell}$$

$$= 0$$

Denominator cannot be 0 if $l \neq 0$

$\therefore X(R, l) = (-1)^R \times 0$

$= 0$ when $l \neq 0$

$$\therefore X(R, l) = \begin{cases} (-1)^R \times 512 & \text{when } l = 0 \\ 0 & \text{when } l \neq 0 \end{cases}$$

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$\therefore$ 2D-DFT

$=$

$$\begin{bmatrix} 512 & 0 & \cdots & 0 \\ -512 & 0 & & \vdots \\ 512 & 0 & & \\ \vdots & \vdots & & \\ -512 & 0 & \cdots & 0 \end{bmatrix}_{512 \times 512}$$