

Q1

r	frequency	pdf	cdf	
0	4	0.25	0.25	} $F(r)$ for input image
1	4	0.25	0.5	
2	4	0.25	0.75	
3	4	0.25	1	

s	frequency	pdf	cdf	
0	4	0.4375	0.4375	} $G(s)$ for target image
1	3	0.1875	0.625	
2	2	0.125	0.75	
3	4	0.25	1	

For histogram matching we map input pixels to output pixels where difference in corresponding cdf value is least i.e. $|F(r) - G(s)|$ is min.

mapping:

$0 \rightarrow 0$	$ F(0) - G(0) = 0.1875$
$1 \rightarrow 0$	$ F(1) - G(0) = 0.0625$
$2 \rightarrow 2$	$ F(2) - G(2) = 0$
$3 \rightarrow 3$	$ F(3) - G(3) = 0$

output matrix after matching:

3	3	0	0
3	2	0	2
2	0	0	0
3	0	0	2

Q2) image =
$$\begin{bmatrix} -1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix}_{3 \times 3}$$

filter =
$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}_{3 \times 1}$$

size of output matrix = $(3+3-1) \times (3+1-1)$
= 5×3

padded image =
$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

rotated filter =
$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

examples of convolution

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \text{output matrix at step 1} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \boxed{1} & 0 & 0 \\ -1 & \boxed{-1} & 2 & -1 \\ 3 & 0 & 1 & \\ -2 & 1 & 2 & \\ 0 & 0 & 0 & \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 2 & -1 \\ 3 & 0 & 1 \\ -2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

output after step 2

Final output after complete convolution =

$$\begin{bmatrix} 1 & -2 & 1 \\ -3 & 0 & -1 \\ 1 & 1 & -3 \\ 3 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix}$$